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Direct Determination of the CKM Matrix from Decays of W Bosons and Top Quarks at High Energy e^+e^- Colliders

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Abstract

At proposed high energy linear e^+e^- colliders a large number of W bosons and top quarks will be produced. We evaluate the potential precision to which the decay branching ratios into the various quark species can be measured, implying also the determination of the respective CKM matrix elements. Crucial is the identification of the individual quark flavours, which can be achieved independent of QCD models. For transitions involving up quarks the accuracy is of the same order of magnitude as has been reached in hadron decays. We estimate that for charm transitions a precision can be reached that is superior to current and projected traditional kinds of measurements. The $t \to b$ determination will be significantly improved, and for the first time a direct measurement of the $t \to s$ transition can be made. In all cases such a determination is complementary to the traditional way of extracting the CKM matrix elements.

1 Introduction

There are nineteen fundamental parameters which cannot be derived from first principles in the Standard Model [1,2] with three fermion generations and massless neutrinos. Three of these are the mixing angles of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [3] which express the probability for a transition between different quark species through charged weak currents. As yet it is unclear how the number and existence of generations can be derived from first principles of an underlying theory. Thus, no prescription exists on how to calculate the values of the CKM matrix elements. Still, it is important to measure their values to the best precision possible. Up to now the CKM matrix elements have been derived from hadron decays using QCD symmetries to extend the theoretical analysis into a low energy scale. The precision of such measurements is therefore often limited by uncertainties reflecting theoretical models and the assumptions invoked. In particular, the transitions involving top quarks can only be indirectly determined, using either $\mathrm{B_d}$, $\mathrm{B_s}$ mixing or $\mathrm{b} \rightarrow \mathrm{s}\gamma$ decays.

The direct observation of the decay products of W bosons and top quarks offers a complementary way to determine the CKM matrix elements. In case of W bosons one may determine the decay widths of W bosons into identified pairs of quarks[1]. Since the branching ratio of the W bosons into a specific pair of quarks (q, q') is proportional to the square of the CKM matrix element $|V_{qq'}|$,

$$B(\mathrm{W} \rightarrow qq') \ \propto \ |V_{qq'}|^2 \ ,$$

the measurement of the decay fractions allows a direct determination of the CKM matrix elements. Similarly one can relate the transitions of $B(\mathrm{t} \rightarrow q\mathrm{W})$ to the CKM matrix elements $|V_{\mathrm{t}q}|$. In practice this requires that all decay modes of the W bosons and top quarks can be reconstructed with a good signal to background ratio, which is the case at $\mathrm{e^+e^-}$ colliders. Experimentally much more complicated is the requirement that all quark species can be individually identified. A first step to determine the CKM matrix elements in W decays has been made at LEP by deriving $|V_{\mathrm{cs}}|$ from the total hadronic W branching ratio [4] or more directly from the measurement of the inclusive decay fraction into charm quarks [5,6]. In both cases the other CKM matrix elements are used as a crucial input to the analysis when its unitarity is assumed. Our method does not rely on these assumptions and provides a determination of the matrix elements that is complementary to traditional means.

Quark tagging for charm and bottom is based on their well understood and unambiguous special properties, particularly their masses and long lifetimes. The identification of light quarks with a precision needed for a meaningful measurement of the CKM matrix elements is much more involved. Several analyses [7] have used light flavour tagging methods based on model assumptions such as those used in JETSET [8] and HERWIG [9] at the price of sizeable uncertainties. A method was suggested in [10] to

[1] To simplify the text, we drop the distinction between particles and antiparticles where the meaning it otherwise clear.

determine individual up, down and strange quark tagging efficiencies from Z^0 data, thus avoiding inherently ambiguous assumptions about the process of hadronisation. Such an analysis was recently carried out by the OPAL collaboration using 2.8 million Z^0 decays recorded at LEP [11].

The main idea for determining the fraction of light flavours is that particles with a large fraction $x_p = 2p/E_{\rm cm}$ of the momentum, p, relative to the centre-of-mass energy, $E_{\rm cm}$, carry information about the primary flavour [12]. At the Z^0 the yields of single tags, where just one jet is tagged by a high x_p particle, and the numbers of double tags, where both jets are tagged, can be used to determine the tagging efficiencies η_q^i with minimal reliance on assumptions about hadronisation. Here η_q^i is the probability that a quark q leads to a particle i that has the highest momentum in an event hemisphere[2].

The efficiencies can be applied almost directly to the decays of W bosons in order to determine the hadronic branching fractions

$$R_{qq'} = \frac{B(\mathrm{W} \to qq')}{B_h} \ ,$$

where B_h is the inclusive branching ratio of the W to hadrons, $B(\mathrm{W} \to \text{hadrons})$. The numbers of single tags and double tags from W boson decays can be expressed by the tagging efficiencies η_q^i obtained from the analysis of Z^0 decays and the $R_{qq'}$. One obtains an over-constrained linear equation system which can be solved for the $R_{qq'}$. With the current samples at LEP of some 20000 W bosons per experiment, the precision of these decay fractions will be fairly limited. The high luminosity at proposed linear e^+e^- colliders both for running at the Z^0 and at high energies above the W pair threshold offers unique possibilities to pursue CKM matrix measurements with substantially higher precision. A similar equation system can be constructed for top decays. In this note we will outline a strategy and estimate the potential precision.

