LC-PHSM-2000-60-TESLA
29th December 2000

Measurement of the Differential Luminosity using Bhabha events in the Forward-Tracking region at TESLA

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Abstract

For most analyses at an e^+e^--linear collider a certain knowledge of the luminosity spectrum after beamstrahlung is needed. The most promising possibility to measure this spectrum is to analyze the acolinearity distribution of a two fermion process. Bhabha scattering in the forward tracking region is such a process with a high enough cross section.
In this note a possible analysis is simulated and the precision reachable at TESLA is studied.

1 Introduction

One of the few unpleasant features of an e^+e^--linear collider is the photon-radiation of an electron[1] in one bunch against the coherent field of the electrons in the colliding bunch. This effect is usually referred to as beamstrahlung. The amount of beamstrahlung and the details of the spectra depend strongly on the machine design, but for current designs the average energy loss of a colliding e^+e^--pair at $\sqrt{s} = 500\,\mathrm{GeV}$ is around 1.5% [1]. Beamstrahlung is in many respects similar to initial state radiation with a large peak at zero energy loss and a long tail, however very hard radiation and large photon angles are cut off by the finite bunch size. Figure 1 shows the expected energy spectrum for colliding electrons at 500 GeV at TESLA. In all physics processes beamstrahlung and normal initial state radiation are adding up. Figure 2 shows the $\sqrt{s'}$-spectra for large angle Bhabha scattering with initial state radiation (ISR) only and with ISR+beamstrahlung. For soft and medium radiation ISR and beamstrahlung are about the same size. For hard radiation the spectrum is completely dominated by ISR. In addition to beamstrahlung the beam energies will be smeared by the usual energy spread of the machine.

In any physics analysis some knowledge of the beamstrahl-spectrum is required. In threshold scans or analyses of missing mass peaks for example, mainly the fraction of events in the peak and close by is needed while for continuum processes mostly the average center of mass energy for the events within the acceptance cuts is relevant.

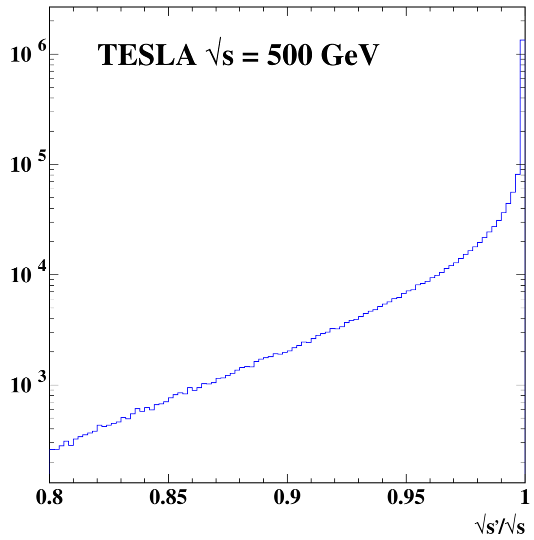

Figure 1: $\sqrt{s'}$ spectrum for $\sqrt{s} = 500\,\mathrm{GeV}$ at TESLA.

In a two fermion process the mass of the $\mathrm{f\bar{f}}$-system can be reconstructed with high precision from the fermion angles, if it is known that one photon is emitted at zero angle. If more photons are emitted occasionally the spectrum can still be reconstructed on a statistical basis. With Bhabha scattering ($e^+e^- \to e^+e^-$) in the forward region such a process is available with high cross section. Bhabha scattering at lowest order proceeds through s- and t-channel photon and Z-exchange (see Fig. 3), where the t-channel photon exchange is divergent for zero scattering angle. The differential cross section for the pure

[1] Unless otherwise stated "electron" is always meant as "electron or positron".

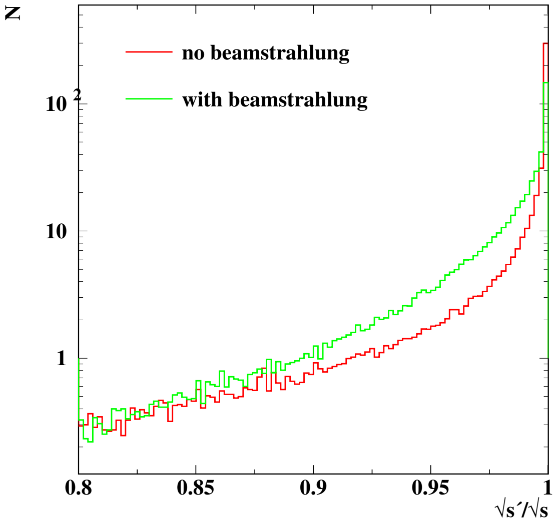

Figure 2: $\sqrt{s'}$ after initial state radiation for Bhabha events with and without beamstrahlung for $\sqrt{s} = 500\,\mathrm{GeV}$.

t-channel process scales as $d\sigma/d\cos\theta \propto 1/\theta^3$. Figure 4 shows the accepted cross section as a function of the low angle cutoff of the detector. For the cutoff of the TESLA detector, $\theta > 7°$, the accepted cross section is 300 pb.

