

# Correction Methods for TPC Operation in Inhomogeneous Magnetic Fields

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## Abstract

The Time Projection Chamber of the International Large Detector is anticipated to operate with the inhomogeneous Anti Detector-Integrated-Dipol magnetic field. The field inhomogeneities will systematically affect the TPC measurements and a correction will be necessary to reach a point resolution of  $\sigma_{\perp} = 100 \mu\text{m}$  in the transversal plane. In this note, two possible correction methods for TPC measurements are presented and illustrated with the example of the Large TPC Prototype setup.

## 1 Introduction and Overview

The Time Projection Chamber (TPC) planned for the International Large Detector (ILD) [1] at the International Linear Collider ILC [2] is anticipated to have a momentum resolution of better than  $\delta p_{\perp}/p_{\perp}^2 \lesssim 10^{-4}/\text{GeV}/c$ . This requires a point resolution of better than  $100 \mu\text{m}$  in the  $r\varphi$  plane. Systematic effects, for example caused by inhomogeneities of the magnetic field in the drift volume of the TPC, can deteriorate the point resolution and should be under control and correctable to a level of better than  $30 \mu\text{m}$ . If this is possible, the resolution degrades by 5% at most, as  $(100 \mu\text{m})^2 + (30 \mu\text{m})^2 \approx (105 \mu\text{m})^2$ .

A correction for the effect of magnetic and electric drift field inhomogeneities on the measurements of a TPC was already performed at the ALEPH experiment [3]. The case of ILD is in particular challenging because it is anticipated to operate the ILD TPC from the beginning in the inhomogeneous Anti Detector-Integrated-Dipol (Anti DiD) field. The field inhomogeneities will be in the few percent regime. The Anti DiD field reduces machine backgrounds which are overlaid to every physics events (e.g. [4]). However, without an appropriate procedure to correct for the impact of field configuration on the TPC measurements, the  $100\text{-}\mu\text{m}$  point resolution will not be achievable.

In this note, two correction methods to cope with inhomogeneities of the magnetic field are discussed. Inhomogeneities of the electric field are not considered, but could be included in a second step. It is shown that an electric field with distortions not larger than  $\Delta E/E \lesssim 10^{-4}$  is sufficiently homogeneous that a correction

on a level of 30- $\mu\text{m}$  is possible. A requirement on the measurement accuracy of the magnetic field of ILD is presented in [5].

The next section presents briefly the mathematical framework to describe the drift of electron clusters in a TPC and the requirement for the electric field homogeneity. Section 3 discusses the correction procedures and in section 4 they are illustrated for the case of the Large TPC Prototype (LP) [6] in the superconducting magnet PCMAG. Finally, in Sections 5 and 6, modifications to the correction procedure to compensate for gas contaminations and a requirement on the positioning accuracy of the TPC are given.

In all calculations throughout the text, the TPC is described by a cylindrical coordinate system with  $\vec{e}_z$  along the cylinder axis. The radial vector  $\vec{e}_r$  and the azimuthal vector  $\vec{e}_\varphi$  lie in the anode plane. Displacements, which a cluster accumulates due to field inhomogeneities while drifting in the TPC volume, are denoted  $\delta(r)$  for the radial direction and  $\delta(r\varphi)$  for the azimuthal direction (see Fig. 1). The correction for these displacements are  $\Sigma(r)$  and  $\Sigma(r\varphi)$ , respectively.

## 2 Drift of Low Electron Clusters in a TPC

The drift velocity  $\vec{v}_{\text{drift}}$  of an electron cluster's center of gravity in a gas volume is described by the solution of the Langevin equation (e.g. [7])

$$\vec{v}_{\text{drift}}(/vecr) = \frac{\mu E}{1 + (\mu B)^2} \left[ \hat{E} + \mu B \hat{E} \times \hat{B} + (\mu B)^2 (\hat{B} \cdot \hat{E}) \cdot \hat{B} \right]. \quad (1)$$

$\vec{E}$  and  $\vec{B}$  are the electric and magnetic field at the point  $\vec{r}$  inside the TPC, respectively, and  $\hat{E}$  and  $\hat{B}$  unit vectors along  $\vec{E}$  and  $\vec{B}$ . The equation is typically given with factors  $\omega\tau$ , instead of  $\mu B$ , where  $\tau$  and  $\omega$  are the mean time between two encounters of a drifting electron with gas molecules and the cyclotron frequency, respectively. These factors are equal to  $\mu B$ . The latter is directly accessible because the electron mobility  $\mu$  of the gas can be determined from simulations [8] or from a measurement of the drift velocity of electrons in the gas.

### 2.1 Case of homogeneous $\vec{B}$ -field

A TPC is typically operated with  $\vec{E}$  and  $\vec{B}$  aligned parallel and perpendicular to the readout plane ( $\vec{E}$  and  $\vec{B}$  parallel to  $\vec{e}_z$ ). Then the term  $\vec{E} \times \vec{B}$  in Equation (1) vanishes and  $\vec{v}_{\text{drift}}$  is

$$\vec{v}_{\text{drift}} = \mu \vec{E} \quad \text{if} \quad \vec{E} \parallel \vec{B}.$$

Inhomogeneities of the electric field  $\Delta \vec{E}_\perp$  ( $\perp \vec{e}_z$ ) cause an  $\vec{E} \times \vec{B}$  term in Equation (1) and a component of  $\vec{v}_{\text{drift}}$  in a direction perpendicular to  $\vec{e}_z$ . A particle which drifts over a distance  $l$  in a region with the non vanishing  $\Delta E_\perp$  gets displaced by  $\delta r_\perp$ . The magnitude can be estimated by

$$\delta r_\perp = \sqrt{\delta(r)^2 + \delta(r\varphi)^2} \lesssim \frac{l}{\sqrt{1 + (\mu B)^2}} \cdot \frac{\Delta E_\perp}{E}. \quad (2)$$

