Track fitting and alignment for a TPC with General Broken Lines

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Abstract

This document describes track fitting with the General Broken Lines method [1] and package [2]. The interface to the Millepede-II algorithm [3, 4] allows for an easy implementation of track based alignment.

1 Introduction

A track fitting method based on General Broken Lines (GBL) has been developed especially for the data taken by the Large Prototype TPC (LPTPC) [5] with pad readout at the DESY test beam [6] and implemented as MarlinTPC [7] processors[1].

In this setup the drift direction perpendicular to the readout plane is along the z-coordinate, the tracks roughly move in the x-direction and the pad rows measure y-coordinates.

The simplest description of the path of a charged particle in a constant magnetic field is a helix or in case of zero magnetic field a straight line. The interaction with the detector material leads to energy loss and deflection due to multiple scattering. In gaseous detectors like a TPC these effects are usually small and are neglected.

The General Broken Lines method allows for a proper description of multiple scattering in material layers described by thin scatterers. It is mathematically equivalent to

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[1]The first version for straight line fitting had been written by Ch. Rosemann, DESY

a Kalman filter, but computationally different. Based on an initial *seed* trajectory all hits are fitted simultaneously. This allows the usage as local (track) fit in the Millepede-II (MP2) algorithm for track based alignment and calibration. The GBL implementation [2] can produce directly the input binary files for MP2. This feature can be explored for the (relative) alignment of the several readout modules of the LPTPC.

In section 2 the basic concepts are presented, section 3 describes the sequence of processing steps and section 4 two other Millepede-II examples. Section 5 explains the steering parameters of the SIMPLEHELIXGENERALBROKENLINEINTERFACEPROCESSOR for constant magnetic field and the STEPWISEHELIXGENERALBROKENLINEINTERFACEPROCESSOR for (slowly) varying magnetic field.

2 Concepts

2.1 Millepede-II

The Millepede-II method [3] allows for track based alignment and calibration of tracking detectors by solving a large linear equation system. It is based on the minimisation of the residuals of the hits to the fitted tracks. For a measurement m_{ij} on a track j the prediction from the track model $f_{ij}(\mathbf{p}, \mathbf{q}_j)$ depends on the *local* (track) parameters $\mathbf{q}_j$ of the track and some *global* parameters $\mathbf{p}$ common to a larger set of tracks. The track model is linearised at initial values of the local ($\mathbf{q}_{0j}$) and global ($\mathbf{p}_0$) parameters:

$$f_{ij}(\mathbf{p}, \mathbf{q}_j) \approx f_{ij}(\mathbf{p}_0, \mathbf{q}_{0j}) + \frac{\partial f_{ij}}{\partial \mathbf{p}} \Delta \mathbf{p} + \frac{\partial f_{ij}}{\partial \mathbf{q}_j} \Delta \mathbf{q}_j \tag{1}$$

Assuming independent measurements with errors σ_{ij} an objective function χ^2 depending on corrections to the global and all local parameters can be constructed:

$$\chi^2(\Delta \mathbf{p}, .., \Delta \mathbf{q}_j, ..) = \sum_j^{\text{tracks}} \sum_i^{\text{hits}} \frac{1}{\sigma_{ij}^2} \left(m_{ij} - f_{ij}(\mathbf{p}_0, \mathbf{q}_{0j}) - \frac{\partial f_{ij}}{\partial \mathbf{p}} \Delta \mathbf{p} - \frac{\partial f_{ij}}{\partial \mathbf{q}_j} \Delta \mathbf{q}_j \right)^2 \tag{2}$$

