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# $\mathcal{O}(\alpha)$ electroweak corrections to the processes $e^+e^- \to \tau^-\tau^+, c\bar{c}, b\bar{b}, t\bar{t}$ – a comparison –*

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**Abstract**

We present the electroweak one-loop corrections to the processes $e^+e^- \to f\bar{f}$, $f = \tau, c, b, t$, at energies relevant for a future linear collider. The results of two independent calculations are compared and agreement is found at a technical-precision level of ten to twelve digits.

## 1 Introduction

With the advent of the next linear collider (LC), center-of-mass energies will rise up to several hundred GeV and the envisioned luminosity will be as high as 300 fb$^{-1}$. Evidently, a new era of precision physics is approaching. The experimental precision which can be achieved at such a machine will by far exceed all current standards and will be a challenge to experimentalists and theoreticians alike. To obtain reliable predictions for the next generation of linear colliders, the inclusion of electroweak one-loop corrections becomes essential.

Two-fermion production processes, such as

$$e^+e^- \to f\bar{f}(\gamma)\,, \tag{1}$$

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play a leading role at typical LC energies as foreseen by [1]. In the late seventies the one-loop correction to muon-pair production was calculated for the first time [2], where the muons were considered to be massless. Ever since, fermion-pair production processes attracted attention and various masses were successively introduced into the calculation. Recently, a high degree of computational precision was achieved in numerically comparing various results on radiative corrections to top-pair production (see [3, 4] and references therein). Such comparisons are invaluable to ensure the establishment of reliable, well-tested codes.

Here, we extend the study [3] to other final states. In this particular comparison we do not include hard bremsstrahlung. This issue has been discussed in detail in [4, 5] and will be calculated for realistic applications by dedicated Monte-Carlo programs for 2- to 6-fermion production [6, 7, 8].

# 2 Cross-section formulae

## 2.1 Notation and conventions

In this section, we will outline the framework to compute electroweak corrections to differential and total cross-sections in $\mathcal{O}(\alpha)$ of the electromagnetic coupling. This includes one-loop amplitudes as well as soft-photon bremsstrahlung.

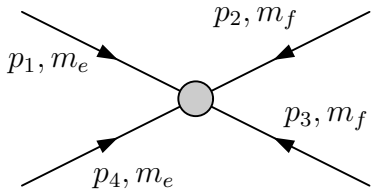



Figure 1: Definitions of the kinematical variables.

In a $2 \to 2$-particle process we follow the momenta and mass convention of Fig. 1:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{1}{32\pi}\frac{\beta_f}{s\beta_e}\sum_{\mathrm{conf}} \left|\mathcal{M}_{ef}\right|^2 , \tag{2}$$

where $\theta$ is the scattering angle. Furthermore we have

$$\beta_i \equiv \sqrt{1-4\frac{m_i^2}{s}} \tag{3}$$

$$s \equiv (p_1+p_4)^2 = E_{\mathrm{CM}}^2 \tag{4}$$

$$t \equiv (p_1+p_2)^2 = -\frac{s}{2}(1-\beta_e\beta_f\cos\theta) + m_e^2 + m_f^2 \tag{5}$$

$$u \equiv (p_1+p_3)^2 = -\frac{s}{2}(1+\beta_e\beta_f\cos\theta) + m_e^2 + m_f^2 \,. \tag{6}$$

## 2.2 Unpolarized cross-section

We consider only the unpolarized cross-section and thus have to average over initial spin configurations ($\sigma_e$), sum over the final ones ($\sigma_f$), and add incoherently the number of colours ($C_f$) which cannot be distinguished:

$$\sum_{\text{conf}} |\mathcal{M}_{ef}|^2 = \frac{1}{4} \sum_{\sigma_e=1}^{4} \sum_{\sigma_f=1}^{4} C_f |\mathcal{M}_{ef}|^2 \,. \tag{7}$$

The invariant transition amplitude $\mathcal{M}_{ef}$ can be expressed in terms of a standard basis of matrix elements $M_i$, containing all the kinematical information of the interaction, and the form factors $F_i$, which account for the pure dynamical part:

$$\mathcal{M}_{ef} = \sum_i M_i F_i \,. \tag{8}$$

## 2.3 Neglecting the electron mass

In this comparative study we are neglecting the electron mass $m_e$ in the purely weak contributions at the diagrammatic level, i.e. we neglect diagrams containing the electron–Higgs Yukawa coupling, which is proportional to the electron mass. This simplifies the final expression significantly and minimizes the number of independent form factors. We do not neglect the electron mass elsewhere so as to safely compute the photonic corrections.

## 2.4 Structure of $\mathcal{O}(\alpha)$ corrections

The hierarchy of contributions in the perturbative expansion of the $2 \to 2$ cross-section reads

$$\begin{aligned} |\mathcal{M}|^2 &= |\mathcal{M}_{ef}^{(0)} + \mathcal{M}_{ef}^{(1)} + \ldots|^2 + |\mathcal{M}_{\gamma}^{(0)} + \ldots|^2 \\ &= \underbrace{\mathcal{M}_{ef}^{(0)^*} \mathcal{M}_{ef}^{(0)}}_{\mathcal{O}(\alpha^2)} + \underbrace{2\,\mathrm{Re}\left(\mathcal{M}_{ef}^{(0)^*} \mathcal{M}_{ef}^{(1)}\right) + \mathcal{M}_{\gamma}^{(0)^*} \mathcal{M}_{\gamma}^{(0)}}_{\mathcal{O}(\alpha^3)} + \underbrace{\ldots}_{\mathcal{O}(\alpha^4)} \end{aligned} \tag{9}$$

Soft-photon contributions are added to remove the infrared singularities of the photonic self-energies, vertices, and boxes.

