A track finding method for a TPC based on triplet chains

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Abstract

This document describes a local track finding algorithm using triplet chains developed for a prototype TPC with pad readout.

1 Introduction

A track finding method based on triplet chains has been developed especially for the data taken by the Large Prototype TPC (LPTPC) [1] with pad readout at the DESY test beam [2] and implemented as a MarlinTPC [3] processor. It is inspired by the pattern recognition for the central jet chamber of the the H1 experiment by V. Blobel (Univ. Hamburg).

In this setup the drift direction perpendicular to the readout plane is along the z-coordinate, the tracks roughly move in the x-direction and the pad rows measure y-coordinates.

The readout plane is segmented into modules. For each module separately triplets of hits are build using both the measurement in the pad row and in the drift direction. Matching triplets form segments and segments of neighbouring modules are combined to track candidates fitted by a simple helix.

In section 2 the basic concepts are presented, section 3 describes the sequence of processing steps, section 4 shows a brief performance test and section 5 explains the steering parameters of the ROWTRIPLETBASEDTRACKFINDERPROCESSOR.

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2 Concepts

2.1 Radial ordering

For the pad based readout modules of the LPTPC the pad rows are on concentric circles in the XY-plane. The position in a row measures an azimuthal coordinate and the row number defines the radius and is used as radial ordering parameter:

- The hits on a track are ordered by row number.

- Tracks may have only a single hit in each row.

- For a set of hits the number of rows defines a size.

- The difference of row numbers defines a distance.

2.2 Triplets

Triplets are constructed from hits in three different equidistant rows ($i < j < k$). The position of the center hit has to be compatible with the prediction from the outer hits both in the azimuthal and longitudinal measurement direction. The azimuthal direction is defined by the local pad row direction φ_m and the longitudinal by the drift direction. The corresponding measurement resolutions are σ_{xy} and σ_z.

The 3D distance between the position of the center hit (j) and the average from the outer hits is projected onto the measurement directions to define a 2D triplet distance $\mathbf{t}_j$:

$$\mathbf{d}_j = \begin{pmatrix} x_j \\ y_j \\ z_j \end{pmatrix} - \frac{1}{2} \begin{pmatrix} x_i + x_k \\ y_i + y_k \\ z_i + z_k \end{pmatrix}, \mathbf{t}_j = \begin{pmatrix} \cos\varphi_{m,j} & \sin\varphi_{m,j} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \mathbf{d}_j = \begin{pmatrix} t_{xy} \\ t_z \end{pmatrix}_j \quad (1)$$

Neglecting the change of the azimuthal measurement direction between the three hits the variance of the triplet distance is a simple sum of the measurement variances:

$$\mathbf{V}_{t,j} = \frac{1}{4} \begin{pmatrix} \sigma^2_{xy,i} & 0 \\ 0 & \sigma^2_{z,i} \end{pmatrix} + \begin{pmatrix} \sigma^2_{xy,j} & 0 \\ 0 & \sigma^2_{z,j} \end{pmatrix} + \frac{1}{4} \begin{pmatrix} \sigma^2_{xy,k} & 0 \\ 0 & \sigma^2_{z,k} \end{pmatrix} = \begin{pmatrix} V_{xy} & 0 \\ 0 & V_z \end{pmatrix}_j \quad (2)$$

Triplets define a line segment with a position and a direction calculated from the average and difference of the positions of the outer hits and measurement directions and a row number taken from the center hit. The positions of the outer hits are transformed into the measurement system of the center hit:

$$\begin{pmatrix} u_l \\ v_l \\ w_l \end{pmatrix} = \begin{pmatrix} \cos\varphi_{m,j} & \sin\varphi_{m,j} & 0 \\ 0 & 0 & 1 \\ \sin\varphi_{m,j} & -\cos\varphi_{m,j} & 0 \end{pmatrix} \cdot \begin{pmatrix} x_l - x_j \\ y_l - y_j \\ z_l - z_j \end{pmatrix} \quad (3)$$

