LC-DET-2005-003
May 11, 2005

Diagnostics of Colliding Bunches from Pair Production and Beam-Strahlung at the IP

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Abstract

The bunches of a future linear collider will be focused at the interaction point to very small dimensions. They are accompanied by huge electric fields. When they collide large amounts of photons and e^+e^--pairs are radiated – the so-called beam-strahlung. The emission is sensitive to the charge distribution of the colliding bunches. In this note it is investigated how the beam-strahlung can be used to extract information about the colliding bunches.

This note is a status report of work done in the frame of the DESY PRC-project: R&D for the TESLA Detector: Instrumentation of the very forward region [1]. The results presented are not considered final[2].

1 Introduction

The electron and positron bunches at a future linear collider are focused at the IP to very small dimensions. The nominal bunch sizes for TESLA are $553\,\mathrm{nm} \times 5\,\mathrm{nm} \times 300\,\mu\mathrm{m}$ in horizontal (x), vertical (y), and longitudinal (z or s) direction. With a charge of $2 \cdot 10^{10}$ e huge electric fields accompany the bunches. When two opposing bunches approach each other at the IP, strong forces are exerted pulling the particles towards the center of the opposing bunch. This so-called pinch effect increases the luminosity by roughly a factor of two. Along with the exertion of forces comes acceleration which stimulates the particles to radiate. This is the origin of the beam-strahlung. As a secondary effect, e^+e^- pairs are produced from the photons of the beam-strahlung. Beam-strahlung as well as pair production depends on the electric fields of the bunches and therefore on their size and shape. Hence by measuring the beam-strahlung and/or pair production we can get information on the geometry of the colliding bunches. This is the basic idea of this note. In the forward calorimeters (BeamCal, see ref. [3]) the energy and distribution of the pairs created are measured and from this measurement the parameters of the colliding bunches are deduced.

The studies presented here are based on the TESLA parameters with zero crossing angle. The parameters of the now considered ILC will not differ much from TESLA except for the crossing angle. It will make a signigicant difference, if it is larger than a few mrad.

[1]This note includes contributions from several DESY summer students: Gunnar Klämke (2001), Marko Ternick (2002), Magdalena Luz (2003), Regina Kwee (2003).

[2]For personal reasons I am leaving the Linear Collider development and cannot continue the work on this project.

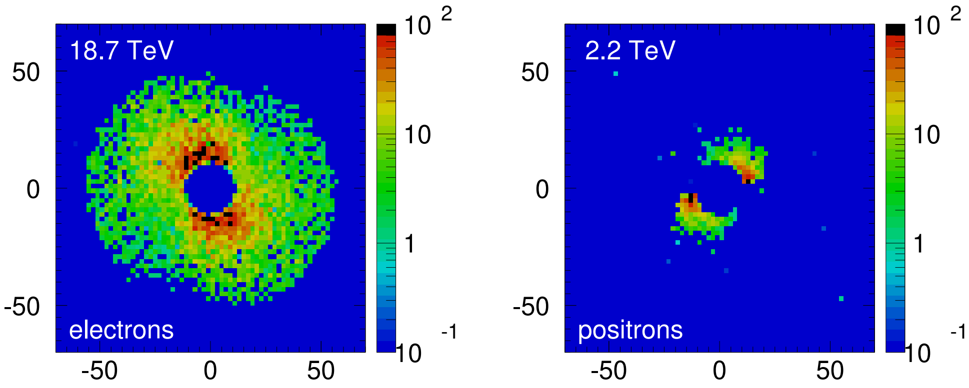

Figure 1: Energy deposition on the BeamCal (GeV/mm^2) in the direction of the electron beam (forward) for nominal beam conditions; 500 GeV center-of-mass energy; 1 bunch crossing. Depositions from electrons and positrons are shown separately. No photons reach BeamCal. Positions are in mm with respect to the beam. The total energy reaching the calorimeter is indicated on the plot.

2 Beam-Strahlung and Pair Production

In a TESLA collision with nominal conditions (500 GeV center-of-mass energy) each particle in a bunch radiates in average 3 to 4 % of its energy adding up to 10^8 TeV per bunch and side. The majority of this energy is emitted by photon radiation. Only a small fraction of the energy is converted into e^+e^- pairs. The energy carried away by the pairs is typically 10^2 TeV per side. The photons are emitted at angles of less than 1 mrad with respect to the beam direction. They exit the detector through the beam pipe. They are absorbed on collimators downstream of the IP. In the TDR design the two main collimators are 130 and 240 meters downstream [2]. In order to detect the photons, these collimators need to be instrumented. For the propagation of the pairs the magnetic field and the forces exerted by the bunch have to be taken into account. In the direction of the electron beam the positrons are attracted by the bunch. Most of the positrons exit the detector through the beam pipe into the final focus magnets. They are over-focused in the magnet due to their low momentum and hit the wall of the beampipe in this region. The electrons are repelled by the force of the bunch. They are deflected to larger radii and a substantial fraction reaches BeamCal. Figure 1 shows the energy distribution on the forward BeamCal front face[3] for one bunch crossing at nominal conditions at 500 GeV. The energy distribution of the photons can be found in fig. 16. More illustrative plots can be found in the appendix. The layout of the forward region is also sketched there in fig. 18 [3].

3 Analysis Model

3.1 General Concept

The colliding bunches are described by beam parameters (see below for a list). Under certain assumptions, these beam parameters are sufficient to define the charge distribution of the colliding bunches as well as their movement. Here I will assume that the bunches are ellipsoidal (???) with Gaussian charge distributions across. Under these assumptions, the beam parameters define uniquely the conditions of the collision. They fix the average luminosity, beam-strahlung and pair production in all their details. From collision to collision statistical fluctuations around these averages will occur. In the real world the laws of nature define the relation between the beam parameters and the beam-strahlung. Here the Monte Carlo program guinea-pig will be used to simulate the collisions [5], including the creations of beam-strahlung and pair production and their interaction with the bunch. Alternatively one could use Caine [6]. A simple analytic calculation

[3]The forward direction is defined as the direction of the electron beam. The forward BeamCal is located at a position $z > 0$.

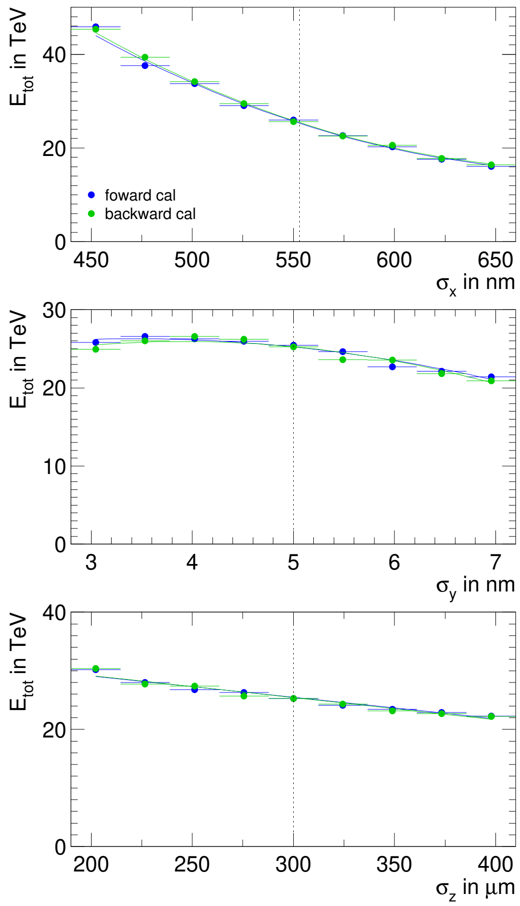

Figure 2: Total energy seen by the forward and backward BeamCal as a function of horizontal (σ_x), vertical (σ_y), and longitudinal (σ_z) bunch sizes. Electron and positron bunch are assumed to be identical (500 GeV center-of-mass energy). The nominal values are 553 nm, 5 nm and 300 μm, respectively.

is used to transport the particles from the interaction region to the front face of BeamCal or a photon collimator. The tracking takes into account the magnetic field of the detector solenoid. Finally energy distributions at the detector are calculated.

The information on the beam parameters of the collisions are encoded in the energy distribution at the detector. To simplify the decoding a set of observables is defined that characterizes the energy distribution. Only the values of these observables enter the analysis, but not the full multidimensional energy distribution. The observables are explained in the following sections. They are thought to contain most of the information about the energy distribution. Any improvements on the observables will directly improve the results given below.

The dependence of the observables on the beam parameters is too complicated to write it down in a single mathematical formula. Instead a Taylor expansion around a default set of beam parameters will be used. The default set will be the nominal beam parameters of the TESLA collider at a center-of-mass energy of 500 GeV [4], unless specified differently. The coefficients of the Taylor series are determined from simulations of collisions with beam parameters varied in the vicinity of their default values. The technical aspects of the determination are described in sec. 4.

The question is then, how the relation from the beam parameters to the observables can be inverted to extract beam parameters from measured values of observables. Two methods are used:

3.1.1 Moore-Penrose Inversion

For this first method the Taylor series is truncated after the first order resulting in a linear relation:

$$(Obs)_{\text{meas}} = \left(\begin{array}{c} \text{1.st order} \\ \\ \text{Taylor} \\ \\ \text{coefficients} \end{array} \right)_{BP=\text{nom}} \cdot \; [(BP)_{\text{act}} - (BP)_{\text{nom}}] \; + \; (Obs)_{\text{nom}} \tag{1}$$

(Obs) is a vector containing the measured values of all observables for forward and backward BeamCal. The index *meas* indicates the values measured for the collision currently under investigation, while the index *nom* labels the values expected to be measured under nominal beam conditions. Correspondingly (BP) is a vector of the beam parameters describing the *actual* and *nominal* electron and positron bunches.

In a situation where the number of observables matches the number of beam parameters the relation can directly be inverted. But in most practical cases the number of observables is larger than the number of beam parameters. The system is over-constrained. The Moore Penrose inverse is a way of inverting the resulting non-quadratic Taylor matrix M. One can show [7] that one of the following two matrices

$$A_1 = \left(M^T \cdot M\right)^{-1} \cdot M^T \qquad \text{or} \qquad A_2 = M^T \cdot \left(M \cdot M^T\right)^{-1} \tag{2}$$

is the desired inverse. More precisely, if inserted into the relation

$$(BP)_{\text{meas}} = \left(\begin{array}{c} \text{Moore} \\ \\ \text{Penrose} \\ \\ \text{inverse} \end{array} \right) \cdot \; [(Obs)_{\text{meas}} - (Obs)_{\text{nom}}] \; + \; (BP)_{\text{nom}} \tag{3}$$

a set of beam parameters is extracted from the data. These beam parameters corresponds to an energy distribution of the beam-strahlung from which observables can be calculated (predicted). These observables will not exactly correspond to the measured observables, but they will have the minimal deviation from the measured observables for any possible set of beam parameters. The deviation is measured as the quadratic sum of the differences between predicted and measured value of all observables:

$$\chi^2 = \sum_i \left(Obs_{i,\text{meas}} - Obs_{i,\text{pred}}\right)^2 . \tag{4}$$

This deviation initially contains a sum over different physical observables expressed in different units. For example one observable, the total energy, might be measured in GeV, while a different observable, let's take the first radial moment is measured in *mm*. To apply the method the different physical units of the different observables have to be matched. All observables will be expressed in units of their standard deviations. A standard deviation is calculated for each observable from a number of statistically independent simulations of collisions with nominal beam parameters. From the scatter of the values the standard deviation is calculated for each observable. For eqn. 3 the deviation of the observables from their nominal values is then normalized to these standard deviations and the Moore-Penrose Inverse is transformed correspondingly.

