Higgs boson mass and new physics

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Abstract

We discuss the lower Higgs boson mass bounds which come from the absolute stability of the Standard Model (SM) vacuum and from the Higgs inflation, as well as the prediction of the Higgs boson mass coming from asymptotic safety of the SM. We account for the 3-loop renormalization group evolution of the couplings of the Standard Model and for a part of two-loop corrections that involve the QCD coupling α_s to initial conditions for their running. This is one step above the current state of the art procedure ("one-loop matching–two-loop running"). This results in reduction of the *theoretical* uncertainties in the Higgs boson mass bounds and predictions, associated with the Standard Model physics, to $1-2\,\mathrm{GeV}$. We find that with the account of existing experimental uncertainties in the mass of the top quark and α_s (taken at 2σ level) the bound reads $M_H \geq M_{\min}$ (equality corresponds to the asymptotic safety prediction), where $M_{\min} = 129 \pm 6\,\mathrm{GeV}$. We argue that the discovery of the SM Higgs boson in this range would be in agreement with the hypothesis of the absence of new energy scales between the Fermi and Planck scales, whereas the coincidence of M_H with $M_{\min}$ would suggest that the electroweak scale is determined by Planck physics. In order to clarify the

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relation between the Fermi and Planck scale a construction of an electron-positron or muon collider with a center of mass energy $\sim 200+200\,\mathrm{GeV}$ (Higgs and t-quark factory) would be needed.

1 Introduction

The mass M_H of the Higgs boson in the Standard Model is an important indicator of the presence of new energy scales in particle physics. It is well known that if $M_{\rm min}^{\rm stability} < M_H < M_{\rm max}^{\rm Landau}$ then the SM is a consistent effective field theory all the way from the Fermi scale up to the (reduced) Planck scale $M_P = 2.44 \times 10^{18}\,\mathrm{GeV}$. The upper limit comes from the requirement that the Landau pole in the scalar self-coupling[1] must not appear at energies below M_P [1–3]. The lower limit comes from the requirement of the stability of the SM vacuum against tunneling to the states with the Higgs field ϕ exceeding substantially the electroweak value 250 GeV [4–6] (see Fig. 1).

The estimates of $M_{\rm max}^{\rm Landau}$ give a number around 175 GeV [1–3, 7] which is in the M_H range excluded (at least in the range $129 - 525\,\mathrm{GeV}$) by the searches for the SM Higgs boson at the LHC and Tevatron [8, 9]. In other words, we already know that the SM is a weakly coupled theory up to the Planck scale. Thus, we will focus on the upgrade of existing computations of $M_{\rm min}^{\rm stability}$ and on the discussion of the significance of the relation between the Higgs boson (to be discovered yet) mass M_H and $M_{\rm min}^{\rm stability}$ for beyond the SM (BSM) physics.

The computation of $M_{\rm min}^{\rm stability}$ has been already done in a large number of papers [10–15]. It is divided into two parts. The first one is the determination of the $\overline{\rm MS}$ parameters from the physical observables and the second one is the renormalization group running of the $\overline{\rm MS}$ constants from the electroweak to a high energy scale. The most advanced recent works [13, 14] use the so-called "one-loop-matching–two-loop-running" procedure. It can determine the Higgs boson mass bounds with the theoretical accuracy of $2-5\,\mathrm{GeV}$ (see the discussion of uncertainties in [14] and below). Meanwhile, the most important terms in the 3-loop running of the gauge and Higgs coupling constants were computed in [16, 17] (we thank K. Chetyrkin and M. Zoller for sharing these results with us prior to publication). The present work accounts for $O(\alpha\alpha_s)$ corrections in the $\overline{\rm MS}$-pole matching procedure, which were not known previously. This allows us to decrease the theoretical uncertainties in the Higgs boson mass prediction/bounds, associated with the SM physics down to $1-2\,\mathrm{GeV}$. This is a new result, based on a superior partial "two-loop-matching–three-loop-running" procedure. These findings are described in Section 2.2. We will see that the experimental errors in the mass of the top-quark and in the value of the strong coupling constant are too large to settle up the question of the stability of the electroweak vacuum, even if the LHC will confirm the evidence for the Higgs signal presented by the ATLAS and CMS collaborations [8, 9] in the region $M_H = 124 - 126\,\mathrm{GeV}$.

In Section 3 we will discuss the significance of the relationship between the true Higgs

[1] To be more precise, the scalar self-coupling is infinite in the one-loop approximation only. If higher order terms are included, it may not become infinite, but move away from the region of the weak coupling.

boson mass M_H and $M_{\rm min}^{\rm stability}$ for BSM physics. We will argue that if $M_H = M_{\rm min}^{\rm stability}$ then the electroweak symmetry breaking is likely to be determined by Planck physics and that this would indicate an absence of new energy scales between the Fermi and gravitational scales. We will also address here the significance of $M_{\rm min}^{\rm stability}$ for the SM with gravity included. Of course, this can only be done under certain assumptions. Specifically, we will discuss the non-minimal coupling of the Higgs field to the Ricci scalar (relevant for Higgs-inflation [14, 18, 19]) and the asymptotic safety scenario for the SM [20].

In Section 4 we present our conclusions. We will argue that if only the Higgs boson with the mass around $M_{\rm min}^{\rm stability}$ and nothing else will be found at the LHC, the next step in high energy physics should be the construction a new electron-positron (or muon) collider—the Higgs and t-factory. It will not only be able to investigate in detail the Higgs and top physics, but also elucidate the possible connection of the Fermi and Planck scales.

Appendix A contains a full set of formulas required for the determination of the $\overline{\rm MS}$ coupling constants from the pole masses of the SM particles, including the corrections of the orders of up to $O(\alpha_s^3)$, $O(\alpha)$, and $O(\alpha\alpha_s)$. The computer code for the matching is made publicly available at `http://www.inr.ac.ru/~fedor/SM/`.

2 The stability bound

The stability bound will be found in the "canonical" SM, without any new degrees of freedom or any extra higher dimensional operators added, see Fig. 2.

2.1 The benchmark mass

It will be convenient for computations to introduce yet another parameter, "benchmark mass", which we will call $M_{\rm min}$ (without any superscript). Suppose that all parameters of the SM, except for the Higgs boson mass, are exactly known. Then $M_{\rm min}$, together with the normalisation point μ_0, are found from the solution of two equations:

$$\lambda(\mu_0) = 0, \quad \beta_\lambda(\lambda(\mu_0)) = 0, \tag{1}$$

where β_λ is the β-function governing the renormalisation group (RG) running of λ. Here we define all the couplings of the SM in the $\overline{\rm MS}$ renormalisation scheme which is used de-facto in the most of the higher-loop computations. Clearly, if any other renormalization scheme is used, the equations $\lambda = \beta_\lambda = 0$ will give another benchmark mass, since the definition of all the couplings are scheme dependent.

The procedure of computing $M_{\rm min}$ is very clean and transparent. Take the standard $\overline{\rm MS}$ definition of all coupling constants of the SM, fix all of them at the Fermi scale given the experimentally known parameters such as the mass of the top quark, QCD coupling, etc., and consider the running Higgs self-coupling $\lambda(\mu)$ depending on the standard t'Hooft-Veltman parameter μ. Then, adjust $M_{\rm min}$ in such a way that equations (1) are satisfied at some μ_0.

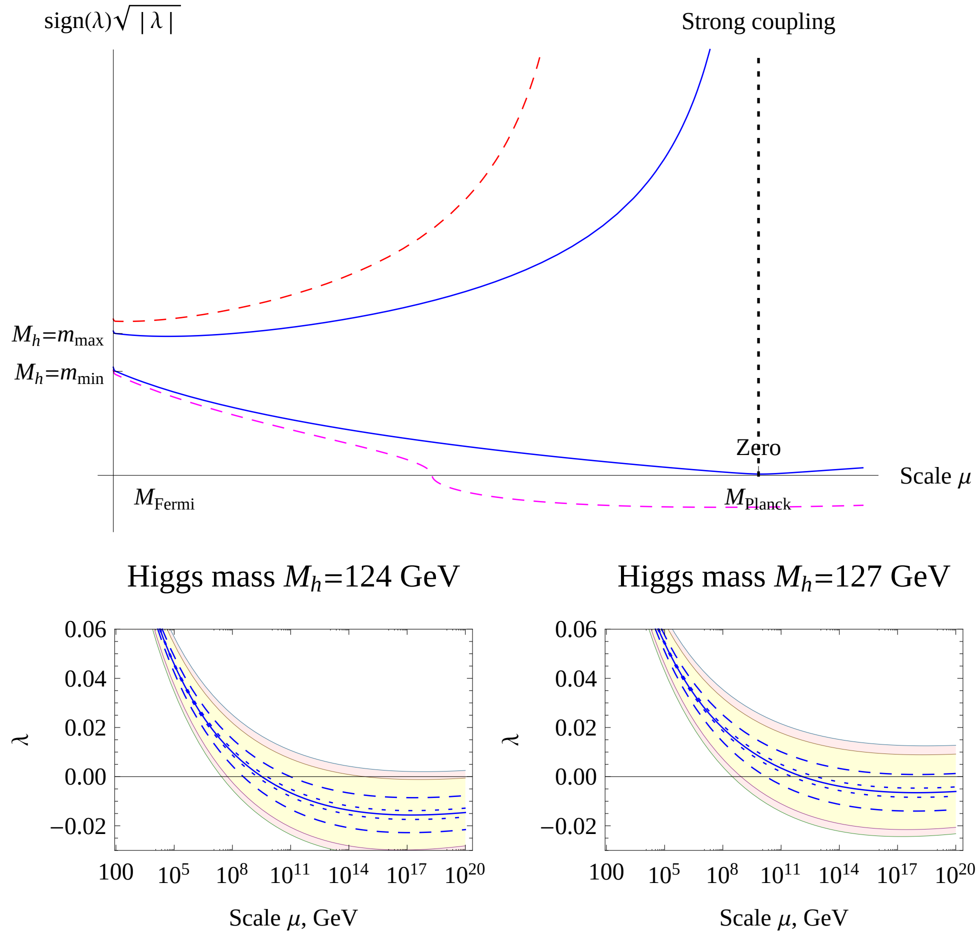

Figure 1: Higgs self-coupling in the SM as a function of the energy scale. The top plot depicts possible behaviors for the whole Higgs boson mass range—Landau pole, stable, or unstable electroweak vacuum. The lower plots show detailed behavior for low Higgs boson masses, with dashed (dotted) line corresponding to the experimental uncertainty in the top mass M_t (strong coupling constant α_s), and the shaded yellow (pink) regions correspond to the total experimental error and theoretical uncertainty, with the latter estimated as 1.2 GeV (2.5 GeV), see section 2 for detailed discussion.

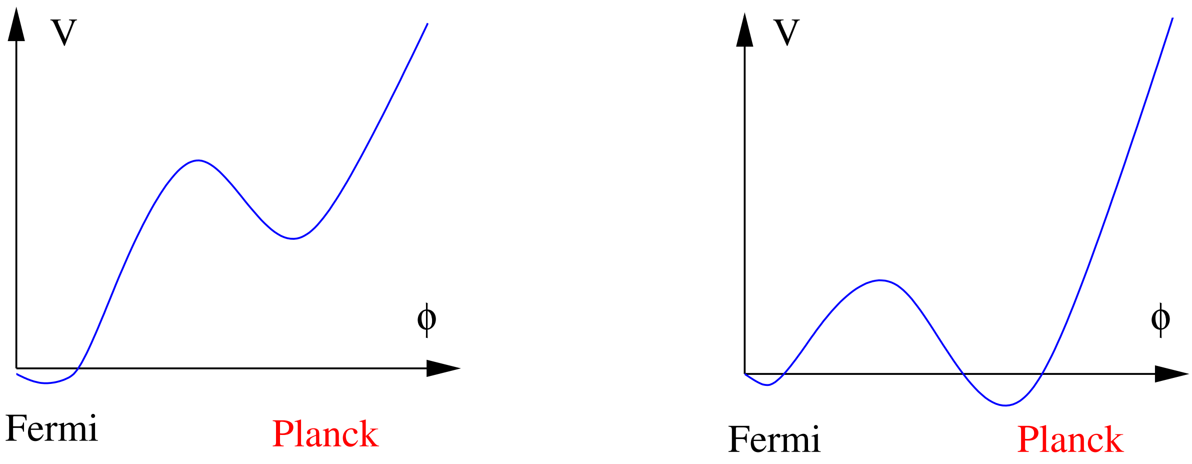

Figure 2: Schematic depiction of the SM effective potential V for the Higgs field for $M_H > M_{\rm min}^{\rm stability}$ (left) and $M_H < M_{\rm min}^{\rm stability}$ (right).