For the Born amplitude, an appropriate basis for the matrix elements is:

$$\begin{array}{rcccccccc} M_1 & \equiv & \bar{v}_e(p_4,\sigma_{e^+}) & \gamma^\mu \mathbb{1} & u_e(p_1,\sigma_{e^-}) & \otimes & \bar{u}_f(-p_2,\sigma_f) & \gamma_\mu \mathbb{1} & v_f(-p_3,\sigma_{\bar{f}}) \\ M_2 & \equiv & \bar{v}_e(p_4,\sigma_{e^+}) & \gamma^\mu \mathbb{1} & u_e(p_1,\sigma_{e^-}) & \otimes & \bar{u}_f(-p_2,\sigma_f) & \gamma_\mu \gamma_5 & v_f(-p_3,\sigma_{\bar{f}}) \\ M_3 & \equiv & \bar{v}_e(p_4,\sigma_{e^+}) & \gamma^\mu \gamma_5 & u_e(p_1,\sigma_{e^-}) & \otimes & \bar{u}_f(-p_2,\sigma_f) & \gamma_\mu \mathbb{1} & v_f(-p_3,\sigma_{\bar{f}}) \\ M_4 & \equiv & \bar{v}_e(p_4,\sigma_{e^+}) & \gamma^\mu \gamma_5 & u_e(p_1,\sigma_{e^-}) & \otimes & \bar{u}_f(-p_2,\sigma_f) & \gamma_\mu \gamma_5 & v_f(-p_3,\sigma_{\bar{f}}) \,. \end{array} \tag{10}$$

The differential Born cross-section finally reads

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right|_{\text{Born}} = \frac{1}{32\pi} \frac{\beta_f}{\beta_e} C_f \Big\{ s(1+\beta_e^2\beta_f^2\cos^2\theta) \left( |F_1^{(0)}|^2 + |F_2^{(0)}|^2 + |F_3^{(0)}|^2 + |F_4^{(0)}|^2 \right)$$

$$
\begin{aligned}
&+ 2s\beta_e\beta_f \cos\theta \left( {F_1^{(0)}}^* F_4^{(0)} + {F_2^{(0)}}^* F_3^{(0)} + {F_3^{(0)}}^* F_2^{(0)} + {F_4^{(0)}}^* F_1^{(0)} \right) \\
&+ 4(m_f^2 + m_e^2) \left( |F_1^{(0)}|^2 - |F_4^{(0)}|^2 \right) + 4(m_f^2 - m_e^2) \left( -|F_2^{(0)}|^2 + |F_3^{(0)}|^2 \right) \\
&+ 16 \frac{m_f^2 m_e^2}{s} \left( -|F_2^{(0)}|^2 - |F_3^{(0)}|^2 + 2|F_4^{(0)}|^2 \right) \Bigg\}, \qquad (11)
\end{aligned}
$$

with the form factors

$$
F_1^{(0)} = ie^2 \left( + V_e V_f \frac{1}{s - M_Z^2 + iM_Z\Gamma_Z} + Q_e Q_f \frac{1}{s} \right) \qquad (12)
$$

$$
F_2^{(0)} = ie^2 \left( - V_e A_f \frac{1}{s - M_Z^2 + iM_Z\Gamma_Z} \right) \qquad (13)
$$

$$
F_3^{(0)} = ie^2 \left( - A_e V_f \frac{1}{s - M_Z^2 + iM_Z\Gamma_Z} \right) \qquad (14)
$$

$$
F_4^{(0)} = ie^2 \left( + A_e A_f \frac{1}{s - M_Z^2 + iM_Z\Gamma_Z} \right). \qquad (15)
$$

The one-loop calculations for the different fermion flavours are very similar: Only the W–W-box diagram is different for different values of the isospin of the final-state fermion (see Fig. 2). These weak box diagrams were suppressed in applications to LEP1 physics but started to become numerically important at LEP2. They were studied systematically e.g. in Section 2.2 of [9] and Section 5.4 of [10], but a comparison with the published numbers is not straightforward.


Figure 2: Electroweak W–W-box diagrams at the one-loop level, where $f$ denotes an isospin-up and $f'$ an isospin-down fermion.

At the one-loop level, with the appearance of vertex and box diagrams, the Lorentz structure of the matrix element is enriched:

$$
\begin{array}{rcccccccc}
M_{1,k} &=& \bar{v}_e(p_4,\sigma_{e^+}) & \gamma^\mu\{\mathbb{1},\gamma_5\} & u_e(p_1,\sigma_{e^-}) & \otimes & \bar{u}_f(-p_2,\sigma_f) & \gamma_\mu\{\mathbb{1},\gamma_5\} & v_f(-p_3,\sigma_{\bar{f}}) \\
M_{2,k} &=& \bar{v}_e(p_4,\sigma_{e^+}) & \not{p}_2\{\mathbb{1},\gamma_5\} & u_e(p_1,\sigma_{e^-}) & \otimes & \bar{u}_f(-p_2,\sigma_f) & \not{p}_4\{\mathbb{1},\gamma_5\} & v_f(-p_3,\sigma_{\bar{f}}) \\
M_{3,k} &=& \bar{v}_e(p_4,\sigma_{e^+}) & \not{p}_2\{\mathbb{1},\gamma_5\} & u_e(p_1,\sigma_{e^-}) & \otimes & \bar{u}_f(-p_2,\sigma_f) & \{\mathbb{1},\gamma_5\} & v_f(-p_3,\sigma_{\bar{f}}) \\
M_{4,k} &=& \bar{v}_e(p_4,\sigma_{e^+}) & \gamma^\mu\{\mathbb{1},\gamma_5\} & u_e(p_1,\sigma_{e^-}) & \otimes & \bar{u}_f(-p_2,\sigma_f) & \gamma_\mu\not{p}_4\{\mathbb{1},\gamma_5\} & v_f(-p_3,\sigma_{\bar{f}}), \qquad (16)
\end{array}
$$

where the index $k$ stands for the four possible combinations of $\{\mathbb{1},\gamma_5\} \otimes \{\mathbb{1},\gamma_5\}$ as in Eq. (10), leading to a basis of 16 elements. The one-loop contribution to the cross-section can be compacted in the following way:

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\Big|_{\mathrm{Born}} + \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\Big|_{1-\mathrm{Loop}} \qquad (17)
$$

$$= \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\Big|_{\mathrm{Born}} + \frac{1}{32\pi}\frac{\beta_f}{\beta_e}C_f 2\,\mathrm{Re}\left(\sum_{i=1}^{4} F_i^{(0)*}\tilde{F}_i^{(1)}\right), \tag{18}$$

with form factors $\tilde{F}_i^{(1)}$ given by

$$\tilde{F}_i^{(1)} \equiv \frac{1}{s}\sum_{j,k=1}^{4} M_{1,i}^{\dagger} M_{j,k} F_{j,k}^{(1)}, \tag{19}$$

that include the corresponding kinematical terms from the product of matrix elements[1] together with the one-loop form factors $F_{j,k}^{(1)}$, carefully defined in [5] and corresponding to the basis (16). The explicit expressions for these form factors $\tilde{F}_i^{(1)}$ are:

$$\begin{aligned}
\tilde{F}_1^{(1)} \equiv &+ 4m_f^2\left(F_{1,1}^{(1)} - F_{3,1}^{(1)} m_f\right) \\
&+ s\Big\{F_{1,1}^{(1)} + \big(F_{3,1}^{(1)} - 2F_{4,1}^{(1)} + 2F_{4,4}^{(1)}\big)m_f - F_{2,4}^{(1)} m_f^2 \\
&\quad + \beta_f\cos\theta\left(2F_{1,4}^{(1)} - F_{2,1}^{(1)} m_f^2\right) + \beta_f^2\cos^2\theta\left(F_{1,1}^{(1)} - F_{3,1}^{(1)} m_f\right)\Big\} \\
&+ \frac{s^2}{4}(1-\beta_f^2\cos^2\theta)\left(F_{2,4}^{(1)} + \beta_f\cos\theta\ F_{2,1}^{(1)}\right) \qquad (20) \\
\tilde{F}_2^{(1)} \equiv &- 4F_{1,2}^{(1)} m_f^2 \\
&+ s\Big\{F_{1,2}^{(1)} - F_{2,3}^{(1)} m_f^2 + \beta_f\cos\theta\left(2F_{1,3}^{(1)} + 2\big(F_{4,2}^{(1)} - F_{4,3}^{(1)}\big)m_f - F_{2,2}^{(1)} m_f^2\right) \\
&\quad + \beta_f^2\cos^2\theta F_{1,2}^{(1)}\Big\} + \frac{s^2}{4}(1-\beta_f^2\cos^2\theta)\left(F_{2,3}^{(1)} + \beta_f\cos\theta\ F_{2,2}^{(1)}\right) \qquad (21) \\
\tilde{F}_3^{(1)} \equiv &+ 4m_f^2\left(F_{1,3}^{(1)} - F_{3,3}^{(1)} m_f\right) \\
&+ s\Big\{F_{1,3}^{(1)} + \big(F_{3,3}^{(1)} + 2F_{4,2}^{(1)} - 2F_{4,3}^{(1)}\big)m_f - F_{2,2}^{(1)} m_f^2 \\
&\quad + \beta_f\cos\theta\left(+2F_{1,2}^{(1)} - F_{2,3}^{(1)} m_f^2\right) + \beta_f^2\cos^2\theta\left(F_{1,3}^{(1)} - F_{3,3}^{(1)} m_f\right)\Big\} \\
&+ \frac{s^2}{4}(1-\beta_f^2\cos^2\theta)\left(F_{2,2}^{(1)} + \beta_f\cos\theta\ F_{2,3}^{(1)}\right) \qquad (22) \\
\tilde{F}_4^{(1)} \equiv &- 4F_{1,4}^{(1)} m_f^2 \\
&+ s\Big\{F_{1,4}^{(1)} - F_{2,1}^{(1)} m_f^2 + \beta_f\cos\theta\left(2F_{1,1}^{(1)} + 2\big(-F_{4,1}^{(1)} + F_{4,4}^{(1)}\big)m_f - F_{2,4}^{(1)} m_f^2\right) \\
&\quad + \beta_f^2\cos^2\theta F_{1,4}^{(1)}\Big\} + \frac{s^2}{4}(1-\beta_f^2\cos^2\theta)\left(F_{2,1}^{(1)} + \beta_f\cos\theta\ F_{2,4}^{(1)}\right). \qquad (23)
\end{aligned}$$

Many technical details of the underlying calculations have been described in [5, 11].

[1] Since the corrections are of $\mathcal{O}(\alpha)$ with respect to the Born cross-section, we neglected the effect of the electron mass here.