For the line connecting the outer hits in this local system the offsets are $u_0 = 0.5 \cdot (u_k + u_i)$ and $v_0 = 0.5 \cdot (v_k + v_i)$. With $\Delta u = u_k - u_i$, $\Delta v = v_k - v_i$ and $\Delta w = w_k - w_i$ the slopes are $u' = \Delta u / \Delta w$ and $v' = \Delta v / \Delta w$ and the arc length in XY is $\Delta s = \sqrt{\Delta u^2 + \Delta w^2}$. The corresponding covariance matrices for offsets and slopes are:

$$\mathbf{V}_u = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{\Delta w^2} \end{pmatrix} (\sigma_{xy,k}^2 + \sigma_{xy,i}^2), \mathbf{V}_v = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{\Delta w^2} \end{pmatrix} (\sigma_{z,k}^2 + \sigma_{z,i}^2) \tag{4}$$

The directions in the global system are $\varphi = \arctan u' + \varphi_{m,j} - \frac{\pi}{2}$ and $\tan \lambda = \Delta z / \Delta s = v' \cdot \cos \beta$ with $\cos \beta = \Delta w / \Delta s$. Their variances $(V_\varphi, V_{\tan \lambda})$ can be calculated using $\frac{\partial \varphi}{\partial u'} = \cos \beta^2$ and $\frac{\partial \tan \lambda}{\partial v'} = \cos \beta$ from equation (4).

In this application the triplets are constructed from consecutive even or odd rows ($j - i = k - j = 2$). This doubles the lever arm for better directional resolution and triplets from adjacent rows are independent.

2.3 Equivalence classes

Equivalence classes are used to combine triplets to segments or segments to tracks. For a set of triplets or segments a *matching* relation is defined for pairs of elements. An equivalence class is the subset of all directly or indirectly related elements. The original set is decomposed in disjunct subsets.

3 Sequence

3.1 Input

Input from each module are the ordered lists of TPC hits with positions, measurement directions and resolutions. The TPC geometry described by GEAR is used to calculate the azimuthal measurement directions and to define neighbouring modules. A constant magnetic field is assumed to describe the tracks by an helix or straight line.

3.2 On module level

3.2.1 Triplet finding

Triplets are looked for in all groups of three adjacent even or odd rows of a module. For all possible combinations of hits in each group the deviations $\mathbf{t}$ of the center to the prediction of the outer hits are calculated according to (1). After a coarse preselection $|t_{xy}| < t_{max}$ and $|t_z| < t_{max}$ candidates are accepted in case of:

$$t_{xy}^2 / V_{xy} < \chi_{trp}^2 \text{ and } t_z^2 / V_z < \chi_{trp}^2 \tag{5}$$

Triplets may be lost due to missing or bad hits.
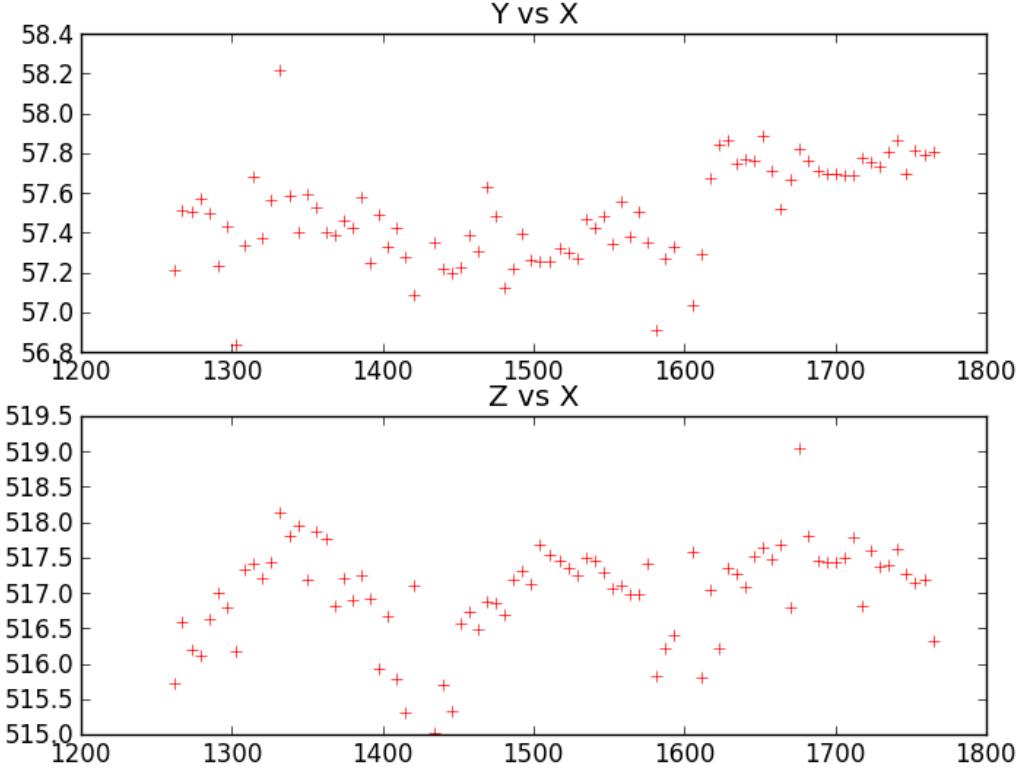
Figure 1: Example track in three DESY GEM modules: hits (crosses) in XY (top) and XZ (bottom) projections. For this uncorrected data some systematic distortions in the Z measurement roughly parabolic in the x-coordinate (or row number) are visible in each module.