This can be minimised by solving a huge linear equation system $\mathbf{C} \cdot \mathbf{x} = \mathbf{b}$ for all this corrections. With block matrix algebra it can be reduced to a system for the global corrections only:

$$\mathbf{C}' \cdot \Delta \mathbf{p} = \mathbf{b}' \tag{3}$$

The Millepede-II implementation [4] consists of two steps. The first called "mille" has to be integrated in the track fitting procedure and produces special binary files containing for all hits from all tracks the residuals ($r_{ij} = m_{ij} - f_{ij}(\mathbf{p}_0, \mathbf{q}_{0j})$) and their errors ($\sigma_{ij}$) and local ($\frac{\partial f_{ij}}{\partial \mathbf{q}_j}$) and global ($\frac{\partial f_{ij}}{\partial \mathbf{p}}$) zero suppressed derivatives. The non zero derivatives are stored in form of the value and a positive integer identifier ("label"). The

second one called "pede" is a standalone fortran90 program reading a steering file and a set of binary files. From those the linear equation system (3) is constructed and solved according to the steering parameters.

2.2 General Broken Lines

The General Broken Lines method [1] provides a refit for a set of measurements and (thin) scatterers based on an initial trajectory. This seed can be from an earlier fit to the data or an external prediction. A GBL trajectory is constructed from a list of points on the initial trajectory ordered by arc-length. For each point a local coordinate system with local track parameters $\mathbf{p}_l$ is defined. Each point may contain a measurement, a scatterer or both or none of those. For the propagation (of track parameter corrections) along the initial trajectory to a point i from the previous point the jacobian $\frac{\partial \mathbf{p}_{l,i}}{\partial \mathbf{p}_{l,i-1}}$ is needed.

As track fit parameters a common curvature correction and (two-dimensional) offsets in the local coordinate system of the first and last point and all points with a scatterer are defined. For each measurement the deviation from the initial trajectory can be described by the interpolation of the two neighbouring points with offsets. For each scatterer kinks are constructed from the offsets at this and the two neighbouring points with a variance depending on the angular multiple scattering error at the center point. With the propagation jacobians the derivatives of kinks and measurements with respect to the fit parameters can be calculated allowing to construct a linear equation system $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$ describing the track fit. The corresponding matrix $\mathbf{A}$ has a bordered band structure. As measurements or kinks only correlate up to three consecutive (two-dimensional) offsets the part of the matrix without the curvature correction is a band matrix with a (maximal) width of five. This part of the linear equation system can be solved by root-free Cholesky decomposition with an effort linear in the number of parameters.

In the General Broken Lines implementation [2] a measurement $\mathbf{m}$ is an arbitrary linear function of the local track parameters:

$$\mathbf{m} = \mathbf{H} \cdot \mathbf{p}_l \tag{4}$$

It has a prediction $\mathbf{f}$ and a covariance matrix $\mathbf{V}_m$. This matrix will be diagonalised internally to produce independent (scalar) measurements m_{ij} with errors σ_{ij} for usage by Millepede-II. Accordingly additional (virtual) measurements are constructed for the (two-dimensional) kinks describing the multiple scattering. The local derivatives $(\frac{\partial \mathbf{f}}{\partial \mathbf{q}_j})$ are already calculated for the construction of the linear equation system of the track fit. Therefore the user only has to supply the labels and values of the global derivatives $(\frac{\partial \mathbf{f}}{\partial \mathbf{p}})$ to produce binary files for MP2 from GBL trajectories.

M-estimators can be applied to down-weight outliers. The number and type of the corresponding iterations of the fit are specified by steering parameters.

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2.3 Propagation in a magnetic field

The propagation along the seeding trajectory is required to calculate the track parameter transformation matrices ("jacobians") which are input to the construction of the linear equation system of the track fit. In a constant magnetic field the track is described by a helix and the jacobians can be calculated analytically as illustrated in [8] (equations (18) to (40)) for the curvilinear track parametrisation. For a (slowly) varying magnetic field an approximate jacobian can be calculated by dividing the propagation step in several smaller steps and using for each step the average value of the magnetic field. The jacobian is in this case the product of the analytical jacobians of all the smaller steps with constant field.