# 3 Numerical results

In this section we present the numerical results for various final states at two typical LC energies: 500 GeV and 1 TeV. We performed two fixed-order calculations, i.e. no higher-order corrections such as photon exponentiation have been taken into account. The MPI Munich group performed a fully automated calculation using *FeynArts* [12, 13] and *FormCalc* [14], where the fermionic structures were evaluated in the Weyl–van-der-Waerden formalism [15] rather than by introducing helicity matrix elements $M_{j,k}$ as outlined before. The numbers of the Zeuthen/CERN group are obtained from a partly automated calculation with DIANA [16] and FORM [17, 18], using a FORTRAN code obtainable from [19]. Both codes use *LoopTools* [14].

We assume the same input values as were used in [3, 4, 5]. They are described in Tab. 1.

| Fermion Masses | | | Boson Masses & Widths | | |
|---|---|---|---|---|---|
| $m_\nu =$ | 0.0 | GeV | | | |
| $m_e =$ | 0.00051099907 | GeV | | | |
| $m_\mu =$ | 0.105658389 | GeV | $m_\gamma =$ | 0.0 | GeV |
| $m_\tau =$ | 1.77705 | GeV | $m_W =$ | 80.4514958 | GeV |
| $m_u =$ | 0.062 | GeV | $m_Z =$ | 91.1867 | GeV |
| $m_c =$ | 1.5 | GeV | $m_H =$ | 120.0 | GeV |
| $m_t =$ | 173.8 | GeV | $\Gamma_W =$ | 0.0 | GeV |
| $m_d =$ | 0.083 | GeV | $\Gamma_Z =$ | 0.0 | GeV |
| $m_s =$ | 0.215 | GeV | $\Gamma_H =$ | 0.0 | GeV |
| $m_b =$ | 4.7 | GeV | | | |

| Other Parameters |
|---|
| $\alpha = 1/137.03599976$ |
| $E^{\max}_{\gamma_{\rm soft}} = \sqrt{s}/10$ |
| $(\hbar c)^2 = 0.38937966 \cdot 10^9 \mathrm{GeV}^2\mathrm{pb}$ |

Table 1: Input parameter set.

The cross-sections shown below depend on the maximum soft-photon energy $E^{\max}_{\gamma_{\rm soft}}$. This dependence should eventually cancel when hard-photon radiation is added, but only for sufficiently small values of $E^{\max}_{\gamma_{\rm soft}}$. The value $E^{\max}_{\gamma_{\rm soft}} = \sqrt{s}/10$, which was used in the numerical evaluation, is by far too large if one aims at a high numerical accuracy after combination with real, hard-photon emission. It has been chosen here nevertheless because it ensures positive cross-section values of a realistic order of magnitude. Even for this large value, however, the numerical change in the combined soft- and hard-photon corrections compared to more realistic values of $E^{\max}_{\gamma_{\rm soft}}$ is at most few per cent at $\sqrt{s} = 500$ GeV and few per mill at $\sqrt{s} = 1$ TeV [4].

The following differential cross-sections are compared:

- $\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right|_{\mathrm{Born}}$: Born cross-section
- $\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right|_{\mathrm{B+weak}}$: Interference of Born with one-loop virtual weak corrections. The running of the electromagnetic coupling is also included in the tables[2]
- $\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right|_{\mathrm{B+w+QED+soft}}$: The QED + soft photon emission (with $E^{\mathrm{max}}_{\gamma_{\mathrm{soft}}} = \sqrt{s}/10$) is added to the previous contributions

The main numerical results are documented in Tabs. 2–9. Compared to [3], the agreement between our calculations for top-pair production has been improved by a factor $10^3$. This has been achieved thanks to a closer contact between both groups and a more methodological programming in the FORTRAN code TOPFIT. The agreement reaches now 11 digits of technical precision, for all flavours studied.

Finally, in Fig. 3, we give an overview of the differential cross-sections for the different flavours at two typical collider energies.

[2]This is not the case for the plots, where the running of the electromagnetic coupling is not included into the weak contributions.

| $e^+e^- \to \tau^+\tau^-$ $\quad \sqrt{s} = 500$ GeV | | | | |
|---|---|---|---|---|
| $\cos\theta$ | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$ /pb | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+weak}}$ /pb | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+w+QED+soft}}$ /pb | Program |
| $-0.9$ | $0.94591\ 02171\ 86329 \cdot 10^{-1}$ | $0.10860\ 60371\ 92303$ | $0.92419\ 02671\ 14061 \cdot 10^{-1}$ | TOPFIT |
| $-0.9$ | $0.94591\ 02171\ 86327 \cdot 10^{-1}$ | $0.10860\ 60371\ 93233$ | $0.92419\ 02671\ 18656 \cdot 10^{-1}$ | FA/FC |
| $-0.5$ | $0.89298\ 53117\ 79858 \cdot 10^{-1}$ | $0.10025\ 68354\ 16001$ | $0.86699\ 48248\ 65248 \cdot 10^{-1}$ | TOPFIT |
| $-0.5$ | $0.89298\ 53117\ 79856 \cdot 10^{-1}$ | $0.10025\ 68354\ 16428$ | $0.86699\ 48248\ 69477 \cdot 10^{-1}$ | FA/FC |
| 0.0 | $0.15032\ 16827\ 75192$ | $0.16418\ 09556\ 08258$ | $0.14359\ 79492\ 08648$ | TOPFIT |
| 0.0 | $0.15032\ 16827\ 75192$ | $0.16418\ 09556\ 07903$ | $0.14359\ 79492\ 08618$ | FA/FC |
| 0.5 | $0.28649\ 90174\ 53525$ | $0.31504\ 05045\ 07441$ | $0.28258\ 86777\ 59811$ | TOPFIT |
| 0.5 | $0.28649\ 90174\ 53525$ | $0.31504\ 05045\ 06135$ | $0.28258\ 86777\ 59161$ | FA/FC |
| 0.9 | $0.44955\ 18970\ 14604$ | $0.50904\ 21673\ 78790$ | $0.47648\ 29191\ 20038$ | TOPFIT |
| 0.9 | $0.44955\ 18970\ 14604$ | $0.50904\ 21673\ 76612$ | $0.47648\ 29191\ 19623$ | FA/FC |