For the stability bound one should find the effective potential $V(\phi)$ and solve the equations

$$V(\phi_{SM}) = V(\phi_1), \quad V'(\phi_{SM}) = V'(\phi_1) = 0, \tag{2}$$

where ϕ_{SM} corresponds to the SM Higgs vacuum, and ϕ_1 correspond to the extra vacuum states at large values of the scalar field. Though the effective potential and the field ϕ are both gauge and scheme dependent, the solution for the Higgs boson mass to these equations is gauge and scheme invariant.

In fact, $M_{\rm min}^{\rm stability}$ is very close to $M_{\rm min}$. Numerically, the difference between them is much smaller, than the current theoretical and experimental precisions for $M_{\rm min}$, see below. The following well known argument explains why this is the case. The RG improved effective potential for large ϕ can be written as [11, 12, 21]

$$V(\phi) \propto \lambda(\phi)\phi^4 \left[1 + O\left(\frac{\alpha}{4\pi}\log(M_i/M_j)\right)\right], \tag{3}$$

where α is here the common name for the SM coupling constants, and M_i are the masses of different particles in the background of the Higgs field. If $O(\alpha)$ corrections are neglected, the equations (2) coincide with (1), meaning that $M_{\rm min} \simeq M_{\rm min}^{\rm stability}$. The numerical evaluation for one loop effective potential gives $\Delta m^{\rm stability} \equiv M_{\rm min}^{\rm stability} - M_{\rm min} \simeq 0.15\,{\rm GeV}$, which can be neglected in view of uncertainties discussed below.

Note that in many papers the stability bound is shown as a function of the cutoff scale Λ (the energy scale up to which the SM can be considered as a valid effective field theory). It is required that $V(\phi) > V(\phi_{SM})$ for all $\phi < \Lambda$. This can be reformulated as $\lambda(\mu) > 0$ for all $\mu < \Lambda$ with pretty good accuracy. Interestingly, if $\Lambda = M_P$, this bound is very close to the stability bound following from eq. (2), having nothing to do with the Planck scale (see also below). Note also that the uncertainties in experimental determinations of M_t and α_s together with theoretical uncertainties, described in the next section, lead to significant changes in the scale Λ. Fig. 1 illustrates that for Higgs boson masses 124 − 127 GeV this scale may vary from 10^8 GeV up to infinity within currently available precisions.

2.2 Value of $M_{\rm min}$

The state of art computation of $M_{\rm min}$ contained up to now the so called one-loop $\overline{\rm MS}$-pole matching, relating the experimentally measured physical parameters to the parameters of the SM in the $\overline{\rm MS}$ subtraction scheme (to be more precise, the two-loop α_s corrections to the top pole mass–$\overline{\rm MS}$ mass relation has been included [13]). Then the results of the first step are plugged into two-loop RG equations and solved numerically.

Before discussing the upgrade of the one-loop-matching–two-loop-running procedure, we will remind of the results already known and their uncertainties. We will make use of our computations of $M_{\rm min}$ presented in [14].[2] A somewhat later paper [22] contains exactly the same numbers for $M_{\rm min}^{\rm stability}$ (note, however, that the theoretical uncertainties were not discussed in [22]). See also earlier computations in [7, 10–13, 23].

In [14] we found:

$$M_{\rm min} = \left[126.3 + \frac{M_t - 171.2\,{\rm GeV}}{2.1\,{\rm GeV}} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5\right] {\rm GeV}, \tag{4}$$

and estimated the theoretical uncertainties as summarized in Table 1 (see also [15]). While repeating this analysis we found some numerical errors which are given at the bottom section of this table (see a detailed discussion below). In total, they shift the value given in eq. (4) up by 0.7 GeV. As for uncertainties, they were estimated as follows. The one-loop matching formulas can be used directly at $\mu = m_t$, or at some other energy scale, e.g. at $\mu = M_Z$, and then the coupling constants at m_t can be derived with the use of RG running. The difference in procedures gives an estimate of two-loop effects in the matching procedure. This is presented by the first two lines in Table 1 (in fact, we underestimated before the uncertainty from λ matching—previously we had here 1.2 GeV and now 1.7 GeV). The next two lines are associated with 3 and 4-loop corrections to the top Yukawa y_t. The 3-loop corrections were computed in [24–26] and the four-loop α_s contribution to the top mass was guessed to be of the order $\delta y_t(m_t)/y_t(m_t) \simeq 0.001$ in [27]. The non-perturbative QCD effects in the top pole mass–$\overline{\rm MS}$ mass matching are expected to be at the same level [28–30]. For 3-loop running we put the typical coefficients in front of the largest couplings α_s and y_t. If these uncertainties are not correlated and can be summed up in squares, the theoretical uncertainty is 2.5 GeV. If they are summed up linearly, then the theoretical error can be as large as ~ 5 GeV.

Now, this computation can be considerably improved. First, in [16] the 3-loop corrections to the running of all gauge couplings has been calculated. Second, in [17] the leading contributions (containing the top Yukawa and α_s) to the running of the top quark Yukawa and the Higgs boson self coupling have been determined. This removes the uncertainty related to 3-loop RG running. In addition, in the present paper, we

[2] The main interest in this paper was the lower bound on the Higgs boson mass in the Higgs-inflation, see below. However, $M_{\rm min}$ has been estimated as well as a by-product of the computation.

Source of uncertainty	Nature of estimate	$\Delta_{\text{theor}} M_{\min}$, GeV
2-loop matching λ	Sensitivity to μ	1.7
2-loop matching y_t	Sensitivity to μ	0.6
3-loop α_s to y_t	known	1.4
4-loop α_s to y_t	educated guess [27]	0.4
confinement, y_t	educated guess $\sim \Lambda_{QCD}$	0.5
3-loop running $M_W \to M_P$	educated guess	0.8
total uncertainty	sum of squares	2.5
total uncertainty	linear sum	5.4
Corrections to [14]		$\Delta M_{\min}$, GeV
Typos in the code used in [14]	error	+0.2
Extra $\delta_{\rm t}^{\rm QED}$ in (A.5) of [13]	error	+0.4
"Exact" formula instead of approximation (2.20) in [31]	clarification	+0.1
Total correction to (7.1) of [14]		+0.7
Total shift to be applied to (7.1) of [14] for comparison		+0.7

Table 1: Theoretical uncertainties and mistakes in the $M_{\min}$ evaluation in [14].

determine the two-loop corrections of the order of $O(\alpha\alpha_s)$ to the matching of the pole masses and the top quark Yukawa and Higgs boson self coupling constants. Also, the known [24–26] three loop QCD correction to the top quark mass relation of the order $O(\alpha_s^3)$ can be included (previously it had been used for estimates of uncertainties). All this considerably decreases the theoretical uncertainties in $M_{\min}$.

The individual contributions of the various new corrections on top of the previous result are summarized in the Table 2. It is clearly seen that there are two new significant contributions—one is the three-loop pure QCD correction to the top quark mass [24–26], and another is the two loop correction $O(\alpha\alpha_s)$ to the Higgs boson mass, found in the present paper. Together the new contributions sum to the overall shift of the previous prediction [14] by $-0.89\,\text{GeV}$, giving the result

$$M_{\min} = \left[128.95 + \frac{M_t - 172.9\,\text{GeV}}{1.1\,\text{GeV}} \times 2.2 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.56\right] \text{GeV}. \tag{5}$$

The new result (5) is less than $0.2\,\text{GeV}$ away from the old one (4) if the same central values for M_t and α_s are inserted. This coincidence is the result of some magic. In the old evaluation several mistakes were present, summarized in Table 1. The largest one was the double counting of $\delta_t^{\rm QED}$ in (A.5) of [13], as compared to the original result [31]. Also, there were minor typos in the computer code for the matching of the Higgs coupling constant, and finally there was a small correction coming from the use of an approximate

Contribution	$\Delta M_{\rm min}$, GeV
Three loop beta functions	-0.23
$\delta y_t \propto O(\alpha_s^3)$	-1.15
$\delta y_t \propto O(\alpha\alpha_s)$	-0.13
$\delta\lambda \propto O(\alpha\alpha_s)$	0.62

Table 2: Contributions to the value of the $M_{\rm min}$.

rather than exact one loop formula for $O(\alpha)$ corrections from [31]. These corrections add 0.7 GeV to the original number in [14]. By chance this almost exactly canceled the -0.89 GeV contribution from the higher loops, Table 2, nearly leading to a coincidence of (5) and (4).

Table 3 summarizes the uncertainties in the new computation. It contains fewer lines. Now we can ignore safely the error from higher order (4-loop) RG corrections for the running up to the Planck scale. The first two lines were derived in the same manner as previously. For the Higgs boson self-coupling we can use the matching formulas (A.42) to get the value of $\lambda(\mu)$ at scale $\mu = M_t$ directly, or to get the value $\lambda(M_Z)$ and then evolve the constants to the scale $\mu = M_t$ with the RG equations. The obtained difference $\delta\lambda(M_t)/\lambda(M_t) \simeq 0.016$ corresponds to the error $\delta m \sim 1.0$ GeV. A similar procedure of comparing evolution between M_t and M_Z using RG equations and direct matching formulas to the order $O(\alpha_s^3, \alpha, \alpha\alpha_s)$ leads for the chang in the top quark Yukawa $\delta y_t/y_t \sim 0.0005$, leading to $\delta M \sim 0.2$ GeV. Note, however, that strictly speaking this test verifies the error of the μ dependent terms in the matching formulas, while the constant ones may lead to larger contributions. We also do not estimate now the contributions of the order $O(\alpha^2)$, where formal order in α may correspond to y_t^4. Thus, this estimate should be better considered as a lower estimate of the error. The 4-loop matching and confinement contributions are the same as before.

As an indication of the dependence on the matching point we present Fig. 3, where the reference Higgs boson mass $M_{\rm min}$ was obtained using the matching formulas at scale μ_0 varying between the Z-boson and top quark masses. One can see that the overall change of the Higgs boson mass is about GeV.

If we assume that these uncertainties are not correlated and symmetric we get a theoretical error in the determination of the critical Higgs boson mass $\delta m_{\rm theor} \simeq 1.2$ GeV. If they are summed up linearly, we get an error of 2.4 GeV. The precision of the theoretical value of $M_{\rm min}$ can be further increased by computing the $O(\alpha^2)$ two-loop corrections to the matching procedure. Numerically, the most important terms are those when α corresponds to y_t^2 and λ.

The result (5) is visualized by Fig. 4. The experimentally allowed regions for the top mass M_t and strong coupling α_s are adopted PDG 2010 edition [32].[3] On top of these

[3]Note however, that the current experimental error estimate is based on averaging over different experimental approaches. In some methods quite a different central values are obtained. See e.g. [33–35] about α_s determination and [36–38] about M_t.

Source of uncertainty	Nature of estimate	$\Delta_{\rm theor} M_{\rm min}$, GeV
3-loop matching λ	Sensitivity to μ	1.0
3-loop matching y_t	Sensitivity to μ	0.2
4-loop α_s to y_t	educated guess [27]	0.4
confinement, y_t	educated guess $\sim \Lambda_{QCD}$	0.5
4-loop running $M_W \to M_P$	educated guess	< 0.2
total uncertainty	sum of squares	1.2
total uncertainty	linear sum	2.3

Table 3: Theoretical uncertainties in the present $M_{\rm min}$ evaluation.