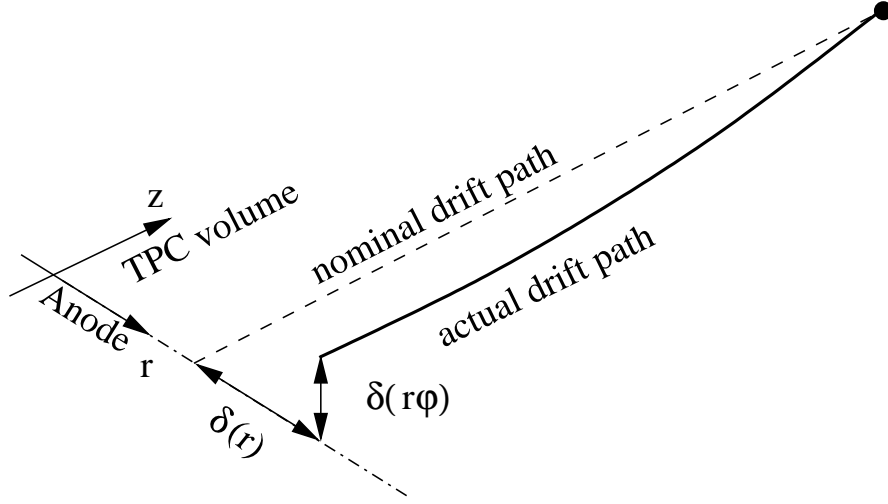


Figure 1: Displacement of a drifting cluster due an inhomogeneous drift field

(see Appendix A.1) If the displacement  $\delta r_{\perp}$  is to stay below  $30 \mu\text{m}$  over a drift distance of 2.15 m in a magnetic field of 3.5 T and a gas with  $\mu \approx 2 \text{T}^{-1}$ , the field distortions must be kept  $\Delta E/E \lesssim 2 \cdot 10^{-4}$  in average. These are possible parameters for the ILD TPC. The same limit is valid for the Large Prototype, because the ratio  $l/B$  is the same for both TPCs - the LP is 60 cm long and operated in a magnetic field of about 1 T.

## 2.2 Case of inhomogeneous $\vec{B}$ -field

If a TPC is placed in an inhomogeneous magnetic field, the  $\vec{E} \times \vec{B}$  contribution in Equation (1) does not vanish. Written in cylindrical coordinates,  $\vec{v}_{\text{drift}}$  is

$$\vec{v}_{\text{drift}} = \frac{\mu E}{1 + (\mu B)^2} \begin{pmatrix} - (\mu B) \hat{B}_{\varphi} + (\mu B)^2 \hat{B}_z \hat{B}_r \\ (\mu B) \hat{B}_r + (\mu B)^2 \hat{B}_z \hat{B}_{\varphi} \\ 1 + (\mu B)^2 \hat{B}_z \hat{B}_z \end{pmatrix} \begin{pmatrix} \vec{e}_r \\ \vec{e}_{\varphi} \\ \vec{e}_z \end{pmatrix}. \quad (3)$$

Here,  $\vec{E}$  is assumed to be parallel to  $\vec{e}_z$ . The drift velocity vector acquires radial and azimuthal components, in the direction of  $\vec{e}_r$  and  $\vec{e}_{\varphi}$ , respectively. Clusters drifting in the TPC volume accumulate displacements  $\delta(r)$  and  $\delta(r\varphi)$  (see Fig. 1).

The evaluation of a correction using a magnetic field map is discussed in the next section. If the electric field also shows inhomogeneities, additional displacements  $\delta r'_{\perp}$  occur, which have a magnitude

$$\delta r'_{\perp} \leq \frac{l}{\sqrt{1 + (\mu B)^2}} \cdot \frac{\Delta E_{\perp}}{E} \cdot f(\mu B, \alpha) \quad (4)$$

(see Appendix A.2). The additional factor  $f$  compared to Equation (2) depends on the average angle between the magnetic and electric field  $\alpha$  and  $\mu B$ . For realistic setups ( $\mu B \lesssim 20$  and  $\alpha \lesssim 5^{\circ}$ ) the factor is  $f \lesssim 2$ . Thus the inaccuracies,

caused by inhomogeneities of the electric field, stay below  $30\ \mu\text{m}$  if the condition  $\Delta E/E \lesssim 10^{-4}$  is fulfilled. In this case, a correction vector field with an accuracy in the  $30\text{-}\mu\text{m}$  regime can be calculated analytically under the assumption of a homogeneous electric field.

### 3 Correction Procedure

A correction for displacements  $\delta(r)$  and  $\delta(r\varphi)$  caused by inhomogeneities of the drift fields can be facilitated with a correction vector map: To reconstruct the origin  $\vec{r}_{\text{origin}}$  of a cluster, first a space point in the TPC,  $\vec{r}_{\text{reco.}}$ , is reconstructed from the measurements of the TPC. This is done irrespective of any field inhomogeneities. Then, a correction vector  $\vec{\Sigma}(\vec{r}_{\text{reco.}})$  is evaluated at the position  $\vec{r}_{\text{reco.}}$  and added. The correction vector points from the reconstructed position  $\vec{r}_{\text{reco.}}$  to the correct origin of the cluster  $\vec{r}_{\text{origin}}$ :

$$\vec{r}_{\text{origin}} = \vec{r}_{\text{reco.}} + \vec{\Sigma}(\vec{r}_{\text{reco.}}).$$

The correction  $\vec{\Sigma}$  cancels  $\delta(r)$  and  $\delta(r\varphi)$ . Inaccuracies can remain after the correction was applied and are given by

$$\Delta = \sqrt{(\delta(r) + \Sigma(r))^2 + (\delta(r\varphi) + \Sigma(r\varphi))^2}. \quad (5)$$

Two ways to calculate the correction vectors  $\vec{\Sigma}(\vec{r})$  are discussed in the following. For this, it is assumed that the magnetic field is known from a measurement and the electric field fulfills the homogeneity requirement of  $\Delta E/E \lesssim 10^{-4}$ . Therefore the electric field is treated as homogeneous  $\vec{E} = E \vec{e}_z$  in the drift volume.