The outline of the paper is as follows. In Section 2 we detail the assumptions on accelerator and detector performance made for this study. The formalism and experimental aspects of the determination of light flavour tagging efficiencies at the Z^0 peak are discussed in Section 3. The method to determine the W boson branching ratios is given in Section 4 and their translation into the CKM matrix elements in Section 5. Direct measurements using top quarks are discussed in Section 6 before concluding in Section 7.

2 Assumptions on Data and Detector Performance

For definiteness we assume in this paper the parameters of the TESLA [13] option for a future e^+e^- linear collider. However, our proposal is in no way restricted to this option

[2] An event hemisphere is defined by the plane perpendicular to the event thrust axis and passing through the origin. In this analysis, we denote hemispheres as representing quark jets, since we are interested in studying the evolution of primary quarks into different hadron types.

	$\epsilon_{\rm c}$	$\epsilon_{\rm b}$	$\epsilon_{\rm uds}$
charm quark tag	0.60	0.20	0.04
bottom quark tag	0.02	0.50	0.0008
light quark tag (W analysis)	0.50	0.10	0.99
light quark tag (top analysis)	n.a.	0.01	0.50

Table 1: Expected efficiencies at TESLA for algorithms optimised to identify charm and bottom quarks. Also listed are the efficiencies assumed for light quark tagging which are different for the W boson and top quark analyses.

and can be pursued at any linear collider allowing for high luminosity measurements at the Z^0 peak (GigaZ) and energies above the top pair threshold. For running at the Z^0 peak we assume an instantaneous luminosity of $\mathcal{L}_{Z^0} = 7 \times 10^{33}$ sec^{-1}cm^{-2} [14]. For a nominal year of running (i.e. 100 days of full efficiency) this implies 70 fb^{-1} of data, or some factor 500 more than collected by each LEP experiment over five years. We will therefore assume for this study a sample of 2×10^9 hadronic Z^0 decays. At centre-of-mass energies of 500 GeV a luminosity of $\mathcal{L}_{HE} = 3 \times 10^{34}$ sec^{-1}cm^{-2} is expected [15].

The detector capabilities which are relevant for our method to determine the CKM matrix elements are in particular hadron and, to a lesser extent, lepton identification. In addition, it is crucial to identify secondary vertices to tag charm and bottom quarks with high efficiency and purity.

Bottom and charm tagging at TESLA has been discussed in detail in [16]. From these considerations we assume efficiencies and purities as given in Table 1. These identification potentials[3] are far better than what has been achieved at LEP because of the smaller beam pipe and the advanced micro vertex detectors foreseen at TESLA. By selecting jets without a prominent secondary vertex one can also increase the purity of a light flavour sample. The optimisation of the purity and efficiency of such tags depends on the process. Different working points are used for the analyses of W boson and top quark production.

No strong emphasis has yet been placed on hadron identification for a TESLA detector. However, the proposed TPC as a central detector offers the possibility to determine the particle species by measuring the ionisation loss. Current estimates assume a $\mathrm{d}E/\mathrm{d}x$ resolution of 4.5%. However, improvements may be possible [18]. The momentum resolution will be substantially better than at LEP, helping to reconstruct invariant masses of resonances. On the other hand, the acceptance for long-lived particles such as $\mathrm{K^0_S}$ and Λ is reduced due to the higher momentum of these hadrons. Their large boost implies that a significant fraction of these will decay outside the tracking detectors. For definiteness we assume the same particle identification capabilities as obtained with the

[3] The study of [16] suggests a dependence of the tagging efficiency and purity on the centre-of-mass energy. However, this is due to the optimisation of the tagging algorithms at Z^0 energies which is then applied to higher energies. Significant improvements can be expected if the algorithms are specifically optimised at high energies [17]. For simplicity we therefore assume the efficiency and purity of the algorithms to be independent of energy.

assigned	true				
	$\pi^\pm$	$\mathrm{K}^\pm$	$\mathrm{p}(\overline{\mathrm{p}})$	K^0_S	$\Lambda(\overline{\Lambda})$
$\pi^\pm$	0.790	0.062	0.003	0.038	0.005
$\mathrm{K}^\pm$	0.146	0.568	0.148	0.017	0.026
$\mathrm{p}(\overline{\mathrm{p}})$	0.040	0.246	0.551	0.014	0.081
K^0_S	0.081	0.030	0.007	0.691	0.026
$\Lambda(\overline{\Lambda})$	0.047	0.024	0.024	0.128	0.696
efficiency	0.487	0.441	0.292	0.155	0.135

Table 2: Fractional compositions of the identified samples (rows) in terms of the true tagging particle, for $x_p > 0.2$, taken from [19]. The rows do not add up to unity because of additional contributions that are not used as tags. The bottom row gives the average efficiency to correctly tag a hemisphere.

OPAL detector [19] at LEP, listed in Table 2, and do not attempt to calculate the potential small differences in particle identification efficiencies.

3 Tagging Efficiencies from Z^0 Decays

3.1 Formalism

In this section we briefly summarise the basic formalism to determine tagging efficiencies from Z^0 data as suggested in [10] and extrapolate a recent LEP analysis [11] to TESLA conditions. As mentioned before, charm and bottom tags can be selected with high efficiency and purity using secondary vertex finding. The determination of the efficiency for light flavours is more complicated. The light flavour tags are based on particle types that are easy to identify, have a significant yield and which carry information about the primary flavour. As discussed in [20] such types are $\pi^\pm$, $\mathrm{K}^\pm$, protons, K^0_S, and Λ. We consider these particles as tagging particles if they have the scaled momentum in an event hemisphere $x_p = 2p/M_{\mathrm{Z}^0} > 0.2$, where the momentum, p, is determined in the Z^0 rest frame. Lifetime information yields an excellent separation of light quark jets from those of bottom and charm origin. Thus, we apply the high x_p tag only to those jets that have no apparent secondary vertex, i.e. are tagged as light flavours. We assume the efficiencies listed in the fourth row of Table 1. In principle jets can fulfil in parallel the requirements for light, charm and bottom tagging. To avoid double counting we assume the following priority among tags according to the achievable purity : bottom, charm and light flavour tags. If a jet is tagged by more than one of those only the higher priority tag is considered.