$$\chi^2 = \sum_i \left(\frac{Obs_{i,\text{meas}}}{\sigma\left(Obs_i\right)} - \frac{Obs_{i,\text{pred}}}{\sigma\left(Obs_i\right)} \right)^2 \tag{5}$$

This method is well suited for an online feedback system. Prior to using the feedback system the time consuming simulations have to be done offline. From the simulations the Taylor matrix

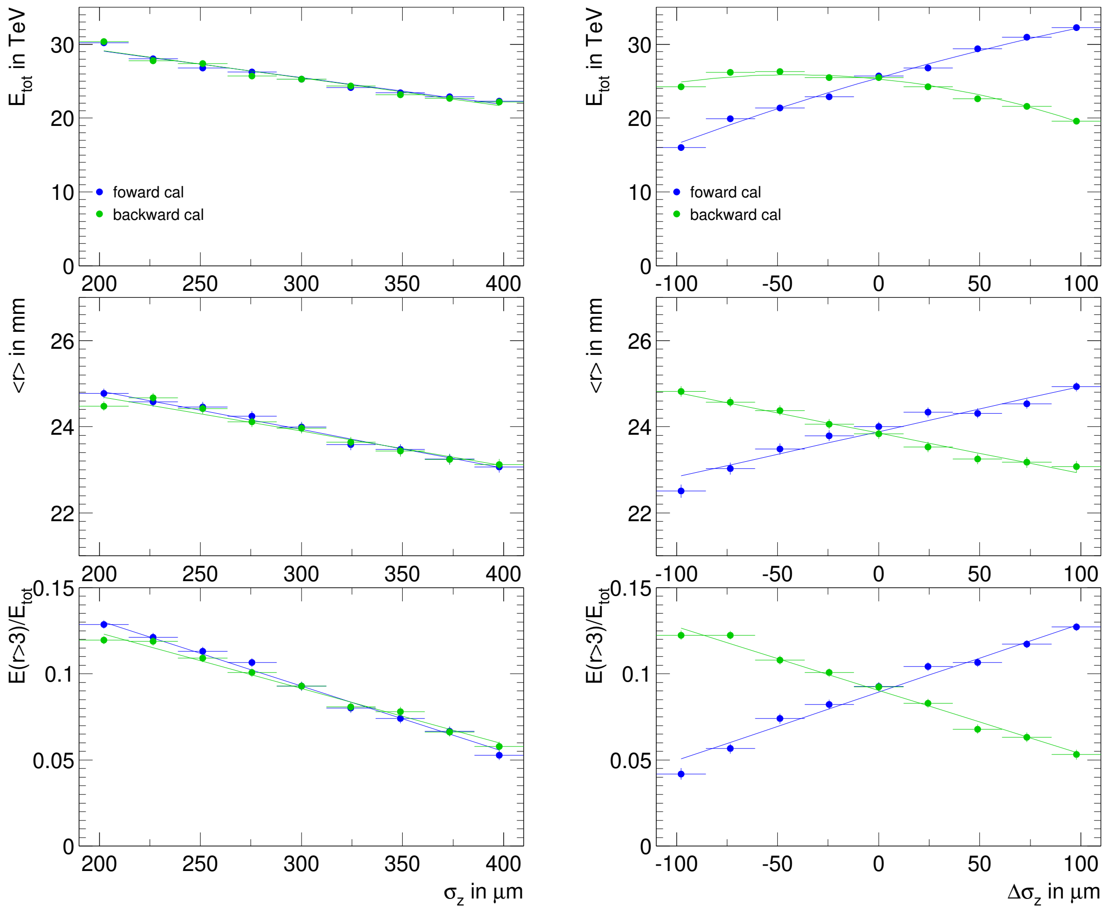

Figure 3: Variation of 3 observables with the longitudinal bunch length σ_z. The observables are explained in section 3.3. The left column shows the situation where the bunch length is modified for both beams in the same direction. For the right column the bunch length is increased and that of the positron beam is decreased for positive $\Delta\sigma_z$. The nominal value is $\sigma_z = 300\ \mu$m for both beams.

has to be determined and the Moore-Penrose inverse is calculated. It depends only on the nominal set of beam parameters, but not on the actual parameters of the collision. The nominal set of beam parameters should be as close to the real beam parameters as one can guess up front. Online the feedback system has to calculate the observables for each collision and multiply them with the precalculated Moore-Penrose inverse. This will give the deviation of the beam parameters from the nominal values for each collision. To increase the statistical precision one can average over several collisions, if one wishes so.

3.1.2 χ^2-Fit

The second method utilizes a χ^2-fit. The dependence of the observables on the beam parameters is parameterized by polynomials of up to fifth order. See fig. 3 (the parameterization is described in sec. 4). The polynomial P_{ij} describes the change of observable i introduced by the deviation of beam parameter j from its nominal value. For a set of beam parameters a prediction is generated for each observable by adding up the deviations due to all beam parameters:

$$Obs_{i,\mathrm{pred}} = Obs_{i,\mathrm{nom}} + \sum_j P_{ij}\left(BP_{j,\mathrm{act}} - BP_{j,\mathrm{nom}}\right) \tag{6}$$

where P_{ij} is the polynomial mentioned above. The prediction is compared to the measured values of the observables and a χ^2 deviation is calculated with the standard deviation of the observables as in eqn. 5. The beam parameters are varied to minimize the χ^2.

If polynomials of first order are sufficient to parameterize all dependencies, this method reproduces exactly the result of the Moore-Penrose inversion. In many cases, however, higher order polynomials are necessary and this method therefore goes beyond the Moore-Penrose inversion, but it is not a full treatment of the 5.th order Taylor expansion. The cross terms (correlations) between different beam parameters are not taken into account. The reason for this simplification is a practical one. Each cross term would need a simulation of a series of points with both relevant beam parameters away from their default values. But with 22 beam parameters there are $22 \times 21 = 462$ cross terms and each point in a series takes about half an hour of CPU time and 30 Mbyte of disk space for the guinea-pig output files. A quantitative estimate of the impact of the correlations has not been done.

The disadvantage of the fit-method over the Moore-Penrose method is the increase in CPU time for the analysis of each bunch collision. It is doubtful that a fit can be performed online in a feedback system for each bunch collision.

3.2 Beam Parameters

The set of beam parameters necessary to fully describe the collisions is not unique. Here the same parameters are used as in guinea-pig. For each parameter there are two values describing the electron and the positron beam, respectively. They are not used directly. Instead the average of the parameter between the value for the electron and the positron beam is used and the difference between the value for the electron beam minus the value for the positron beam. The latter ones are labeled with a Δ. The 22 beam parameters are:

- $\boldsymbol{\sigma_{\mathrm{x}}}$ and $\boldsymbol{\Delta\sigma_{\mathrm{x}}}$
  The horizontal width of the bunches. Nominal value is 553 nm.

- $\boldsymbol{\sigma_{\mathrm{y}}}$ and $\boldsymbol{\Delta\sigma_{\mathrm{y}}}$
  The vertical height of the bunches. Nominal value is 5 nm.

- $\boldsymbol{\sigma_{\mathrm{z}}}$ and $\boldsymbol{\Delta\sigma_{\mathrm{z}}}$
  The longitudinal bunch length. Nominal value is 300 μm.

- $\boldsymbol{\varepsilon_{\mathrm{x}}}$ and $\boldsymbol{\Delta\varepsilon_{\mathrm{x}}}$
  The normalized horizontal emittance. Nominal value is $10 \cdot 10^{-6}$ mm mrad.

- $\boldsymbol{\varepsilon_{\mathrm{x}}}$ and $\boldsymbol{\Delta\varepsilon_{\mathrm{x}}}$
  The normalized vertical emittance. Nominal value is $0.03 \cdot 10^{-6}$ mm mrad.

- $\boldsymbol{d_{\mathrm{x}}}$
  The offset of the center of the electron from the center of the positron bunch in the horizontal plane at the interaction point. Nominally zero. A common shift of both bunches in the horizontal plane is not taken into account. It has no measurable impact on the energy depositions in BeamCal.

- $\boldsymbol{d_{\mathrm{y}}}$
  The offset of the center of the electron from the center of the positron bunch in the vertical plane at the interaction point. Nominally zero. A common shift of both bunches in the horizontal plane is not taken into account. It has no measurable impact on the energy depositions in BeamCal.

- $\boldsymbol{w_{\mathrm{x}}}$
  The horizontal waist shift, i.e. the distance of the focal points of the horizontal betatron-oscillation between the electron beam and the positron beam. Each focal point is separated from the interaction point by $w_{\mathrm{x}}/2$. For positive values the beams are already focused before they reach the interaction point, for negative values the focal point is behind the IP. Nominal value is zero.

- $\boldsymbol{w_{\mathrm{y}}}$
  Same as w_{x}, but for the focus of the vertical beam envelop. Nominal value: 360 μm.

- $\boldsymbol{\alpha_h}$ and $\boldsymbol{\Delta\alpha_h}$
  The rotation of the charge distribution of the bunches in the horizontal plane, i.e. around a vertical axis. For α_h the rotation is in the same direction and the bunch profiles remain parallel. Nominally there is no rotation.

- $\boldsymbol{\alpha_v}$ and $\boldsymbol{\Delta\alpha_v}$
  The rotation of the charge distribution of the bunches in the vertical plane, i.e. around a horizontal axis. Nominally there is no rotation.

- $\boldsymbol{\phi_l}$ and $\boldsymbol{\Delta\phi_l}$
  Rotation of the charge distribution of the bunches around the longitudinal axis of the bunches. For ϕ_l the rotation is in the same direction and the bunch profiles remain parallel. Nominally there is no rotation.

- $\mathbf{N_e}$ and $\boldsymbol{\Delta}\mathbf{N_e}$
  The number of particles per bunch. Nominal value is $2 \cdot 10^{10}$.

Both beams are assumed to move exactly parallel to the nominal beam axis (z-axis), i.e. no crossing angles and no misalignment of the beam axis to the detector axis. Any such rotation or crossing angle in the horizontal or vertical plane is practically undistinguishable from the angles α_h, α_v, $\Delta\alpha_h$, and $\Delta\alpha_v$, unless they are unreasonably large.

A three-dimensional ellipsoidal shape of the bunches is assumed. The three axis of the bunches are aligned with the detector axis, if none of the angles α or ϕ are different from zero. The charge distributions are assumed to be Gaussian in cross section with the widths σ_x, σ_y, and σ_z.

In reality the charge distribution of the bunches can be more complicated. Unfortunately, I do not see a possibility to determine an arbitrary charge distribution from the beam-strahlung and/or pair production monitoring. Some model is necessary for a meaningful measurement. Here the simplest model of ellipsoidal bunches is used. Of particular interest is the extension to bunches which are bend longitudinally (banana shape). Wake fields in the cavities can produce such distortions of the bunches. Guinea-pig is able to simulate such bunches, but I didn't find the time to do the work on the analysis.

3.3 Observables

A number of observables is used to describe the energy distribution measured in BeamCal. They are described below. Each observable appears twice, once for the forward and once for the backward calorimeter.

- $\boldsymbol{E_{tot}}$: The total energy measured in the calorimeter calculated as

  $$E_{\text{tot}} = \sum_l E_l \tag{7}$$

  where the sum runs over all cells of the calorimeter with the energy deposition E_l in the cell.

- $\langle \boldsymbol{r} \rangle$: The first radial moment or average radius of the energy deposition calculated as

  $$\langle r \rangle = \left(\sum_l r_l \, E_l \right) / E_{\text{tot}} \tag{8}$$

  where r_l is the radial position of the center of the cell.