3.2.2 Triplet matching

Each triplet is compared with all triplets from the next even and odd row. For each pair of triplets first the directions have to be compatible:

$$\frac{(\varphi_1 - \varphi_2)^2}{V_{\varphi,1} + V_{\varphi,2}} < \chi^2_{dir} \text{ and } \frac{(\tan \lambda_1 - \tan \lambda_2)^2}{V_{\tan \lambda,1} + V_{\tan \lambda,2}} < \chi^2_{dir} \tag{6}$$

Now both triplets are extrapolated to the point of closest approach to their midpoint (by Δw_{mid}). The distance $\mathbf{d}$ between this two points is projected on the average measurement directions similar to (1) yielding $(\Delta u_{mid}, \Delta v_{mid})$. They have to fulfil with the

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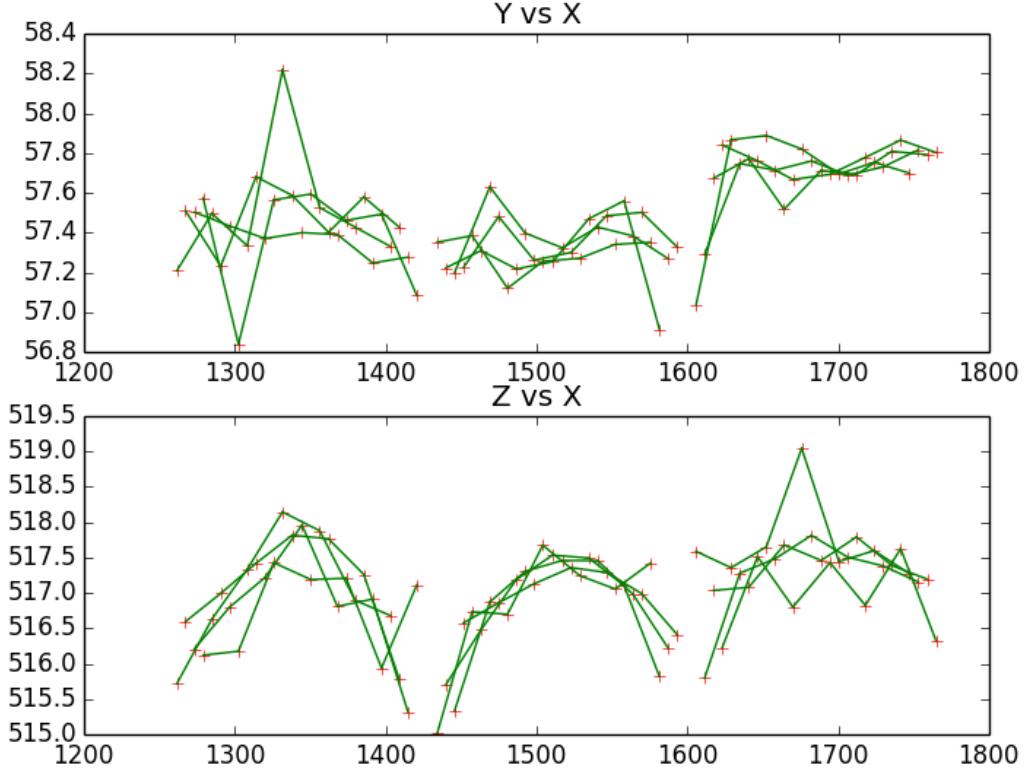
Figure 2: Example track in three DESY GEM modules: hits (crosses) and triplets (solid lines connecting outer hits) in XY (top) and XZ (bottom) projections.