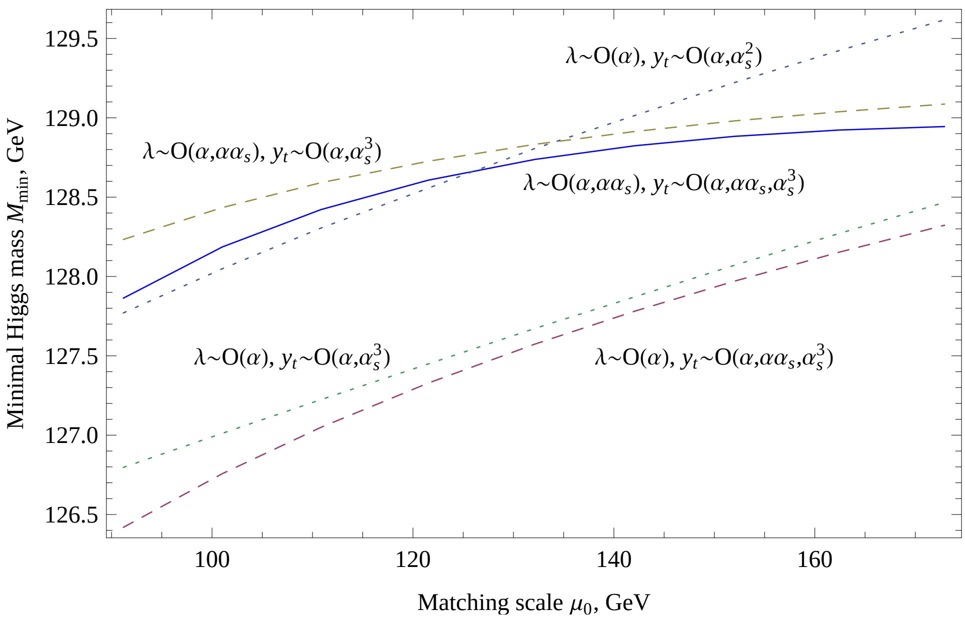

Figure 3: The dependence of the reference Higgs boson mass $M_{\rm min}$ on the matching scale μ_0 (the $\overline{\rm MS}$ constants are obtained by matching formulas at scale μ_0 and then used for the solution of the equations (1)). The solid line corresponds to the full matching formulas $\lambda \sim O(\alpha, \alpha\alpha_s)$, $y_t \sim O(\alpha_s^3, \alpha, \alpha\alpha_s)$; the dashed and dotted lines correspond to using matching formulas of lower order. Here $M_t = 172.9\,$GeV and $\alpha_s = 0.1184$.

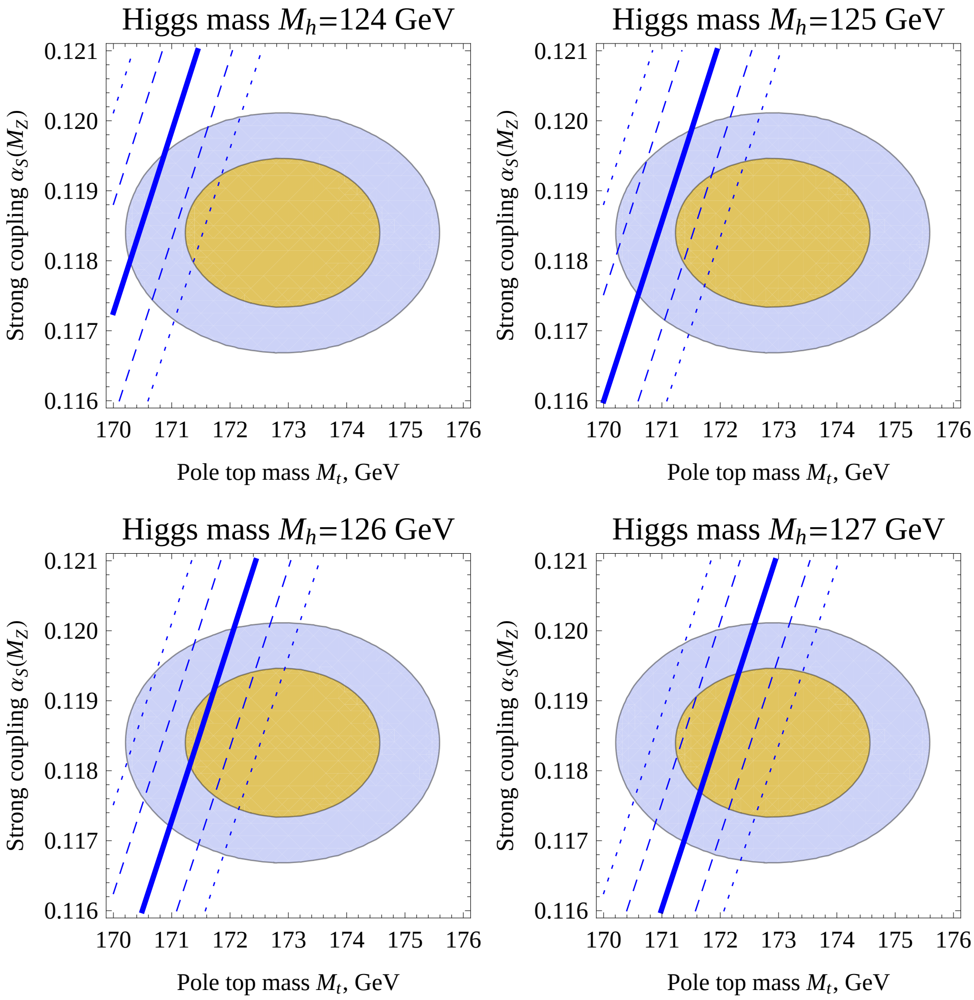

Figure 4: The values of the strong coupling constant α_s and top mass M_t corresponding to several minimal Higgs boson mass $M_{\rm min}$. The 68% and 95% experimentally allowed regions for α_s and M_t are given by shaded areas. The dashed (dotted) lines correspond to 1.2 GeV (2.45 GeV) uncertainty in the $M_{\rm min}$ theoretical determination.

allowed regions the bands corresponding to the reference values of the Higgs boson mass $M_{\rm min}$ being equal to $124, 125, 126, 127\,\mathrm{GeV}$ are shown, with the dashed and dotted lines corresponding to quadratically or linearly added estimates of theoretical uncertainties.

One can see that the accuracy of theoretical computations and of the experimental measurements of the top and the Higgs boson masses does not allow yet to conclude with confidence whether the discovery of the Higgs boson with the mass $124-127\,\mathrm{GeV}$ would indicate stability or metastability of the SM vacuum. All these reference values of Higgs masses are compatible within 2σ with current observations.

3 $M_{\rm min}$ and BSM physics

Our definition of the "benchmark" Higgs boson mass consists of the solution of the two equations (1) and gives, in addition to $M_{\rm min}$, the value of the scale μ_0 at which the scalar self-coupling and its β-function vanish simultaneously. The central value for μ_0 is $2.9\times10^{18}\,\mathrm{GeV}$ and is quite stable if m_t and α_s are varied in their confidence intervals (see Fig. 5). One can see that there is a remarkable coincidence between μ_0 and the (reduced) Planck scale $M_P = 2.44\times10^{18}\,\mathrm{GeV}$. The physics input in the computation of μ_0 includes the parameters of the SM only, while the result gives the gravity scale. A possible explanation may be related to the asymptotic safety of the SM, see [20] and below.[4] It remains to be seen if this is just the random play of the numbers or a profound indication that the electroweak symmetry breaking is related to Planck physics. If real, this coincidence indicates that there should be no new energy scales between the Planck and Fermi scales, as they would remove this coincidence unless some conspiracy is taking place.

We will discuss below two possible minimal embeddings of the SM to the theory of gravity and discuss the significance of $M_{\rm min}$ in them.

3.1 Asymptotic safety

The asymptotic safety of the SM [20], associated with the asymptotic safety of gravity [41], is strongly related to the value of the Higgs boson mass. Though General Relativity is non-renormalizable by perturbative methods, it may exist as a field theory non-perturbatively, exhibiting a non-trivial ultraviolet fixed point (for a review see [42]). If true, all other coupling of the SM (including the Higgs self-interaction) should exhibit an asymptotically safe behaviors with the gravity contribution to the renormalisation group running included.

The prediction of the Higgs boson mass from the requirement of asymptotic safety of the SM is found as follows [20]. Consider the SM running of the coupling constants and add to the β-functions extra terms coming from gravity, deriving their structure from

[4] Yet another one is the "multiple point principle" of [39, 40], requiring the degeneracy between the SM vacuum and an extra one appearing at the Planck scale.

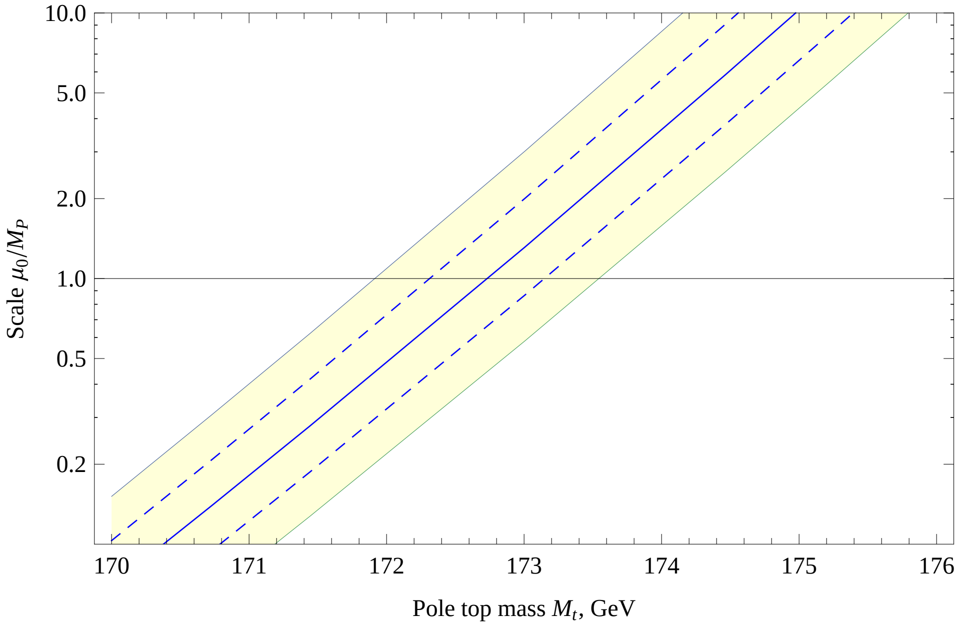

Figure 5: The scale μ_0 (solution of (1)) depending on the top mass M_t. The dashed lines correspond to 1σ uncertainty in the α_s. The yellow shaded region corresponds to adding the α_s experimental error and the theoretical uncertainty in the matching of the top Yukawa y_t and top pole mass.

dimensional analysis:

$$\beta_h^{\mathrm{grav}} = \frac{a_h}{8\pi}\frac{\mu^2}{M_P^2(\mu)}h, \tag{6}$$

where a_1, a_2, a_3, a_y, and a_λ are some constants (anomalous dimensions) corresponding to the gauge couplings of the SM g, g', g_s, the top Yukawa coupling y_t, and the Higgs self-coupling λ. In addition,

$$M_P^2(\mu) \simeq M_P^2 + 2\xi_0\mu^2 \tag{7}$$

is the running Planck mass with $\xi_0 \approx 0.024$ following from numerical solutions of functional RG equations [43–45]. Now, require that the solution for all coupling constants is finite for all μ and that λ is always positive. The SM can only be asymptotically safe if a_1, a_2, a_3, a_y are all negative, leading to asymptotically safe behavior of the gauge and Yukawa couplings. For $a_\lambda < 0$ we are getting the interval of admissible Higgs boson masses, $M_{\mathrm{min}}^{\mathrm{safety}} < M_H < M_{\mathrm{max}}^{\mathrm{safety}}$. However, if $a_\lambda > 0$, as follows from computations of [44, 45], only one value of the Higgs boson mass $M_H = M_{\mathrm{min}}^{\mathrm{safety}}$ leads to asymptotically safe behavior of λ. As is explained in [20], this behavior is only possible provided $\lambda(M_P) \approx 0$ and $\beta_\lambda(\lambda(M_P)) \approx 0$. And, due to miraculous coincidence of μ_0 and M_P, the difference $\Delta m^{\mathrm{safety}} \equiv M_{\mathrm{min}}^{\mathrm{safety}} - M_{\mathrm{min}}$ is extremely small, of the order 0.1 GeV. The evolution of the Higgs self-coupling for the case of $a_h < 0$ is shown in Fig. 6, and for the case $a_h > 0$ in Fig. 7.