In determining the efficiencies $\eta^i_{\mathrm{u,d,s}}$ no assumption is made about the details of the hadronisation process like hardness, shape of the fragmentation functions, light flavour composition in the hadronisation phase or the amount of resonance production. No such information from QCD models like JETSET [8] or HERWIG [9] is needed. The only assumption that is invoked is that the branching ratios of the Z^0 into fermion pairs are as predicted by the Standard Model [21]. This is consistent with the high precision results obtained at LEP and can, at least for some flavours, be very accurately tested at GigaZ.

As detailed in [10, 20] the η_q^i are determined by using tags in event hemispheres of a hadronic Z^0 decay. Each event is separated into two hemispheres using the plane perpendicular to the thrust axis containing the interaction point. In each hemisphere a secondary vertex is searched for and in case of a light flavour tag the highest momentum particle, labelled i, subject to the requirement $x_p > x_{\text{min.}}$, some minimum value. What can be directly observed are the number of "single-tagged hemispheres" tagged as type i, labelled N_i, and the number of "double-tagged events" containing a tag in both hemispheres, labelled N_{jk}, where j and k are the tagging particle types. The tagging probability η_q^i is then given by

$$\eta_q^i = \frac{N_{q\to i}}{N_q}.$$

for a number, N_q, of hemispheres which originate from a quark of type q and a number, $N_{q\to i}$, of these with tags of type i. The event counts are related to the tagging probabilities by:

$$\frac{N_i}{N_{\mathrm{Z}^0}^{\mathrm{had}}} = 2 \sum_{q=\mathrm{d,u,s,c,b}} \eta_q^i \, R_q \tag{1}$$

$$\text{and} \quad \frac{N_{jk}}{N_{\mathrm{Z}^0}^{\mathrm{had}}} = (2-\delta_{ij})\rho_{\mathrm{Z}^0} \sum_{q=\mathrm{d,u,s,c,b}} \eta_q^j \, \eta_q^k \, R_q, \tag{2}$$

where $\delta_{jk} = 1$ if $j = k$ and zero otherwise and $N_{\mathrm{Z}^0}^{\mathrm{had}}$ is the number of hadronic Z^0 decays. Note that the η_q^i include possible distortions due to detector effects. The parameter ρ_{Z^0} will not be unity if there are correlations between the tagging probabilities in opposite hemispheres, due to kinematical or geometrical effects, for example. R_q is the hadronic branching fraction of the Z^0 to quarks q:

$$R_q = \frac{\Gamma_{\mathrm{Z}^0\to q\bar{q}}}{\Gamma_{\mathrm{had}}}.$$

They are taken to be the Standard Model values [21].

In [10] several so-called "hadronisation relations" based on approximate SU(2) symmetries between the different η_q^i were proposed, such as $\eta_{\mathrm{s}}^{\mathrm{K}^-} \approx \eta_{\mathrm{s}}^{\mathrm{K}^0}$. These extra constraints are necessary to solve the system of equations in the case of limited statistics, as at LEP. At a high luminosity e^+e^- collider running at the Z^0 peak a solution can be found with fewer hadronisation relations, although $\eta_{\mathrm{d}}^{\pi^\pm} \approx \eta_{\mathrm{u}}^{\pi^\pm}$ has to be kept. Note however that a systematic uncertainty of up to 2% should be assigned to this relation, following studies with QCD models [10].

Although the relation $\eta_{\mathrm{d}}^{\pi^\pm} \approx \eta_{\mathrm{u}}^{\pi^\pm}$ is fairly well motivated, it uncertainty would become a limiting factor in the determination of the branching ratios of the W boson and would also make the measurement somewhat model dependent. We prefer to abandon the use of the relation by also taking into account the W determination. The W decays provide a separation of up and down quarks. At the Z^0 the main handle to separate $\eta_{\mathrm{d}}^{\pi^\pm}$ and $\eta_{\mathrm{u}}^{\pi^\pm}$ is to tag an up or down event by a high x_p pion in one jet and to find a charged kaon (indicating an up quark) or a neutral kaon (indicating a down quark) in the opposite jet. Given the dilution of the signal by decays of strange vector mesons and strange quark events, the discrimination power is only marginal. The resulting uncertainty in the η_q^i limits the accuracy with which the CKM matrix elements can be determined. On the other hand, since the charged W boson can only decay into a restricted number of quark combinations, these decays discriminate between up and down quarks. For example, a charm quark can only be associated with a down quark, but not with an up quark. As a consequence, a combined fit of W and Z^0 decays for the η_q^i and the W branching ratios allows one to find a solution without any assumptions about QCD symmetries.