- $\langle \frac{\mathbf{1}}{\boldsymbol{r}} \rangle$: the radial moment to the power -1 calculated as

  $$\langle r \rangle = \left(\sum_l \frac{1}{r_l} \, E_l \right) / E_{\text{tot}} \tag{9}$$

- $\boldsymbol{\phi_{thrust}}$: The direction of the thrust axis. As can be seen from fig. 1 the energy deposition in BeamCal is not rotationally symmetric. An axis along the main energy distribution is calculated with an algorithm similar to what is used in jet-physics to describe the main direction of energy flow in multi-jet events. If the vector $\hat{e}_{\text{thrust}}$ is a unit-vector in the x-y plane (the plane of the BeamCal) and $\hat{e}_l$ is a unit-vector pointing to the calorimeter cell l. A quantity E_{T} is defined as

$$E_{\text{T}} = \sum_l \left(\hat{e}_{\text{thrust}} \cdot \hat{e}_l\right) E_l. \tag{10}$$

The sum runs over all cells of one calorimeter. The thrust axis is the axis that minimizes this quantity. Its angle ϕ_{thrust} with respect to the x-axis is the observable. To avoid ambiguities it is chosen in the interval 0 to π.

- ***T***: The thrust value calculated as

$$T = E_{\text{T,min}}/E_{\text{tot}} \tag{11}$$

with the minimal value of E_{T} from the calculation above.

- ***δφ***: The angular spread of the energy deposition. It is calculated as

$$\delta\phi = \left(\sum_{\text{rings}} \sum_l \left(\phi_{\text{thrust,ring}} - \phi_l\right) \cdot E_l\right) / E_{\text{tot}}. \tag{12}$$

where the calculation of the thrust direction is repeated for each ring, taking into account only the cells in this ring, but not the whole calorimeter.

- **U-D**: The vertical energy imbalance. The calorimeter is separated into four quadrants along the thrust axis and an axis perpendicular to it (see fig. 4. The energy deposition in the four quadrants up-right (E_{UR}), up-left (E_{UL}), etc. is calculated. The observable U-D is given by

$$\text{U} - \text{D} = \left[\left(E_{\text{UR}} + E_{\text{UL}}\right) - \left(E_{\text{DR}} + E_{\text{DL}}\right)\right] / E_{\text{tot}}. \tag{13}$$

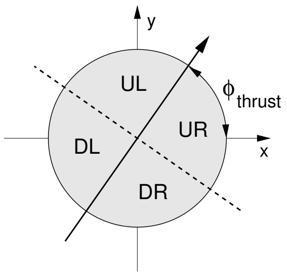

Figure 4: Definition of the four quadrants (see text).

- **R-L**: The vertical energy imbalance calculated as

$$\text{R} - \text{L} = \left[\left(E_{\text{UR}} + E_{\text{DR}}\right) - \left(E_{\text{UL}} + E_{\text{DL}}\right)\right] / E_{\text{tot}}. \tag{14}$$

- **Diag**: The diagonal energy imbalance calculated as

$$\text{Diag} = \left[\left(E_{\text{UR}} + E_{\text{DL}}\right) - \left(E_{\text{UL}} + E_{\text{DR}}\right)\right] / E_{\text{tot}}. \tag{15}$$

- ***E*out**: The energy deposition in the outer rings starting from ring number 4 normalized to the total energy:

$$E_{\text{out}} = \sum_{\text{ring}>3} E_l / E_{\text{tot}} \tag{16}$$

.

- $\boldsymbol{T_1}$ **and** $\boldsymbol{T_2}$: These two observables try to measure how much the energy deposition varies around the circumference of the calorimeter. Two regions with high energy deposition (E_1) and (E_2) are defined and two regions with low activity (E_3) and (E_4) with respect to the thrust axis, each 3 to 4 cells wide. The two observables are calculated as

$$T_1 = (E_1 + E_2) / (E_3 + E_4) \tag{17}$$
$$T_2 = E_3 / E_4 \tag{18}$$

3.4 Angular Beam Parameters

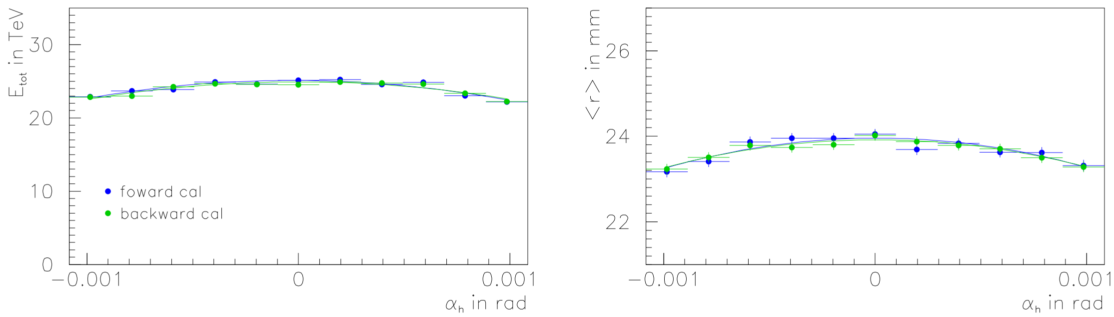

Figure 5: Variation of E_{tot} and $\langle r \rangle$ with $\Delta\alpha_h$, the relative angle between the two bunches in the horizontal plane. Both observables (and all others) reach a maximum at the nominal value $\Delta\alpha_h = 0$.

For some of the beam parameters the nominal value represents a point with maximum luminosity. This is the case for example for all the angular parameters (α_h, α_v, and ϕ_l). In this situation also the observables take external values for the nominal value of the beam parameter and the first Taylor coefficient vanishes. In that case it is not possible to extract this beam parameter with the Moore-Penrose method. The χ^2-fit can still work in this situation, but the sensitivity will be reduced and the fit has problems determining the sign of a possible deviation from the nominal value, as the leading term is quadratic in the deviation and contains no information on the sign. An example is shown in fig. 5.

4 Technical Aspects of the Analysis

Currently I analyze the beam-strahlung from individual bunch collisions. The results achieved (see sec. 5) show a very interesting statistical precision already at that level. Such single bunch measurements are interesting for example, if one wants to do parameter scans during a bunch train. Evidently the statistical precision can be increased by averaging the energy depositions in BeamCal over several bunches. But this cannot be pushed too far. It only makes sense to average over many bunches, if the stability of the accelerator is better than the precision of the diagnostic results. Furthermore, every analysis produces statistical and systematic errors. The systematic errors have not been investigated here. I believe they can be reduced below the level of the statistical precision from one bunch crossing, but they will dominate, if one averages over many bunches.

At the current status of the work no simulation of the showers induced by the particles hitting BeamCal has been included. The BeamCal is segmented into individual cells. When a particle reaches the front face of BeamCal, the cell of impact is determined and the energy of the particle is simply added to this cell. The detector has an inner radius of 12 mm and an outer radius of 80 mm. In default segmentation there are 7 radial rings. Each ring is further segmented in azimuth into cells which are in average 20 % larger in azimuth than in radial direction. The detector forseen is a silicon-tungsten sandwich calorimeter with a Moliére radius of approx. 1.0 cm.

There is also no detector resolution included in the analysis. The design studies for the BeamCal [8] show that an energy resolution can be achieved that is substantially better than the bunch to bunch fluctuations expected from the beam-strahlung. Therefore no reduction in the precision of the results is expected from the detector resolution.

Also the tracking used here is quite simple. The Monte Carlo program guinea-pig [5] simulates the collision and the generation of the beam-strahlung including pair production. It determines the momentum and the position of each particle leaving the interaction. The electric forces between the particles are taken into account. After the particle leaves the interaction a helical trajectory is calculated analytically to transport it to a distance corresponding to a z-coordinate of $z = \pm 3650mm$ where the front faces of the forward and backward BeamCal are located. From there the impact cell is determined. A magnetic field of 4 Tesla is assumed. No inhomogeneities of the magnetic field or misalignments of field or detector are taken into account. These effects are expected to be small.

The parameterization of 3 observables by a polynomial can be seen in fig. 3. The fitting of the polynomials is fully automized. The same fits are used for the parameterizations for the χ^2-fit (eqn. 6) and to calculate the Taylor coefficients for the Moore-Penrose inversion (eqn. 1)[4]. Events are generated with nominal beam parameters and then for each beam parameter with a number of values in the vicinity of the nominal value. 6 to 12 such additional points are typically used (in fig. 3 there are 8 + nominal). The observables are calculated and each is parameterized by a polynomial of up to fifth order.

In order to fit the observables a meaningful error has to be associated with every point. The error is determined from the variation of the observables for a number of bunch crossings, simulated with the same beam parameters. Typically 10 to 30 bunch crossings are used. It is impractical to do this for every point (there are about 500 such points). In fact it is only done at the nominal point. The error calculated at the nominal point is then used to estimate the errors at the other points. In a first attempt it was assumed that the error is the same at every point. But this fails for some beam parameters. Mistuning a beam parameter can lead to a strong reduction in the total amount of pair production from beam-strahlung. Then the number of particles detected by BeamCal strongly decreases and the statistical errors of the observables increase substantially. It is now assumed that the statistical error of the observables scales like $1/\sqrt{E_{\rm tot}}$, where $E_{\rm tot}$ is the energy detected in the corresponding BeamCal.

The fitting procedure is the following: First a polynomial of zeroth order (a constant) is fitted to the points. The resulting χ^2 is investigated. If the corresponding probability is larger than 0.5 % the fit is accepted. If not, the order of the polynomial is increased by one and the procedure is repeated until a probability of more than 0.5 % is found. If a fifth order polynomial is still not sufficient an error message is issued and the fit is accepted nevertheless.

The analysis presented here was developed for beam diagnostics from the pairs detected in BeamCal, but it is not limited to this detector. In principle it is possible to include other observables measured by other diagnostics devices into the analysis. All one needs is a bunch by bunch value of the observable and an estimate of its statistical error. In this way the measurement of the beam-strahlung photons on one of the collimators downstream of the IP could be included. There are some beam parameters which are more easily and with higher precision determined from other diagnostics devices. They can also be included. I have tested two such external measurements: the measurement of beam offsets from the fast feedback system and the measurement of the bunch charges from a current loop. In these cases the observables are directly the values of the corresponding beam parameter. For example for the fast feedback system I assume that it directly gives a value for the beam offset at the IP with an error of 2 nm in the horizontal plane and 0.5 nm in the vertical plane. The measurement of the bunch charge has an assumed error of 5 %. If the set of beam parameters to be determined is increased by these two and the external measurements are used in addition to the observables from BeamCal, there is no reduction in resolution on the parameters determined before. This is not astonishing from the physics point of view, but it demonstrates the technical feasibility of a global beam parameter analysis.