extrapolated variances $V_{mid} = (1 \quad \Delta w_{mid}) \cdot \mathbf{V} \cdot (1 \quad \Delta w_{mid})^t$:

$$\frac{\Delta u_{mid}^2}{V_{u,mid,1} + V_{u,mid,2}} < \chi_{pos}^2 \text{ and } \frac{\Delta v_{mid}^2}{V_{v,mid,1} + V_{v,mid,2}} < \chi_{pos}^2 \tag{7}$$

Pairs of triplets passing this cuts are added to the set of matching triplets.

3.2.3 Segment building

Track segments are constructed from the equivalence classes of the set of matching triplets. Each hit could be part of several triplets which could be elements of different classes and is assigned to the largest class. Therefore the classes are ordered according to the row numbers of the corresponding triplets by decreasing size (number of rows) and length (distance between last and first row). According to this order for each class the still unused hits from all triplets are selected and ordered by row. The hits from all

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rows with exactly one hit in this selection define a segment and are marked as used. Segments with too few hits are discarded. Due to missing triplets single tracks may be broken up into several segments.
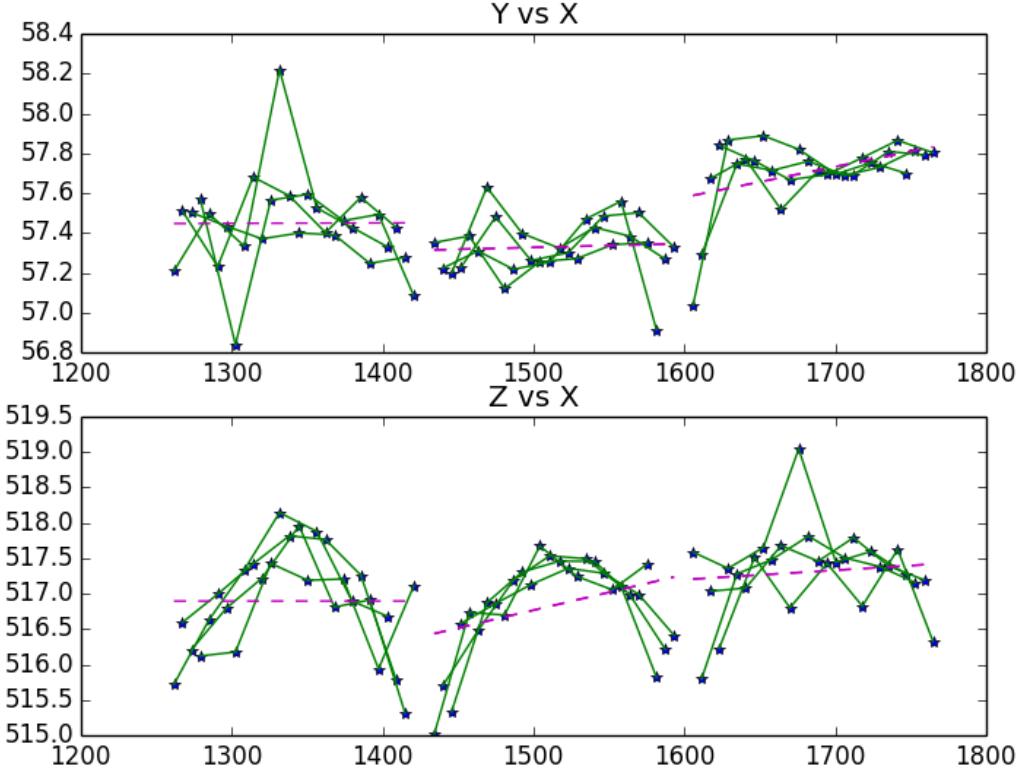
Figure 3: Example track in three DESY GEM modules: hits (stars) used (by a segment), triplets (solid lines) and segments (dashed lines) in XY (top) and XZ (bottom) projections.