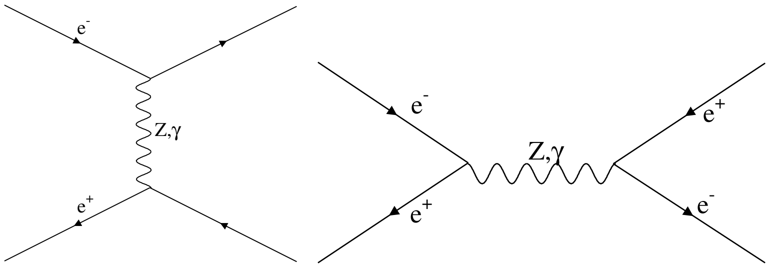

Figure 3: Feynman diagrams for Bhabha scattering.

In the following it will be shown, how this process can be used to reconstruct the beamstrahl spectra at a linear e^+e^--collider like TESLA.

2 $\sqrt{s'}$ reconstruction

If in a process $e^+e^- \to f\bar{f}$ one photon is radiated at zero angle with respect to one beam[2] the effective collision energy can be obtained from the fermion angles only:

$$\frac{\sqrt{s'}}{\sqrt{s}} = \sqrt{1 - 2\frac{\sin(\theta_1 + \theta_2)}{\sin(\theta_1 + \theta_2) - \sin\theta_1 - \sin\theta_2}} \tag{1}$$

where θ_1, θ_2 are the fermion angles relative to the assumed photon direction (i.e. $\theta_1 + \theta_2 > \pi$, see Fig. 5).

[2]More exact: all photons are radiated in the same direction

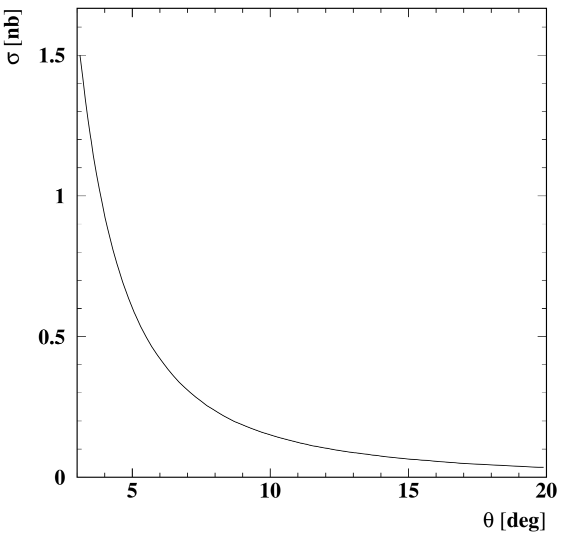

Figure 4: Bhabha cross section at $\sqrt{s} = 500\,\mathrm{GeV}$ as a function of the low angle cutoff of the detector.

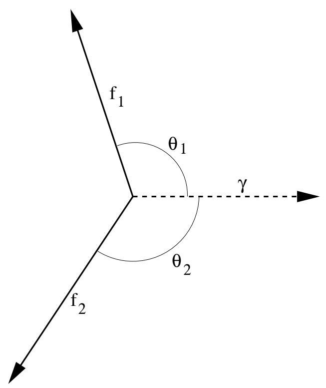

Figure 5: Principle of the s' reconstruction.

As an example Fig. 6 shows the reconstructed versus the true $\sqrt{s'}$ for Bhabha scattering with initial state radiation only. One can see that for most events $\sqrt{s'}$ is reconstructed well, however there is a significant tail to larger $\sqrt{s'}$ stemming from events where photons in both directions have been radiated.