2.4 Variance of residuals

A proper covariance matrix for the measured distance of a hit to a track ("residual") is an important ingredient to the track fit. It is based on the covariance matrix $\mathbf{V}_{\text{hit}}$ of the position measured by the hit. For a TPC two coordinates are measured in the readout plane and the third by the drift time. For a pad based readout with the longer pad side parallel to the x-direction $\mathbf{V}_{\text{hit}}$ can be described by:

$$\mathbf{V}_{\text{hit}} = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_a^2 & 0 \\ 0 & 0 & \sigma_l^2 \end{pmatrix} \tag{5}$$

Here σ_r is the radial error defined by the pad size (and transversal diffusion), σ_a and σ_l are the azimuthal and longitudinal *measurement* errors as result of the hit finding procedure.

For the track fit the component of the residual perpendicular to the track is important, the parallel component corresponds to the movement to another point on the track. With the track direction $\mathbf{t}$ represented by azimuth Φ and dip angle λ the curvilinear system is defined by two directions $\mathbf{u}$ and $\mathbf{v}$ perpendicular to the track ([8] section 2):

$$\mathbf{t} = \begin{pmatrix} \cos\Phi\cos\lambda \\ \sin\Phi\cos\lambda \\ \sin\lambda \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\sin\Phi \\ \cos\Phi \\ 0 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} -\cos\Phi\sin\lambda \\ -\sin\Phi\sin\lambda \\ \cos\lambda \end{pmatrix}, \tag{6}$$

The transform of the covariance matrix of the hit position in the uv-plane is:

$$\mathbf{V}_{\text{uv}} = \begin{pmatrix} \mathbf{u} & \mathbf{v} \end{pmatrix}^t \cdot \mathbf{V}_{\text{hit}} \cdot \begin{pmatrix} \mathbf{u} & \mathbf{v} \end{pmatrix}$$
$$= \begin{pmatrix} \sin^2\Phi\,\sigma_r^2 + \cos^2\Phi\,\sigma_a^2 & \sin\lambda\sin\Phi\cos\Phi(\sigma_r^2 - \sigma_a^2) \\ \sin\lambda\sin\Phi\cos\Phi(\sigma_r^2 - \sigma_a^2) & \sin^2\lambda(\cos^2\Phi\,\sigma_r^2 + \sin^2\Phi\,\sigma_a^2) + \cos^2\lambda\,\sigma_l^2 \end{pmatrix} \tag{7}$$

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The projection matrix into the pad row (at $\varphi_p = 0$) and drift distance measurement system is given by ([8] section 3):

$$\mathbf{P}_u = \begin{pmatrix} \dfrac{1}{\cos\Phi} & \tan\lambda\tan\Phi \\ 0 & \dfrac{1}{\cos\lambda} \end{pmatrix} \tag{8}$$

Therefore the variance of the distance perpendicular to the track in the measurement system is:

$$\mathbf{V}_{m\perp} = \mathbf{P}_u \cdot \mathbf{V}_{uv} \cdot \mathbf{P}_u^t = \begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_l^2 \end{pmatrix} + \sigma_r^2 \begin{pmatrix} \tan^2\Phi & \dfrac{\tan\lambda\tan\Phi}{\cos\Phi} \\ \dfrac{\tan\lambda\tan\Phi}{\cos\Phi} & \dfrac{\tan^2\lambda}{\cos^2\Phi} \end{pmatrix} \tag{9}$$

The radial error σ_r adds a dependence on the track direction and a correlation to the measurement errors. In the general case of a non zero pad direction φ_p in equation (9) Φ has to be replaced by $(\Phi - \varphi_p)$.

2.5 Rigid body alignment

The alignment of a rigid body is described by three displacements and three rotations. For the alignment with Millepede-II the (global) derivatives of the measurement prediction $\mathbf{f}$ with respect to this six parameters have to be calculated.