#### Displacement Integrals (Fig. 2)

The displacements  $\delta(r)$  and  $\delta(r\varphi)$  can be calculated from Equation (3) via

$$\delta(r)(\vec{r}_{\text{reco.}}) = \int_{\vec{r}_0}^{\vec{r}_{\text{origin}}} \frac{\vec{v}_{\text{drift}} \cdot \vec{e}_r}{|\vec{v}_{\text{drift}}|} dl \quad \text{and} \quad \delta(r\varphi)(\vec{r}_{\text{reco.}}) = \int_{\vec{r}_0}^{\vec{r}_{\text{origin}}} \frac{\vec{v}_{\text{drift}} \cdot \vec{e}_\varphi}{|\vec{v}_{\text{drift}}|} dl.$$

Both integrals span over the drift path of the cluster. The components of the correction vector are

$$\Sigma(r)(\vec{r}_{\text{reco.}}) = -\delta(r)(\vec{r}_{\text{reco.}}) \quad \text{and} \quad \Sigma(r\varphi)(\vec{r}_{\text{reco.}}) = -\delta(r\varphi)(\vec{r}_{\text{reco.}}).$$

To calculate the integrals, the absolute of  $\vec{v}_{\text{drift}}$ ,

$$|\vec{v}_{\text{drift}}| = \frac{\mu E}{1 + (\mu B)^2} \sqrt{1 + (\mu B)^2 + ((\mu B)^2 + (\mu B)^4)(\hat{E} \cdot \hat{B})^2},$$

is approximated by

$$|\vec{v}_{\text{drift}}| \approx \mu E (\hat{E} \cdot \hat{B}). \quad (6)$$

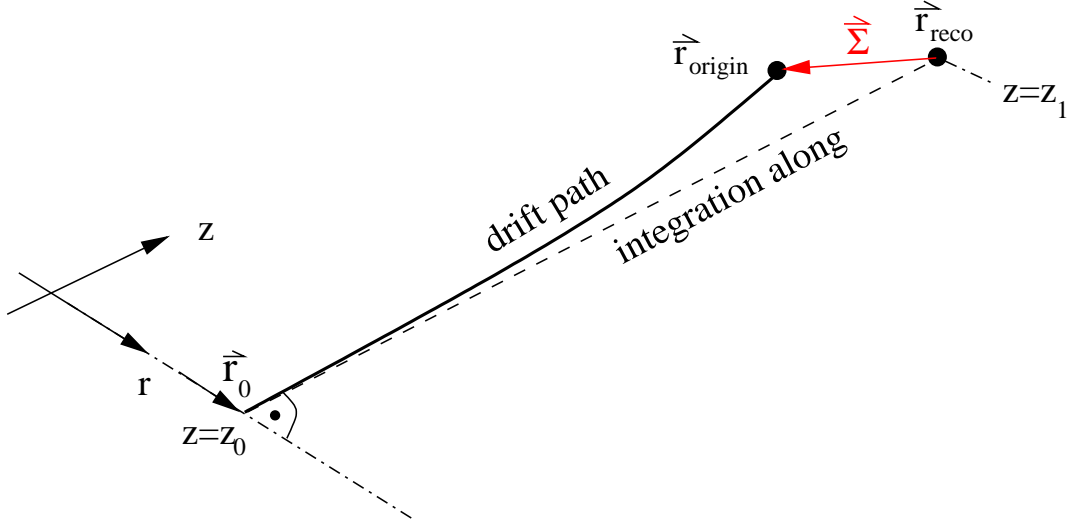


Figure 2: Calculation of a correction vector by the integration method

This assumes small angles between  $\hat{B}$  and  $\hat{E}$  ( $\angle(\vec{E}, \vec{B}) \lesssim 30^\circ$ ) which is the case in realistic setups. As a second approximation, the integration along the drift path is replaced by an integration parallel to the  $z$  axis (see Fig. 2). This is accurate, if  $\hat{B}_r$  and  $\hat{B}_\varphi$  do not show large variations between the actual drift path and the approximated one.

Hence,  $\vec{\Sigma}(\vec{r}_{\text{reco.}})$  for a cluster which has been detected at a position  $\vec{r}_0$  on the anode is calculated by evaluating the integrals

$$\Sigma(r)(\vec{r}_{\text{reco.}}) \approx - \int_{z_0}^{z_{\text{reco.}}} \frac{1}{1 + (\mu B)^2} \frac{1}{\hat{B}_z} \left( -\mu B \hat{B}_\varphi + (\mu B)^2 \hat{B}_z \hat{B}_r \right) dz \quad (7)$$

$$\Sigma(r\varphi)(\vec{r}_{\text{reco.}}) \approx - \int_{z_0}^{z_{\text{reco.}}} \frac{1}{1 + (\mu B)^2} \frac{1}{\hat{B}_z} \left( \mu B \hat{B}_r + (\mu B)^2 \hat{B}_z \hat{B}_\varphi \right) dz \quad (8)$$

from  $z_0 = 0$  to  $z_{\text{reco.}} = \vec{r}_{\text{reco.}} \cdot \vec{e}_z$ . Here  $\hat{B} \cdot \hat{E} = \hat{B}_z$  was used.

### Inverse drift (Fig. 3)

Alternatively,  $\Sigma(r)$  and  $\Sigma(r\varphi)$  can be calculated by an inverse drift procedure: Starting from  $\vec{r}_0$  on the anode, where a cluster was registered, the drift path in the TPC volume is reconstructed. This is done with an inverse tracking procedure (Fig. 3). Iteratively, points  $\vec{r}_i$  on the drift path of the cluster are calculated by

$$\vec{r}_i = \vec{r}_{i-1} + \frac{\vec{v}_{\text{drift}}(\vec{E}, \vec{B}(\vec{r}_{i-1}))}{|\vec{v}_{\text{drift}}(\vec{E}, \vec{B}(\vec{r}_{i-1}))|} \cdot \delta l \quad \text{and} \quad t_i = t_{i-1} + \frac{\delta l}{|\vec{v}_{\text{drift}}(\vec{E}, \vec{B}(\vec{r}_{i-1}))|}, \quad (9)$$

with  $\delta l$  being the step width of the iterative procedure. In parallel, the drift time  $t_i$  is summed up. Before a step ( $i \rightarrow i+1$ ) is performed, the direction of  $\vec{v}_{\text{drift}}$  at the position  $\vec{r}_{i-1}$  is calculated from the electric field  $\vec{E}$  and the local magnetic field  $\vec{B}(\vec{r}_{i-1})$ .