In fact, in the discussion of the asymptotic safety of the SM one can consider a more general situation, replacing the Planck mass in eq. (7) by some cutoff scale $\Lambda = \kappa M_P$.

Figure 6: Schematic depiction of the behavior of the scalar self-coupling if $a_h < 0$ for $M_{\rm min}^{\rm safety} < M_H < M_{\rm max}^{\rm safety}$ (left) and $M_H < M_{\rm min}^{\rm safety}$ (right). In both cases gravity leads to asymptotically free behavior of the scalar self-coupling. Negative λ lead to instability and thus excluded.

Indeed, if the Higgs field has non-minimal coupling with gravity (see below), the behavior of the SM coupling may start to change at energies smaller than M_P by a factor $1/\xi$, leading to an expectation for the range of κ as $1/\xi \lesssim \kappa \lesssim 1$. Still, the difference between $M_{\rm min}$ and $M_{\rm min}^{\rm safety}$ remains small even for $\kappa \sim 10^{-4}$, where $M_{\rm min}^{\rm safety} \simeq 128.4\,{\rm GeV}$, making the prediction $M_H \simeq M_{\rm min}$ sufficiently stable against specific details of Planck physics within the asymptotic safety scenario.

3.2 $M_{\rm min}$ and cosmology

It is important to note that if the mass of the Higgs boson is smaller than the stability bound $M_{\rm min}$, this does not invalidate the SM. Indeed, if the life-time of the metastable SM vacuum exceeds the age of the Universe (this is the case when $M_H > M_{\rm meta}$, with $M_{\rm meta} \simeq 111\,{\rm GeV}$ [13]) then finding a Higgs boson in the mass interval $M_{\rm meta} < M_H < M_{\rm min}$ would simply mean that we live in the metastable state with a very long lifetime. Of course, if the Higgs boson were discovered with a mass below $M_{\rm meta}$, this would prove that there *must be* new physics between the Fermi and Planck scales, stabilizing the SM vacuum state. However, the latest LEP results, confirmed recently by LHC, tell us that in fact $M_H > M_{\rm meta}$, and, therefore, that the presence of a new energy scale is not required, if only the metastability argument is used.

The bound $M_H > M_{\rm meta}$ can be strengthened if thermal cosmological evolution is considered [13]. After inflation the universe should find itself in the vicinity of the SM vacuum and stay there till present. As the probability of the vacuum decay is temperature dependent, the improved Higgs boson mass bound is controlled by the reheating temperature after inflation (or maximal temperature of the Big Bang). The latter is model dependent, leading to the impossibility to get a robust bound much better than $M_{\rm meta}$. For example, in R^2 inflation [46, 47] the reheating temperature is rather low, $T \sim 10^9\,{\rm GeV}$ [47], leading to the lower bound $116\,{\rm GeV}$ [48] on the Higgs boson mass, which exceeds $M_{\rm meta}$ only by $4\,{\rm GeV}$.

However, if no new degrees of freedom besides those already present in the SM are introduced and the Higgs boson plays the role of inflaton, the bound $M_H \gtrsim M_{\rm min}$

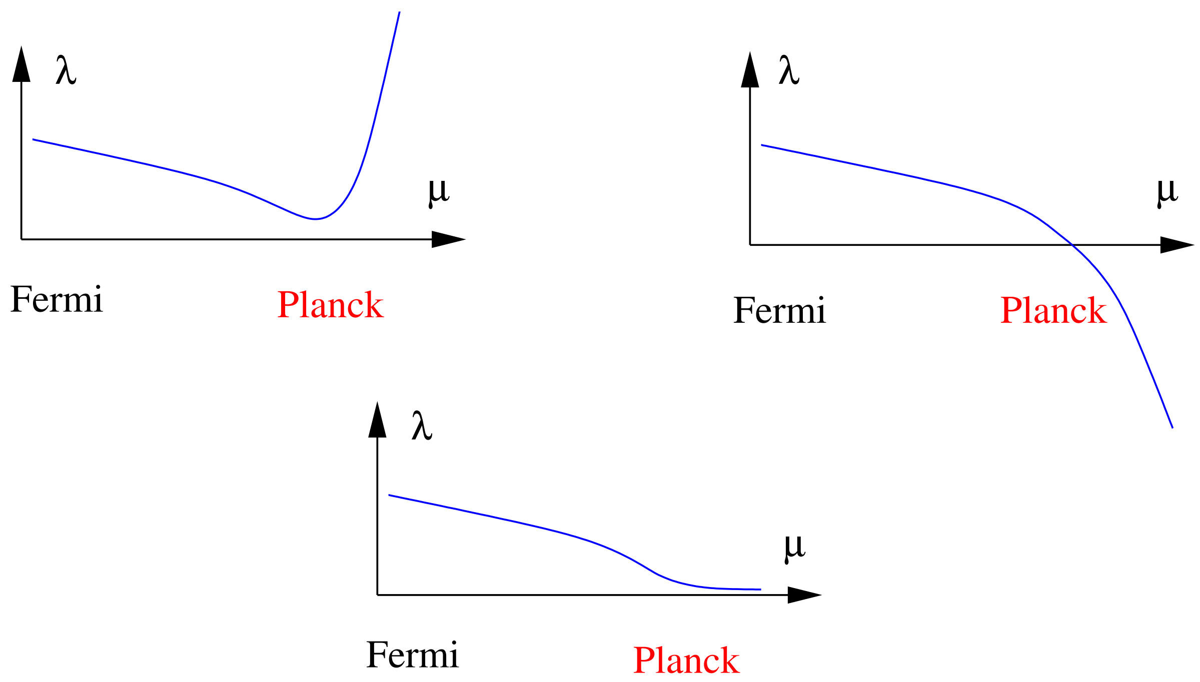

Figure 7: Schematic depiction of the behavior of the scalar self-coupling if $a_h > 0$ for $M_H > M_{\rm min}^{\rm safety}$, leading to Landau-pole behavior (left), $M_H > M_{\rm min}^{\rm safety}$, leading to instability (right) and $M_H = M_{\rm min}^{\rm safety}$, asymptotically safe behavior (middle). Only this choice is admissible.

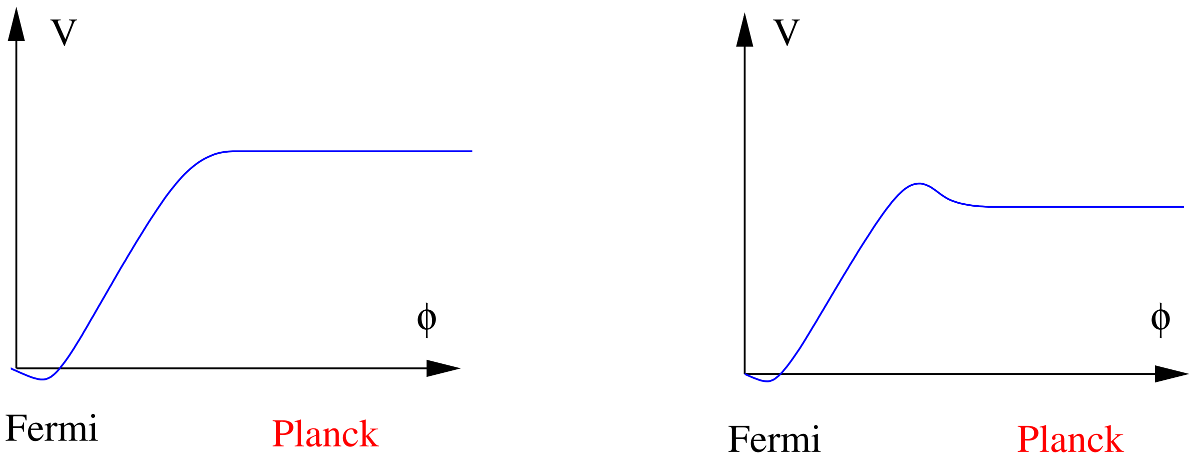

Figure 8: Schematic depiction of the effective potential V for the Higgs field in the Higgs-inflationary theory in the Einstein frame for $M_H > m_{\rm min}^{\rm inflation}$ (left) and $M_H < m_{\rm min}^{\rm inflation}$ (right).

reappears, as is discussed below.

3.3 Higgs inflation

The inclusion of a non-minimal interaction of the Higgs field with gravity, given by the Lagrangian $\xi|\phi|^2 R$, where R is the Ricci scalar, changes drastically the behavior of the Higgs potential in the region of large Higgs fields $\phi > M_{\rm inflation} \simeq M_P/\sqrt{\xi}$ [18]. Basically, the potential becomes flat at $\phi > M_{\rm inflation}$, keeping the value it acquired at $\phi \simeq M_P/\sqrt{\xi}$. This feature leads to a possibility of Higgs-inflation: if the parameter ξ is sufficiently large, $700 < \xi < 10^5$, [14] the Higgs boson of the SM can make the Universe flat, homogeneous and isotropic, and can produce the necessary spectrum of primordial fluctuations. The possibility of the Higgs inflation is also strongly related to the value of the Higgs boson mass: the successful inflation can only occur if $M_{\rm min}^{\rm inflation} < M_H < M_{\rm max}^{\rm inflation}$. The upper limit $M_{\rm max}^{\rm inflation}$ comes from the requirement of the validity of the SM up to the inflation scale $M_{\rm inflation}$. Near $M_{\rm min}^{\rm inflation}$ the behavior of the effective potential in the Einstein frame changes as shown in Fig. 8: if $M_H < M_{\rm min}^{\rm inflation}$ the "bump" in the Higgs potential prevents the system to go to the SM vacuum state. As in the previous case, these bounds can be formulated with the use of the Higgs self-coupling λ. Basically, it must be perturbative and positive for all energy scales below $M_{\rm inflation}$. Though any Higgs boson mass in the interval $M_{\rm min}^{\rm inflation} < M_H < M_{\rm max}^{\rm inflation}$ can lead to successful inflation, the value $M_{\rm min}^{\rm inflation}$ is somewhat special. For the lower part of the admitted interval the value of the non-minimal coupling ξ reaches its minimal value $\xi \simeq 700$, extending the region of applicability of perturbation theory [14, 49, 50].

The computation of the lower bound on the Higgs boson mass from inflation is more complicated. It is described in detail in [14, 19]. Basically, one has to compute the Higgs potential in the chiral electroweak theory associated with large values of the Higgs field and find when the slow-roll inflation in this potential can give the large-scale perturbations observed by the COBE satellite. The outcome of these computations, however, can be formulated in quite simple terms: for inflationary bound find $M_{\rm min}^{\rm inflation}$ from the condition $\lambda(\mu) > 0$ for all $\mu < M_{\rm inflation}$ [14]. A priori, the inflationary bound could have been very different from $M_{\rm min}$ and thus from $M_{\rm min}^{\rm stability}$. Indeed, both $M_{\rm min}$ and $M_{\rm min}^{\rm stability}$ know *nothing* about the Planck scale and are defined entirely within the SM, whereas the inflationary bound does use M_P. However, the remarkable numerical coincidence, between μ_0 and M_P, makes $M_{\rm min}$ and $M_{\rm min}^{\rm inflation}$ practically the same. The coupling constant λ evolves very slowly near the Planck scale, so that the regions for the Higgs boson mass following from the conditions $\lambda(\mu) > 0$ for $\mu < M_P$ and $\mu < M_{\rm inflation}$ are almost identical. This leads to the result that $\Delta m^{\rm inflation} \equiv M_{\rm min}^{\rm inflation} - M_{\rm min} \simeq -0.1 - 0.2\,\text{GeV}$. This number is derived within the SM without addition of any higher dimensional operators.