3.2.4 Segment fitting

Assuming a constant magnetic field in Z-direction the segments are fitted with a circle [4] in the XY- and a straight line in the ZS-projection using the average of the positions of the first and last hit as reference point. The five track parameters $\mathbf{p}$ with covariance matrix $\mathbf{V}_p$ defined there are the curvature and for both projections the directions at and distances to the point of closest approach in XY. For zero magnetic field straight lines are fitted in both projections.

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3.2.5 Segment matching

Each segment is compared with all other segments which don't overlap (hits in common rows) and the row gap in-between is less than the average segment size. For both segments the (n_{par}) parameters and the covariance matrices are propagated to the midpoint of their reference points (as new reference point) and compared there:

$$(\mathbf{p}_{mid,1} - \mathbf{p}_{mid,2}) \cdot (\mathbf{V}_{p,mid,1} + \mathbf{V}_{p,mid,2})^{-1} \cdot (\mathbf{p}_{mid,1} - \mathbf{p}_{mid,2})^t / n_{par} < \chi^2_{seg} \quad (8)$$

Pairs of segments passing this cuts are added to the set of matching segments.

3.2.6 Segment combination

Segments are combined from the equivalence classes of matching segments. For each class the list of segments is replaced by a single segment with the combined list of hits and the track parameters calculated from the weighted average at the new reference point.

3.2.7 Addition of hits

Hits which could not have been part of a triplet due to missing matching hits are picked up now. For each segment for all rows without a hit single unused hits are searched for in two steps from the center row of the segment to the first and to the last row of the module. This allows the segment to grow in length. Only hits with the row inside the segment or not too far away from the current first or last row are considered. For each such hit the position and azimuthal measurement direction defines a line in the XY-plane. The segment is propagated to the intersection with this line defining the predicted hit position. The distance $\mathbf{d}$ between the measured and predicted position is projected on the measurement directions of the hit similar to (1) yielding ($\Delta u_{hit}, \Delta v_{hit}$). With the propagated variances $\mathbf{V}_{u,hit}$ and $\mathbf{V}_{v,hit}$ the hits have to pass:

$$\frac{\Delta u^2_{hit}}{\sigma^2_{xy} + V_{u,hit}} < \chi^2_{hit} \text{ and } \frac{\Delta v^2_{hit}}{\sigma^2_z + V_{v,hit}} < \chi^2_{hit} \quad (9)$$

Rows with a single matching hit are added to the segment and the hits marked as used. The segment parameters are not updated with the additional hits as this usually would provide only a minor improvement.