3.2 Experimental aspects of running at the Z^0

In addition to secondary vertex finding, the main selection requirements at the Z^0 are to identify hadrons. In order to have sufficient particle separation power with the $\mathrm{d}E/\mathrm{d}x$ measurement and for the V^0 reconstruction of $\mathrm{K}_{\mathrm{S}}^0$ and Λ identification, we require jets and tagging particles to be within the central part of the detector. Specifically we require that the polar angle of the thrust axis with respect to the beam direction, θ_{Thrust}, satisfy $|\cos\theta_{\mathrm{Thrust}}| < 0.8$ and that the tagging particles have a polar angle of the momentum, θ_p, within the range $|\cos\theta_p| < 0.9$. The detailed reconstruction criteria will depend on the performance of the tracking system.

For the luminosities expected at TESLA at the Z^0, we estimate from a JETSET simulation the number of single- and double-tagged events listed in Table 3. As discussed in [10, 11] and mentioned in the previous section, the tagging efficiencies η_q^i can be determined from these measurements with errors as given in Table 4. Also shown in Table 4 are the contributions from a $\pm 2\%$ systematic uncertainty in the hadronisation relation $\eta_{\mathrm{d}}^{\pi^\pm} \approx \eta_{\mathrm{u}}^{\pi^\pm}$. In omitting this relation but including data from W decays, the results are independent of any assumption about hadronisation and represent a considerable improvement over the precision reached at LEP. In general the correlations between the various elements are small. The most important correlations are between the η_q^i of the same tagging particle type. For example, $\eta_{\mathrm{d}}^{\pi^\pm}$ and $\eta_{\mathrm{u}}^{\pi^\pm}$ are almost fully anticorrelated.

3.3 Systematic uncertainties

In view of the unprecedented number of Z^0 decays available at GigaZ a detailed evaluation of the systematic uncertainties is unrealistic at this stage. We will just mention some potential distortions and suggest how to estimate them. From past experience we assume

particle type	tagged hemispheres/1000	double-tagged events /1000						
		$\pi^\pm$	$K^\pm$	$p(\overline{p})$	K^0_S	$\Lambda(\overline{\Lambda})$	charm tag	bottom tag
$\pi^\pm$	461395	41890	35442	8667	4442	1962	29183	949
$K^\pm$	222445		10043	4212	2793	1227	16361	546
$p(\overline{p})$	50187			490	542	240	3283	110
K^0_S	29304				204	178	2137	69
$\Lambda(\overline{\Lambda})$	12747					39	831	27
charm tag	587044						124575	51628
bottom tag	450683							108898

Table 3: Numbers of tagged event hemispheres and double-tagged events, scaled down by a factor of 10^{-3}, for $x_p > 0.2$ in an event sample of 2×10^9 Z^0 events from a JETSET simulation.

that the huge amount of data will provide enough cross checks to keep all these sources of uncertainty under control.

One crucial element of the analysis is charm and bottom quark tagging based on secondary vertices. As shown at LEP and SLD many contributions to uncertainties of their efficiencies and purities can be derived from data.

As discussed in [11] the major systematic uncertainties in the light quark sector in the LEP analysis are due to the efficiencies and purities of the hadron identification. At LEP these are estimated from relatively pure samples of particles from $K^0_S \to \pi^+\pi^-$ and $D^0 \to K^-\pi^+$ decays or photon conversions into an e^+e^- pair, for example. Such cross checks can also be performed at GigaZ where one can expect the much higher statistics to lead to a sizeable improvement of the systematic uncertainty compared to LEP.

Another major uncertainty in the LEP analysis comes from the hadronisation relations, briefly mentioned in Section 3.1. These relations were needed to obtain a stable solution of the equation system. As discussed above, with the higher statistics at GigaZ and the use of W decays these hadronisation relations are no longer needed.

Depending on the actual detector performance it may also be possible to use additional high x_p particle types like $\phi(1020)$ mesons, which are very likely to originate from a strange quark, to further constrain the tagging efficiencies.

4 Determination of the W branching ratios

4.1 Formalism

Neglecting experimental effects, the tagging efficiencies in the rest frame of the W and Z^0 bosons are almost identical, since the masses are so similar. Therefore, the η^i_q determined

quantity	value	error fitting only Z^0 $\eta_d^{\pi^\pm} \approx \eta_u^{\pi^\pm}$		error fitting Z^0 and W no had. rel.
$\eta_d^{\pi^\pm}$	0.209478	0.000045	0.002568	0.000800
$\eta_u^{\pi^\pm}$	0.208519	0.000045	0.002568	0.001058
$\eta_s^{\pi^\pm}$	0.129708	0.000089	0.000076	0.000087
$\eta_c^{\pi^\pm}$	0.028584	0.000014		0.000014
$\eta_b^{\pi^\pm}$	0.000472	0.000003		0.000003
$\eta_d^{K^\pm}$	0.056490	0.000276	0.001558	0.000475
$\eta_u^{K^\pm}$	0.074605	0.000437	0.002761	0.000819
$\eta_s^{K^\pm}$	0.122413	0.000089	0.000594	0.000175
$\eta_c^{K^\pm}$	0.019890	0.000011		0.000011
$\eta_b^{K^\pm}$	0.000247	0.000002		0.000002
η_d^{p}	0.014970	0.000077	0.000166	0.000091
η_u^{p}	0.024974	0.000107	0.000207	0.000113
η_s^{p}	0.019905	0.000043	0.000594	0.000095
η_c^{p}	0.003399	0.000005		0.000005
η_b^{p}	0.000058	0.000001		0.000001
$\eta_d^{K_S^0}$	0.007233	0.000089	0.000445	0.000146
$\eta_u^{K_S^0}$	0.005826	0.000119	0.000514	0.000176
$\eta_s^{K_S^0}$	0.019489	0.000021	0.000047	0.000024
$\eta_c^{K_S^0}$	0.002573	0.000004		0.000004
$\eta_b^{K_S^0}$	0.000029	0.000001		0.000001
η_d^{Λ}	0.003048	0.000052	0.000183	0.000069
η_u^{Λ}	0.002893	0.000072	0.000228	0.000091
η_s^{Λ}	0.008498	0.000015	0.000007	0.000014
η_c^{Λ}	0.000858	0.000002		0.000002
η_b^{Λ}	0.000014	0.000001		0.000001