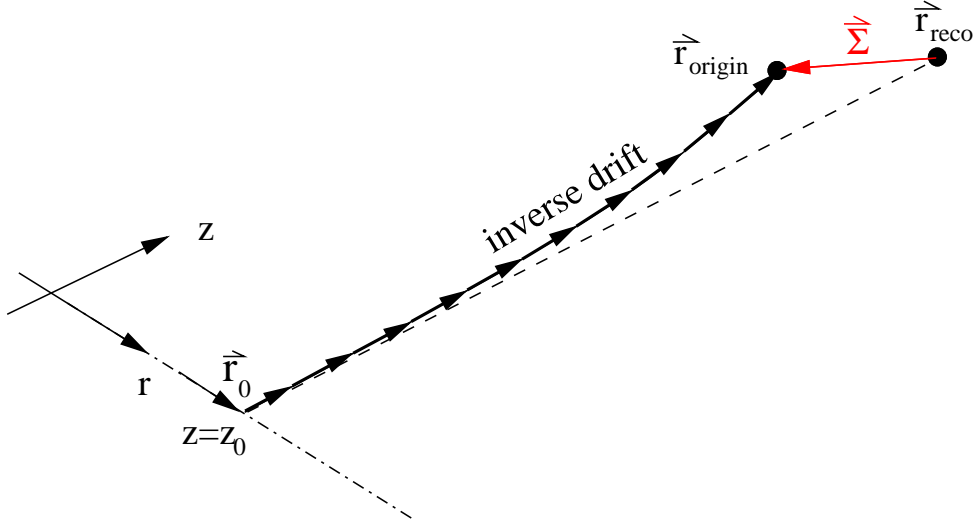


Figure 3: Calculation of a correction vector by the reverse drift

After a step has been performed, the correction vector  $\vec{\Sigma}(\vec{r}_{\text{reco.}})$  is calculated by

$$\vec{\Sigma}(\vec{r}_{\text{reco.}}) = \vec{\Sigma}(\vec{r}_0 + t_i \cdot \underbrace{\mu E \vec{e}_z}_{v_{\text{drift}}}) = [\vec{r}_i - (\vec{r}_0 + t_i \cdot \underbrace{\mu E \vec{e}_z}_{v_{\text{drift}}})].$$

The term  $\vec{r}_0 + t_i \cdot \mu E \vec{e}_z$  is equal to  $\vec{r}_{\text{reco.}}$  – the origin of the cluster, which will be reconstructed when the field inhomogeneities are not considered. The corrected origin of the cluster  $\vec{r}_{\text{origin}}$  is

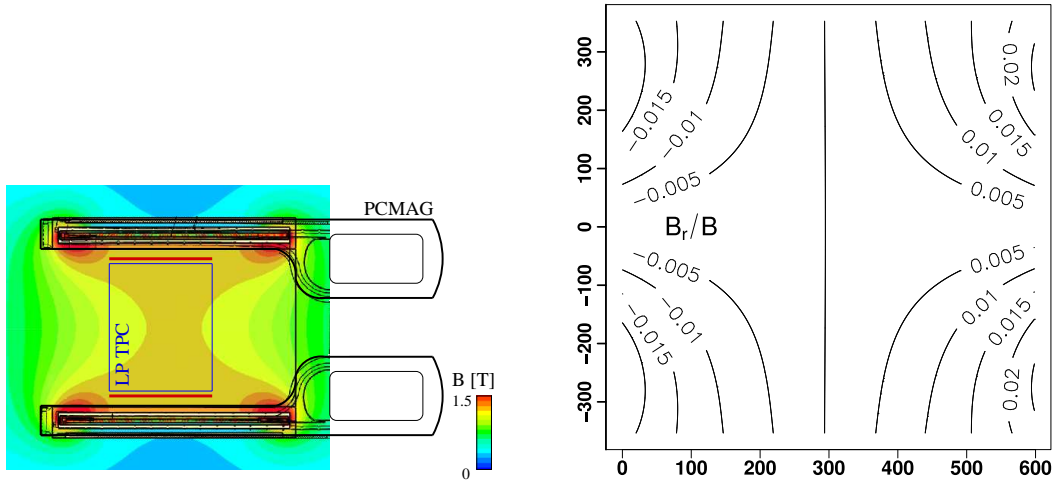
$$\vec{r}_{\text{origin}} = \underbrace{\vec{r}_0 + t \cdot \mu E \vec{e}_z}_{\text{reconstructed position } \vec{r}_{\text{reco.}}} + \underbrace{\vec{\Sigma}(\vec{r}_0 + t \cdot \mu E \vec{e}_z)}_{\text{correction vector}} \quad (t : \text{drift time}).$$

The reconstruction of the drift path of clusters is able to give accurate results for any possible configuration of the magnetic field inside the TPC volume. This holds even in the presence of significant field gradients, if the step width  $\delta l$  is chosen small enough that neither  $B_r$  nor  $B_\phi$  show large variations over  $\delta l$ . Electric field inhomogeneities can be considered in Equation (9) by respecting the dependency of  $\vec{v}_{\text{drift}}$  on  $\vec{E}$ . For this a position dependent description of the electric field  $\vec{E}(\vec{r})$  is needed.

## 4 Large TPC Prototype inside PCMAG

The application of both correction methods is illustrated with the example of the Large Prototype (LP) setup at the DESY electron test beam.

The LP has a drift distance of 600 mm, a radius of 360 mm and is operated inside the superconducting solenoid magnet PCMAG (Fig. 4). The magnet has an inhomogeneous magnetic field which was measured precisely in 2007 [9]. In the following, it is assumed that the 61-cm long LP is operated in the center of the magnet. For simplicity, a calculated magnetic field map of PCMAG is used,



(a) magnetic field strength distribution of PCMAG

(b)  $B_r/B$  of the magnetic field inside the LP for the central position in PCMAG

Figure 4: Magnetic field of PCMAG inside the LP

which is assumed to be rotational symmetric with a vanishing azimuthal component ( $B_\varphi \equiv 0$ ). Due to the symmetry, the calculation of the correction vectors could be restricted to a two dimensional plane cut through the center of the LP. In this plane, the magnetic field map was sampled in bins of  $200 \mu\text{m} \times 200 \mu\text{m}$ . For the electron drift mobility,  $\mu = 2 \text{T}^{-1}$  was used, which is the value of TDR gas. As a cross check, all calculations were also performed with a value of  $\mu_{\text{P5}} = 4 \text{T}^{-1}$  for P5 gas. The results do vary only slightly and are not significantly different.