Table 2: Differential cross-sections for selected scattering angles for $\tau$-production at $\sqrt{s} = 500$ GeV. The three columns contain the Born cross-section, Born including only the weak $\mathcal{O}(\alpha)$ corrections, and Born including the weak and photonic $\mathcal{O}(\alpha)$ corrections. For each angle, the first row represents the TOPFIT result of the Zeuthen group while the second stands for the *FeynArts/FormCalc* calculation of the Munich group.

| $e^+e^- \to \tau^+\tau^-$ $\quad \sqrt{s} = 1$ TeV | | | | |
|---|---|---|---|---|
| $\cos\theta$ | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$ /pb | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+weak}}$ /pb | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+w+QED+soft}}$ /pb | Program |
| $-0.9$ | $0.24337\ 58691\ 13477 \cdot 10^{-1}$ | $0.27641\ 21664\ 58412 \cdot 10^{-1}$ | $0.23440\ 03881\ 68909 \cdot 10^{-1}$ | TOPFIT |
| $-0.9$ | $0.24337\ 58691\ 13477 \cdot 10^{-1}$ | $0.27641\ 21664\ 60671 \cdot 10^{-1}$ | $0.23440\ 03881\ 70852 \cdot 10^{-1}$ | FA/FC |
| $-0.5$ | $0.22648\ 34522\ 34421 \cdot 10^{-1}$ | $0.25087\ 88401\ 11477 \cdot 10^{-1}$ | $0.21435\ 50246\ 92009 \cdot 10^{-1}$ | TOPFIT |
| $-0.5$ | $0.22648\ 34522\ 34421 \cdot 10^{-1}$ | $0.25087\ 88401\ 12536 \cdot 10^{-1}$ | $0.21435\ 50246\ 93075 \cdot 10^{-1}$ | FA/FC |
| 0.0 | $0.37338\ 94309\ 20687 \cdot 10^{-1}$ | $0.40075\ 04507\ 03072 \cdot 10^{-1}$ | $0.34538\ 81564\ 13972 \cdot 10^{-1}$ | TOPFIT |
| 0.0 | $0.37338\ 94309\ 20687 \cdot 10^{-1}$ | $0.40075\ 04507\ 02276 \cdot 10^{-1}$ | $0.34538\ 81564\ 13421 \cdot 10^{-1}$ | FA/FC |
| 0.5 | $0.70698\ 59649\ 23715 \cdot 10^{-1}$ | $0.76863\ 25654\ 09100 \cdot 10^{-1}$ | $0.68181\ 23407\ 81333 \cdot 10^{-1}$ | TOPFIT |
| 0.5 | $0.70698\ 59649\ 23714 \cdot 10^{-1}$ | $0.76863\ 25654\ 06057 \cdot 10^{-1}$ | $0.68181\ 23407\ 78805 \cdot 10^{-1}$ | FA/FC |
| 0.9 | $0.11082\ 80391\ 95421$ | $0.12645\ 00486\ 28998$ | $0.11773\ 76209\ 15053$ | TOPFIT |
| 0.9 | $0.11082\ 80391\ 95421$ | $0.12645\ 00486\ 28487$ | $0.11773\ 76209\ 14679$ | FA/FC |

Table 3: The same as Tab. 2 for $\sqrt{s} = 1$ TeV.

| $e^+e^- \to b\bar{b}$ $\quad \sqrt{s} = 500$ GeV | | | | |
|---|---|---|---|---|
| $\cos\theta$ | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{Born}}$/pb | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{B+weak}}$/pb | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{B+w+QED+soft}}$/pb | Program |
| $-0.9$ | $0.35947\,21020\,03927 \cdot 10^{-1}$ | $0.42347\,36269\,56878 \cdot 10^{-1}$ | $0.37629\,38061\,51582 \cdot 10^{-1}$ | TOPFIT |
| $-0.9$ | $0.35947\,21020\,03927 \cdot 10^{-1}$ | $0.42347\,36269\,50374 \cdot 10^{-1}$ | $0.37629\,38061\,44883 \cdot 10^{-1}$ | FA/FC |
| $-0.5$ | $0.52846\,99142\,94595 \cdot 10^{-1}$ | $0.55564\,40895\,92051 \cdot 10^{-1}$ | $0.49542\,16119\,64096 \cdot 10^{-1}$ | TOPFIT |
| $-0.5$ | $0.52846\,99142\,94594 \cdot 10^{-1}$ | $0.55564\,40895\,84646 \cdot 10^{-1}$ | $0.49542\,16119\,57136 \cdot 10^{-1}$ | FA/FC |
| 0.0 | 0.13444 84372 56821 | 0.13513 90019 99522 | 0.12117 62087 02347 | TOPFIT |
| 0.0 | 0.13444 84372 56821 | 0.13513 90019 97996 | 0.12117 62087 00907 | FA/FC |
| 0.5 | 0.28324 62378 51991 | 0.29122 72277 53244 | 0.26454 12363 95596 | TOPFIT |
| 0.5 | 0.28324 62378 51991 | 0.29122 72277 50185 | 0.26454 12363 92671 | FA/FC |
| 0.9 | 0.45066 58537 60950 | 0.48256 44834 85869 | 0.44708 31668 19343 | TOPFIT |
| 0.9 | 0.45066 58537 60950 | 0.48256 44834 81057 | 0.44708 31668 15091 | FA/FC |