5 Results

In the simplest of all situations I assume that all beam parameters are perfectly known except for a single one. The value of this parameter is extracted from the beam-strahlung with the two methods

[4]The 1st order Taylor coefficient is simply the coefficient of the linear term in the polynomial.

beam parameter	unit	nominal value	reconstructed method 1	method 2
σ_{x}	nm	553	553.5 ± 1.5	553.5 ± 1.5
$\Delta\sigma_{\mathrm{x}}$	nm	0	0.4 ± 2.1	0.4 ± 2.2
σ_{y}	nm	5.0	5.10 ± 0.17	5.10 ± 0.15
$\Delta\sigma_{\mathrm{y}}$	nm	0.0	0.27 ± 0.47	0.26 ± 0.23
σ_{z}	μm	300	303.9 ± 4.3	304.0 ± 4.6
$\Delta\sigma_{\mathrm{z}}$	μm	0	-1.7 ± 2.7	-1.7 ± 2.7
ε_{x}	mm mrad	10.0	$— \pm —$	$— \pm —$
$\Delta\varepsilon_{\mathrm{x}}$	mm mrad	0.0	$— \pm —$	0.2 ± 1.0
ε_{x}	μm mrad	30.0	29.2 ± 1.2	29.0 ± 1.6
$\Delta\varepsilon_{\mathrm{x}}$	μm mrad	0.0	0.7 ± 2.0	1.3 ± 2.9
d_{x}	nm	0	8.3 ± 6.2	9.4 ± 6.8
d_{y}	nm	0.0	-1.2 ± 0.4	-0.1 ± 0.3
W_{x}	μm	0	$— \pm —$	$— \pm —$
W_{y}	μm	360	326 ± 24	280 ± 73
α_{h}	mrad	0	$— \pm —$	$— \pm —$
$\Delta\alpha_{\mathrm{h}}$	mrad	0	$— \pm —$	227 ± 56
α_{v}	mrad	0	$— \pm —$	$— \pm —$
$\Delta\alpha_{\mathrm{v}}$	mrad	0	$— \pm —$	-56 ± 71
ϕ_{l}	mrad	0	$— \pm —$	$— \pm —$
$\Delta\phi_{\mathrm{l}}$	mrad	0	$— \pm —$	$— \pm —$
$\mathrm{N_e}$	10^{10}	2.0	2.01 ± 0.01	2.01 ± 0.01
$\Delta\mathrm{N_e}$	10^{10}	0.0	-0.01 ± 0.01	-0.01 ± 0.01

Table 1: Results of the single parameter analysis. Method 1 is the Moore-Penrose inversion, method 2 the χ^2-fit. For more details see text.

described in sec. 3. The methods are applied to ten different events, all simulated with nominal beam parameters. In table 1 the averages of the reconstructed values from these 10 events are quoted together with the spread around the average. The spread gives an estimate of the precision of the reconstruction. One can see from the table that excellent results are achieved for the size of the two bunches in any direction (σ and $\Delta\sigma$), the beam offsets (d_{x} and d_{y}) and the bunch charges N_{e}. Measurements of ε_{y} and w_{y} are possible with moderate precision. The beam offsets and the bunch charges can also be measured with high precision from other beam diagnostics instruments[5]. This leads to the conclusion, that the beam-strahlung can be used as a tool to determine the size of the colliding bunches. On top of that a moderate measurement of the vertical waist shift might be possible. With the second method some of the angular variables can also be extracted with precisions of approx. 0.1 rad.

It has to be mentioned, that this simple situation of single parameter measurements produces optimistic results and is not very realistic. I report it to set the scale. In more realistic scenarios the results cannot get better than the table 1.

In a more realistic scenario several beam parameters are measured at the same time. Table 2 gives the achievable resolution for several such scenarios. Each vertical column of the table gives one situation. In the column labeled 1-par. the results from the single parameter analysis are repeated for reference (see tab. 1). Then three scenarios with a combined measurement of two bunch sizes are given and three scenarios with four bunch sizes. The second but last column gives the resolution if all three bunch sizes of both beams are measured simultaneously. The precision decreases if more parameters are to be measured, as expected. But even with all six beam sizes the precision on the horizontal and longitudinal bunch sizes are still around 2 % and around 20 % for the small vertical bunch sizes. Finally the vertical waist shift is added as a seventh parameter to be measured. The precision on the bunch sizes doesn't change much. The waist shift can be measured with 275 μm. It's nominal value is 360 μm.

[5] the BPMs of the fast feedback system measure the offsets and the bunch charges can be measured from current loops.

beam parameter	unit	achievable resolution 1-par.	2-par.			4-par.			6-par.	7-par.
σ_x	nm	1.5	1.7			5.3		3.1	10.0	17.4
$\Delta\sigma_x$	nm	2.1	2.3			5.5		5.7	6.2	6.6
σ_y	nm	0.17		0.18		0.55	0.29		0.81	0.79
$\Delta\sigma_y$	nm	0.47		0.56		1.21	1.21		1.33	1.56
σ_z	μm	4.3			4.5		4.8	6.5	9.5	7.2
$\Delta\sigma_z$	μm	2.7			2.8		5.6	5.6	6.2	6.2
w_y	μm									275

Table 2: Results from simultaneous measurements of several beam parameters. Each column corresponds to one simultaneous measurement (method 1).

2-par.	σ_x	$\Delta\sigma_x$
σ_x	1	-0.61
$\Delta\sigma_x$	-0.61	1

2-par.	σ_y	$\Delta\sigma_y$
σ_y	1	-0.09
$\Delta\sigma_y$	-0.09	1

2-par.	σ_z	$\Delta\sigma_z$
σ_z	1	-0.19
$\Delta\sigma_z$	-0.19	1

4-par.	σ_x	$\Delta\sigma_x$	σ_y	$\Delta\sigma_y$
σ_x	1	-0.27	-0.98	0.11
$\Delta\sigma_x$	-0.27	1	0.24	-0.93
σ_y	-0.98	0.24	1	-0.13
$\Delta\sigma_y$	0.11	-0.93	-0.13	1

4-par.	σ_y	$\Delta\sigma_y$	σ_z	$\Delta\sigma_z$
σ_y	1	-0.63	-0.64	0.73
$\Delta\sigma_y$	-0.63	1	0.07	-0.94
σ_z	-0.64	0.07	1	-0.20
$\Delta\sigma_z$	0.73	-0.94	-0.20	1

4-par.	σ_x	$\Delta\sigma_x$	σ_z	$\Delta\sigma_z$
σ_x	1	-0.56	-0.82	0.46
$\Delta\sigma_x$	-0.56	1	0.26	-0.95
σ_z	-0.82	0.26	1	-0.28
$\Delta\sigma_z$	0.46	-0.95	-0.28	1

6-par.	σ_x	$\Delta\sigma_x$	σ_y	$\Delta\sigma_y$	σ_z	$\Delta\sigma_z$
σ_x	1	-0.19	-0.94	0.14	-0.87	-0.24
$\Delta\sigma_x$	-0.19	1	0.08	-0.47	0.15	-0.48
σ_y	-0.94	0.08	1	-0.27	0.72	0.46
$\Delta\sigma_y$	0.14	-0.47	-0.27	1	-0.06	-0.48
σ_z	-0.87	0.15	0.72	-0.06	1	0.10
$\Delta\sigma_z$	-0.24	-0.48	0.46	-0.48	0.10	1

Table 3: Correlation coefficients for different scenarios (method 1).

All results are achieved from the analysis of the beam-strahlung of single collisions. The measured values show strong correlations among each other. The correlation coefficients for some situations are given in table 3. There is no problem separating the size of the electron from the size of the positron bunch. The correlation coefficients between σ and $\Delta\sigma$ are small, typically 0.1 to 0.2 only. On the other hand the correlations between the different dimensions x, y, and z, are generally strong and negative, reaching even -0.98.

To this point bunch collisions with nominal beam parameters were analyzed to determine the achievable precision. The simulations show, that it is possible to verify that the accelerator is indeed operating with nominal beam parameters. But one wants to go beyond that. If the accelerator is nor running with nominal beam parameters, we want to be able to detect which parameter is out of spec and by how much. Figure 6 shows that this is indeed possible. For each of the six plots collisions have been simulated where the beam parameter indicated on the plot deviates from its nominal value. All other parameters have nominal values. This parameter is then reconstructed in a single parameter analysis. All six plots show good linearity over a substantial range. The most critical one is the small vertical bunch size. Here the point at $\Delta\sigma_y = -2$ nm already shows a strong deviation.

Figure 7 shows a test where several beam parameters (bunch sizes) deviate from there nominal values. The bunch crossing is analyzed in a six parameter analysis with σ_x, $\Delta\sigma_x$, σ_y, $\Delta\sigma_y$, σ_z, and $\Delta\sigma_z$ as parameters. The blue dot indicates the nominal value and the red dot the input values for the simulation of this bunch crossing. The brown dot with the error bars is the 2-dimensional projection of the reconstructed value. One can see that the result is consistent with the input and

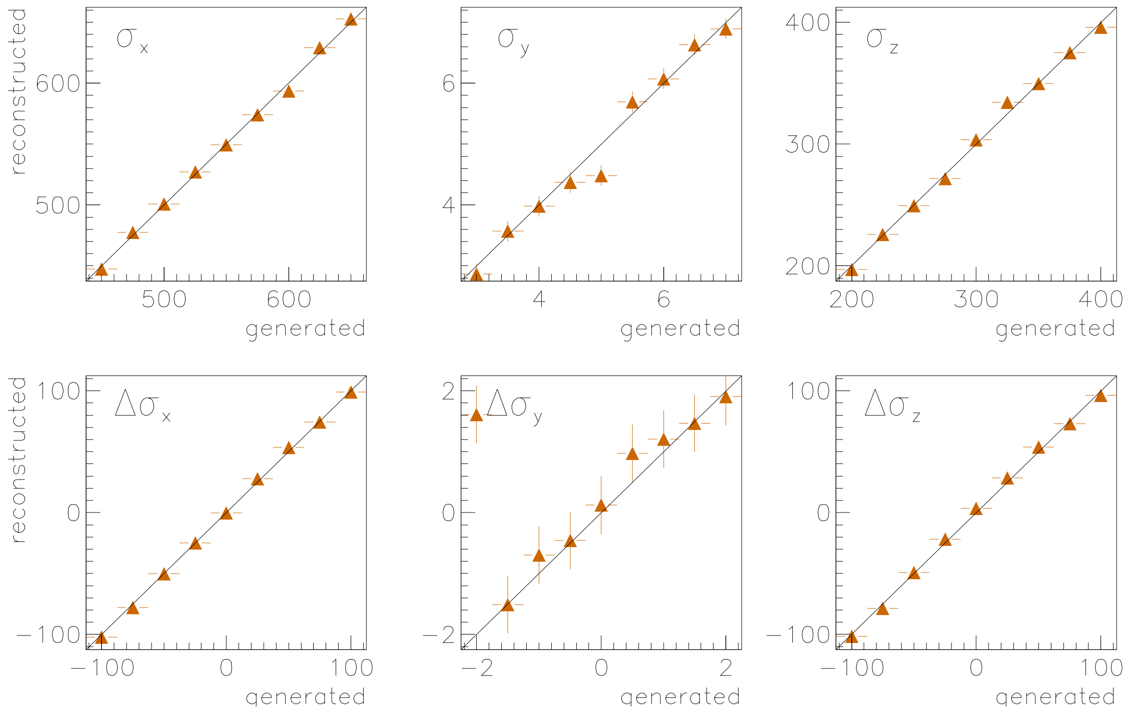

Figure 6: Test of linearity for single parameter analysis (method 1). Simulation of bunch collisions with nominal beam parameters except for the one indicated on each plot.

that the errors are reasonably estimated.

It is interesting to see which variables contribute much to an analysis and which don't. Figure 8 attempts to do that. It is extracted from single parameter analysis of each beam parameter. The figure shows a matrix of variables versus beam parameters. Each variable is listed twice, once for the forward and once for the backward BeamCal. A big box indicates a large contribution of a variable to a beam parameter. The table really shows the first order Taylor coefficient of variable i to beam parameter j. The Taylor coefficients are normalized to units of standard deviations of the variables and the achieved resolutions on the beam parameters. In case the whole weight of a beam parameter analysis is carried by a single observable, the Taylor coefficient should be 1 in these units. If the coefficient is smaller, the dependence of observable i on parameter j is less pronounced and the observable carries less weight.