#### 4.1 Calculation of the Systematic Displacements

In the first step, the displacements  $\delta(r)$  and  $\delta(r\varphi)$  for the magnetic field of PCMAG were calculated similar to the evaluation of the correction vector field discussed above (Sec. 3 and Fig 3): Starting from a cell of a  $200 \mu\text{m} \times 200 \mu\text{m}$  grid of the LP volume, the drift path of an electron cluster was extrapolated iteratively up to the anode. The accumulated  $\delta(r)$  and  $\delta(r\varphi)$  were then evaluated from the end point of a cluster on the anode and the cluster's origin. This was done successively with every cell on the grid to cover the complete volume of the LP. The electric field was assumed to be homogeneous  $\vec{E} = E \vec{e}_z$  for this calculation. Hence also  $\mu$  was set constant.

The results are shown in Figure 5. Displacements along  $z$  are negligibly small – typically some  $10 \mu\text{m}$  – and therefore omitted.

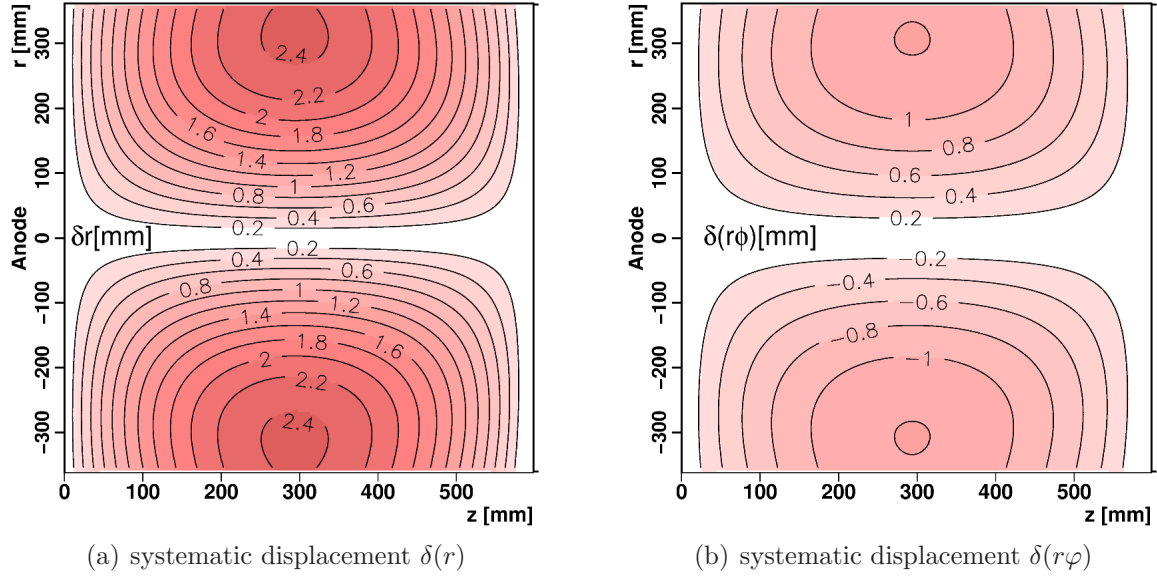


Figure 5: Calculated systematic displacements  $\delta(r)$  and  $\delta(r\varphi)$  for the LP at the center position inside PCMAG (see Fig. 4).

## 4.2 Application of the Correction

### Calculation of Displacement Integrals for PCMAG

For the case of a vanishing azimuthal component of the magnetic field ( $B_\varphi \equiv 0$ ) the calculation of the corrections using Equations (7) and (8) simplifies to

$$\Sigma(r)(z) \approx - \int_{z_0}^{z_1} \frac{(\mu B)^2}{1 + (\mu B)^2} \frac{B_r}{B} dz' \approx - \sum_{z_0}^{z_1} \frac{(\mu B)^2}{1 + (\mu B)^2} \frac{B_r}{B} \Delta z$$

$$\Sigma(r\varphi)(z) \approx - \int_{z_0}^z \frac{\mu B}{1 + (\mu B)^2} \frac{B_r}{B_z} dz' \approx - \sum_{z_0}^{z_1} \frac{\mu B}{1 + (\mu B)^2} \frac{B_r}{B_z} \Delta z.$$

The integrals were replaced by a summations with  $\Delta z = 200 \mu\text{m}$ , because the magnetic field was not available as an analytic expression. In Figures 6(a) and 6(b), the calculated components of the correction vector field are shown.

The remaining inaccuracies  $\Delta(\vec{r})$  (see Eq. (5)), after applying the correction, have a magnitude of less than  $10 \mu\text{m}$  (see Fig. 6(c)). The drift path of clusters being created in the according regions pass areas in front of the anode where the radial component of the magnetic field,  $\hat{B}_r$ , shows the largest variations in the radial direction (see Fig. 4(b)). (The contour lines in this plot become perpendicular to the anode). In these areas, the integrals underestimate  $\hat{B}_r$  and  $\hat{B}_\varphi$ , which has an effect for larger  $z$ . Going from the center towards the cathode,  $\Delta$  is reduced, because the magnetic field is symmetric to the center of the LP at  $z = 300 \text{ mm}$ , except the sign of  $\hat{B}_r$ . Displacements accumulated in the front part of the LP ( $z < 300 \text{ mm}$ ) are compensated in the back part ( $z > 300 \text{ mm}$ ).