Table 4: The same as Tab. 2 for $b$-production at $\sqrt{s} = 500$ GeV.

| $e^+e^- \to b\bar{b}$ $\quad \sqrt{s} = 1$ TeV | | | | |
|---|---|---|---|---|
| $\cos\theta$ | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{Born}}$/pb | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{B+weak}}$/pb | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{B+w+QED+soft}}$/pb | Program |
| $-0.9$ | $0.85256\,94949\,38769 \cdot 10^{-2}$ | $0.98313\,19956\,72613 \cdot 10^{-2}$ | $0.86113\,09362\,51944 \cdot 10^{-2}$ | TOPFIT |
| $-0.9$ | $0.85256\,94949\,38768 \cdot 10^{-2}$ | $0.98313\,19956\,58270 \cdot 10^{-2}$ | $0.86113\,09362\,37511 \cdot 10^{-2}$ | FA/FC |
| $-0.5$ | $0.12689\,55586\,65297 \cdot 10^{-1}$ | $0.12711\,32506\,71243 \cdot 10^{-1}$ | $0.11163\,82185\,03862 \cdot 10^{-1}$ | TOPFIT |
| $-0.5$ | $0.12689\,55586\,65297 \cdot 10^{-1}$ | $0.12711\,32506\,69579 \cdot 10^{-1}$ | $0.11163\,82185\,02235 \cdot 10^{-1}$ | FA/FC |
| 0.0 | $0.32532\,44660\,76073 \cdot 10^{-1}$ | $0.31258\,65157\,55267 \cdot 10^{-1}$ | $0.27674\,41895\,03390 \cdot 10^{-1}$ | TOPFIT |
| 0.0 | $0.32532\,44660\,76072 \cdot 10^{-1}$ | $0.31258\,65157\,51750 \cdot 10^{-1}$ | $0.27674\,41894\,99947 \cdot 10^{-1}$ | FA/FC |
| 0.5 | $0.68639\,85356\,49626 \cdot 10^{-1}$ | $0.69302\,03325\,89997 \cdot 10^{-1}$ | $0.62501\,12097\,14961 \cdot 10^{-1}$ | TOPFIT |
| 0.5 | $0.68639\,85356\,49626 \cdot 10^{-1}$ | $0.69302\,03325\,82884 \cdot 10^{-1}$ | $0.62501\,12097\,07973 \cdot 10^{-1}$ | FA/FC |
| 0.9 | 0.10923 62308 06567 | 0.12127 77274 48650 | 0.11240 86957 39236 | TOPFIT |
| 0.9 | 0.10923 62308 06567 | 0.12127 77274 47528 | 0.11240 86957 38153 | FA/FC |

Table 5: The same as Tab. 2 for $b$-production at $\sqrt{s} = 1$ TeV.

| $e^+e^- \to c\bar{c}$ $\quad \sqrt{s} = 500$ GeV | | | | |
|---|---|---|---|---|
| $\cos\theta$ | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{Born}}$/pb | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{B+weak}}$/pb | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{B+w+QED+soft}}$/pb | Program |
| $-0.9$ | $0.78403\,69156\,96992 \cdot 10^{-1}$ | $0.91244\,84607\,87569 \cdot 10^{-1}$ | $0.83668\,39315\,90920 \cdot 10^{-1}$ | TOPFIT |
| $-0.9$ | $0.78403\,69156\,96992 \cdot 10^{-1}$ | $0.91244\,84607\,99371 \cdot 10^{-1}$ | $0.83668\,39316\,04269 \cdot 10^{-1}$ | FA/FC |
| $-0.5$ | $0.10411\,12875\,82399$ | $0.11650\,15689\,39071$ | $0.10590\,20427\,16561$ | TOPFIT |
| $-0.5$ | $0.10411\,12875\,82399$ | $0.11650\,15689\,39412$ | $0.10590\,20427\,16692$ | FA/FC |
| $0.0$ | $0.24770\,82888\,45901$ | $0.26255\,80017\,68786$ | $0.23448\,15990\,25778$ | TOPFIT |
| $0.0$ | $0.24770\,82888\,45900$ | $0.26255\,80017\,67528$ | $0.23448\,15990\,23961$ | FA/FC |
| $0.5$ | $0.51515\,25192\,73431$ | $0.53094\,95526\,19036$ | $0.46371\,41775\,17198$ | TOPFIT |
| $0.5$ | $0.51515\,25192\,73431$ | $0.53094\,95526\,15566$ | $0.46371\,41775\,12847$ | FA/FC |
| $0.9$ | $0.81827\,79086\,13557$ | $0.83043\,43356\,61887$ | $0.70026\,97050\,29472$ | TOPFIT |
| $0.9$ | $0.81827\,79086\,13556$ | $0.83043\,43356\,56199$ | $0.70026\,97050\,21870$ | FA/FC |

Table 6: The same as Tab. 2 for $c$-production at $\sqrt{s} = 500$ GeV.