Up to this point all observables are derived from the energy distributions measured in BeamCal. It is possible to build a detector that also measures the spacial distribution of the arriving particles, where each particle contributes with the same signal independent of its energy. This could be realized, for example, in a sampling calorimeter. The signals from the first sensor plane give approximately the distribution in numbers, while after summing the signals in depth one gets the distribution in energy. Table 4 shows how much one would gain, if the distribution in numbers is used on top of the distribution in energy. One new variable N/E is defined. It is the average energy of all particles hitting the calorimeter (more correctly it is the inverse of this number). This is probably the most sensitive variable that can be constructed from the number distribution. The results for a six-parameter analysis (bunch sizes for both beams) is given in table 4. The resolution improves by approximately 20 %, the correlations get slightly stronger (compare to table 3. I leave it to the reader to decide whether this is worth the additional effort in hardware. I would like to point out, however, that in my experience the energy distribution is more sensitive than the distribution in numbers. A detector that only counts the pairs, but does not measure their energy, will give worse results.

Not only the e^+e^- pairs produced in the collisions carry information on the charge distribution of the colliding bunches. Also the radiated photons do. Figures 16 and 17 in the appendix show a series of simulations with varying bunch sizes. As in the case of the e^+e^- pairs the photons show

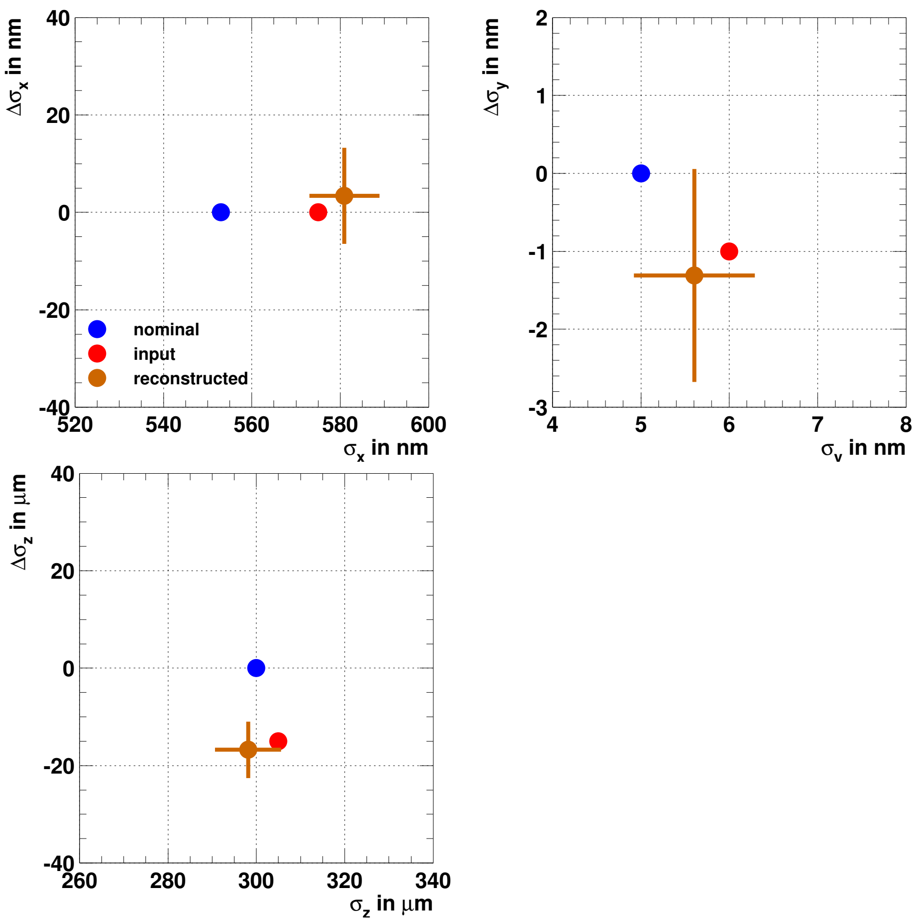

Figure 7: Test of linearity for single parameter analysis (method 1). Simulation of bunch collisions with nominal beam parameters except for the one indicated on each plot.

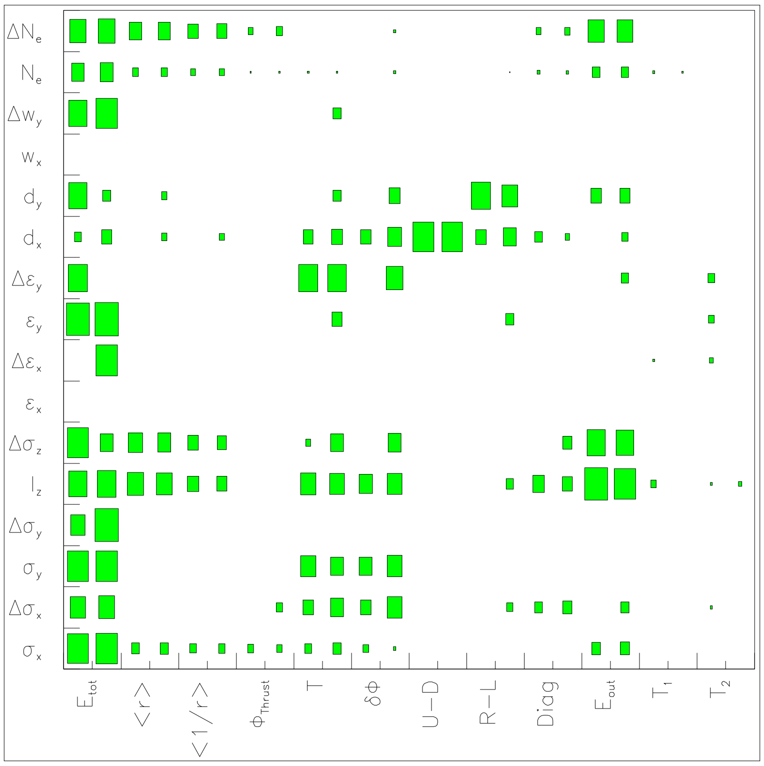

Figure 8: The weight of the different observables (columns) to the single parameter analysis (method 1) of different beam parameters (rows). A full box would indicate a weight of 1.

beam parameter	unit	resolution no N/E	resolution with N/E
σ_x	nm	10.0	8.3
$\Delta\sigma_x$	nm	6.2	6.0
σ_y	nm	0.81	0.61
$\Delta\sigma_y$	nm	1.33	0.93
σ_z	μm	9.5	9.4
$\Delta\sigma_z$	μm	6.2	6.1

6-par.	σ_x	$\Delta\sigma_x$	σ_y	$\Delta\sigma_y$	σ_z	$\Delta\sigma_z$
σ_x	1	-0.37	-0.93	0.22	-0.91	-0.05
$\Delta\sigma_x$	-0.37	1	0.15	-0.11	0.29	-0.77
σ_y	-0.93	0.15	1	-0.35	0.74	0.33
$\Delta\sigma_y$	0.22	-0.11	-0.35	1	-0.11	-0.46
σ_z	-0.91	0.29	0.74	-0.11	1	-0.03
$\Delta\sigma_z$	-0.05	-0.77	0.33	-0.46	-0.03	1

Table 4: Comparison of a six parameter analysis from the energy distribution alone (no N/E) and from an analysis including the variable N/E from the number distribution (method 1). The upper table gives the resolutions, the lower one the correlation coefficients.

a strong dependence on the horizontal and the longitudinal beam size. But in contrast to the pairs there is no visible dependence on the vertical beam size. A variation of σ_y between 4 and 7 nm produces a change in the radiated energy of less than 5 %. The work to characterize the photon distributions by appropriate variables to include them in the analysis has not been done yet. But it is already clear from the plots, that the photon distributions will help a lot to reduce the strong correlations between the vertical bunch size on one hand and the horizontal and longitudinal sizes on the other hand.

Such a behavior is expected from theory. Pair production (and luminosity) is proportional to

$$E\left(\mathrm{e}^{+}\,\mathrm{e}^{-}\right) \propto \frac{1}{\sigma_{\mathrm{x}} \cdot \sigma_{\mathrm{y}}} \tag{19}$$

while the photon radiation is proportional to

$$E\left(\gamma\right) \propto \frac{1}{\sqrt{\sigma_{\mathrm{x}}^{2}+\sigma_{\mathrm{y}}^{2}}} \approx \frac{1}{\sigma_{\mathrm{x}}} \tag{20}$$

as σ_{y} is much smaller than σ_{x}[6].

The photons of the beam-strahlung are emitted with very small angles. They can only be detected a substantial distance downstream of the IP when their separation from the beam axis has become large enough to place a detector. Such a detector would also act as a photon collimator. Or one could say that an instrumented photon collimator could also act as the photon detector.

6 Hardware Implementation

It is the goal to use the analysis of the beam-strahlung to optimize the luminosity of TESLA in a slow feedback loop. The basic idea is sketched in fig. 9. Measurements of the detector observables are taken while the accelerator is in stable running (collisions). The observables are analyzed in real-time with the Moore-Penrose method to extract beam parameters. The beam parameters give the actual luminosity. In order to optimize the luminosity correction signals for various beam elements are generated. It is beyond the scope of this note to discuss how appropriate correction signals can be found. These corrections signals are send to the accelerator elements. New bunches with modified parameters collide. This closes the feedback loop. If everything works correctly the new beam parameters lead to an improved luminosity. For this note the guinea-pig simulation replaces the real accelerator and the loop is only followed to the determination of the beam parameters.

During machine development times one might want to follow a different approach. No optimization is done. In order to understand the influence of the correction elements on the beam sizes correction signals are generated with a known pattern (for example linear scans). The beam-strahlung is used to analyze the changes in beam parameters.

Figure 10 presents a sketch of the necessary electronics blocks and a very rough estimate of the turn-around time of the feedback loop. The creation of signals in the BeamCal sensors is very fast (<1 nsec). The analog electronics has to shape and store the signals which should be possible in 20 nsec. Then the signals have to be digitized. There are in the order of 1000 channels. With enough parallel processing this can probably be done in another 20 nsec. In the next step a digital processor on each of the two detectors has to calculate the values of the observables from the signals. Each observable is a sum over the energy of all the cells with appropriate weighting factors. For example in the case of E_{tot} the weighting factor is 1 for each cell, while for $\left\langle\frac{1}{r}\right\rangle$ it is the radius of the center of the cell. It is assumed that the weighting factors are precalculated and stored in lookup tables. For each observable a processor has to get take the signal of each cell multiply it with the number in the lookup table and determine the sum over all cells. This might take 100 nsec. To this point all processing has been done in the front-end electronics of each of the two BeamCals. Now the digital values of the observables have to a central processor that can access the observables from both BeamCals (and the photon detectors, if they are to be included). Assuming 50 meters of cable this will take roughly 150 nsec. In the central processor the vector of observables is multiplied with the precalculated Moore-Penrose inverse which might take another 100 nsec. Now the beam parameters are ready and the optimizer can calculate the correction signals. I cannot give an estimate how much time this might need. 100 nsec is a wild guess. Finally the correction signals have to be transported to the correction elements on the accelerator. These might be hundreds of meters away and the transition time of the correction signals from

[6] The suppression of beam-strahlung with flat beams is based on this behavior. With a relatively large σ_{x} a reduction of σ_{y} increases the luminosity, but not the beam-strahlung.