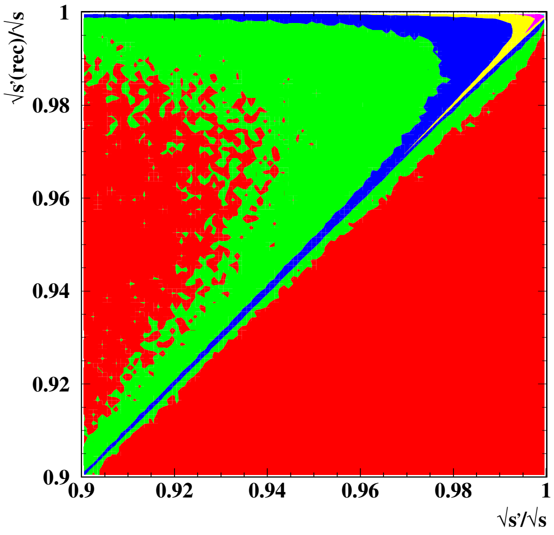

Figure 6: Reconstructed versus true $\sqrt{s'}$ for Bhabha scattering.

The picture is complicated by the beam energy spread. Per construction one has $\sqrt{s'} < \sqrt{s}$ while beam energy spread is approximately symmetric around the mean value. Figure 7 shows the change in $\sqrt{s'}$ for events without and with hard radiation with the assumption of a Gaussian beamspread. In the case of no radiation the beamspread spectrum is simply folded at $\sqrt{s'}/\sqrt{s} = 1$. Due to this feature it is impossible to separate the two effects in a model independent way. Instead one has to make assumptions on the shapes of the spectra in the fit.

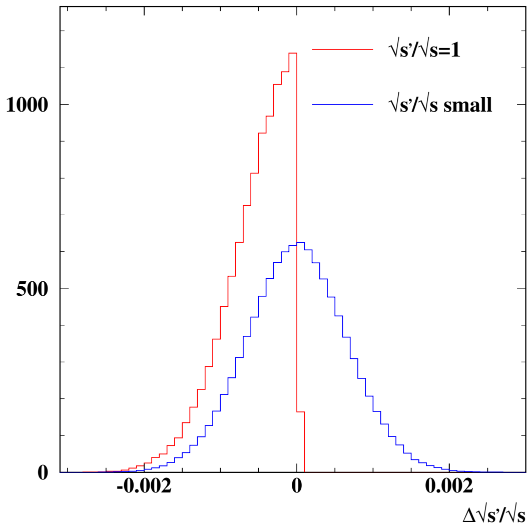

Figure 7: Effect of beamspread on reconstructed $\sqrt{s'}$.

In the limit of small radiation equation (1) reduces to $\sqrt{s'}/\sqrt{s} = 1 - \frac{1}{2}\frac{\Delta\theta}{\sin\theta}$ with $\Delta\theta = \theta_1 + \theta_2 - \pi$ and $\theta = \frac{1}{2}(\theta_1 - \theta_2 + \pi)$[3]. Detector resolution has the same effect on the measured $\sqrt{s'}$ spectra as the energy spread. In order to be able to measure the energy spread of the order of 0.1% from the acolinearity distributions the detector resolution has thus to be much better than $10^{-3} \times \sin 7^\circ \approx 10^{-4}$. As can be seen from Fig. 8 this will be achieved with the silicon trackers alone at the proposed TESLA detector.

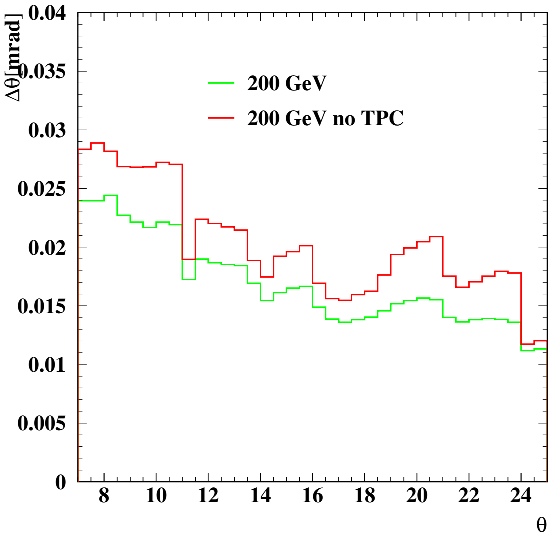

Figure 8: Polar angle resolution of the TESLA detector.

3 Fit method

On the microscopic level beamstrahlung can be simulated with the program GuineaPig [2] which simulates the details of the bunch crossing using as input the beam parameters of the electron and positron beam. To have a fast simulation of beamstrahlung the program CIRCE [3] has been written which parameterizes beamstrahlung according to

$$f(x) \;=\; a_0\delta(1-x) + a_1 x^{a_2}(1-x)^{a_3}, \tag{2}$$

where the assumptions are made that the beamstrahlung in the two beams is equal and uncorrelated between the beams. In this analysis it is assumed that the true spectrum is described by this functional form (eq. (2)). a_0, a_2 and a_3 are then fitted to simulated $\sqrt{s'}$ spectra, while a_1 is given by the normalization condition $\int f(x)\mathrm{d}x = 1$. The default parameters for $\sqrt{s} = 500\,\mathrm{GeV}$, which have also been used in Fig. 1 are

$$\begin{aligned} a_0 &= 0.5274 \\ a_2 &= 13.895 \\ a_3 &= -0.6314. \end{aligned}$$

Figure 9 shows the variation of the beamstrahl spectrum, when the three parameters are varied individually.