Table 4: Values of the η_q^i and their expected precision for $x_p > 0.2$. Column three gives the statistical uncertainties from a fit to 2×10^9 Z^0 decays assuming the hadronisation relation $\eta_d^{\pi^\pm} \approx \eta_u^{\pi^\pm}$. The fourth column shows the effect of a $\pm 2\%$ systematic uncertainty in the hadronisation relation. Column five gives the results of the combined fit to the Z^0 and W decays without invoking a hadronisation relation.

	up	charm
down	2375000	126000
strange	124800	2370000
bottom	200	4000

Table 5: Number of produced W boson decays into specific quark pairs for a total of five million accepted hadronically decaying W bosons.

at the Z^0 peak can be used to measure flavour production in W decays in the W rest frame. To determine the branching ratios of the W bosons we use both single and double tags. A singly tagged W is where a tag of type k is found in just one of the jets. A doubly tagged W is a candidate where particle types i and j are tagged in each jet belonging to a W boson. This leads to the generic equations:

$$N_k = N_{\mathrm{W}}^{\mathrm{had}} \cdot \sum_{qq'} [\eta_q^k (1 - \sum_l \eta_{q'}^l) + (1 - \sum_l \eta_q^l) \eta_{q'}^k] \cdot R_{qq'} \quad (3)$$

$$N_{ij} = N_{\mathrm{W}}^{\mathrm{had}} \cdot (1 - 0.5\delta_{ij}) \sum_{qq'} [\eta_q^i \eta_{q'}^j + \eta_q^j \eta_{q'}^i] \cdot R_{qq'} \ , \quad (4)$$

where $\delta_{ij} = 1$ if $i = j$, and zero otherwise, which avoids double counting if identical tags are required, and where $N_{\mathrm{W}}^{\mathrm{had}}$ is the number of selected hadronically decaying W candidates. Here we neglected experimental effects, hemisphere correlations, background from non-W events, and assumed the right assignment of particles and jets to W bosons. In a real experiment none of the conditions holds exactly. The full equations which account for these complications are given in the Appendix. The number of 31 possible equations over-constrain the six unknown branching ratios, which can be obtained from a χ^2 fit.

As discussed in Section 3.1, to minimise the uncertainties originating from the limited knowledge of the η_q^i, we perform a simultaneous fit for the η_q^i and the W branching ratios using both Z^0 and W decays.

4.2 Events with W Bosons at TESLA

The high luminosity of TESLA leads to a substantial yield of W bosons. The main production processes are $e^+e^- \rightarrow e\nu_e W$, with a cross section of some 5 pb at $\sqrt{s} =$ 500 GeV, and W pair production of some 8 pb. In addition, W bosons are produced in top decays, but because of the lower cross section and some complications arising from the multi-jet environment we will not consider them. In total some seven million W bosons will be produced in a nominal TESLA year.

We restrict this analysis to W bosons that are scattered into the central part of a future TESLA detector because of the experimental requirements of good hadron identification and efficient and pure heavy quark tagging. Within the polar angles θ_{W} of the W boson

$|\cos\theta_W\ | < 0.8$ some 700000 hadronically decaying W bosons can be retained for W pair production, while some 450000 single hadronically decaying W bosons are kept. We will base the following discussion on 5 million usable hadronic W decays, which could be collected in a few years of high energy data taking. Assuming the current knowledge of the CKM matrix[4], the expected yields of the different decay modes are listed in Table 5.

4.3 Experimental Procedure for W Bosons

The basic strategy is to boost the decay products of W bosons into the W rest frame, search in each jet for a secondary vertex or for a light flavour tag. In the latter case we identify the type of the highest momentum particle in each hemisphere, and finally assign the probability η_q^i obtained from Z^0 decays that such a tag stems from a certain primary quark flavour. To achieve this one has to first find the proper association of particles to W bosons, reconstruct the W boson energy and momentum and identify the tag.

In case of single W production the association of particles to the W boson is unambiguous. This is also true for W pair production where one W decays hadronically and the other into a pair of leptons, denoted "semi-hadronic". If both W bosons from a pair decay hadronically, the jet assignment is more difficult. After grouping the particles into four jets, three different jet pairings are possible. Our simulation studies using PYTHIA [8] show that at least for LEP energies this pairing is correct for about 85% of all W pairs[5].

In W pair events the energies and momenta of the W bosons can be rather precisely reconstructed by fitting the observed momenta and energies of jets and leptons to obey energy and momentum conservation and to combine to have the W mass. This has been shown by the LEP experiments, which use total centre-of-mass energy as given by the accelerator and the fact that the W pairs are produced at rest at e^+e^- colliders. Similarly in single W events the two jets can be constrained to have a mass identical to the mass of the W, M_W.

To identify the primary quark flavour, each jet is searched for a secondary vertex and classified either as bottom, charm or, if it does not have a secondary vertex, as a light flavour candidate. The efficiencies for the charm, bottom and light flavour tags are listed in Table 1. For the light flavour tags the tracks and clusters assigned to each W boson are then boosted along the reconstructed four momentum of the W candidate into its rest frame. The particles are grouped into two hemispheres with respect to the direction of the thrust axis and in each of these the particle with the highest momentum is identified.