3.3 Combination of modules

Segments from individual modules are combined to track candidates similar to 3.2.5 and 3.2.6. Each segment is compared to all segments of neighbouring modules. Track candidates are build from the equivalence classes of matching segments.
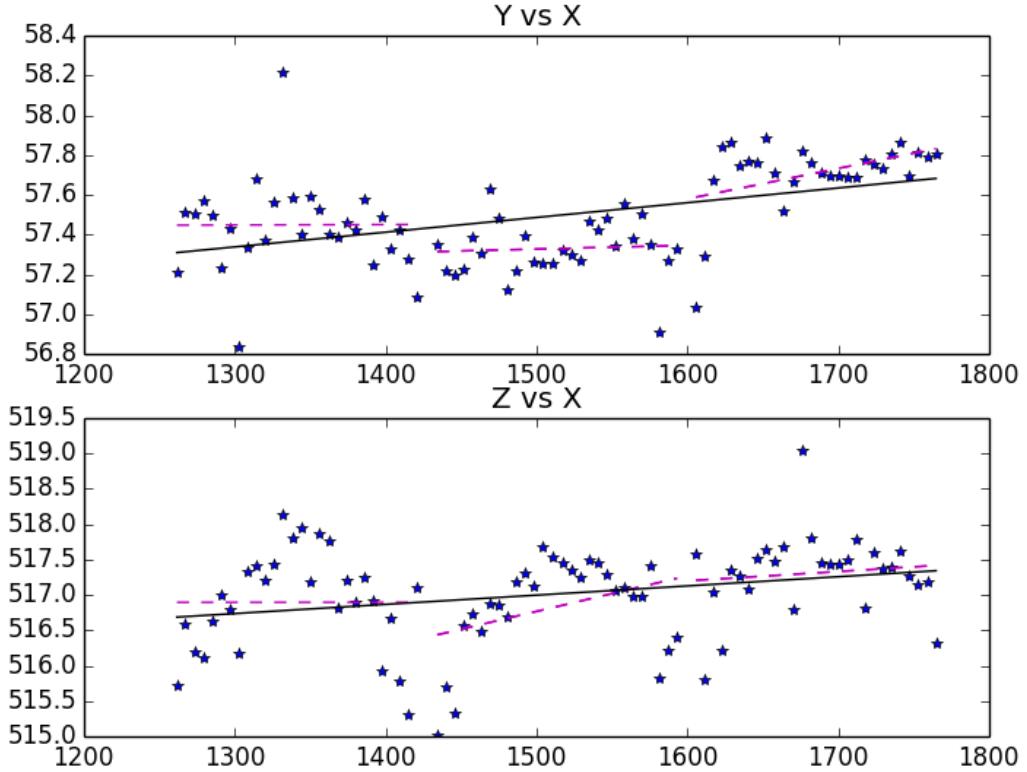
Figure 4: Example track in three DESY GEM modules: hits (stars) used (by a segment), segments (dashed lines) and track candidate (solid line) in XY (top) and XZ (bottom) projections.

3.4 Output

For each track candidate a TPC track is constructed from the parameters and hit list. The track parameters and the covariance matrix are transformed to the convention used by LCIO (different order and signs). As reference point the position of the first hit is used. This ensures a proper value of the magnetic field. Alternatively the point of closest approach (PCA) to the origin in the global frame can be used. For the test beam setup this could be outside of the magnet where a curvature parameter makes no sense anymore.

4 Performance test

The performance of this triplet finder and of the general purpose topological TPC pattern recognition CLUPATRA [5] has been compared briefly for a set of track with three DESY GEM modules in a magnetic field of 1 Tesla (LPTPC run 19103). Figure 5 shows the distributions of the number of hits found per track which are in rough agreement. In detail with this method less very short tracks are reconstructed and for long tracks penetrating all three modules slightly more hits are associated. The track finding (and fitting) is about a factor 15 faster compared to CLUPATRA with default steering parameters.