| $e^+e^- \to c\bar{c}$ $\quad \sqrt{s} = 1$ TeV | | | | |
|---|---|---|---|---|
| $\cos\theta$ | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{Born}}$/pb | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{B+weak}}$/pb | $\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\right]_{\mathrm{B+w+QED+soft}}$/pb | Program |
| $-0.9$ | $0.20476\,82671\,10479 \cdot 10^{-1}$ | $0.23804\,15350\,74367 \cdot 10^{-1}$ | $0.21460\,20354\,03294 \cdot 10^{-1}$ | TOPFIT |
| $-0.9$ | $0.20476\,82671\,10479 \cdot 10^{-1}$ | $0.23804\,15350\,77280 \cdot 10^{-1}$ | $0.21460\,20354\,06337 \cdot 10^{-1}$ | FA/FC |
| $-0.5$ | $0.26302\,86046\,48394 \cdot 10^{-1}$ | $0.29192\,27449\,28377 \cdot 10^{-1}$ | $0.26283\,52825\,19898 \cdot 10^{-1}$ | TOPFIT |
| $-0.5$ | $0.26302\,86046\,48394 \cdot 10^{-1}$ | $0.29192\,27449\,29292 \cdot 10^{-1}$ | $0.26283\,52825\,20679 \cdot 10^{-1}$ | FA/FC |
| $0.0$ | $0.61063\,66375\,83921 \cdot 10^{-1}$ | $0.63092\,30352\,27478 \cdot 10^{-1}$ | $0.55698\,44755\,91055 \cdot 10^{-1}$ | TOPFIT |
| $0.0$ | $0.61063\,66375\,83920 \cdot 10^{-1}$ | $0.63092\,30352\,24633 \cdot 10^{-1}$ | $0.55698\,44755\,87819 \cdot 10^{-1}$ | FA/FC |
| $0.5$ | $0.12635\,58682\,75626$ | $0.12548\,22393\,89320$ | $0.10778\,82066\,27453$ | TOPFIT |
| $0.5$ | $0.12635\,58682\,75626$ | $0.12548\,22393\,88519$ | $0.10778\,82066\,26582$ | FA/FC |
| $0.9$ | $0.20057\,22407\,70464$ | $0.19463\,36446\,48183$ | $0.16019\,87823\,32139$ | TOPFIT |
| $0.9$ | $0.20057\,22407\,70464$ | $0.19463\,36446\,46866$ | $0.16019\,87823\,30647$ | FA/FC |

Table 7: The same as Tab. 2 for $c$-production at $\sqrt{s} = 1$ TeV.

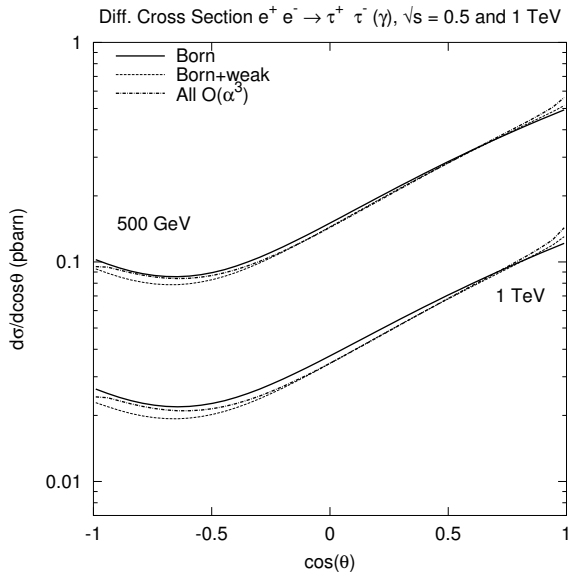
| $e^+e^- \to t\bar{t}$ $\quad \sqrt{s} = 500$ GeV | | | | |
|---|---|---|---|---|
| $\cos\theta$ | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$ /pb | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+weak}}$ /pb | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+w+QED+soft}}$ /pb | Program |
| −0.9 | 0.10883 91940 76039 | 0.12425 90371 32943 | 0.11408 40955 77861 | TOPFIT |
| −0.9 | 0.10883 91940 76039 | 0.12425 90371 33664 | 0.11408 40955 78964 | FA/FC |
| −0.5 | 0.14227 50693 93371 | 0.15684 83718 76069 | 0.14308 12051 65511 | TOPFIT |
| −0.5 | 0.14227 50693 93371 | 0.15684 83718 76250 | 0.14308 12051 65581 | FA/FC |
| 0.0 | 0.22547 04640 33559 | 0.24026 68040 30724 | 0.21718 80097 67412 | TOPFIT |
| 0.0 | 0.22547 04640 33559 | 0.24026 68040 30032 | 0.21718 80097 66323 | FA/FC |
| 0.5 | 0.35466 64703 33217 | 0.36888 65069 94389 | 0.32933 72739 51692 | TOPFIT |
| 0.5 | 0.35466 64703 33217 | 0.36888 65069 92599 | 0.32933 72739 49095 | FA/FC |
| 0.9 | 0.49114 37157 67761 | 0.50333 75116 05520 | 0.44290 81673 51494 | TOPFIT |
| 0.9 | 0.49114 37157 67761 | 0.50333 75116 02681 | 0.44290 81673 46094 | FA/FC |

Table 8: The same as Tab. 2 for $t$-production at $\sqrt{s} = 500$ GeV.