[4]Reference [22] gives directly measured values and the ranges from a unitarity constraint CKM matrix. For our purposes we adjusted these values such that the individual decay modes add up to exactly five million W bosons. We also assumed the decay into up quarks to be exactly as frequent as the one to charm quarks. For the purpose of this analysis these adjustments are unimportant.

[5]No detailed study exists yet for energies of 500 GeV. We therefore assume for the following evaluation the purity observed at LEP.

particle type	tagged hemispheres	double-tagged events						
		$\pi^{\pm}$	$\mathrm{K}^{\pm}$	$\mathrm{p}(\overline{\mathrm{p}})$	$\mathrm{K}^0_{\mathrm{S}}$	$\Lambda(\overline{\Lambda})$	charm tag	bottom tag
$\pi^{\pm}$	829270	116657	84621	23297	9287	4089	240508	7639
$\mathrm{K}^{\pm}$	349183		17061	8556	4035	1714	190195	6300
$\mathrm{p}(\overline{\mathrm{p}})$	87307			1116	997	429	33204	1077
$\mathrm{K}^0_{\mathrm{S}}$	41863				235	198	29319	980
$\Lambda(\overline{\Lambda})$	18241					41	12775	426
charm tag	1124876						62888	4517
bottom tag	36701							81

Table 6: Numbers of single and double tagged candidates obtained from a sample of five million W decays from a PYTHIA simulation.

These particles are retained if they are in the geometrical acceptance range, fulfil the criteria to identify their species and have a scaled momentum in the W boson rest frame $x_p = 2p/M_{\mathrm{W}} > 0.2$. The expected numbers of single and double tags in five million W decays are listed in Table 6.

To apply the Z^0 tagging efficiencies to the W decays, several corrections have to be applied, even though the selection and reconstruction resembles closely the one used to determine the η_q^i at the Z^0.

- The mass of the W is some 10 GeV below the Z^0 mass. For particle tags this leads to a small difference of the energy spectra due to QCD scaling violations. The resulting differences in the tagging efficiencies at these two energy scales were estimated with $\mathrm{e}^+\mathrm{e}^-$ continuum events generated with JETSET at centre-of-mass energies equivalent to the Z^0 and the W masses. The fractional changes of the η_q^i relative to those at the Z^0 are shown in Figure 1. For $x_p > 0.2$ the η_q^i were found to be 1.7% higher at the W mass. These scaling violations are rather independent of the primary quark flavour. Some dependence on the tagging particle is observed. Note that the latter can be rather precisely determined from data by summing over all flavours.

Furthermore, differences in the detection efficiencies have to be accounted for:

- The high x_p particles and those from the secondary vertices from W and Z^0 decays have different momentum and polar angle distributions in the lab system. This can be inferred from Figures 2 and 3 where the jet energies in single W and W pair events are shown. There is a broad spectrum of jet energies, mostly larger than the jet energy of ~ 45.5 GeV in Z^0 decays. This potentially leads to different efficiencies and purities of secondary vertex finding and, for the highest x_p particle, momentum resolution and $\mathrm{d}E/\mathrm{d}x$ efficiencies.

	Hadronic branching ratios $R_{qq'}$		CKM matrix elements $\lvert V_{qq'}\rvert$	
	up	charm	up	charm
down	0.4750±0.0027	0.0252 ±0.0016	0.9747±0.0028	0.2245 ±0.0072
strange	0.0250±0.0027	0.4740 ±0.0017	0.2234±0.0124	0.9737 ±0.0017
bottom	0.00004±0.00010	0.00080±0.00005	0.0089±0.0114	0.0400 ±0.0011

Table 7: Expected precision on the hadronic branching ratios of W bosons and CKM matrix elements. Note that R_{ub} is constrained by $R_{\mathrm{ub}} = 1 - \sum_{qq'} R_{qq'}$, where the sum ranges over all other hadronic branching ratios.

- In addition possible misassignments of particles to W bosons and distortions in the reconstruction of the momentum of the W bosons have to be taken into account.

The necessary corrections can be estimated using detector simulations. In many cases significant cross checks with data can be performed. In applying the η_q^i to fully hadronically decaying W pairs, there could emerge in principle a further distortion due to Bose-Einstein or colour reconnection effects. However, as already known from the discussions on the colour reconnection for W pairs at LEP, this affects in particular low momentum particles, whereas the leading particles are rather undistorted [24].

In addition background from non-W production processes have to be considered. The major backgrounds in the W pair sample with fully hadronic decays are due to e^+e^- continuum quark production with two hard gluons, Z^0 pair production and top pair production. Early studies at 500 GeV [23] indicate that these backgrounds can be kept below the 5% level. Background is significantly smaller for semi-hadronic events.

5 W Branching Ratios and CKM Matrix Elements

5.1 Statistical precision

Following the procedure outlined above we estimate the precisions for the hadronic branching ratios, $R_{qq'}$, using a χ^2 fit to the observed tagged yields in W and Z^0 decays. In the following we assume that the W decays only into the known leptons and quarks, except the top quark which is inaccessible because of its mass. If additional decays contribute they probably will either modify the true $R_{qq'}$ or even imply that $\sum R_{qq'} \neq 1$. As default we assume the Standard Model case in this analysis and constrain $R_{\mathrm{ub}} = 1 - \sum_{qq'} R_{qq'}$, where the sum ranges over all other hadronic branching ratios. The result of this fit is listed in Table 7. We just note that by omitting such a constraint we can determine $\sum_{qq'} R_{qq'}$ with a statistical precision of $\sim 5 \times 10^{-4}$.