On the technical side the triplet finder contains in contrast to CLUPATRA a straight line fit for data without magnetic field and can cope with dispaclements of the modules in arbitrary directions to account for misalignment.
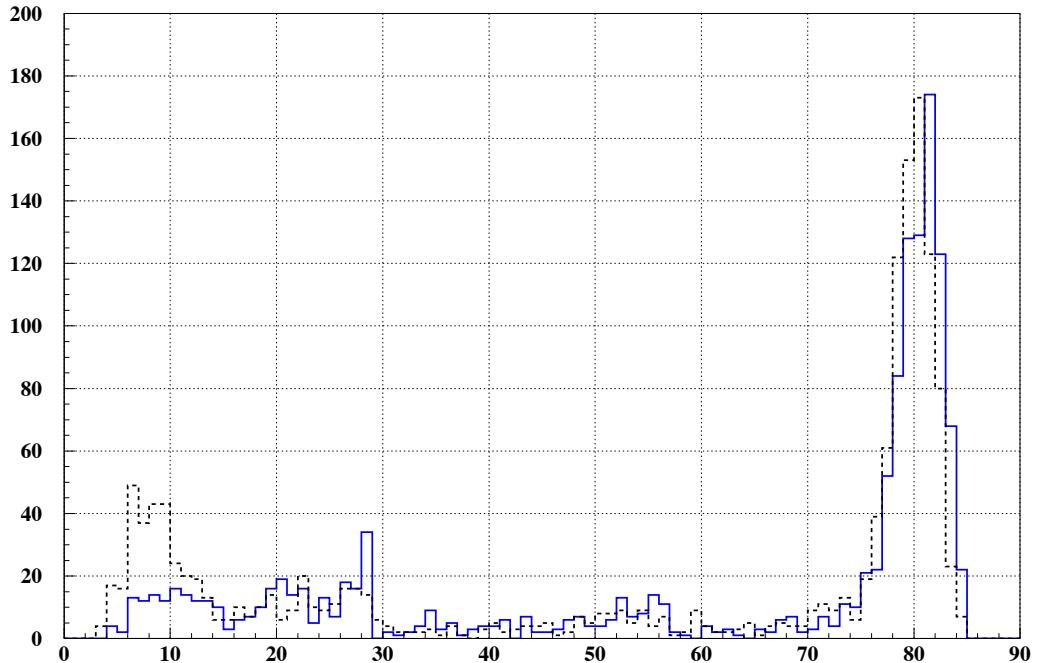
Figure 5: Comparison of number of hits on track found by CLUPATRA (dashed) and this triplet finder (solid).

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5 Steering parameters

The steering parameters with the defaults indicated in parentheses are:

InputHits ("TPCHits"): The name of the input collection of TPC hits .

OutputTracks ("TripletTracks"): The name of the output collection with the found tracks.

BFieldScaleFactor (1.0): Scale factor for the magnetic field (map), allows to switch off the magnetic field (section 3.2.4).

TripletPreSelectionDistCut (10.): Coarse cut on XY and Z residuals for triplet preselection (t_{max} in section 3.2.1).

TripletDefinitionChi2Cut (20.): χ^2 cut for triplet definition on XY and Z residuals (χ^2_{trp} in section 3.2.1).

TripletPositionMatchingChi2Cut (20.): χ^2 cut for triplet position matching in XY and Z (χ^2_{pos} in section 3.2.2).

TripletDirectionMatchingChi2Cut (20.): χ^2 cut for triplet direction matching in XY and Z (χ^2_{dir} in section 3.2.2).

SegmentMatchingChi2Cut (30.): χ^2 cut normalised to number of track parameters (5 for magnetic field on, 4 for off) for segment matching (χ^2_{seg} in section 3.2.5).

UnusedHitMatchingChi2Cut (20.): χ^2 cut for matching unused hits in XY and Z (χ^2_{hit} in section 3.2.7).

UnusedHitMaxGapCut (4): Maximal (row) gap for matching unused hits in XY and Z (section 3.2.7).

EncodedModuleID (true): Flag for encoding of module ID in CellID0 (section 3.1).

ReferencePointAtPca (false) Flag for using the point of closest approach (PCA) as track reference point instead of position of first hit (section 3.4).

6 Summary

A simple and fast local track finding method based on triplet chains in individual modules for curved or straight tracks has been presented. It relies on good single hit efficiency and reasonable hit errors. For the LPTPC test beam data it shows a similar performance as the standard track finder for a complete TPC while being significantly faster. An implementation in MarlinTPC is available as the ROWTRIPLETBASED-TRACKFINDERPROCESSOR.

References

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[2] A. Münnich, Performance of DESY GEM Module in Testbeam Measurements, Talk given at LCWS 2013, Tokyo, November 2013, http://www.icepp.s.u-tokyo.ac.jp/lcws13

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[5] F. Gaede, Pattern recognition in a TPC, Talk given at AIDA WP2 Meeting 2011, CERN, October 2011, http://indico.cern.ch/event/159340