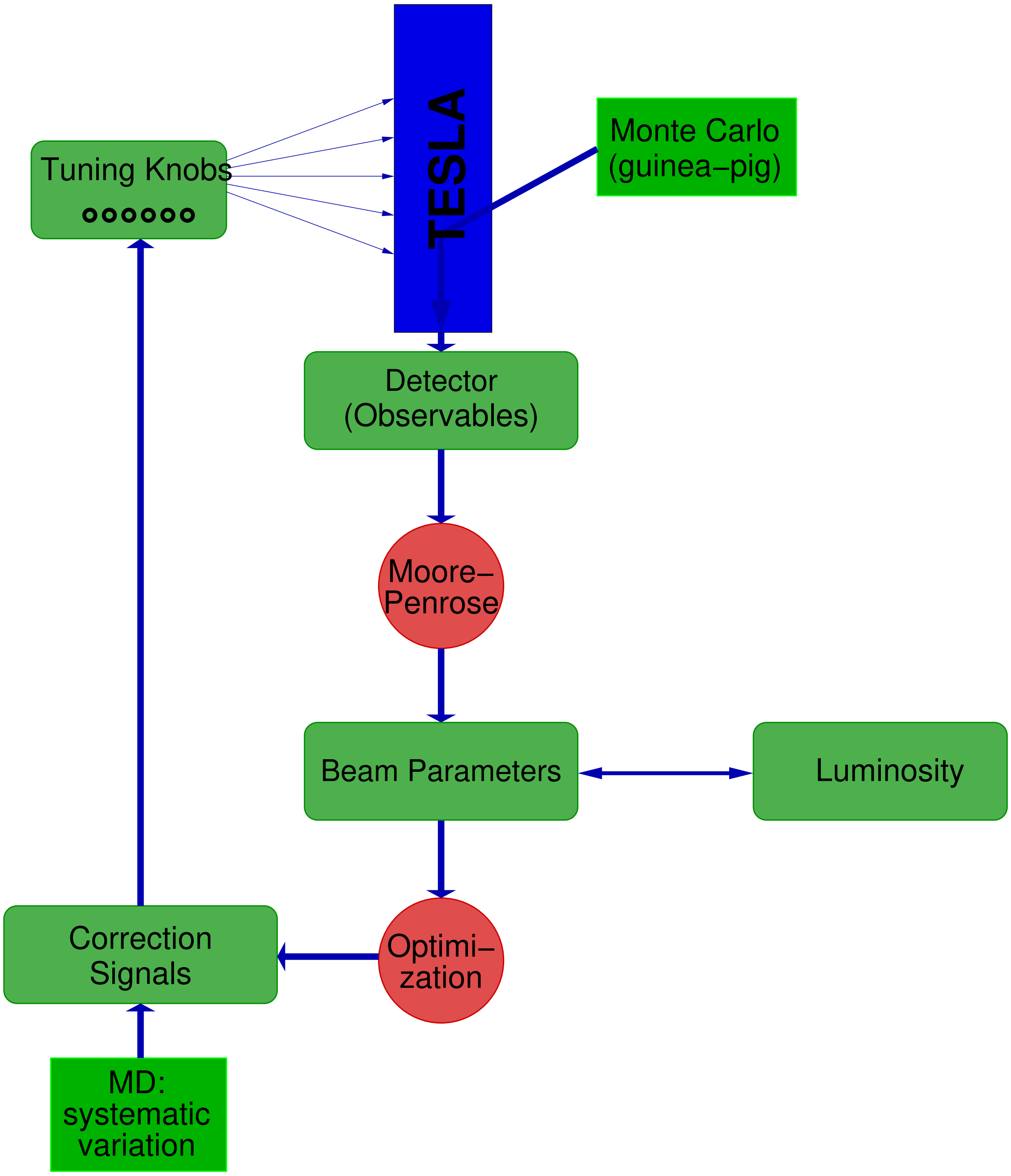

Figure 9: Logical scheme for the slow feedback at TESLA.

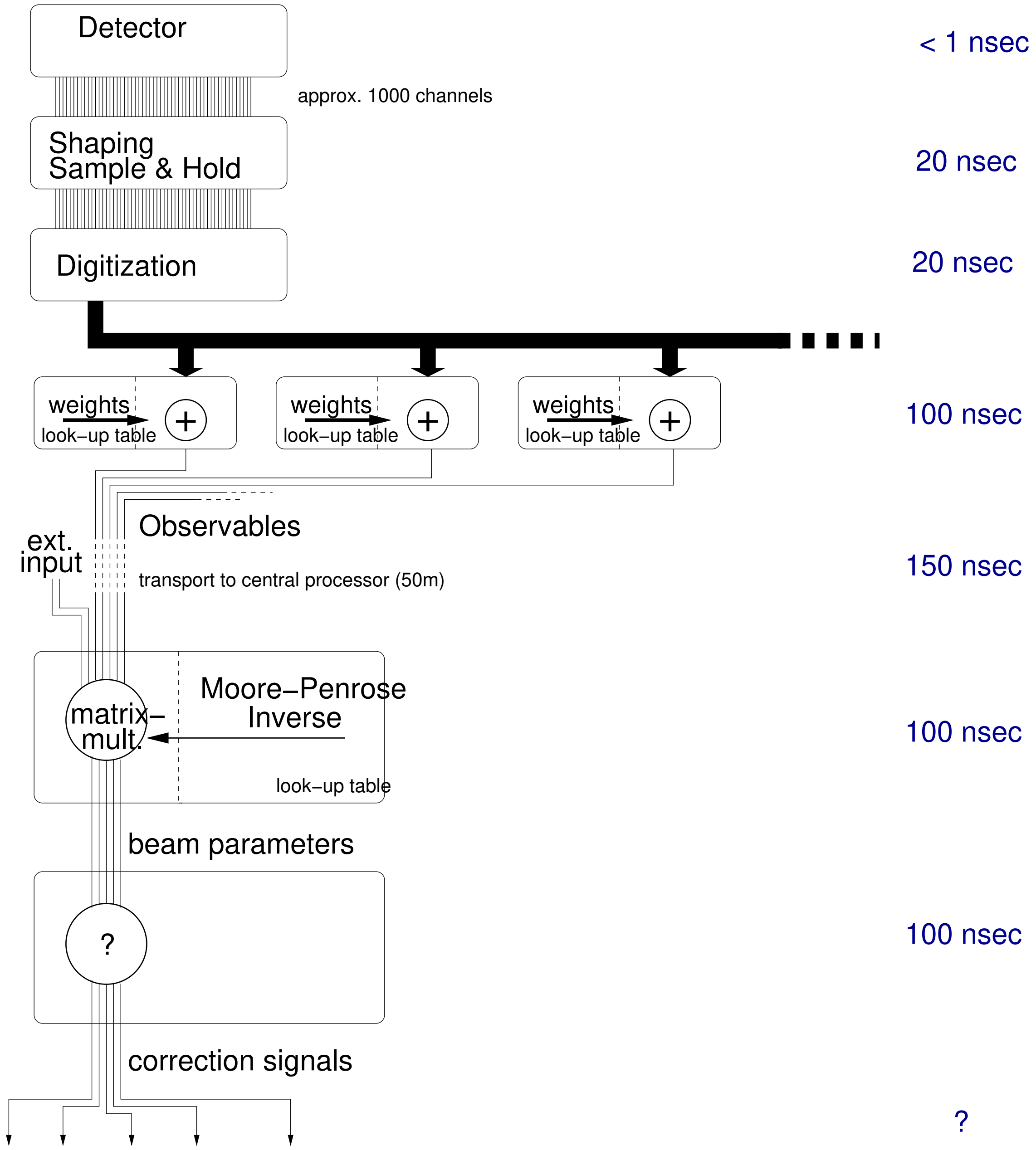

Figure 10: Rough time flow in the slow feedback loop.

the central processor to the correction elements might reach microseconds and limit the speed of this feedback loop. If the signals from the photon radiation are included the transition time of the corresponding observables to the central processor might also be a limiting factor depending how far from the IP the photon detector is located.

In a more conservative scenario one might want to average 10 beam crossings and expect another 10 beam crossings (3 μsec) to do the corrections. This is still sufficient for a slow feedback loop.

7 Outlook

This note is only a status report. Intermediate results are presented. More work is needed to fully exploit the potential of the beam-strahlung for beam diagnostics and to understand the implications on the design of the detector. But already at that stage one can see, that the detection of $e^+ e^-$ pairs and photons from the beam-strahlung will provide very interesting information on the colliding bunches. The beam-strahlung is sensitive to the electric fields of the colliding bunches and is therefore ideal to measure the size of the bunches. The studies presented here indicate that it is possible to measure the small vertical bunch size to 20 % from a single bunch crossing and the horizontal and longitudinal bunch sizes to a few percent. Some smaller experimental effects are not yet included in the studies and the results are probably a bit too optimistic.

In the current status of the analysis only the information from the $e^+ e^-$ pairs is used. A first glance at the radiated photons looks quite optimistic. A measurement of the photons will probably help to reduce the strong correlations observed between the measurements of σ_x, σ_y, and σ_z. The work, however, still has to be done.

Throughout the note, it has been assumed that the bunches are of ellipsoidal shape in all three dimensions. That is an ideal situation. In a real accelerator wake field created in the cavities by the head of the bunches might interact with their tails and deflect the tails from the axis (banana effect). The resulting bunches resemble the shape of a banana. It will be interesting to find out, whether this bunch deformation can be detected from the beam-strahlung. If so, a feedback loop might be able to correct for the banana effect. The Monte Carlo program guinea-pig is able to simulated arbitrarily shaped bunches, but I have not done the work. First a beam parameter needs to be defined, that quantifies the amount of banana deformation. Second, sensitive observables need to be identified and finally one can simulate the collisions and determine the expected resolution.

In the current note all results were derived from single bunch collisions. In the real experiment this is probably not necessary and one will average over several bunches. But this is only meaningful, if the bunch-to-bunch stability of the accelerator is better than the resolution of the beam diagnostics. Real beam simulations including the effect of the most important ground motions and the beam steering of the fast feedback system[7] are available. It will be interesting to see, how the diagnostics developed here, reacts to real bunch simulations.

References

[1] DESY proposal, PRC November 2002, *R&D for the TESLA Detector: Instrumentation of the very forward region*, (http://www-zeuthen.desy.de/~astahl/Collaboration.html).

[2] TESLA Technical Design Report, DESY 2001-011, ECFA 2001-209, March 2001. See part II, section 7.6.2.

[3] K. Büßer, A. Stahl, *Detector Concept for the Forward Regions*, LC-DET-2004-034.

[4] TESLA Technical Design Report, DESY 2001-011, ECFA 2001-209, March 2001. For the forward regions see Vol. IV, sec. 3.5 and 7.1.

[5] Guinea-pig web-page: http://www-sldnt.slac.stanford.edu/nlc/programs/guinea_pig/gp_index.html

[7] The feedback system measures the deflection of the bunches when they collide from beam position monitors behind the interaction point and corrects it to zero. That essentially guarantees, that the center-of-charge of the bunches collide without offsets.

[6] K. Yokoya, *A Computer Simulation Code for Beam-Beam Interaction in Linear Colliders*, KEK report 85-9, Oct. 1985.
http://www-acc-theory.kek.jp/members/cain/default.html

[7] See for example: Eric W. Weisstein, CRC Consise Encyclopedia of mathematics or http://mathworld.wolfram.com/Moore-PenroseMatrixInverse.html

[8] H. Abramowicz et al., IEEE Trans. Nucl. Sci. **51** (2004) 2983.