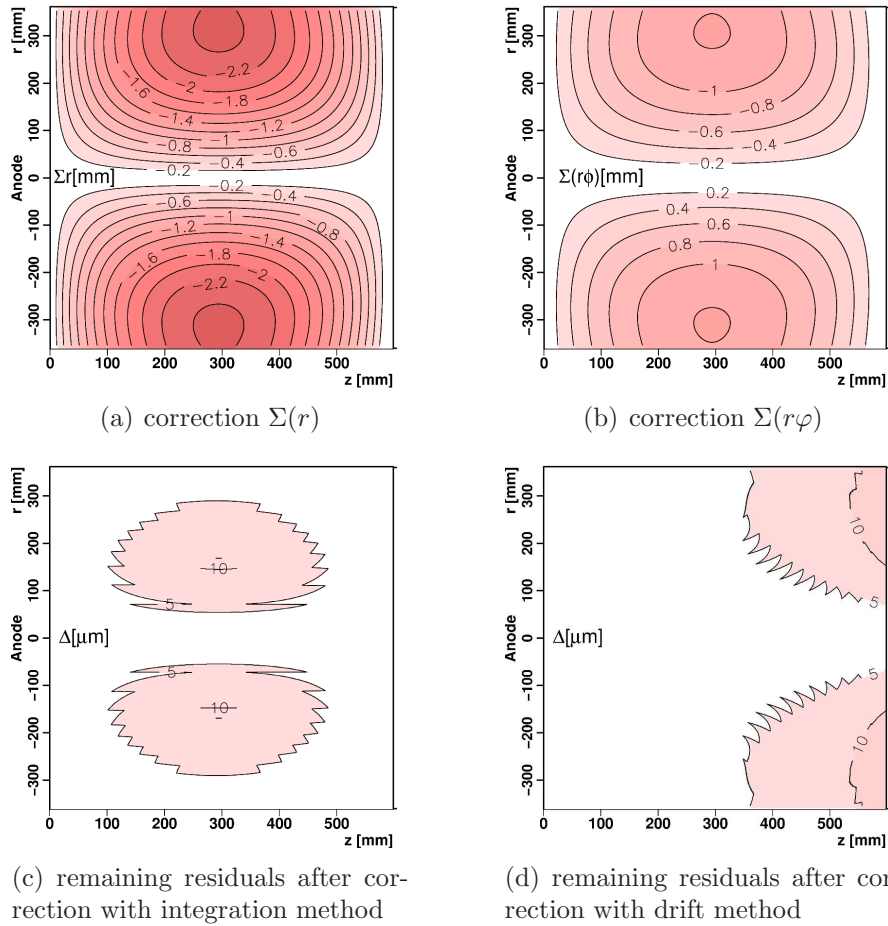


Figure 6: Components of the correction vector map and remaining residuals after application of the correction

### Inverse drift

The calculation of the correction vector map with the inverse drift method was as well performed with a step width of  $\delta l = 200 \mu\text{m}$ . The results are qualitatively equal to the ones shown in Figures 6(a) and 6(b). Differences have a magnitude of less than  $10 \mu\text{m}$ . For this case, Figure 6(d) shows the remaining residuals  $\Delta$  after the correction was applied. Here,  $\Delta$  is less than  $10 \mu\text{m}$  in a large part of the LP volume, but is maximal close to the cathode. The correction is expected to become less accurate for long drift distances, as inaccuracies in the reconstruction of a drift path, for example due to binning effects, accumulate towards the cathode.

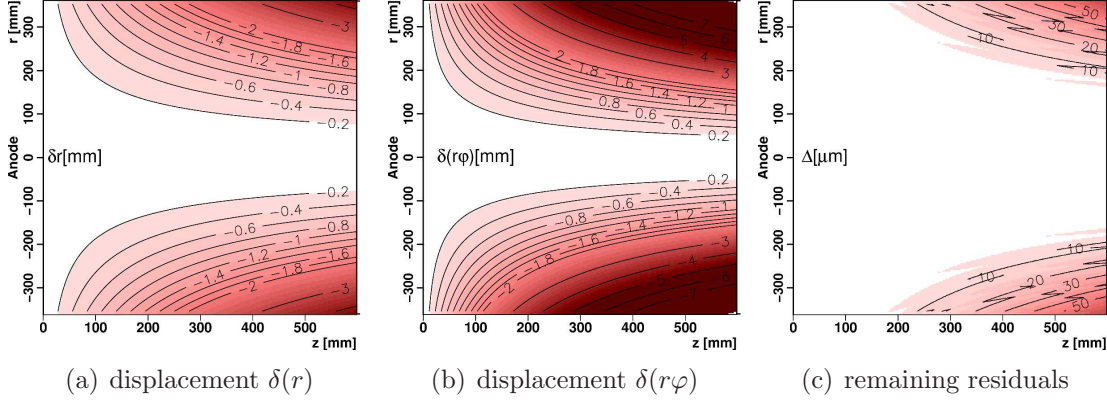


Figure 7: Systematic displacements and residuals after correction with integral method for the analytic  $\vec{B}$  field (Eq. 10).

### 4.3 Analytic Magnetic Field

To study a case without potential binning effects, the correction procedure was applied for the case of the analytically constructed magnetic field

$$\vec{B} = B_0 \left( \frac{z [\text{mm}] - 300 \text{ mm}}{1.02 \cdot 300 \text{ mm}} \right)^2 \begin{pmatrix} r^2/r_{\text{field Cage}}^2 \\ r^2/r_{\text{field Cage}}^2 \\ \sqrt{1 - 2 \cdot r^4/r_{\text{field Cage}}^4} \end{pmatrix} \begin{pmatrix} \vec{e}_r \\ \vec{e}_\varphi \\ \vec{e}_z \end{pmatrix}. \quad (10)$$

The constant defining the maximum field strength is  $B_0 = 1.2 \text{ T}$  and  $r_{\text{field cage}}$  is the inner radius of 360 mm of the LP field cage.

Figure 7(a) shows the displacements, calculated with a step width of  $1 \mu\text{m}$ . The correction maps were calculated with the magnetic field again binned in  $200 \mu\text{m}$  steps as before.