Without assuming unitarity, the CKM matrix elements can be obtained from the partial widths of the W bosons, in a way similar to what has been used in [6]:

$$\Gamma(\mathrm{W} \to qq') = \frac{C_{\mathrm{QCD}} G_{\mathrm{F}} M_{\mathrm{W}}^3}{6\sqrt{2}\pi} |V_{qq'}|^2 \,, \tag{5}$$

where G_F is the Fermi constant and

$$C_{\mathrm{QCD}} = 3\left\{1 \,+\, \frac{\alpha_{\mathrm{s}}(M_{\mathrm{W}})}{\pi} \,+\, 1.409\left(\frac{\alpha_{\mathrm{s}}(M_{\mathrm{W}})}{\pi}\right)^2 \,-\, 12.77\left(\frac{\alpha_{\mathrm{s}}(M_{\mathrm{W}})}{\pi}\right)^3\right\} \tag{6}$$

expresses the QCD radiative corrections. The partial width is related to the hadronic branching fractions by:

$$\Gamma(\mathrm{W} \to qq') \;=\; \Gamma_{\mathrm{W}} \; B_h \; R_{qq'} \tag{7}$$

with Γ_{W} the total W boson width and B_h the inclusive branching ration of the W into hadrons.

The CKM matrix elements can thereby be determined using the measured values of the basic quantities. However, such a measurement may finally be limited by the knowledge of Γ_{W}. Given by the very precise measurement of G_F in μ decays and lepton universality measurements in τ decays [25], the latter can be substituted by the measurement of the electronic or muonic partial width of the W leading to

$$\frac{\Gamma(\mathrm{W} \to qq')}{\Gamma(\mathrm{W} \to (\mathrm{e},\mu)\nu)} = \frac{\Gamma_{\mathrm{W}} B_h R_{qq'}}{\Gamma_{\mathrm{W}} B_{(\mathrm{e},\mu)}} = C_{\mathrm{QCD}} \cdot |V_{qq'}|^2 \,, \tag{8}$$

where $B_{(\mathrm{e},\mu)}$ is the electronic or muonic decay fraction of the W. Thus, the CKM matrix elements can be related to the measured fraction of $\mathrm{W} \to qq'$ decays using the QCD correction factor and the well-measurable ratio $B_h/B_{(\mathrm{e},\mu)}$

$$|V_{qq'}| = \sqrt{\frac{1}{C_{\mathrm{QCD}}} \frac{B_h}{B_{(\mathrm{e},\mu)}} R_{qq'}} \,. \tag{9}$$

Currently the ratio $B_h/B_{(\mathrm{e},\mu)}$ is known experimentally from LEP data to 1%, but the error should significantly decrease to $\mathcal{O}(0.05\%)$ due to the higher yield of W bosons at TESLA. The QCD radiative correction is calculated to third order in α_{S}; however, the strong coupling is currently only known to a few percent. Given two billion Z^0 events the uncertainty could be rather significantly reduced. In fact C_{QCD} should be almost identical in Z^0 and W decays. Both these error contributions are smaller than the uncertainties in $R_{qq'}$. We obtain the matrix elements given in Table 7, and their correlations are listed in Table 8. We do not show the negligible correlations between the $|V_{qq'}|$ and the η_q^i resulting from the combined fit. The results are consistent with the input values.

At this stage we have completely neglected any possible theoretical corrections and uncertainties due to mass effects, electroweak contributions etc.

	$\lvert V_{\rm ud}\rvert$	$\lvert V_{\rm us}\rvert$	$\lvert V_{\rm ub}\rvert$	$\lvert V_{\rm cd}\rvert$	$\lvert V_{\rm cs}\rvert$	$\lvert V_{\rm cb}\rvert$
$\lvert V_{\rm ud}\rvert$	1.000	-0.983	0.087	-0.590	0.566	-0.014
$\lvert V_{\rm us}\rvert$		1.000	-0.174	0.581	-0.603	0.006
$\lvert V_{\rm ub}\rvert$			1.000	-0.117	0.139	-0.107
$\lvert V_{\rm cd}\rvert$				1.000	-0.964	0.014
$\lvert V_{\rm cs}\rvert$					1.000	-0.014
$\lvert V_{\rm cb}\rvert$						1.000

Table 8: Correlations between CKM matrix elements.

5.2 Systematic Uncertainties

For the envisaged precision and the huge amount of data, a detailed analysis of the potential systematic uncertainties is impossible at this stage. Here we discuss some major uncertainties and indicate ways to estimate their importance. In all cases we believe that they can be kept under control.

- **Errors and correlations for η_q^i from $\mathbf{Z^0}$.** The uncertainties of the efficiencies η_q^i from the billion Z^0 bosons produced at a high luminosity GigaZ propagate into an uncertainty in the branching ratios of the W. This is taken into account by the result of the combined fit.

  The principal problem will be to keep the systematic uncertainties of the η_q^i under control to this high level of precision. Some potential uncertainties were mentioned in Section 3.3. Ideally the data at GigaZ and at high energies would be collected with the same detector. In this case many of the potential uncertainties are the same for the Z^0 and the W boson analyses and can be neglected. Otherwise the relative uncertainties between two different detectors in flavour tagging have to be taken into account. Given the large statistics of data to cross check the detector performance, it appears possible to maintain a sufficiently high precision.