As is explained in [49], adding to the SM higher-dimensional operators with a Higgs-field dependent cutoff modifies the lower bound on the Higgs boson mass in Higgs inflation. If these operators are coming with "natural" power counting coefficients (for exact definition see [49]) the sensitivity of the Higgs boson mass bound to unknown details of ultraviolet physics is rather small $\Delta M_{\rm min}^{\rm inflation} \simeq 0.6\,\text{GeV}$ [49]. At the same time, it is certainly not excluded that the change of $M_{\rm min}^{\rm inflation}$ can be larger.

4 Conclusions

If the SM Higgs boson will be discovered at LHC in the remaining mass interval $115.5 < M_H < 127\,\text{GeV}$ not excluded at 95% [8, 9], there is no necessity for a new energy scale between the Fermi and Planck scales. The EW theory remains in a weakly coupled region all the way up to M_P, whereas the SM vacuum state lives longer than the age of the Universe. If the SM Higgs boson mass will be found to *coincide* with $M_{\rm min}$ given by (5), this would put a strong argument in favor of the *absence* of such a scale and indicate that the electroweak symmetry breaking may be associated with the physics at the Planck scale.

The experimental precision in the Higgs boson mass measurements at the LHC can eventually reach 200 MeV and thus be much smaller than the present theoretical ($\sim 1-2\,\text{GeV}$) and experimental ($\sim 5\,\text{GeV}$, 2σ) uncertainties in determination of $M_{\rm min}$. The largest uncertainty comes from the measurement of the mass of the top quark. It does not look likely that the LHC will substantially reduce the error in the top quark mass determination. Therefore, to clarify the relation between the Fermi and Planck scales a construction of an electron-positron or muon collider with a center-of-mass energy of $\sim 200+200\,\text{GeV}$ (Higgs and t-quark factory) would be needed. This would be decisive for setting up the question about the necessity for a new energy scale besides the two ones already known—the Fermi and the Planck scales. In addition, this will allow to study in detail the properties of the two heaviest particles of the Standard Model, potentially most sensitive to any types on new physics.

Surely, even if the SM is a valid effective field theory all the way up the the Planck scale, it cannot be complete as it contradicts to a number of observations. We would like to use this opportunity to underline once more that the confirmed observational signals in favor of physics beyond the Standard Model which were not discussed in this paper (neutrino masses and oscillations, dark matter and baryon asymmetry of the Universe) can be associated with new physics *below* the electroweak scale, for reviews see [51, 52] and references therein.[5] The minimal model—νMSM, contains, in addition to the SM particles, three relatively light singlet Majorana fermions. These fermions could be responsible for neutrino masses, dark matter and baryon asymmetry of the Universe. The νMSM predicts that the LHC will continue to confirm the Standard Model and see no deviations from it. At the same time, new experiments at the high-intensity frontier, discussed in [55], may be needed to uncover the new physics below the Fermi scale. In addition, new observations in astrophysics, discussed in [52], may shed light to the nature of Dark Matter. As the running of couplings in the νMSM coincides with that in the SM, all results of the present paper are equally applicable to the νMSM.

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[5] As for the dark energy, it may be related to a massless dilaton realizing spontaneously broken scale invariance [53, 54].

A $O(\alpha\alpha_s)$ electroweak corrections to the top Yukawa and Higgs self couplings in Standard Model

The evaluation of radiative corrections to the relations between $\overline{\mathrm{MS}}$ parameters (coupling constants) and masses of particles includes two steps: evaluation of radiative corrections between the Fermi constant G_F and its $\overline{\mathrm{MS}}$ counterpart [57] (see [58–60] for recent reviews) and the evaluation of the radiative corrections between $\overline{\mathrm{MS}}$ and pole masses.

The one-loop electroweak corrections $\mathcal{O}(\alpha)$ to the relation between the self-coupling $\lambda(\mu^2)$ and the pole mass of the Higgs boson was obtained in [61] and to the relation between the Yukawa coupling y_t and the pole mass of top quark was found in [31]. The corresponding ingredients for the 2-loop mixed electroweak-QCD corrections were evaluated in [56, 62–66], but has never been assembled. We performed independent (re)calculations of all $O(\alpha)$ and $O(\alpha\alpha_s)$ contributions. In the following we will denote the on-shell masses by capital M and the $\overline{\mathrm{MS}}$ masses by lowercase m.

A.1 $O(\alpha\alpha_s)$ corrections to the relation between on-shell and $\overline{\mathrm{MS}}$ Fermi constant

The relation between the Fermi coupling constant and the bare parameters is as follows [57]:

$$\frac{G_F}{\sqrt{2}} = \frac{g_0^2}{8m_{W,0}^2}\left\{1+\Delta R_0\right\}, \tag{A.8}$$

where ΔR_0 includes unrenormalized electroweak corrections and $g_0, m_{W,o}^2$ are the SU(2) coupling constant and the bare W boson mass (see for details [58–60]). After performing $\overline{\mathrm{MS}}$ renormalization this relation has the following form:

$$\frac{G_F}{\sqrt{2}} = \frac{G_F(\mu^2)}{\sqrt{2}}(1+\Delta_{G_F,\alpha}+\Delta_{G_F,\alpha\alpha_s}+\cdots). \tag{A.9}$$

where on the r.h.s. all masses and coupling constants are taken in the $\overline{\mathrm{MS}}$ renormalization scheme. The one-loop coefficient, $\Delta_{G_F,\alpha}$, is known from [57] and for $N_c = 3$, $C_F = 4/3$ and $m_b = 0$ has the following form:

$$\begin{aligned}
\Delta_{G_F,\alpha} = \frac{g^2}{16\pi^2}\Bigg\{ & \frac{m_t^4}{m_W^2 m_H^2}\left(6-6\ln\frac{m_t^2}{\mu^2}\right)+\frac{m_t^2}{m_W^2}\left(-\frac{3}{4}+\frac{3}{2}\ln\frac{m_t^2}{\mu^2}\right)+\frac{m_H^2}{m_W^2}\left(-\frac{7}{8}+\frac{3}{4}\ln\frac{m_H^2}{\mu^2}\right) \\
& +\frac{m_Z^4}{m_H^2 m_W^2}\left(-\frac{1}{2}+\frac{3}{2}\ln\frac{m_Z^2}{\mu^2}\right)+\frac{m_W^2}{m_H^2}\left(-1+3\ln\frac{m_Z^2}{\mu^2}\right)-\frac{m_W^2}{m_H^2-m_W^2}\ln\left(\frac{m_W^2}{m_H^2}\right) \\
& +\frac{m_Z^2}{m_W^2}\left(\frac{5}{8}+\frac{17}{4}\ln\frac{m_W^2}{\mu^2}-5\ln\frac{m_Z^2}{\mu^2}\right)-\frac{3}{4}\ln\frac{m_W^2}{\mu^2}-\frac{3}{4}\ln\frac{m_H^2}{\mu^2} \\
& -\frac{m_Z^2}{m_W^2\sin^2\theta_W}\ln\left(\frac{m_W^2}{m_Z^2}\right)+\frac{5}{4}+\frac{7}{2\sin^2\theta_W}\ln\left(\frac{m_W^2}{m_Z^2}\right)\Bigg\},
\end{aligned} \tag{A.10}$$

Here, $\sin^2\theta_W$ is defined in the $\overline{\rm MS}$ scheme as

$$\sin^2\theta_W \equiv \sin^2\theta_W^{\overline{MS}}(\mu^2) = \frac{g'^2(\mu^2)}{g^2(\mu^2)+g'^2(\mu^2)} = 1 - \frac{m_W^2(\mu^2)}{m_Z^2(\mu^2)}, \tag{A.11}$$

where $g'(\mu^2)$ and $g(\mu^2)$ are the U(1) and SU(2) $\overline{\rm MS}$ gauge coupling constants, respectively. The matching conditions between the $\overline{\rm MS}$ parameter, defined by Eq. (A.11), and its on-shell version, [57], follows from identification

$$\sin^2\theta_W^{OS} = 1 - \frac{M_W^2}{M_Z^2}, \tag{A.12}$$

where M_Z and M_W are the pole masses of the gauge bosons (see detailed discussion in [67–69]). The evaluation of the mixed QCD-EW coefficient, $\Delta_{G_F,\alpha\alpha_s}$, is reduced to the evaluation of the $O(\alpha\alpha_s)$ corrections to the W boson self-energy at zero momenta transfer and may be written in the following way [70–74]:

$$\begin{aligned}\Delta_{G_F,\alpha\alpha_s} &\equiv 2g_R^2 Z_{g,\alpha\alpha_s} - \left[Z_{W,\alpha\alpha_s} - Z_{m_t^2,\alpha_s} m_t^2 \frac{\partial}{\partial m_t^2}\frac{\Pi_{WW,\alpha}(0)}{m_W^2(\mu^2)} - \frac{\Pi_{WW,\alpha\alpha_s}(0)}{m_W^2(\mu^2)}\right] \qquad (A.13)\\ &= C_f N_c \frac{g^2 g_s^2}{(16\pi^2)^2}\frac{m_t^2}{m_W^2}\left[20\frac{m_t^2}{m_H^2} - \frac{13}{8} + \zeta_2\right.\\ &\qquad \left. + \left(1-20\frac{m_t^2}{m_H^2}\right)\ln\left(\frac{m_t^2}{\mu^2}\right) - \left(\frac{3}{2}-12\frac{m_t^2}{m_H^2}\right)\ln^2\left(\frac{m_t^2}{\mu^2}\right)\right],\end{aligned}$$

where for $Z_{g,\alpha\alpha_s}$, $Z_{g,\alpha\alpha_s}$ and $Z_{W,\alpha\alpha_s}$ we used the results[6] of [69] and n_F is the number of fermion families (n_F is equal to 3 in the SM).

Using the fact, that G_F is RG invariant, i.e. $\mu^2\frac{d}{d\mu^2}G_F = 0$, the μ-dependent terms in Eq. (A.13) can be evaluated explicitly from the one-loop correction and explicit knowledge of anomalous dimension γ_{G_F}. As was shown in [67–69, 75, 76], the anomalous dimension γ_{G_F} can be extracted (i) via the beta-function β_λ of the scalar self-coupling and the anomalous dimension of the mass parameter m^2 (in unbroken phase) or (ii) via the β-function of the SU(2) gauge coupling g and the anomalous dimension of the W

[6] There are typos in Eq. (4.41) of [69]: in all $\overline{\rm MS}$ renormalization constants, $Z_W^{\alpha_s}$ and $Z_Z^{\alpha_s}$, "m_t^2/m_W^2" should be replaced by "m_t^2/m_H^2"

$$\begin{aligned} Z_W^{\alpha_s} &= 1 + \frac{g^2}{(16\pi^2)}\frac{\alpha_s}{4\pi}N_c C_f\left[\frac{1}{\varepsilon}\left(4\frac{m_t^4}{m_H^2 m_W^2} - \frac{5}{4}\frac{m_t^2}{m_W^2} + \frac{1}{2}n_F\right) + \frac{1}{\varepsilon^2}\left(-12\frac{m_t^4}{m_H^2 m_W^2} + \frac{3}{2}\frac{m_t^2}{m_W^2}\right)\right]\\ Z_Z^{\alpha_s} &= 1 + \frac{g^2}{(16\pi^2)}\frac{\alpha_s}{4\pi}N_c C_f\left[\frac{1}{\varepsilon}\left(4\frac{m_t^4}{m_H^2 m_W^2} - \frac{5}{4}\frac{m_t^2}{m_W^2} + \frac{10}{9}n_F\frac{m_W^2}{m_Z^2} + \frac{11}{18}n_F\frac{m_Z^2}{m_W^2} - \frac{11}{9}n_F\right)\right.\\ &\qquad \left. + \frac{1}{\varepsilon^2}\left(-12\frac{m_t^4}{m_H^2 m_W^2} + \frac{3}{2}\frac{m_t^2}{m_W^2}\right)\right],\end{aligned}$$

boson (in broken phase):

$$\gamma_{G_F} \equiv \mu^2 \frac{\partial}{\partial \mu^2} \ln G_F(\mu^2) = \frac{\beta_\lambda}{\lambda} - \gamma_{m^2} = 2\frac{\beta_g}{g} - \gamma_W. \tag{A.14}$$