The drift method can correct the displacements to a level better than  $30 \mu\text{m}$  in the complete volume. However, for this magnetic field, the integral correction method becomes inaccurate in areas where the radial displacements are larger than 3 mm. For this  $\vec{B}$ -field, the correction is sufficiently accurate only if

$$\int_0^z \frac{B_r}{B} dz \lesssim 3 \text{ mm}.$$

## 5 Gas Contaminations

The calculations of the correction vector maps were performed with a nominal value for the electron mobility  $\mu$ . Gas contaminations, in particular water pollutions, modify the value of  $\mu$  in a range of  $\pm 10\%$  (see e.g. [10]). A variation of  $\mu$  has an influence of the drift velocities  $\vec{v}_{\text{drift}}(\vec{r})$  in the TPC volume (see Eq. (1)) and the displacements accumulated by the drifting electron clusters. The correction vectors need to be adjusted to the modified scenario before the correction procedure is applied.

In case of the correction method using the calculated displacement integrals (Sec. 3), the adjustment can be performed by the scaling the prefactors in Equations (7) and (8) according to

$$\frac{(\mu B)^2}{1 + (\mu B)^2} \rightarrow \frac{\mu'^2}{\mu^2} \frac{1 + (\mu B)^2}{1 + (\mu' B)^2} \quad \text{and} \quad \frac{\mu B}{1 + (\mu B)^2} \rightarrow \frac{\mu'}{\mu} \frac{1 + (\mu B)^2}{1 + (\mu' B)^2}. \quad (11)$$

$\mu'$  is the modified value of  $\mu$ . For PCMAG,  $B$  can be assumed to be a constant mean value, although the magnetic field of the PCMAG shows variations in strength of  $\pm 3\%$ . The remaining residuals after the correction are of the order  $15 \mu\text{m}$ , if  $\mu'$  does not differ from  $\mu$  by more than  $\pm 10\%$ .

The scaling factors derived from the adaption procedure can also be used to adapt a correction field which was evaluated with the drift method.

## 6 Positioning Accuracy of the TPC

To derive a condition for the positioning accuracy which is required for the LP inside the PCMAG that the correction works accurately, it is assumed that the LP is displaced in the  $r$ -direction by  $\Delta r \neq 0$  with respect to its nominal position. In this case, the correction vector for a cluster, which has been reconstructed at  $\vec{r}_{\text{reco.}}$  is found at the modified position  $\vec{r}_{\text{reco.}} + \Delta r \cdot \vec{e}_r$ . If  $\Delta r$  is assumed to vanish, the correction is inaccurate by

$$\Delta\Sigma(r) = \sqrt{\left(\frac{\partial}{\partial r}\Sigma(r)\right)^2 + \left(\frac{\partial}{\partial r}\Sigma(r\varphi)\right)^2} \cdot \Delta r$$

The same argument for the longitudinal component yields the inaccuracy

$$\Delta\Sigma(z) = \sqrt{\left(\frac{\partial}{\partial z}\Sigma(r)\right)^2 + \left(\frac{\partial}{\partial z}\Sigma(r\varphi)\right)^2} \cdot \Delta z$$

For the LP/PCMAG setup, the magnitude of the largest derivatives can be estimated from the correction vector map in Figures 6(a) and 6(b). These are

$$\begin{aligned} \frac{\partial}{\partial r}\Sigma(r) \Big|_{\text{max}} &\approx 0.010 & \text{and} & & \frac{\partial}{\partial r}\Sigma(r\varphi) \Big|_{\text{max}} &\approx 0.005 \\ \frac{\partial}{\partial z}\Sigma(r) \Big|_{\text{max}} &\approx 0.016 & \text{and} & & \frac{\partial}{\partial z}\Sigma(r\varphi) \Big|_{\text{max}} &\approx 0.008. \end{aligned}$$

$\Delta\Sigma(r)$  and  $\Delta\Sigma(z)$  become

$$\Sigma(r) \approx 0.011 \Delta r \quad \text{and} \quad \Sigma(z) \approx 0.018 \Delta z.$$

The additional inaccuracy of the correction is limited to  $10 \mu\text{m}$ , if  $\Delta z$  and  $\Delta r$  are both smaller than  $0.5 \text{ mm}$ .

## 7 Next Steps

The correction methods presented in this note can be applied to the data taken with the Large TPC Prototype (LP) infrastructure at DESY. As said before, the LP is operated inside the well-measured magnetic field of the superconducting magnet PCMAG. Particle tracks are produced inside the volume of the LP by electrons from the DESY test beam.

By the end of 2010 it is planned to operate the LP with a large readout structure of about a quarter of square meter. In parallel, a set of silicon detectors will be prepared and installed directly on the outside of the LP. These additional detectors will perform precise measurements of passing test beam electrons and deliver independent reference points of the particle trajectories. In the data analysis, it will be possible to compare corrected trajectories with the external reference points.

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## A Derivation of Equations (2) and (4)

### A.1 Displacement due to electric field inhomogeneities in a homogeneous magnetic field

To derive Equation (2), it is assumed that the magnetic field in the TPC is  $\vec{B} = B\vec{e}_z$ . In a point of the TPC volume, the electric field shall have a field

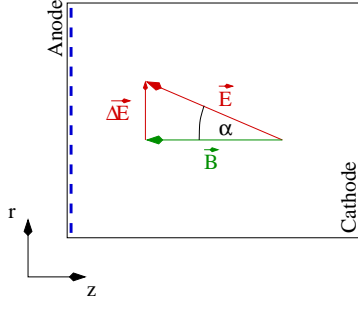


Figure 8: Definition of  $\alpha = \sphericalangle(\vec{E}, \vec{B})$

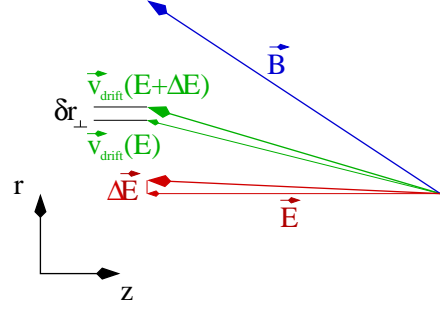


Figure 9: Definition of  $\vec{v}_{\text{drift}}(\Delta E \neq 0)$  and  $\vec{v}_{\text{drift}}(\Delta E = 0)$

component in the radial direction, namely  $\vec{E} = E_z \vec{e}_z + E_r \vec{e}_r$ . Introducing the angle  $\alpha = \sphericalangle(\vec{E}, \vec{e}_z)$  (see Figure 8), this can be rewritten as

$$\vec{E} = E(\cos \alpha \vec{e}_z + \sin \alpha \vec{e}_r).$$

In this point, the drift velocity vector is

$$\vec{v}_{\text{drift}} = \frac{\mu E}{1 + (\mu B)^2} [\sin \alpha \vec{e}_x + (\mu B) \sin \alpha \vec{e}_y + \cos \alpha \cdot (1 + (\mu B)^2) \vec{e}_z] \quad (12)$$

and has the absolute value

$$|\vec{v}_{\text{drift}}| = \frac{\mu E}{1 + (\mu B)^2} \sqrt{1 + (1 + \cos^2 \alpha) \cdot (\mu B)^2 + (\mu B)^4 \cos^2 \alpha} \approx \frac{\mu E \cos \alpha}{\sqrt{1 + (\mu B)^2}}. \quad (13)$$