A Appendix

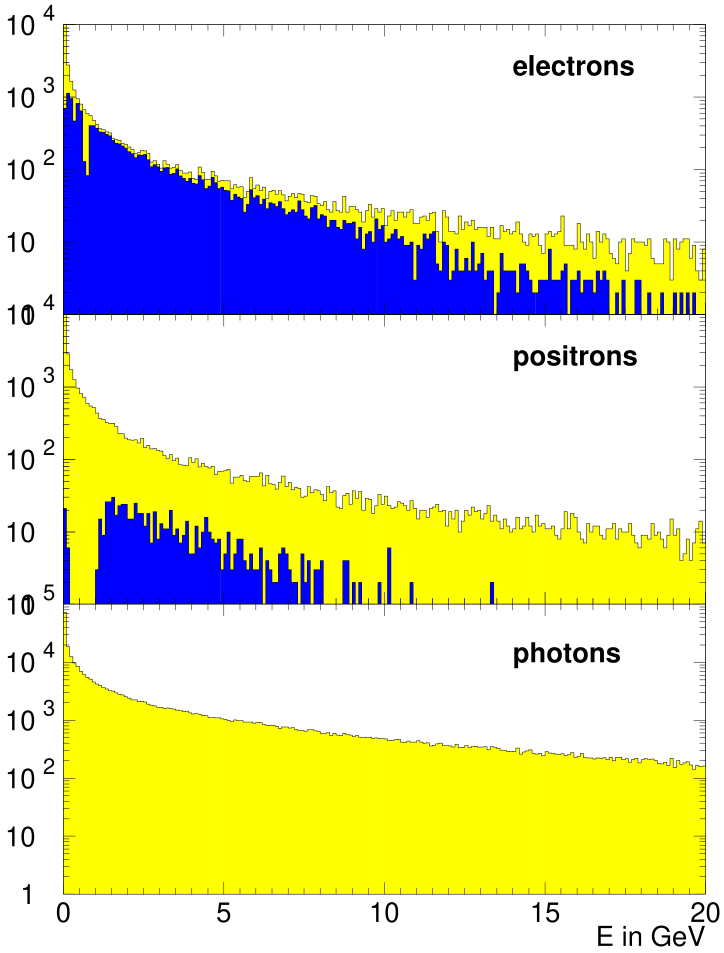

Figure 11: Energy spectrum of electrons, positrons, and photons radiated in 1 bunch crossing at 500 GeV center-of-mass energy at nominal conditions in the forward direction. The dark (blue) histogram shows the particles that reach BeamCal, the lighter (yellow) histograms show all particles.

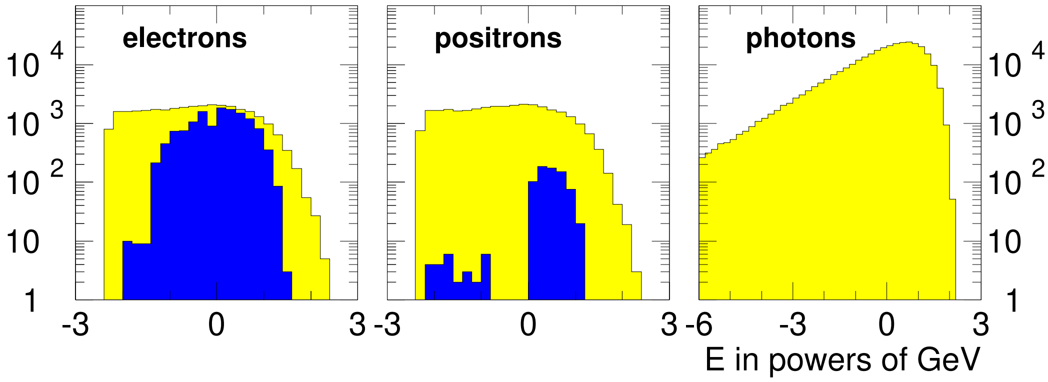

Figure 12: Same as above, but logarithmic energy scale.

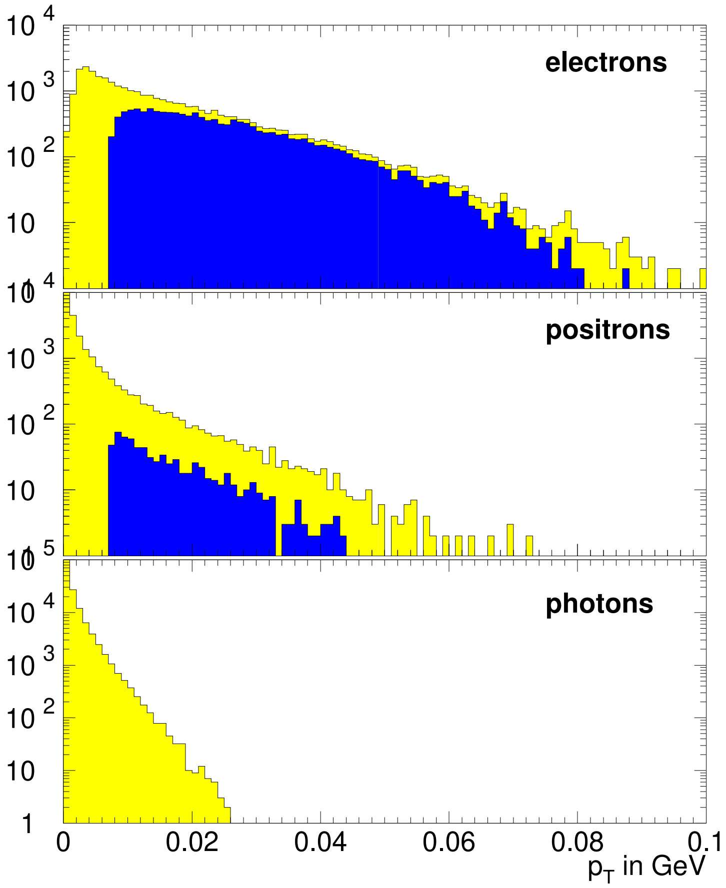

Figure 13: Transverse momentum of electrons, positrons, and photons radiated in 1 bunch crossing at 500 GeV center-of-mass energy at nominal conditions in the forward direction. The dark (blue) histogram shows the particles that reach BeamCal, the lighter (yellow) histograms show all particles.

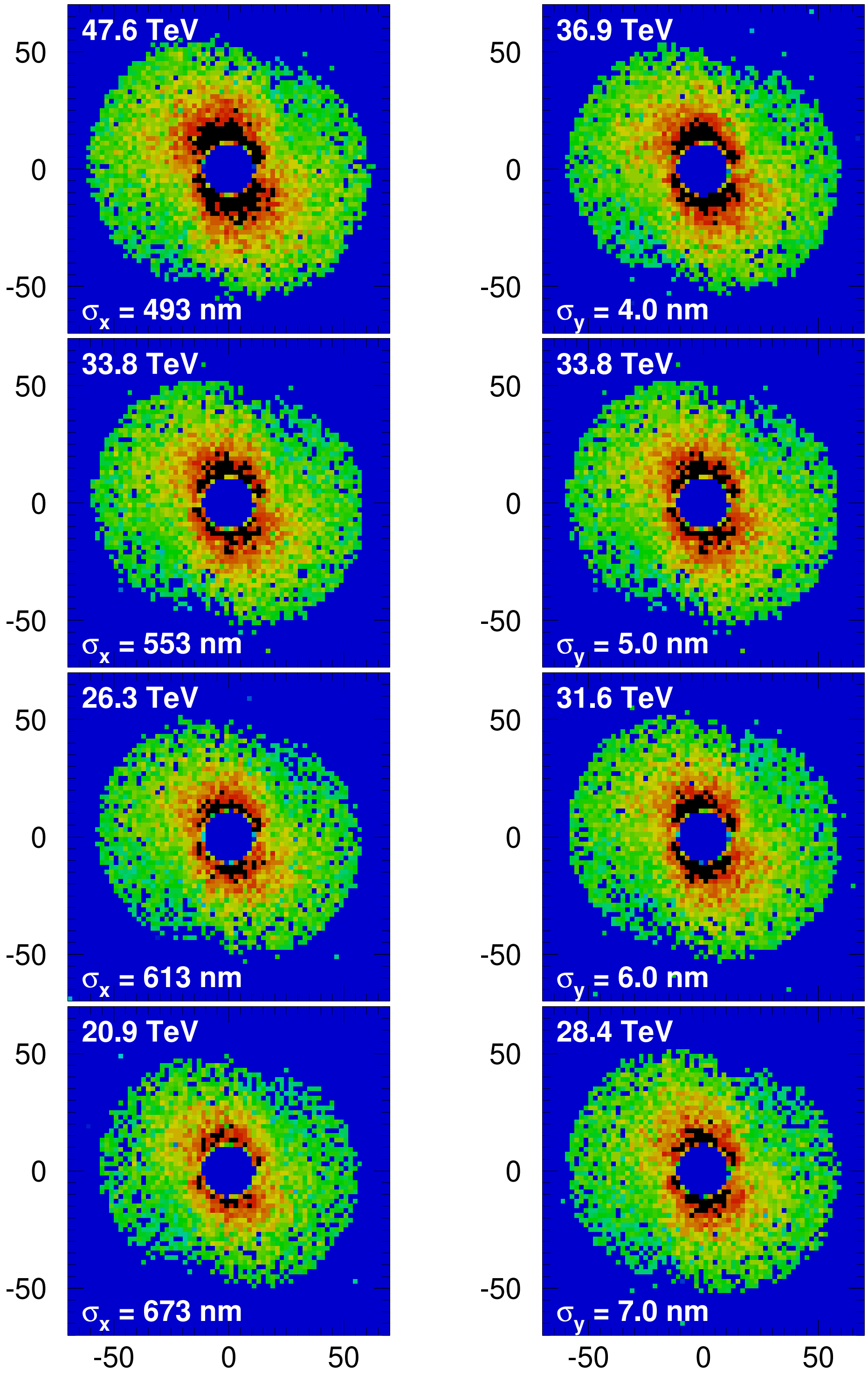

Figure 14: Energy distribution on forward BeamCal for bunches with different parameters. Positions in mm from beam. Left column: horizontal bunch size varies (nominal 553 nm). Right column: vertical bunch size varies (nominal 5 nm). Next page: bunch length varies (nominal 300 μm). All other parameters nominal (500 GeV).

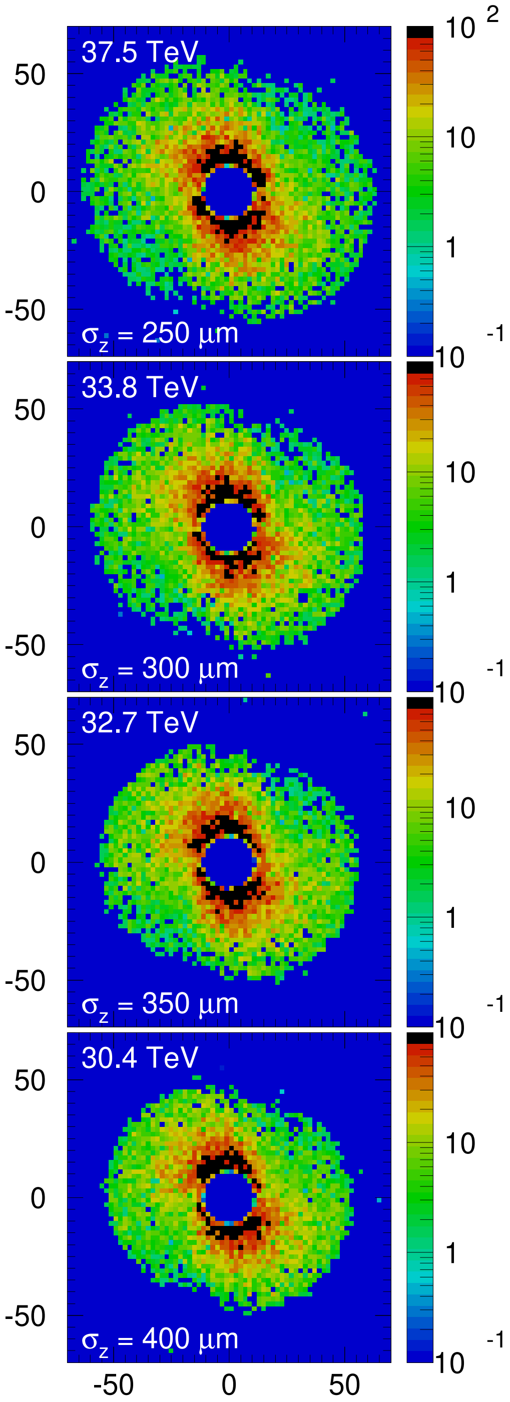

Figure 15: See previous page.