The following assumes that the detector measures a position in a plane with normal vector $\mathbf{n}$. A prediction $\mathbf{f}_0 = (x_p, y_p, z_p)$ changes due to the effect of displacements $(\Delta x, \Delta y, \Delta z)$ along and small rotations $(\Delta\alpha, \Delta\beta, \Delta\gamma)$ around the axes of the coordinate system to:

$$\tilde{\mathbf{f}} = \mathbf{f}_0 + \Delta\mathbf{f}, \ \ \Delta\mathbf{f} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + \Delta\alpha \begin{pmatrix} 0 \\ -z_p \\ y_p \end{pmatrix} + \Delta\beta \begin{pmatrix} z_p \\ 0 \\ -x_p \end{pmatrix} + \Delta\gamma \begin{pmatrix} -y_p \\ x_p \\ 0 \end{pmatrix} \tag{10}$$

Of interest is the prediction $\mathbf{f}_{int}$ at the intersection of the track with the measurement plane (defined by $\mathbf{f}_0$ and $\mathbf{n}$). Using the track direction $\mathbf{t}$ to linearise the track model $\mathbf{f}(\Delta s) = \tilde{\mathbf{f}} + \mathbf{t} \cdot \Delta s$ the arc-length correction Δs is defined by:

$$0 = (\mathbf{f}(\Delta s) - \mathbf{f}_0) \cdot \mathbf{n} = (\tilde{\mathbf{f}} - \mathbf{f}_0) \cdot \mathbf{n} + \mathbf{t} \cdot \mathbf{n} \cdot \Delta s \ \Rightarrow \ \Delta s = -\frac{\Delta\mathbf{f} \cdot \mathbf{n}}{\mathbf{t} \cdot \mathbf{n}} \tag{11}$$

Resulting in:

$$\mathbf{f}_{int} = \mathbf{f}_0 + \Delta\mathbf{f} - \mathbf{t} \cdot \frac{\Delta\mathbf{f} \cdot \mathbf{n}}{\mathbf{t} \cdot \mathbf{n}} \tag{12}$$

$$\frac{\partial \mathbf{f}_{\text{int}}}{\partial \Delta \mathbf{f}} = \mathbf{1} - \frac{\mathbf{t} \cdot \mathbf{n}^t}{\mathbf{t} \cdot \mathbf{n}} = \left(\delta_{ij} - \frac{t_i \cdot n_j}{\mathbf{t} \cdot \mathbf{n}} \right) \tag{13}$$

The global derivatives $\frac{\partial \mathbf{f}_{\text{int}}}{\partial \mathbf{p}}$ can be calculated with the chain rule:

$$\frac{\partial \mathbf{f}_{\text{int}}}{\partial \mathbf{p}} = \frac{\partial \mathbf{f}_{\text{int}}}{\partial \Delta \mathbf{f}} \frac{\partial \Delta \mathbf{f}}{\partial \mathbf{p}} \tag{14}$$

$$\frac{\partial \Delta \mathbf{f}}{\partial \mathbf{p}} = \begin{pmatrix} 1 & 0 & 0 & 0 & z_p & -y_p \\ 0 & 1 & 0 & -z_p & 0 & x_p \\ 0 & 0 & 1 & y_p & -x_p & 0 \end{pmatrix} \tag{15}$$

$$\mathbf{p} = \begin{pmatrix} \Delta x & \Delta y & \Delta z & \Delta \alpha & \Delta \beta & \Delta \gamma \end{pmatrix}^t \tag{16}$$

3 Sequence

3.1 Input

Input for a track are the associated TPC hits with positions, errors and measurement directions and an initial set of track parameters at a reference point e.g. from the track finding. A constant magnetic field (from the LPTPC center) is applied to describe the track by a simple helix (or straight line) or the complete map of the magnetic field is used for a step-wise helix.

3.2 GblPoints

For each track the hits are ordered by the arc-length on the initial trajectory. For each hit a "GblPoint" is created based on the propagation jacobian from the previous hit (section 2.3). As local system and track parametrisation the curvilinear system and parameters are used ([8] section 2). As two-dimensional measurement the distance of the hit position to the initial trajectory in the plane defined by the (pad row and drift distance) measurement directions are used. The measurement, its (inverse) covariance matrix (section 2.4) and the transformation matrix (8) from the local to the measurement system (as $\mathbf{H}$ in (4)) are added to the GblPoint. For the rigid body alignment of the readout modules the global derivatives based on the measurement plane and the track direction (section 2.5) have to be supplied too (by helper class TRACKERALCACALCULATOR). There are six alignment parameters per module. They are labeled with the module number times ten plus the parameter number (from one to six). As multiple scattering inside the TPC is being ignored no scatterers are added to the GblPoints.