Eq. (A.9) can be written as

$$\begin{aligned} \frac{G_F}{G_F(\mu^2)} &= 1 - \frac{g^2}{16\pi^2}\left[\gamma_{G_F,\alpha} L - \Delta X^{(1)}_{G_F,\alpha}\right] \\ &\quad + \frac{g^2 g_s^2}{(16\pi)^2}\left[\Delta X^{(2)}_{G_F,\alpha\alpha_s} + C^{(2,2)}_{G_F,\alpha\alpha_s} L^2 - C^{(2,1)}_{G_F,\alpha\alpha_s} L\right], \end{aligned} \tag{A.15}$$

where $L = \ln \frac{\mu^2}{m_t^2}$ and the coefficients $C^{(2,2)}_{G_F,\alpha\alpha_s}$ and $C^{(2,1)}_{G_F,\alpha\alpha_s}$ are defined via the RG equations:

$$2C^{(2,2)}_{G_F,\alpha\alpha_s} = Z_{m_t^2,\alpha_s} \frac{\partial}{\partial m_t^2} \gamma_{G_F,\alpha} = -6C_f \left[\frac{3}{2}\frac{m_t^2}{m_W^2} - 12\frac{m_t^4}{m_W^2 m_H^2}\right]\Bigg|_{N_c=3}, \tag{A.16}$$

$$C^{(2,1)}_{G_F,\alpha\alpha_s} = \gamma_{G_F,\alpha\alpha_s} + Z_{m_t^2,\alpha_s} m_t^2 \frac{\partial}{\partial m_t^2} \Delta X^{(1)}_{G_F,\alpha} + Z_{m_t^2,\alpha_s} \gamma_{G_F,\alpha}, \tag{A.17}$$

with

$$\gamma_{G_F,\alpha\alpha_s} = \left[2\frac{\beta_{g,\alpha\alpha_s}}{g} - 2Z_{W,\alpha\alpha_s}\right] = \frac{1}{2} N_c C_f \left[5\frac{m_t^2}{m_W^2} - 16\frac{m_t^4}{m_W^2 m_H^2}\right]. \tag{A.18}$$

Collecting all terms in Eq. (A.17) we get

$$C^{(2,1)}_{G_F,\alpha\alpha_s}\Big|_{N_c=3, C_f=\frac{4}{3}} = 4\frac{m_t^2}{m_W^2} - 80\frac{m_t^4}{m_W^2 m_H^2}. \tag{A.19}$$

At the end of this section we again point out that the anomalous dimension of the vacuum expectation value $v^2(\mu^2) = 1/(\sqrt{2} G_F(\mu^2))$ within the diagram technique is defined by Eq. (A.14) and it is not equal to the anomalous dimension of the scalar field as in the effective potential approach [77]. Another important property of Eq. (A.14) is the appearance of an inverse power of the coupling constant λ due to the explicit inclusion of the tadpole contribution. As consequence, the limit of zero Higgs mass, $m_H^2 = 0$, does not exist within the perturbative approach. The importance of the inclusion of the tadpole contribution to restore gauge invariance of on-shell counterterms was recognized a long time ago [78] and was explicitly included in the one-loop electroweak corrections to the matching conditions [31, 61]. The RG equations for the mass parameters were discussed in [67–69, 75, 76].

A.2 $O(\alpha\alpha_s)$ corrections to the relation between the $\overline{\rm MS}$ and pole masses of the top quark

The detailed discussion and explicit evaluation[7] have been presented in [66]. For our analysis is enough to write the following symbolic relation between the $\overline{\rm MS}$ and pole masses of the top quark:

$$\frac{m_t(\mu^2)}{M_t} \quad = \quad 1+\sigma_\alpha+\sigma_{\alpha_s}+\sigma_{\alpha_s^2}+\sigma_{\alpha_s^3}+\sigma_{\alpha\alpha_s}+\cdots\,, \tag{A.20}$$

where σ_α and $\sigma_{\alpha\alpha_s}$ are defined by Eq.(5.54) or Eq.(5.57) of [66].

The pure QCD corrections can be found in [24–26] (only the value of $\sigma_{\alpha_s}(M_t)$ is given there, but the expression for other μ values can be readily reconstructed from the beta functions).

A.3 $O(\alpha\alpha_s)$ corrections to the relation between the $\overline{\rm MS}$ and pole masses of the Higgs boson

At the two-loop level the relation between the pole and $\overline{\rm MS}$ masses is defined as follows:

$$\begin{aligned} s_P &= m_0^2-\Pi_0^{(1)}-\Pi_0^{(2)}-\Pi_0^{(1)}\Pi_0^{(1)\prime}-\left[\sum_j(\delta m_{j,0}^2)^{(1)}\frac{\partial}{\partial m_{j,0}^2}+\sum_j(\delta g_{j,0})^{(1)}\frac{\partial}{\partial g_{j,0}}\right]\Pi_0^{(1)} \\ &= m_a^2-\left\{\Pi_a^{(1)}\right\}_{\overline{\rm MS}}-\left\{\Pi_a^{(2)}+\Pi_a^{(1)}\Pi_a^{(1)\prime}\right\}_{\overline{\rm MS}}\,, \end{aligned} \tag{A.21}$$

where the sum runs over all species of particles, $g_j=\alpha,\ g_s,\ (\delta g_{j,0})^{(1)}$ and $(\delta m_{j,0}^2)^{(1)}$ are the one-loop counterterms for the charges and physical masses in the $\overline{\rm MS}$ scheme and after differentiation we put all parameters equal to their on-shell values. The derivatives in Eq. (A.21) correspond to the subtraction of sub-divergences. The genuine two-loop mass counterterm comes from the shift of the m_0^2 term. The relation between the bare and $\overline{\rm MS}$ masses of the Higgs boson has the form

$$\begin{aligned} \left(m_H^B\right)^2 = \left(m_H^R(\mu^2)\right)^2\Bigg[1&+\frac{g^2}{16\pi^2\varepsilon}Z_{H,\alpha} \\ &+\frac{g^4}{(16\pi^2)^2}\left(\frac{1}{\varepsilon}Z_{H,\alpha^2}^{(2,1)}+\frac{1}{\varepsilon^2}Z_{H,\alpha^2}^{(2,2)}\right)+\frac{g_s^2g^2}{(16\pi^2)^2}\left(\frac{1}{\varepsilon}Z_{H,\alpha\alpha_s}^{(2,1)}+\frac{1}{\varepsilon^2}Z_{H,\alpha\alpha_s}^{(2,2)}\right)\Bigg], \end{aligned} \tag{A.22}$$

where g is the SU(2) $\overline{\rm MS}$ renormalized coupling constant.

[7] There is typo in Eq. (4.46) of [66]: the common factor C_f was lost. The correct result is

$$=\frac{\alpha_s}{4\pi}\frac{e^2}{16\pi^2\sin^2\theta_W}C_f\left(\frac{1}{C_f}C_{\alpha\alpha_s}^{(2,2)}\ln^2\frac{m_t^2}{\mu^2}+\frac{1}{C_f}C_{\alpha\alpha_s}^{(2,1)}\ln\frac{m_t^2}{\mu^2}+\text{without modifications}\right).$$

However, all plots, the Eq. (5.57) and the Maple program [79] are correct.

The exact analytical result for the $O(\alpha\alpha_s)$ two-loop quark contribution to the Higgs-boson self-energy was calculated in [65, 72, 72]. The bare two loop mixed QCD-EW contribution (with explicit inclusion of the tadpole) for the quark with mass m_q reads:

$$\begin{aligned}
\Pi^{(2)}_{0,m_H^2,\alpha\alpha_s,q} &= \frac{g^2 g_s^2}{(16\pi^2)^2} N_c C_f \frac{m_q^2}{m_W^2} \Bigg\{ -J_{0qq}(1,1,1;m_H^2)(n-3) \\
&+ J_{0qq}(1,1,2;m_H^2)\left[m_H^2 - 4m_q^2\right] \frac{(n^2-5n+8)}{(n-4)(n-3)} \\
&+ A_0(m_q^2) B_0(m_q^2,m_q^2;m_H^2) \frac{(n-2)}{(n-3)(n-4)} \\
&\quad \times \left[(n^3-8n^2+19n-16) + \frac{m_H^2}{2m_q^2}(n^2-5n+8)\right] \\
&+ \left[B_0(m_q^2,m_q^2;m_H^2)\right]^2 \left[m_H^2 \frac{(n-2)^2}{2(n-4)} - m_q^2 \frac{2(n^2-4n+2)}{(n-4)}\right] \\
&+ \left[A_0(m_q^2)\right]^2 \frac{(n-2)(n^2-5n+8)}{2m_q^2(n-4)(n-3)} + \left[A_0(m_q^2)\right]^2 \frac{3}{2} \frac{(n-1)(n-2)^2}{m_q^2(n-3)} \Bigg\}, \qquad \text{(A.23)}
\end{aligned}$$

where the last terms come from the tadpole, n is the dimension of space-time [80] and

$$\begin{aligned}
J_{0qq}(a,b,c;m^2) &= \int \frac{d^n(k_1 k_2)}{[(k_1+k_2-p)^2]^a [k_1^2+m_q^2]^b [k_2^2+m_q^2]^c}\Bigg|_{p^2=-m^2}, \\
B_0(m_1^2,m_2^2,m^2) &= \int \frac{d^n k_1}{[k_1^2+m_1^2][(k_1-p)^2+m_2^2]}\Bigg|_{p^2=-m^2}, \\
A_0(m^2) &= \int \frac{d^n k_1}{k_1^2+m^2} \equiv \frac{4(m^2)^{\frac{n}{2}-1}}{(n-2)(n-4)}. \qquad \text{(A.24)}
\end{aligned}$$

In accordance with Eq. (A.21), the coefficient $\Delta_{m_H^2,\alpha_s\alpha,q}$ of order $O(\alpha\alpha_s)$ relating the pole and $\overline{\text{MS}}$ masses of the Higgs boson, $s_p - m_H^2$, can be written as

$$\begin{aligned}
&\Delta_{m_H^2,\alpha\alpha_s,q} = \qquad \text{(A.25)} \\
&\lim_{\varepsilon\to 0}\left(\frac{g^2 g_s^2}{(16\pi^2)^2}\left[\frac{1}{\varepsilon} Z^{(2,1)}_{H,\alpha\alpha_s,q} + \frac{1}{\varepsilon^2} Z^{(2,2)}_{H,\alpha\alpha_s,q}\right] - \frac{g_s^2}{16\pi^2}\frac{1}{\varepsilon} Z_{m_q^2,\alpha_s} m_q^2 \frac{\partial}{\partial m_q^2}\Pi^{(1)}_{0,H,\alpha} - \Pi^{(2)}_{0,m_H^2,\alpha\alpha_s,q}\right),
\end{aligned}$$

where

$$\frac{\partial}{\partial m_q^2}\Pi^{(1)}_{0,H,\alpha} = \frac{N_c}{m_W^2}\frac{g^2}{16\pi^2}\left\{ B_0(m_q^2,m_q^2,m_H^2)\left[\frac{m_H^2 - 2m_q^2(n+1)}{2}\right] - \frac{(3n-2)}{2} A_0(m_q^2)\right\}. \qquad \text{(A.26)}$$

As result of our calculation we find:

$$Z^{(2,1)}_{H,\alpha\alpha_s,q} = \frac{g^2 g_s^2}{(16\pi^2)^2} N_c C_f \frac{5}{4}\frac{m_q^2}{m_W^2}, \quad Z^{(2,2)}_{H,\alpha\alpha_s,q} = -\frac{g^2 g_s^2}{(16\pi^2)^2} N_c C_f \frac{3}{2}\frac{m_q^2}{m_W^2}. \qquad \text{(A.27)}$$