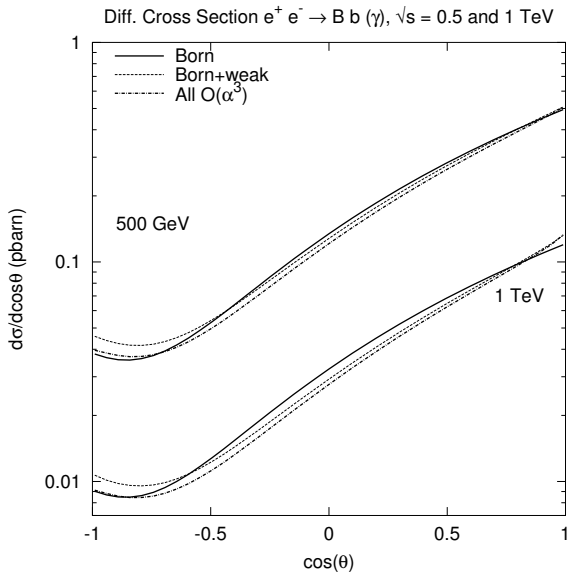
| $e^+e^- \to t\bar{t}$ $\quad \sqrt{s} = 1$ TeV | | | | |
|---|---|---|---|---|
| $\cos\theta$ | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$ /pb | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+weak}}$ /pb | $\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+w+QED+soft}}$ /pb | Program |
| −0.9 | $0.22785\,42327\,32090 \cdot 10^{-1}$ | $0.25521\,28532\,98051 \cdot 10^{-1}$ | $0.23101\,70508\,05040 \cdot 10^{-1}$ | TOPFIT |
| −0.9 | $0.22785\,42327\,32090 \cdot 10^{-1}$ | $0.25521\,28533\,00748 \cdot 10^{-1}$ | $0.23101\,70508\,07714 \cdot 10^{-1}$ | FA/FC |
| −0.5 | $0.29782\,13110\,31861 \cdot 10^{-1}$ | $0.31863\,48943\,59857 \cdot 10^{-1}$ | $0.28823\,01902\,00931 \cdot 10^{-1}$ | TOPFIT |
| −0.5 | $0.29782\,13110\,31861 \cdot 10^{-1}$ | $0.31863\,48943\,60711 \cdot 10^{-1}$ | $0.28823\,01902\,01653 \cdot 10^{-1}$ | FA/FC |
| 0.0 | $0.61180\,06742\,25039 \cdot 10^{-1}$ | $0.61591\,61295\,77963 \cdot 10^{-1}$ | $0.54950\,88904\,88739 \cdot 10^{-1}$ | TOPFIT |
| 0.0 | $0.61180\,06742\,25038 \cdot 10^{-1}$ | $0.61591\,61295\,75474 \cdot 10^{-1}$ | $0.54950\,88904\,85894 \cdot 10^{-1}$ | FA/FC |
| 0.5 | 0.11774 69498 88318 | 0.11404 76860 51226 | $0.99417\,00898\,39905 \cdot 10^{-1}$ | TOPFIT |
| 0.5 | 0.11774 69498 88318 | 0.11404 76860 50527 | $0.99417\,00898\,32292 \cdot 10^{-1}$ | FA/FC |
| 0.9 | 0.18112 20970 86446 | 0.17134 61927 22790 | 0.14426 23325 41248 | TOPFIT |
| 0.9 | 0.18112 20970 86446 | 0.17134 61927 21645 | 0.14426 23325 40061 | FA/FC |

Table 9: The same as Tab. 2 for $t$-production at $\sqrt{s} = 1$ TeV.

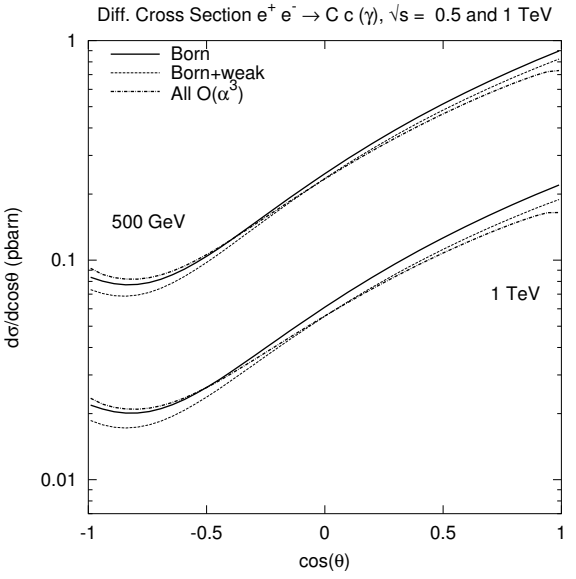
a) $\tau$ production.




b) $b$ production.




c) $c$ production.




d) $t$ production.

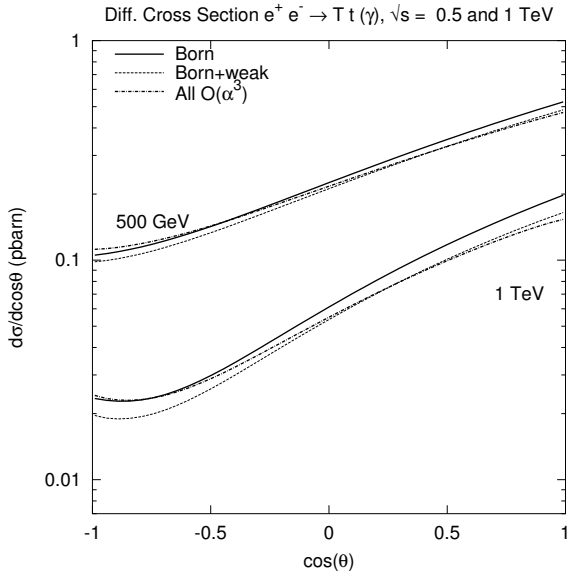



Figure 3: Comparison of differential cross-sections. Solid line stands for Born, dashed for Born+weak (without running coupling), and dashed-dotted for complete $\mathcal{O}(\alpha)$ (i.e. Born+weak+QED+soft).

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