3.3 GblTrajectory

A "GblTrajectory" is created for each input track from the (ordered) list of GblPoints. In case the reference point is not a hit an additional GblPoint without measurement at the point of closest approach is added at the proper arc-length as only from GblPoints corrections for the (local) track parameters can be obtained. The GblTrajectory is fitted according to the steering parameters for outlier down-weighting (section 2.2). Without scatterers the fit parameters are two offsets at the first and two offsets at the last hit and a curvature correction (unless the magnetic field is zero).

3.4 Output

For each input track a new TPC track is created from the fitted parameters of the GblTrajectory and the list of hits. At the reference point the parameters and covariance matrix are converted to the LCIO parametrisation. Optionally the trajectory including the global derivatives are written to a Millepede-II binary file.

3.5 Pede

After all tracks from all events have been fitted the Millepede-II binary file is ready for use. With the "pede" executable the alignment parameters can be determined from this or a larger set of binary files. The following describes an alignment example for data taken in the LPTPC with three DESY GEM modules without magnetic field (run 19084).

The modules used for this run have the numbers 0, 3 and 5. Therefore the binary file contains derivatives for the global parameters with labels 1–6, 31–36 and 51–56. As no external reference is available the center module is used as internal reference (to determine the relative alignment of the outer modules). This can be realised by fixing the parameters 31–36 to zero. This starting conditions, the name of the binary files and options for the solution of the corresponding linear equation system by pede have to be specified in a text steering file [4]:

```
Cfiles ! C-type binary files
milleBinaryAlign2.dat ! alignment with pad row and drift measurement

Parameter ! Starting conditions
    4 0. -1.  ! fix rotation around x
    5 0. -1   ! fix rotation around y
! fix all parameters of center module
   31 0. -1.  ! offset in x
   32 0. -1.  ! offset in y
   33 0. -1.  ! offset in z
   34 0. -1.  ! rotation around x
```

```
   35 0. -1.  ! rotation around y
   36 0. -1.  ! rotation around z
!
   54 0. -1.  ! fix rotation around x
   55 0. -1.  ! fix rotation around y

outlierdownweighting 3  ! 3 iterations for outlier down-weighting
method inversion  3  0.10  ! solution by inversion

end
```

In addition the rotations around the x- and y-axis have been fixed as they are sensitive to systematic distortions in the drift distance measurement. From the original 18 parameters only 8 remain free. With about 1200 tracks from thousand events the results for those are:

```
!   label   final value   initial error    correction and  error from MP2
        1      0.85422        0.0000          0.85422        0.14696
        2      5.0177         0.0000          5.0177         0.11682
        3      0.34625        0.0000          0.34625        0.74496E-02
        4      0.0000        -1.0000
        5      0.0000        -1.0000
        6     -0.30487E-02    0.0000         -0.30487E-02    0.72996E-04
       31      0.0000        -1.0000
       32      0.0000        -1.0000
       33      0.0000        -1.0000
       34      0.0000        -1.0000
       35      0.0000        -1.0000
       36      0.0000        -1.0000
       51     -0.80873        0.0000         -0.80873        0.16429
       52      0.80207        0.0000          0.80207        0.10402
       53      0.12050        0.0000          0.12050        0.74539E-02
       54      0.0000        -1.0000
       55      0.0000        -1.0000
       56     -0.43124E-03    0.0000         -0.43124E-03    0.72607E-04
```

The displacements in the xy-plane are in the order of millimetres and the rotations around the z-axis of milliradians with errors ten times smaller. As the offsets are defined at the center of the coordinate system more than a meter away from the modules there is some significant correlation with the rotations due this large lever arm.