The contributions of other quarks with non-zero mass are additive. Exploring the ε expansion for the master integral J_{0qq} from [81], we have for t-quark contribution ($q = t$):

$$
\begin{aligned}
\Delta_{m_H^2,\alpha_s\alpha} \equiv \Delta_{m_H^2,\alpha_s\alpha,t} = & \frac{g_s^2 g^2}{(16\pi^2)^2} N_c C_f \frac{m_t^4}{m_W^2} \left\{ \frac{4(z-2)(z-4)}{z} F(y) - \frac{4(1+y)^3}{y(1-y)} G(y) \right. \\
& + \frac{3+20y+16y^2-4y^3-9y^4}{2y(1-y)^2} \ln^2 y + \frac{(1+y)}{(1-y)} \frac{(17+88y+17y^2)}{2y} \ln y \\
& \left. + \frac{(131+258y+131y^2)}{8y} - 6\zeta_3 \frac{(1+y)^2(1+y^2)}{y(1-y)^2} \right\} \\
& + C^{(2,2)}_{H,\alpha\alpha_s} \ln^2\left(\frac{m_t^2}{\mu^2}\right) + C^{(2,1)}_{H,\alpha\alpha_s} \ln\left(\frac{m_t^2}{\mu^2}\right),
\end{aligned} \tag{A.28}
$$

where

$$
z = \frac{m_H^2}{m_t^2}, \quad y = \frac{1-\sqrt{\frac{z}{z-4}}}{1+\sqrt{\frac{z}{z-4}}}, \quad z = -\frac{(1-y)^2}{y}, \quad 4m_t^2 - m_H^2 = m_t^2 \frac{(1+y)^2}{y}, \tag{A.29}
$$

and we have introduced the two functions $F(y)$ and $G(y)$ (see also [64, 65]) defined as[8]

$$
\begin{aligned}
F(y) &= 3\left[\mathrm{Li}_3(y) + 2\mathrm{Li}_3(-y)\right] - 2\ln y \left[\mathrm{Li}_2(y) + 2\mathrm{Li}_2(-y)\right] \\
&\quad - \frac{1}{2}\ln^2 y \left[\ln(1-y) + 2\ln(1+y)\right], \\
G(y) &= \left[\mathrm{Li}_2(y) + 2\mathrm{Li}_2(-y)\right] + \ln y \left[\ln(1-y) + 2\ln(1+y)\right],
\end{aligned} \tag{A.31}
$$

and

$$
\ln\left(\frac{m_H^2}{m_t^2}\right) = 2\ln(1-y) - \ln y + i\pi. \tag{A.32}
$$

In Eq. (A.28) we explicitly factorized the RG logarithms, $C^{(2,2)}_{H,\alpha\alpha_s}$ and $C^{(2,1)}_{H,\alpha\alpha_s}$, which may be calculated also from the one-loop result and the mass anomalous dimensions (see [75, 76] for the general case). From the parametrization

$$
\begin{aligned}
M_H^2 &= m_H^2 + \frac{g^2}{16\pi^2}\left[\Delta X^{(1)}_{H,\alpha} - C^{(1)}_{H,\alpha} L\right] + \frac{g^2 g_s^2}{(16\pi^2)^2}\left[\Delta X^{(2)}_{H,\alpha\alpha_s} + C^{(2,2)}_{H,\alpha\alpha_s} L^2 - C^{(2,1)}_{H,\alpha\alpha_s} L\right] \\
&= m_H^2 + \Delta_{m_H^2,\alpha} + \Delta_{m_H^2,\alpha\alpha_s},
\end{aligned} \tag{A.33}
$$

[8] We cross checked, that Eq. (A.28) minus tadpole contribution coincides with results of Ref. [64, 72–74] after the following substitutions:

$$
\begin{aligned}
& r = \frac{z}{4}, \quad 1 - r = \frac{(1+y)^2}{4y}, \quad r_+ = 1/\sqrt{y}, \quad r_- = \sqrt{y}, \\
& f = -\frac{1}{2}\ln y, \quad g = \ln(1-y) - 1/2\ln y, \quad h = \ln(1+y) - 1/2\ln y.
\end{aligned} \tag{A.30}
$$

where $L = \ln \frac{\mu^2}{m_t^2}$, and using the fact that pole mass is RG invariant, we have:

$$C^{(1)}_{H,\alpha} = m_H^2 Z_{H,\alpha}, \quad \gamma_{m_t^2,\alpha_s} = Z_{m_t^2,\alpha_s} = -6C_f, \tag{A.34}$$

$$2C^{(2,2)}_{H,\alpha\alpha_s} = Z_{m_t^2,\alpha_s} m_t^2 \frac{\partial}{\partial m_t^2} m_H^2 Z_{H,\alpha} = -3m_H^2 C_f N_c \frac{m_t^2}{m_W^2}, \tag{A.35}$$

$$C^{(2,1)}_{H,\alpha\alpha_s} = m_H^2 \gamma_{H,\alpha\alpha_s} + Z_{m_t^2,\alpha_s} C^{(1)}_{H,\alpha} + Z_{m_t^2,\alpha_s} m_t^2 \frac{\partial}{\partial m_t^2} \Delta X^{(1)}_{H,\alpha}, \tag{A.36}$$

where

$$Z_{H,\alpha} = -\frac{3}{2} - \frac{3}{4}\frac{m_Z^2}{m_W^2} + \frac{3}{4}\frac{m_H^2}{m_W^2} + \sum_{\text{lepton}} \frac{1}{2}\frac{m_l^2}{m_W^2} + N_c \sum_{\text{u}} \frac{1}{2}\frac{m_u^2}{m_W^2} + N_c \sum_{\text{d}} \frac{1}{2}\frac{m_d^2}{m_W^2}. \tag{A.37}$$

In terms of the variable y, defined by Eq. (A.29), the final result reads:

$$C^{(2,1)}_{H,\alpha\alpha_s} = -C_f N_c \frac{m_t^4}{m_W^2}\left[3\frac{(1+y)(1+8y+y^2)}{y(1-y)}\ln y + \frac{(17+38y+17y^2)}{2y}\right]. \tag{A.38}$$

A.4 $O(\alpha\alpha_s)$ corrections to the top Yukawa and Higgs self couplings

The relation between the top Yukawa (Higgs) coupling and the Fermi constant G_F is obtained from Eqs. (A.9), (A.20) and (A.33) as:

$$\frac{y_t^2(\mu^2)}{2\sqrt{2}G_F M_t^2} = \frac{m_t^2(\mu^2)}{M_t^2}\frac{G_F(\mu^2)}{G_F}, \tag{A.39}$$

$$\frac{\lambda(\mu^2)}{\sqrt{2}G_F M_H^2} = \frac{m_H^2(\mu^2)}{M_H^2}\frac{G_F(\mu^2)}{G_F}, \tag{A.40}$$

and the relation between the Higgs coupling constant $\lambda \equiv h_{\text{Sirlin}}$ used in [61] and the parametrization of [66–69] follows from the comparison of the RG functions: $h_{\text{Sirlin}} = \lambda_{\text{Jegerlehner}}(\mu^2)/6$.

The $O(\alpha\alpha_s)$ result for the top-Yukawa coupling reads (see Eq. (21) in [56] and [82])

$$\begin{aligned}
&\sqrt{\frac{y_t^2(\mu^2)}{2\sqrt{2}G_F M_t^2}} - 1 = (1+\sigma_\alpha+\sigma_{\alpha_s}+\sigma_{\alpha\alpha_s}) \\
&\times \left(1 - \Delta_{G_F,\alpha} - \Delta_{G_F,\alpha\alpha_s} - \sum_f \left[m_f^2 - M_f^2\right]_{\alpha_s} \frac{\partial}{\partial m_f^2}\Delta_{G_F,\alpha}\right)^{\frac{1}{2}}\Bigg|_{m_j^2=M_J^2, e^2=e_{OS}^2} - 1 \\
&= \left(\sigma_\alpha - \frac{1}{2}\Delta_{G_F,\alpha} + \sigma_{\alpha_s}\right)\Bigg|_{m_j^2=M_J^2, e^2=e_{OS}^2} \\
&+ \left(\sigma_{\alpha\alpha_s} - \frac{1}{2}\Delta_{G_F,\alpha\alpha_s} - \frac{1}{2}\sigma_{\alpha_s}\Delta_{G_F,\alpha} - \frac{1}{2}\sum_f \left[m_f^2 - M_f^2\right]_{\alpha_s}\frac{\partial}{\partial m_f^2}\Delta_{G_F,\alpha}(m_t^2)\right)\Bigg|_{m_j^2=M_J^2, e^2=e_{OS}^2}
\end{aligned} \tag{A.41}$$

where σ_X are defined in Eq. (A.20). The $O(\alpha\alpha_s)$ result for the Higgs coupling is

$$\frac{\lambda(\mu^2)}{\sqrt{2}G_F M_H^2} - 1 = + \left(-\Delta_{G_F,\alpha} - \frac{\Delta_{m_H^2,\alpha}}{M_H^2}\right)\Bigg|_{m_j^2=M_J^2.e^2=e_{OS}^2} \tag{A.42}$$

$$+ \left(-\Delta_{G_F,\alpha\alpha_s} - \frac{\Delta_{m_H^2,\alpha\alpha_s}}{M_H^2} - \left[m_t^2 - M_t^2\right]_{\alpha_s} \frac{\partial}{\partial m_t^2}\left[\Delta_{G_F,\alpha} + \frac{\Delta_{m_H^2,\alpha}}{M_H^2}\right]\right)\Bigg|_{m_j^2=M_J^2.e^2=e_{OS}^2},$$

where

$$\left[m_t^2 - M_t^2\right]_{\alpha_s} = -2M_t^2 C_f \frac{g_s^2}{16\pi^2}\left(4 - 3\ln\frac{M_t^2}{\mu^2}\right),$$

and the sum runs over all quarks.

For completeness, we present also the explicit expressions for the derivatives (for $N_c = 3$, $C_F = 4/3$ and $m_b = 0$):

$$m_t^2 \frac{\partial}{\partial m_t^2}\Delta_{m_H^2,\alpha} = \frac{g^2}{16\pi^2}\frac{3m_t^4}{m_H^2 m_W^2}\left[\frac{1+4y+y^2}{y} + \frac{(1+y)(1+8y+y^2)}{2y(1-y)}\ln y + \frac{1}{2}\frac{m_H^2}{m_t^2}\ln\left(\frac{m_t^2}{\mu^2}\right)\right], \tag{A.43}$$

$$m_t^2 \frac{\partial}{\partial m_t^2}\Delta_{G_F,\alpha} = \frac{g^2}{16\pi^2}\left\{\frac{m_t^4}{m_W^2 m_H^2}\left(6 - 12\ln\frac{m_t^2}{\mu^2}\right) + \frac{m_t^2}{m_W^2}\left(\frac{3}{4} + \frac{3}{2}\ln\frac{m_t^2}{\mu^2}\right)\right\}. \tag{A.44}$$

Terms of the order $O(\alpha)$, $O(\alpha_s)$ in Eq. (A.41) and Eq. (A.42) correspond to [31] and [61], respectively. Terms of the order $O(\alpha\alpha_s)$ in Eq. (A.41) and Eq. (A.42) are the mixed electroweak-QCD corrections and $\Delta_{G_F,\alpha\alpha_s}$, $\sigma_{\alpha\alpha_s}$, $\Delta_{m_H^2,\alpha\alpha_s}$ are defined by Eq. (A.13), Eq. (A.28), and Eq.(5.54) or Eq.(5.57) of [66].