Combining Equations (12) and (13), the displacement  $\delta_{\perp}$ , which the cluster sums up while drifting a distance  $l$  in the vicinity of the point, calculates to

$$\delta r_{\perp} = \frac{\sqrt{(\vec{v}_{\text{drift}} \cdot \vec{e}_r)^2 + (\vec{v}_{\text{drift}} \cdot \vec{e}_{\varphi})^2}}{|\vec{v}_{\text{drift}}|} \cdot l \approx \frac{l}{\sqrt{1 + (\mu B)^2}} \frac{\sin \alpha}{\cos \alpha}.$$

With the approximation  $\tan \alpha \approx \sin \alpha = \Delta E / E$  for small  $\alpha$ , this is

$$\delta r_{\perp} \approx \frac{l}{\sqrt{1 + (\mu B)^2}} \frac{\Delta E}{E}.$$

If  $\Delta E$  has its maximum in the considered point,  $\delta_{\perp}$  for the maximum drift distance  $L$  of the TPC can be estimated by replacing  $l$  by  $L$  in this equation.

## A.2 Inaccuracy of a correction vector map due to electric field inhomogeneities

Equation (2) is an estimation on the inaccuracy of a correction for an inhomogeneous drift field caused by electric field inhomogeneities. It is derived under

the assumption that the magnetic field  $\vec{B}$  is measured and the electric field assumed to be homogeneous  $\vec{E} = E \vec{e}_z$ . In this case, the drift velocity vector  $\vec{v}_{\text{drift}} = \vec{v}_{\text{drift}}(\Delta E = 0)$  is given by (3). If a small additional electric field homogeneity  $\Delta \vec{E} = \Delta E \vec{e}_r$  is present in a point of the TPC, the local  $\vec{v}_{\text{drift}}$  is

$$\vec{v}_{\text{drift}}(\Delta E \neq 0) \approx \frac{\mu E}{1 + (\mu B)^2} \begin{pmatrix} \Delta E/E & - & (\mu B) \hat{B}_\varphi & + & (\mu B)^2 (\Delta E/E \hat{B}_r + \hat{B}_z) \hat{B}_r \\ & (\mu B) (\hat{B}_r - \Delta E/E \hat{B}_z) & + & (\mu B)^2 (\Delta E/E \hat{B}_r + \hat{B}_z) \hat{B}_\varphi \\ 1 & + & (\mu B) (\Delta E/E \hat{B}_\varphi) & + & (\mu B)^2 (\Delta E/E \hat{B}_r + \hat{B}_z) \hat{B}_z \end{pmatrix}. \quad (14)$$

The inaccuracy, perpendicular to the z-axis, of the correction which is accumulated in the vicinity  $l$  of the point is (see Figure 9)

$$\delta r_\perp = \frac{|\vec{v}_{\text{drift}}(\Delta E \neq 0) - \vec{v}_{\text{drift}}(\Delta E = 0)| \cdot (\vec{e}_r + \vec{e}_\varphi)}{|\vec{v}_{\text{drift}}|} \cdot l$$

where  $v_{\text{drift}}(\Delta E \neq 0) \approx v_{\text{drift}}(\Delta E = 0)$  is assumed, that means  $|\vec{v}_{\text{drift}}|$  does not changed significantly by the small  $\Delta E$ . With equations (3) and (14) and the approximation (6), this calculates to

$$\delta r_\perp \approx \frac{1}{\sqrt{1 + (\mu B)^2}} \frac{\Delta E}{E} \frac{\sqrt{1 + (\mu B)^2 + ((\mu B)^2 + (\mu B)^4) \hat{B}_r^2}}{\sqrt{1 + (\mu B)^2} \hat{B} \cdot \hat{E}} \quad (15)$$

where  $\hat{B}_r^2 + \hat{B}_\varphi^2 + \hat{B}_z^2 = 1$  was used.

For the special case  $\vec{B} = B(\cos \alpha \vec{e}_z + \sin \alpha \vec{e}_r)$ , which is realized in the model of PCMAG in Sec 4,  $\delta r_\perp$  is

$$\delta r_\perp \approx \frac{1}{\sqrt{1 + (\mu B)^2}} \frac{\Delta E}{E} \underbrace{\frac{\sqrt{1 + (\mu B)^2 + ((\mu B)^2 + (\mu B)^4) \sin^2 \alpha}}{\sqrt{1 + (\mu B)^2} \cos \alpha}}_{f(\mu B, \alpha)}.$$

In this case  $f \approx 2$  for  $B = 1 \text{ T}$ ,  $\mu = 4 \text{ T}^{-1}$  and  $\alpha = 25^\circ$  and decreases equally with any of the three. For the LP operated in the center of PCMAG,  $\alpha \lesssim 2^\circ$  in the whole volume, as can be calculated from Figure 4(b), thus  $f \approx 1$ . In case of the ILD TPC  $f \approx 2$  for  $B = 3.5 \text{ T}$ ,  $\mu = 4 \text{ T}^{-1}$  and  $\alpha = 7^\circ$ .

The derivation above is restricted for the case  $\Delta \vec{E} = \Delta E \vec{e}_r$ , but the result is the same for the case  $\Delta \vec{E} = \Delta E \vec{e}_\varphi$ . Then  $\hat{B}_r$  is replaced by  $\hat{B}_\varphi$  in Equation (15).