4   Other Millepede examples

A simple MP2 application is to determine offsets for the measurements as function of some variable. In this case the global derivatives are equal to one. The available range of the variable can be divided in a reasonable number of bins and for each bin with suffiicient content an offset is fitted with Millepede-II.

4.1 Offsets per row

The same data (run 19084) as for the alignment example has been used to determine offsets from the track fit residuals in the pad row and drift measurement directions for each of the 28 rows of each of the three modules. As there is no external reference for the tracks and without magnetic field a track has four degrees of freedom the offsets for two rows (14, 70) in both measurement directions have been fixed to zero. For data with magnetic field a third row has to be fixed in the bending plane.

The remaining 160 free parameters have been fitted by Millepede-II from thousand events. For the pad row measurement direction the offsets are in the order of few 100 μm with errors of about 25 μm and for the drift direction of few mm with errors of about 50 μm.
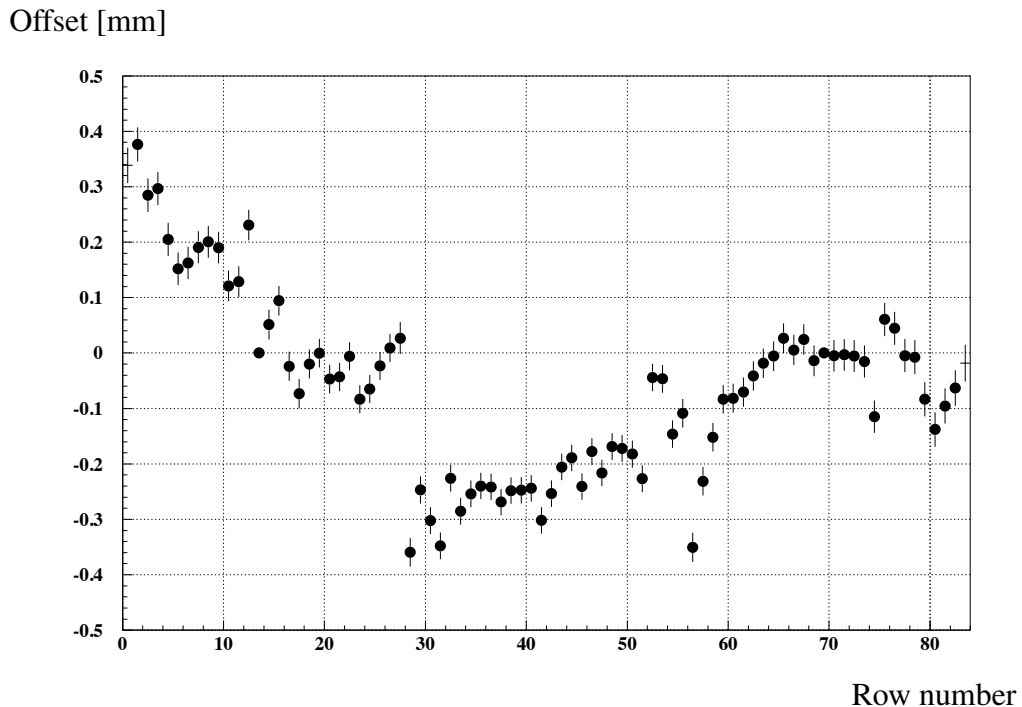
Offset [mm]

Row number

Figure 1: Offset in pad row measurement direction per row for LPTPC run 19084 with three DESY GEM modules of 28 rows each. Rows 14 and 70 are fixed as reference. The first module shows a significantly different slope in agreement with the rotation around the z-axis of about 3 mrad as determined by the alignment.