[3]Note that for back to back events $\theta_1 + \theta_2 = \pi$ so that $\Delta\theta$ is the acolinearity.

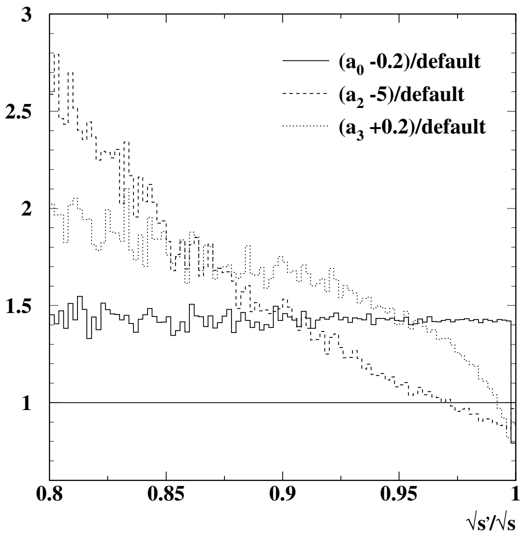

Figure 9: Ratio of the beamstrahl spectrum with modified parameters to the default one.

In a first analysis the beamspread has been neglected. As input "measured" $\sqrt{s'}$-spectra have been generated at $\sqrt{s} = 500\,\mathrm{GeV}$ using the generator Bhwide [4] where Beamstrahlung has been added with the TESLA default. The outgoing electrons have been smeared according to the resolution of the detector. In the analysis it is assumed that the effect of Beamstrahlung is linear in the parameters for some range around the default value in each bin of $\sqrt{s'}$. Thus four simulated $\sqrt{s'}$ spectra are needed, one with default parameters (MC_{def}) and one each with a_0, a_2 and a_3 varied by Δa_i (MC_i).

The data are then fitted, minimizing the χ^2 function

$$\chi^2(a_0, a_2, a_3) \;=\; \sum_{j=\mathrm{bins}} \left(\frac{N^{(j)}_{\mathrm{data}} - N^{(j)}_{\mathrm{pred}}(a_0, a_2, a_3)}{\sqrt{N^{(j)}_{\mathrm{data}}}} \right)^2$$

with

$$N^{(j)}_{\mathrm{pred}}(a_0, a_2, a_3) \;=\; MC^{(j)}_{\mathrm{def}} + \sum_i \frac{a_i - a_i^{(\mathrm{def})}}{\Delta a_i} \left(MC^{(j)}_i - MC^{(j)}_{\mathrm{def}} \right).$$

It has been verified that the true values of a_i are reproduced by the fit for several standard deviations around the default value. For an integrated luminosity of $3\,\mathrm{fb}^{-1}$ corresponding to 10^6 Bhabha events the results reported in Table 1 have been obtained.

Par.	default	error	correlation matrix		
a_0	0.5274	0.0014	1.000	0.430	0.708
a_2	13.895	0.082	0.430	1.000	0.755
a_3	-0.6314	0.0021	0.708	0.755	1.000

Table 1: Results of the fit without beamspread for $\sqrt{s} = 500\,\mathrm{GeV}$ and $\mathcal{L} = 3\,\mathrm{fb}^{-1}$.

Integrating over the full spectrum the error on every single parameter corresponds to an error in the mean $\sqrt{s'}$ of $\Delta \left\langle \frac{\sqrt{s'}}{\sqrt{s}} \right\rangle \sim 6 \cdot 10^{-5}$. Taking into account correlations the total error on $\sqrt{s'}$ is $\Delta \left\langle \frac{\sqrt{s'}}{\sqrt{s}} \right\rangle = 4 \cdot 10^{-5}$.

In a second step the beamspread has been included into the analysis. For the simulated data the individual events have been smeared by the assumed beamspread. For the prediction the $\sqrt{s'}$ spectra without the beamspread have been folded with the beamspread function during the fit:

$$\begin{aligned} f_{\mathrm{BS}}(x) &= \int_0^1 \mathrm{d}x' f(x') BS'(x'-x,x) \\ BS'(\Delta,x) &= BS(\Delta) + BS(2-2x-\Delta) \end{aligned}$$

with $x = \sqrt{s'}/\sqrt{s}$. The function BS describes the shape of the beamspread. The second term in BS' accounts for the fact that $\sqrt{s'}/\sqrt{s}$ is per construction less than one. Adding a Gaussian beamspread of 0.1% the results summarized in Table 2 have been obtained.