For completeness we present also the the coefficient $\Delta_{m_H^2,\alpha}$. We divide all corrections into bosonic (diagrams without any fermions) and fermionic (diagrams exhibiting a fermion loop) ones: $\Delta_{m_H^2,\alpha} = \frac{g^2}{16\pi^2} m_H^2 \left(\Delta_{m_H^2,\alpha,\text{boson}} + \Delta_{m_H^2,\alpha,\text{fermion}}\right)$. Using the notations of [67–69] we may write the one-loop corrections in the following form[9]

$$\begin{aligned}
\Delta_{m_H^2,\alpha,\text{boson}} &= \frac{1}{2} - \frac{1}{2}\ln\frac{m_W^2}{\mu^2} - B(m_W^2, m_W^2; m_H^2) \\
&+ \frac{m_H^2}{m_W^2}\left(-\frac{3}{2} + \frac{9}{8}\frac{\pi}{\sqrt{3}} + \frac{3}{8}\ln\frac{m_H^2}{\mu^2} + \frac{1}{4}B(m_W^2, m_W^2; m_H^2) + \frac{1}{8}B(m_Z^2, m_Z^2; m_H^2)\right) \\
&+ \frac{m_Z^2}{m_W^2}\left(\frac{1}{4} - \frac{1}{4}\ln\frac{m_Z^2}{\mu^2} - \frac{1}{2}B(m_Z^2, m_Z; m_H^2)\right) \\
&+ \frac{m_W^2}{m_H^2}\left(3 - 3\ln\frac{m_W^2}{\mu^2} + 3B(m_W^2, m_W; m_H^2)\right) \\
&+ \frac{m_Z^4}{m_W^2 m_H^2}\left(\frac{3}{2} - \frac{3}{2}\ln\frac{M_Z^2}{\mu^2} + \frac{3}{2}B(m_Z^2, m_Z; m_H^2)\right),
\end{aligned} \tag{A.45}$$

[9]For simplicity we assume a diagonal Cabibbo-Kobayashi-Maskawa matrix.

$$
\begin{aligned}
\Delta_{m_H^2,\alpha,\text{fermion}} &= \frac{1}{2}\frac{m_l^2}{m_W^2}\sum_{lepton}\left[B_0(m_l^2,m_l^2;m_H^2)\left(1-4\frac{m_l^2}{m_H^2}\right)-4\frac{m_l^2}{m_H^2}\left(1-\ln\frac{m_l^2}{\mu^2}\right)\right] \\
&+ \frac{1}{2}\frac{m_q^2}{m_W^2}N_c\sum_{quark}\left[B_0(m_q^2,m_q^2;m_H^2)\left(1-4\frac{m_q^2}{m_H^2}\right)-4\frac{m_q^2}{m_H^2}\left(1-\ln\frac{m_q^2}{\mu^2}\right)\right],
\end{aligned}
\tag{A.46}
$$

where (see Eq. (E.6) in [81])

$$
\begin{aligned}
B(m^2,m^2;m_H^2) &= \int_0^1 dx \ln\left(\frac{m^2}{\mu^2}x+\frac{m^2}{\mu^2}(1-x)-\frac{m_H^2}{\mu^2}x(1-x)-\mathrm{i}0\right) \\
&= \ln\frac{m^2}{\mu^2}-2-\frac{1+Y}{1-Y}\ln Y,
\end{aligned}
\tag{A.47}
$$

with

$$
Y=\frac{1-\sqrt{\frac{r}{r-4}}}{1+\sqrt{\frac{r}{r-4}}}, \quad r=\frac{m_H^2}{m^2}.
$$

All results are collected in the Maple code of Ref. [79].

B Beta functions

Two loop SM beta functions above the top mass are collected in [13] (see [83–92] for original works). The three loop beta functions can be read off [16, 17].

Below the top mass the one loop beta functions for the gauge couplings were used to evolve the PDG values from M_Z to M_t. For example, for the $\alpha(\mu)$

$$
\alpha(\mu)=\frac{\alpha(M_Z)}{1+\frac{11}{6\pi}\alpha(M_Z)\log(\frac{\mu}{m_Z})}. \tag{B.48}
$$

The higher loop corrections are not important numerically for the electroweak constants for the small energy range between M_Z and M_t.

For the strong coupling $\alpha_s \equiv g_S^2/(4\pi)$ the RG equation up to order $O({\alpha_s}^3)$ is used

$$
\frac{d\alpha_s}{d\log\mu}=-(11-\frac{2}{3}N_f)\frac{{\alpha_s}^2}{2\pi}-(51-\frac{19}{3}N_f)\frac{{\alpha_s}^3}{4\pi^2}-(2857-\frac{5033}{9}N_f+\frac{325}{27}N_f^2)\frac{{\alpha_s}^4}{64\pi^3}, \tag{B.49}
$$

and $N_f=5$ is the number of flavors below the top quark. Strictly speaking, the value of $\alpha_s(M_t)$ obtained from this equation should be also shifted to the 6-quark value by

$$
\alpha_{S,N_f=6}(M_t)=\alpha_{S,N_f=5}(M_t)-\frac{11}{72\pi^2}\alpha_{S,N_f=5}^3(M_t), \tag{B.50}
$$

but this introduces a negligible effect ($<0.1\,\mathrm{GeV}$) for the Higgs mass.

In all the formulas of the Appendix A we use the values of α and α_s at the matching scale μ.

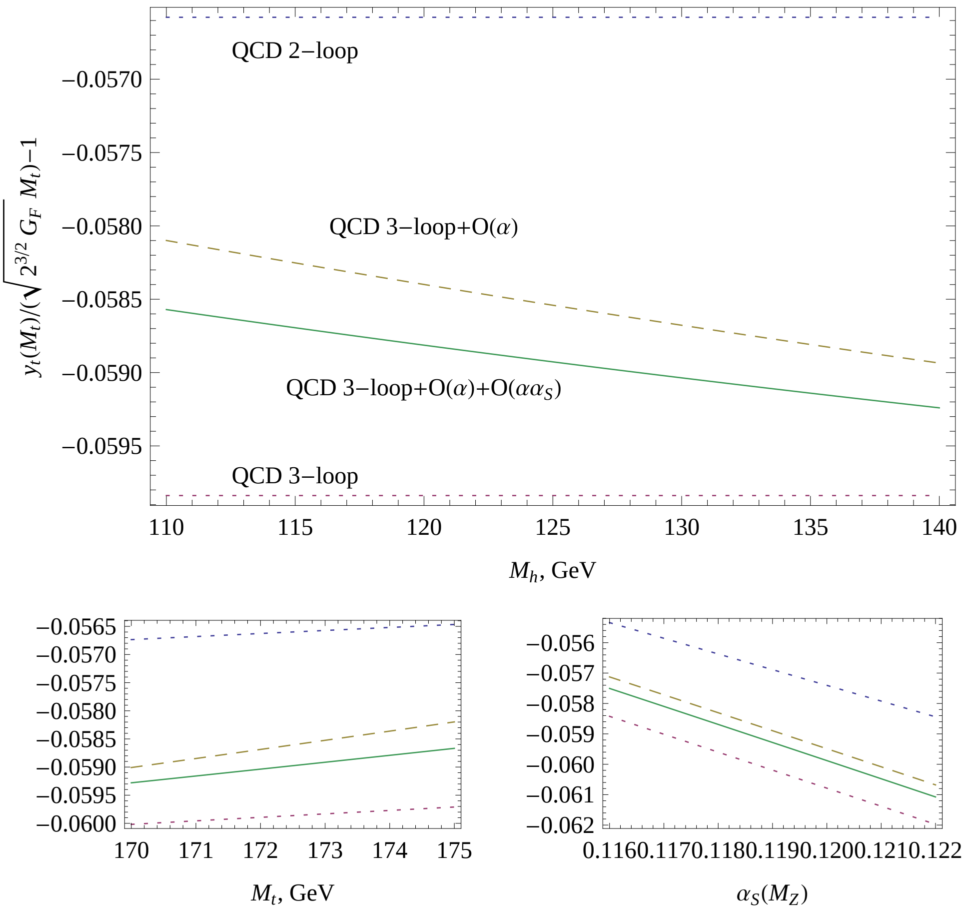

Figure 9: Contributions to the top Yukawa constant from QCD up to 2 loops, up to 3 loops, QCD and 1 loop EW corrections $O(\alpha)$, and QCD with $O(\alpha)+O(\alpha\alpha_s)$ corrections. One parameter is vrying, the two others are chosen from $M_t = 172.9\,\mathrm{GeV}$, $\alpha_s = 0.1184$, $M_h = 125\,\mathrm{GeV}$.

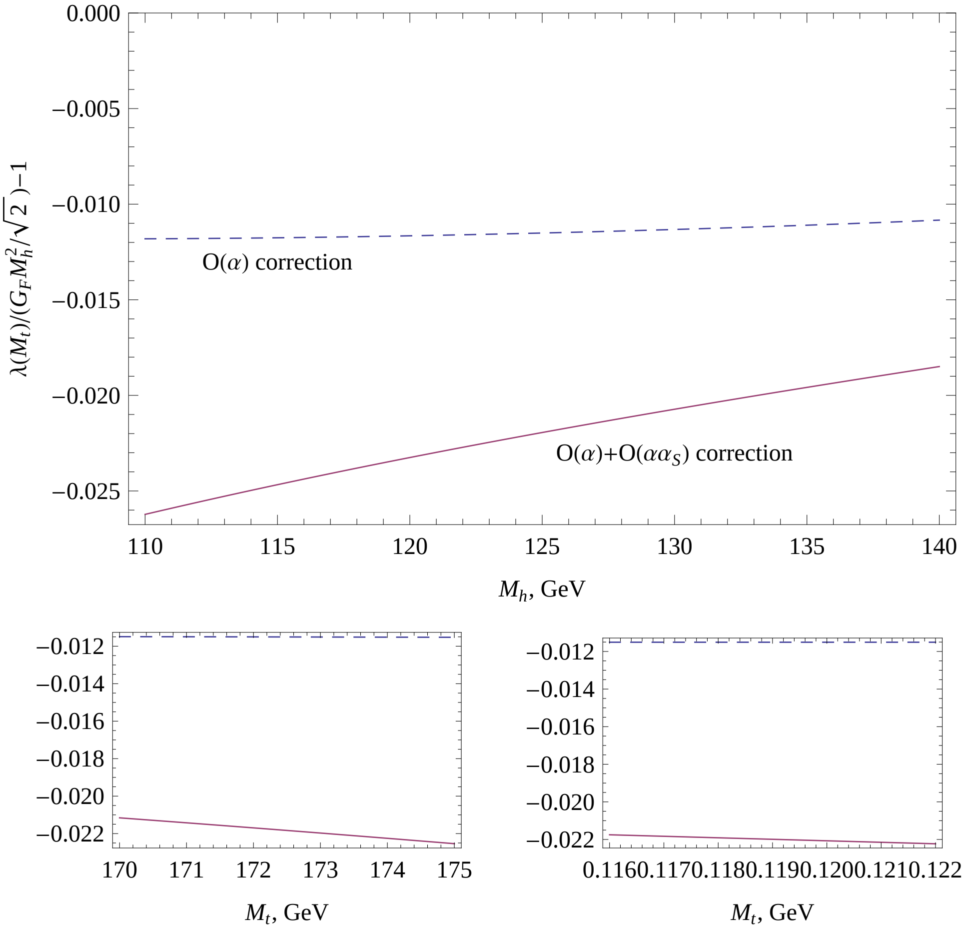

Figure 10: Contributions to the Higgs self coupling constant of the order $O(\alpha)$ and $O(\alpha) + O(\alpha\alpha_s)$. One parameter is vrying, the two others are chosen from $M_t = 172.9\,\mathrm{GeV}$, $\alpha_s = 0.1184$, $M_h = 125\,\mathrm{GeV}$.

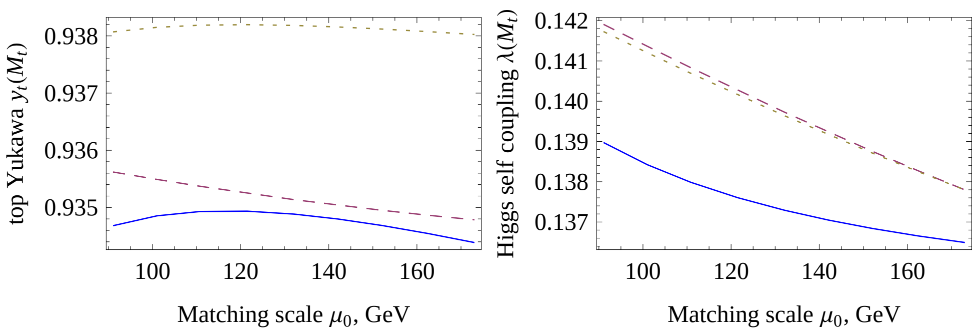

Figure 11: Top Yukawa (left) and Higgs coupling (right) at scale M_t. The constants are extracted by using matching formulas at scale μ and then evolving to the scale M_t by RG equations. Blue solid line corresponds to using full $O(\alpha, \alpha\alpha_s, {\alpha_s}^3)$ matching formulas, dashed line is matching with $O(\alpha, {\alpha_s}^3)$, dotted is matching with $O(\alpha, {\alpha_s}^2)$.

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