Another popular method is to the determine the row offsets from the average distance of a row from the fit with all other rows. This is an ill-defined estimator as the
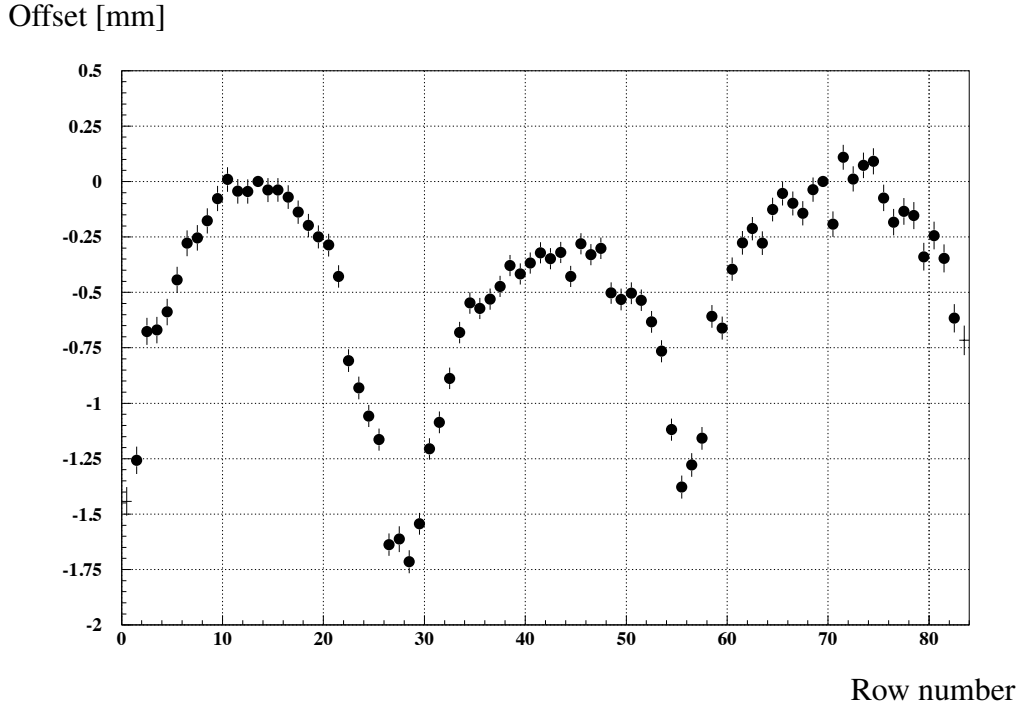
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Figure 2: Offset in drift direction per row for LPTPC run 19084 with three DESY GEM modules of 28 rows each. Rows 14 and 70 are fixed as reference. All modules show a parabolic shape with an amplitude of about 1.5 mm.

offsets from the other rows are contributing due to the track fit to the offset for the row under study. Therefore only a mixture of the true offsets is determined with the (singular) mixing matrix depending on the set of hits used. In the Millepede-II ansatz the undefined degrees of freedom are fixed first. For the remaining problem this defines external references preventing mixing. For the row offsets this corresponds to calculate the prediction from the track only from the fixes rows.

4.2 Charge dependent offsets

The offsets in the drift distance measurement as function of the charge of the hits (Q) have been studied with Millepede-II for two runs with three DESY GEM modules close to the anode and close to the cathode. The hit finder used takes the drift time from the largest pulse in a hit. The hits have typically a charge between ten and thousand counts. For this range $\lg(Q)$ is divided in 40 bins and for each bin a separate offset is fitted. As reference the offset for the bin with charges of about hundred counts is fixed to zero.

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Additionally different sets of offsets are determined for hits with or without pulses in the overflow of the amplitude measurement.