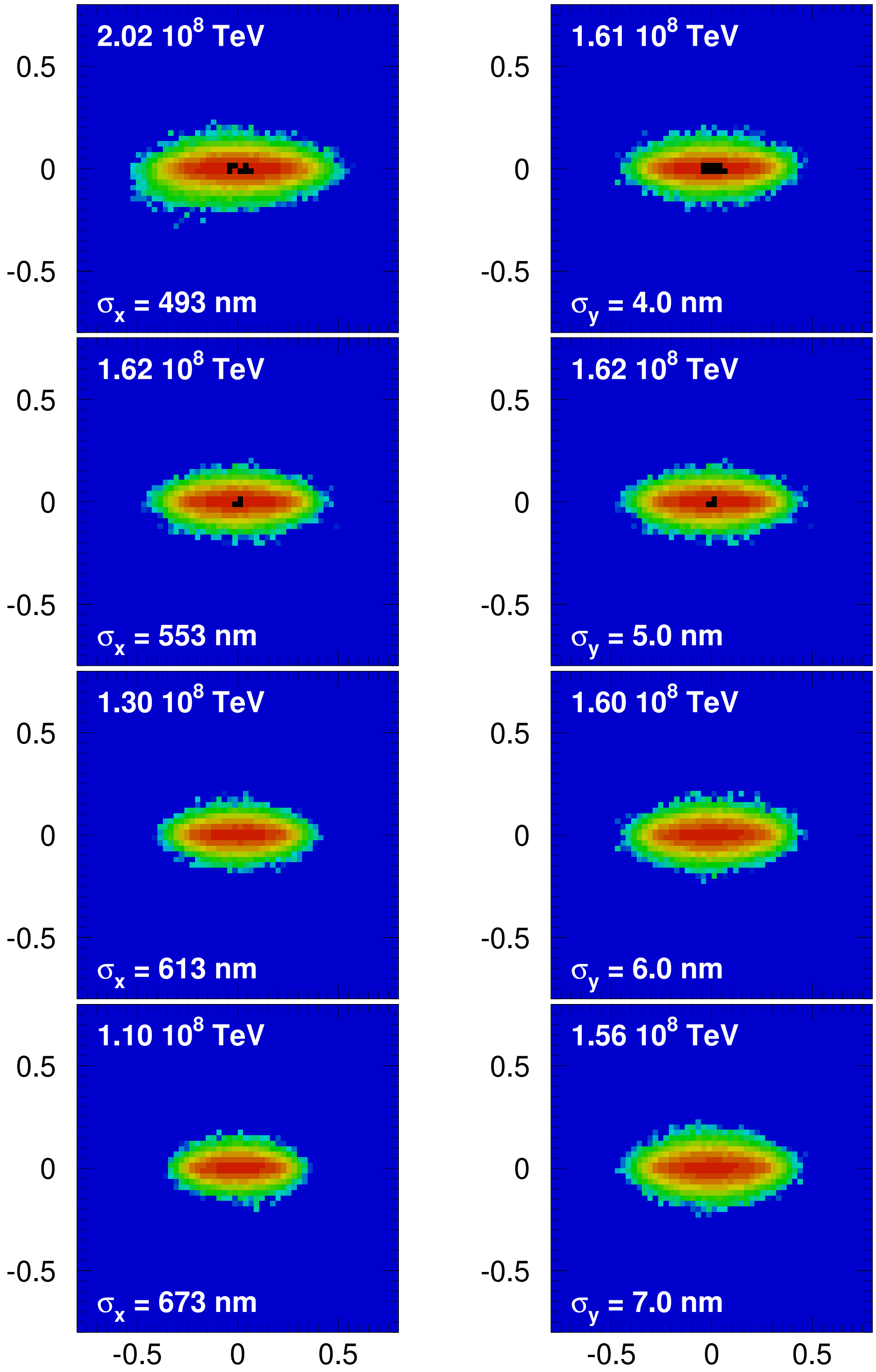

Figure 16: Energy distribution of photons with respect to the beam direction. The distance to the beam is given in angles (mrad). This can be translated into mm Once the position of the photon detector is fixed. Left column: horizontal bunch size varies (nominal 553 nm). Right column: vertical bunch size varies (nominal 5 nm). Next page: bunch length varies (nominal 300 μm). All other parameters nominal (500 GeV).

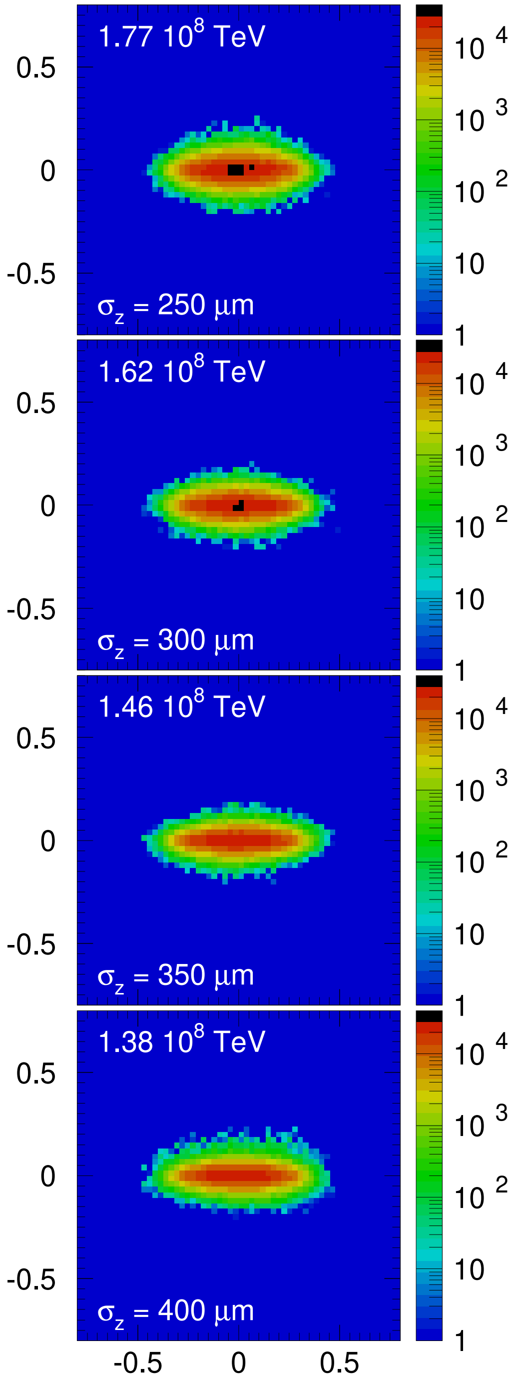

Figure 17: See previous page.

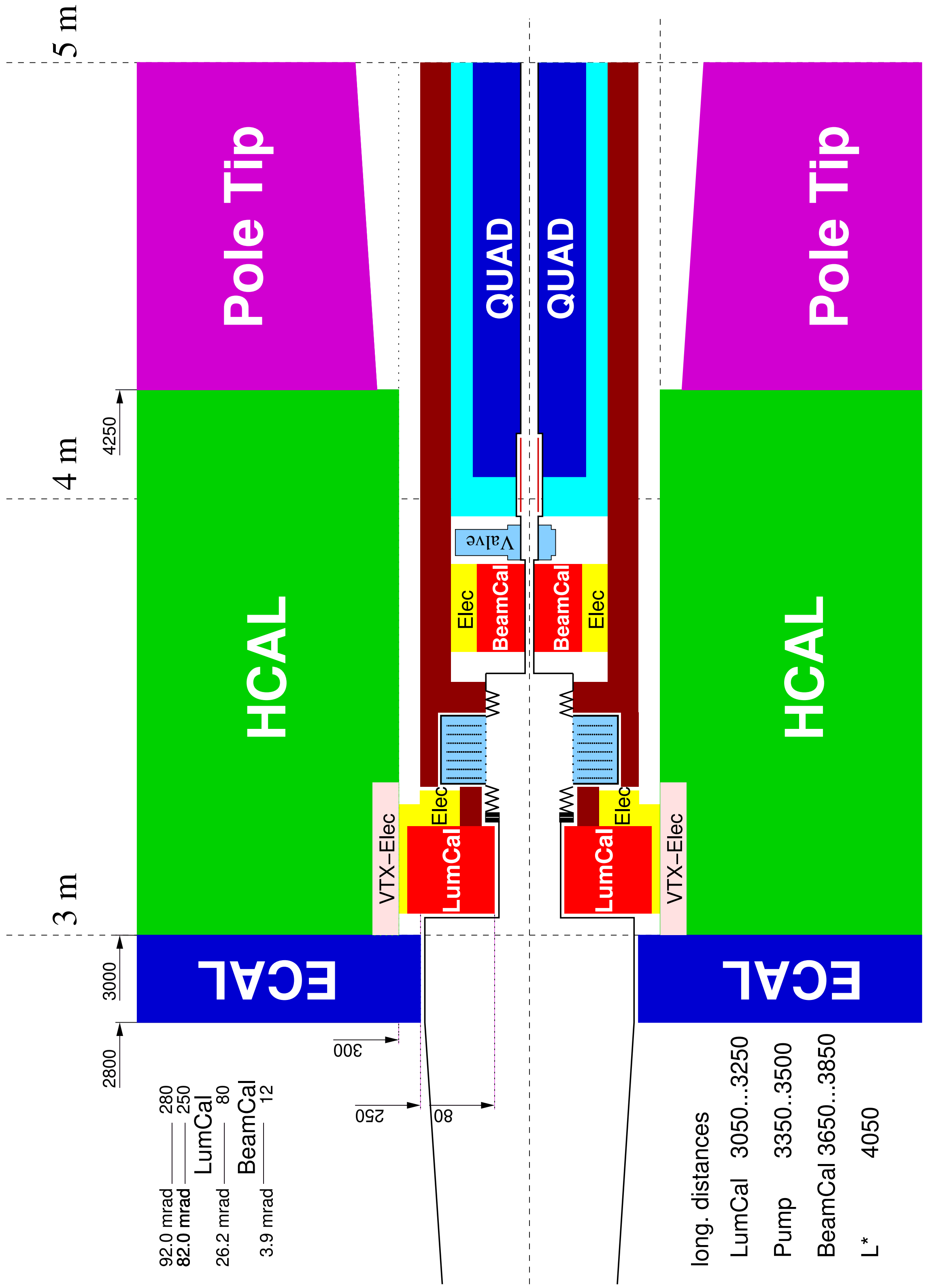

Figure 18: A vertical cut through the forward region. ECAL and HCAL are the endcaps of the main electromagnetic and hadronic calorimeters. LumCal and BeamCal are the two forward calorimeters foreseen in the new design.