The error in each of the beamstrahl parameters corresponds to $\Delta \left\langle \frac{\sqrt{s'}}{\sqrt{s}} \right\rangle \sim 15 \cdot 10^{-5}$. However due to the large correlations the total error in $\left\langle \frac{\sqrt{s'}}{\sqrt{s}} \right\rangle$ is only $5 \cdot 10^{-5}$.

Par.	default	error	correlation matrix			
a_0	0.5274	0.0048	1.000	0.819	0.963	0.544
a_2	13.895	0.14	0.819	1.000	0.903	0.338
a_3	-0.6314	0.0061	0.963	0.903	1.000	0.474
BS	$\sim 10^{-3}$	$0.5 \cdot 10^{-5}$	0.544	0.338	0.474	1.000

Table 2: Results of the fit including beamspread for $\sqrt{s} = 500\,\mathrm{GeV}$ and $\mathcal{L} = 3\,\mathrm{fb}^{-1}$.

To get a feeling for the sensitivity to the beamspread shape a rectangular beamspread has been generated which then has been fitted with a Gaussian. The resulting changes in the beamstrahl parameters are:

$$\begin{aligned} \Delta a_0 &= 0.015 \\ \Delta \left\langle \frac{\sqrt{s'}}{\sqrt{s}} \right\rangle &= 10^{-4} \end{aligned}$$

However the χ^2 if the fit gets much worse, so that some free parameters for the beamspread shape should be possible.

3.1 Correlations

In the analysis up to now it was assumed that the beamstrahlung is uncorrelated between the two beams. However there should be correlations between the two beams at least because of two reasons: The field seen by an electron depends on its transverse position in the bunch. This position has to be equal for the two colliding electrons leading to a correlation. At the beginning of the bunch crossing the colliding electrons had only very little chance to radiate while at the end they have already traversed the whole opposite bunch before they annihilate and thus have radiated with a high probability. Also this effect causes correlations.

To test the effect of correlations the GuineaPig output has been used directly instead of using Circe. In a first run the colliding electrons have been taken directly from the GuineaPig events while in a second run the electron and positron have been taken from different events eliminating correlations. To enlarge the significance the beamspread has been set to zero. Both event samples have been fitted with the described procedure and the following results have been found:

$$\begin{aligned}
\Delta a_0 &= 0.012 \\
\Delta a_2 &= 0.12 \\
\Delta a_3 &= 0.0024 \\
\Delta \left\langle \frac{\sqrt{s'}}{\sqrt{s}} \right\rangle &= 3 \cdot 10^{-4}
\end{aligned}$$

The effect for a_2 and a_3 is about the size of the statistical error. However for both runs the same electron beamstrahlung and the the same Bhabha events have been used and only the positron beamstrahlung was different, so that the true statistical error is probably much smaller. In any case the change in a_0 is highly significant, resulting in an effect on $\left\langle \frac{\sqrt{s'}}{\sqrt{s}} \right\rangle$ corresponding to 8σ with $3\,\mathrm{fb}^{-1}$. With Monte Carlo studies one thus needs to understand correlations on the few percent level. The correlation study has been repeated including beamspread. The conclusions on the correlations stayed the same.

4 Conclusions

The possibility to measure the luminosity spectrum and the beamspread using Bhabha scattering in the forward region of the TESLA detector has been studied. With an integrated luminosity of $3\,\mathrm{fb}^{-1}$ at $\sqrt{s} = 500\,\mathrm{GeV}$ the mean energy loss due to beamstrahlung can be measured with a statistical precision of $5 \cdot 10^{-5}$ and the beamspread to $0.5 \cdot 10^{-5}$. Systematic errors also seem to be controllable at this level. The very good detector resolution is mainly important to measure the beam energy spread, however, since the exact shape of the beamspread and the detector resolution affects the results for the luminosity spectrum it helps also to reduce systematic errors.

Acknowledgments

I'd like to thank Thorsten Ohl for useful discussions about Circe and Karsten Büßer for producing the GuineaPig spectra needed for the correlation studies.

References

[1] See e.g. contributions by R. Brinkmann, T. Tauchi and K.A. Thompson et al. in Proceedings of the Worldwide Study on Physics and Experiments with Future Linear e^+e^--colliders, Sitges, April 1999, Vol. 2.

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