Offsets up to 3 mm have been observed with a weak slope for small charges probably due to threshold effects and a stronger slope for large charges due to overflows. The onset of the overflow effects is different for different drift distances, but the shapes are similar.
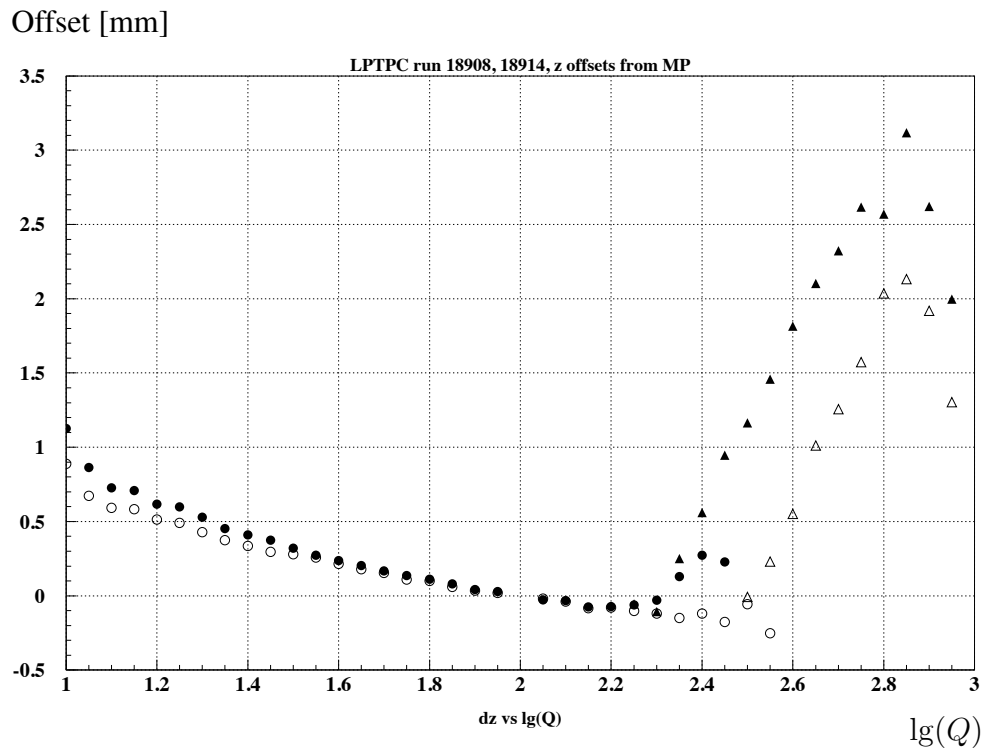
Offset [mm]

Figure 3: Offset in drift direction as function of the hit charge Q for LPTPC run 18914 close to the anode (solid symbols) and run 18908 close to the cathode (open symbols) with three DESY modules. The circles indicate hits without and the triangles with pulses in the overflow of the amplitude measurement.

5 Steering parameters

The steering parameters with the defaults indicated in parentheses are:

InputSeedTracks (”TPCSeedTracks”): The name of the input collection of track candidates.

OutputTracks (”TPCTracks”): The name of the output collection with the fitted tracks.

WriteOutputToStorage (true): Flag for writing the output to storage.

BFieldScaleFactor (1.0): Scale factor for the magnetic field (map), allows to switch off the magnetic field (section 2.3).

MaxStep (10.0): Maximal step size (with constant magnetic field) (section 2.3).

FitOptions (””): Defines multiple passes with ”h” for huber, ”t” for tukey or ”c” for cauchy function for outlier down-weighting (section 3.3).

WriteMillepedeBinary (false): Flag for writing of trajectories to MP2 binary file (section 3.2).

MillepedeCalcMethod (0): Selector for parameter sets implemented in TRACKERALCACALCULATOR (0: module alignment, 1: row offsets).

MillepedeFilename (””): Name of the MP2 binary file (section 3.4).

EncodedModuleID (true): Flag for encoding of module ID in CellID0 (section 3.1).

6 Summary

A simple and fast track refitting implementation based on General Broken Lines for curved or straight tracks has been presented. The interface to Millepede-II allows for easy determination of systematic effects in the position measurement like misalignment. An implementation in MarlinTPC is available as the SIMPLEHELIXGENERALBROKENLINEINTERFACEPROCESSOR for constant magnetic field and the STEPWISEHELIXGENERALBROKENLINEINTERFACEPROCESSOR for (slowly) varying magnetic field.

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