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IS A LASER WIRE A NON-INVASIVE METHOD ? *

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Abstract

A tightly focused laser beam (laser wire) is used for measurement of transverse electron beam sizes in storage rings and linear colliders. It is assumed that the laser beam does nothing with the electron beam except Compton scatterings which happen with a rather small probability. In reality, electrons crossing the laser beam get kicks (with 100% probability) proportional to the square of the laser field and inversely proportional to the beam energy. In practical cases of beam diagnostics this effect is negligible.

1 INTRODUCTION

A laser beam (wire) is used for measurement of electron beam sizes at storage rings and linear colliders [1-6]. In this method, a laser beam with a diameter of about one wavelength moves across the electron beam and Compton scattered photons are detected. Typically, the probability of Compton scattering per one electron is about 10^{-7}. There is a common belief that only scattered electrons are lost and nothing happens with the other electrons which cross the laser beam, though they oscillate inside the laser field, but the resulting kick is zero.

On the other hand, the diameter of the laser beam is about λ, the laser field E_L during the electron crossing is almost unipolar and one can expect the kick $P_{\perp}c \sim eE_L\lambda$. [1] Using the standard formula for the Gaussian laser beam and performing integration of the Lorentz force along the unperturbed electron trajectory one would also get a non-zero result, though somewhat lower than given above, but again $P_{\perp} \propto E_L$.

Below we show that the kick is non-zero and depends on the laser field quadratically.

$$P_{\perp}c \sim \frac{mc^2k}{\alpha\gamma} \propto E_L^2, \quad (1)$$

where k is Compton scattering probability, m the electron mass, c the speed of light, $\gamma = \mathcal{E}_e/mc^2$, $\mathcal{E}_e$ is the electron energy, $\alpha = e^2/\hbar c = 1/137$ and all numerical coefficients are omitted. In most practical applications of the laser wire this effect can be neglected, except for low energy beams and large conversion coefficients.

Physics of this non-zero result is very interesting and instructive. This subject was actively discussed during the last decade in physics community working on laser acceleration of electrons and this discussion is still continued.

2 INTERACTION OF ELECTRONS WITH A LASER BEAM

Below we consider electron-laser interactions by two methods that allow us to understand better the origin of a non-zero energy-momentum exchange between electrons and laser beams.

2.1 *Considerations based on conservation of the energy*

The electron energy and momentum can change only if something happens with the laser beam (beside Compton scattering where all is clear). The electron static field is the same before and after crossing the laser beam. If the laser field is also unchanged then the electron kinetic energy should be conserved. Let us take now into account the electron radiation field. Integrating the energy density of the electromagnetic field before and after the interaction one gets a net change of the electron energy:

$$\Delta\mathcal{E}_e \sim \int (\vec{E}_L + \vec{E}_r)^2 dV - \int E_L^2 dV$$
$$\approx \int (2\vec{E}_L\vec{E}_r + E_r^2) dV, \quad (2)$$

where E_L is the laser field, E_r the electron radiation field which interferes with the laser field.

In the presence of matter near the interaction region, then the electron can radiate without presence of the laser beam. In this case E_r is independent of the laser field and therefore $\Delta\mathcal{E}_e \propto E_rE_L \propto E_L$. Namely this mechanism explains the energy gain in linear accelerators where the accelerating gradient is just proportional to the RF field. After acceleration of the electron bunch, the field in the linac becomes weaker. The electron bunch radiates to many cavity modes, but the mode which coincides with the accelerator field gives the main contribution due to the interference of a rather weak radiated field with a strong RF accelerator

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[1] In this paper we omit all numerical coefficients.

field. Depending on the phase it may be acceleration or deceleration of the beam. All other radiated modes just give some small energy loss $\Delta\mathcal{E}_e \propto E_r^2$. This picture is well known in the accelerator community.

In the case of the open space we are interested in, the electron radiates only due to the acceleration in the laser field and therefore $E_r \propto E_L$ and $\Delta\mathcal{E}_e \propto E_L^2$.

Let us estimate $\Delta\mathcal{E}_e$ for the case when the laser beam diameter is about one wavelength and the electron intersects it perpendicular. The radiation field in the dipole approximation at large angles (only such radiation can interfere with the laser beam) is equal

$$E_r \sim \frac{ea}{rc^2}, \quad a \sim \frac{eE_L}{\gamma m} \quad \Rightarrow \quad E_r \sim \frac{e^2 E_L}{\gamma mc^2 r}, \tag{3}$$

where a is the electron acceleration and r the distance from the electron. As the laser is focused on the spot with a diameter $\sim \lambda$ the characteristic volume is $\Delta V \sim \lambda^3$, $r \sim \lambda$. As result we get

$$\Delta\mathcal{E}_e \sim E_L E_r \Delta V \sim \frac{e^2 E_L^2 \lambda^2}{mc^2\gamma} \sim \frac{mc^2\xi^2}{\gamma}, \tag{4}$$

where

$$\xi^2 = \frac{e^2\langle E_L^2\rangle}{m^2c^2\omega^2} \tag{5}$$

is the parameter characterizing the nonlinear effects in Compton scattering (or the undulator parameters in wigglers). The energy exchange between the electron and the laser wave $\Delta\mathcal{E}_e$ corresponds to absorption of laser photons and emission to the laser wave without Compton scattering. Due to the laser divergence, absorption and emission of photons with different directions may result in the kick of the electron in the direction perpendicular to the laser beam. Therefore (4) is also the estimate of the transverse momentum kick $P_\perp$.

A free electron can not absorb or emit one photon but virtual absorption and reemission (to some other direction) is allowed and gives a net kick. The two-photon nature of the considered effect is seen from the quadratic dependence of the force on the field strength.

2.2 *Action of a ponderomotive force*

Let us consider the same effect in a different way, in the language of ponderomotive forces. In the non-uniform laser field the electron undergoes fast oscillations and drifts to the region with the lower field. The latter can be understood in the following way. The energy of the relativistic electron oscillating in a weak laser field is [2]

$$\mathcal{E}_e \sim \sqrt{P^2c^2 + P_x^2c^2 + m^2c^4} \sim \mathcal{E}_{e0} + \frac{P_x^2c^2}{2\mathcal{E}_{e0}}, \tag{6}$$

where $P_x = (eE_L/\omega)\cos\omega t$, ω is the laser frequency. Substituting P_x and averaging the energy over the fast oscillations we get

$$\mathcal{E}_e = \mathcal{E}_{e0} + \frac{mc^2\xi^2}{2\gamma}. \tag{7}$$

The second term can be considered as the potential energy. The corresponding force is the well known ponderomotive force [7, 8] which pushes out the electron from the laser field

$$F_p = \frac{mc^2}{2\gamma}\nabla\xi^2. \tag{8}$$

Taking $\nabla\xi^2 \sim \xi^2/r$ (r is the radius of the laser beam) we find the kick

$$\Delta P_\perp c \sim F_p r \sim \frac{mc^2\xi^2}{\gamma}. \tag{9}$$

This estimate coincides with (4) obtained in a completely different way. It is more transparent and in addition gives a well defined direction of the force. Eqs.(8),(9) are valid for $r \gg \lambda$ (for many oscillations) but can be used down to $r \sim \lambda$ as an estimate.

Considering the ponderomotive force we did not mention radiation fields. They are hidden in the fast oscillations of the electron. If one would neglect the electron motion under influence of the laser field, then the integration of the Lorentz force along the straight electron trajectory would give a zero result.

The non-zero result for the Gaussian laser beams mentioned in the beginning of the paper is connected with the fact that a Gaussian description of the field near the laser focus is just a paraxial approximation corresponding to the case when the laser diameter is much larger than the wavelength. In the general case of large diffraction angles, the Gaussian description is not valid because it does not obey the Maxwell equations. There are papers where high order corrections to the Gaussian beams are found [9]. But even without formulas it is clear that integration of the force along any trajectory (ignoring electron motion in the field) can give only a result which is proportional to the field strength (because the Lorentz force is proportional to the field strength). But the linear term in the case of free space is forbidden by simple energy considerations arguments discussed above. This statement is known as the Lawson-Woodward theorem [10].

3 ESTIMATION OF THE ELECTRON KICK BY THE LASER WIRE

Let us express (4,9) in terms of the Compton scattering probability (or the conversion coefficient k) which is

$$k = n_\gamma \sigma_c l \sim \frac{e^4 E_L^2}{\hbar\omega^2 m^2 c^3}, \tag{10}$$

where $n_\gamma \sim E_L^2/(\hbar\omega)$ is the density of laser photons, $\sigma_c \sim r_e^2 = e^4/m^2c^4$ is the Compton cross section, $l \sim \lambda$ is the

[2] For simplicity we assume that the laser electrical field is perpendicular to the electron direction, a general case gives similar result.

diameter of the laser beam. Using (5,10) we can rewrite (9) as

$$\Delta P_{\perp} c \sim \frac{kmc^2}{\alpha\gamma}. \tag{11}$$

This kick should be compared with the rms transverse momenta of electrons in the beam which are by definition

$$P_{\perp} c = mc^2 \sqrt{\frac{\epsilon_n \gamma}{\beta}}, \tag{12}$$

where ϵ_n is the normalized emittance and β the beta function. The relative value of the transverse momentum spread is

$$\frac{\Delta P_{\perp}}{P_{\perp}} \sim \frac{k}{\alpha\gamma}\sqrt{\frac{\beta}{\epsilon_n \gamma}}. \tag{13}$$

For $\epsilon_n \sim 10^{-6}$ cm (the minimum vertical normalized emittance considered at damping rings for linear colliders), $\gamma \sim 5\times 10^3$ (damping ring), $\beta = 300\sqrt{\mathcal{E}_e(\mathrm{GeV})}$ cm ~ 500 cm, $k \sim 10^{-7}$ we get

$$\frac{\Delta P_{\perp}}{P_{\perp}} \sim 10^{-6}, \tag{14}$$

which is negligible.

4 CONCLUSION

Electrons passing the laser wire do not only undergo Compton scattering (with a small probability), but in addition all electrons receive transverse kicks with 100% probability . Fortunately, these kicks are rather small and can be ignored in most applications.

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