- **W boson reconstruction**. The accuracy of the measurements of the branching ratios, $R_{qq'}$, depends on how reliably particles and jets can be assigned to W bosons and how well this is understood. Related is the correctness of the W boson reconstruction from a kinematic fit. These affect the determination of the x_p values of the tagging particle in the W rest frame.

  In part potential misassignments can be estimated from other processes at high energies, like $e^+e^- \rightarrow Z^0Z^0$. Our simulation studies suggest that the corresponding error will be unimportant.

- **Differences in tagging efficiencies**. As discussed in Section 4.3, several subtleties have to be taken into account in translating the tagging efficiencies at the Z^0 to those for the W bosons. The differences can be minimised by applying rather similar selection requirements in the two cases.

Here we list potential corrections:

- relative capability for charm, bottom and light flavour tagging (including particle identification capability) in W pair production at high energies compared to the Z^0 data,
- QCD scaling violations which slightly change the η_q^i,
- effects of the different angular acceptance and kinematical and geometrical correlations.

None of these uncertainties appears to impose a substantial problem. Systematic uncertainties can be derived from data.

- **Backgrounds**. Most of the overall yield from potential background processes will be measured to very high precision and their contribution to the sample of W bosons can be well determined. Somewhat more uncertain is their contribution to the light flavour tags. This requires a knowledge of the particle content. For example, the number of $e^+e^- \rightarrow q\bar{q}$ continuum events with two hard gluons will probably be quite well known. However, to account for their contribution to the W sample, the leading particle distributions in gluon jets also needs to be understood. A possible way to determine their impact is to study the particle composition in these processes for kinematic properties that cannot be confused with W pair production and extrapolate it to the relevant kinematic configuration.

5.3 Some possible improvements

- **Particle separation.** A better detector performance to separate the hadron species will lead to a better purity of the individual light flavours.

- **Tags including charge.** As already discussed in [10], including the charge sign of the tags allows one to improve the flavour purity of the η_q^i from Z^0 decays. Charge dependent η_q^i are even more interesting for both W production processes because of the significant charge asymmetry in $\cos\theta$.

- **Polarisation.** W pair production at TESLA will lead to polarised W bosons, the degree of polarisation depending on the scattering angle, leading down-type jets to be in general more energetic than up-type jets. This effect can be seen in the accumulation of jets with high and low energies in Figure 2, for example. One may use this property to statistically separate between up- and down-type quarks. Ambiguities from the high x_p tagging between charged kaons in up and strange quark events, for example, can thereby be reduced. Such separation could become even more powerful if in addition the charge of the tagged particle is used.

6 Top quark decays

As yet the CKM matrix elements have only been determined indirectly from processes such as $b \to s\gamma$ and $B^0\bar{B}^0$ mixing. In both cases the transition involving top quarks occur at the one loop level. A first direct measurement of the fraction of top decays into bottom quarks can be found in [26]. The high statistics and clean environment at a high energy e^+e^- collider allow more comprehensive and precise direct measurements.

6.1 Formalism

To extract the relevant branching ratios of top quarks into the various species of down - type quarks one has to tag just the jet that is produced together with the W. An equation system can be constructed relating the number of events, N_k, identified with a tag k, out of a number of top decays, N_{top},

$$N_k = N_{\mathrm{top}} \sum_{q=\mathrm{d,s,b}} \eta_q^k B(\mathrm{t} \to q\mathrm{W}) \tag{10}$$

which can be easily solved for the branching ratios $B(\mathrm{t} \to q\mathrm{W})$. In the following we assume the absence of any non-standard decay. As was discussed for W decays, such an exotic contribution may become apparent by measuring the branching ratios with high precision.

6.2 Top Yields at TESLA

With the luminosity given in Section 2, some 400,000 top quarks will be produced each year. To determine the transition $\mathrm{t} \to q\mathrm{W}$ it is least ambiguous to demand leptonic W decays, thus avoiding any confusion in jet assignments. With such a requirement, some 120000 top quarks per year will be kept. However, it may also be possible to use at least a part of the fully hadronic top decays, providing a significantly larger sample. We will base our discussion on one million well-tagged and reconstructed top quarks in the $\mathrm{W} \to l\nu$ mode. The expected yields of the various decays are listed in Table 9.

6.3 CKM Matrix Elements from Top Quark Decays

The branching ratios of the top quark are related to the CKM matrix elements by including higher order corrections summarised in [27]:

$$\Gamma(\mathrm{t} \to q\mathrm{W}) \;\sim\; |V_{\mathrm{t}q}|^2 \;\times\; 1.42 \text{ GeV} \; . \tag{11}$$

	$\lvert V_{\mathrm{t}q}\rvert$	number of top decays
down	0.006	40
strange	0.04	1600
bottom	0.999	998360

Table 9: Number of expected top quark decays into down type quarks. Here we assume one million top decays. Also shown is the CKM matrix element assumed. As for the estimate on W decays we fixed the CKM matrix elements to be consistent with unitarity.

It should be noted that, in contrast to all other known quarks, the top quark is expected to decay before it hadronises [28]. This renders the determination of the CKM matrix elements independent of uncertainties inherent in hadron physics.

The partial width $\Gamma(\mathrm{t} \to q\mathrm{W})$ can be related to the branching ratios $B(\mathrm{t} \to q\mathrm{W})$ by

$$\Gamma(\mathrm{t} \to q\mathrm{W}) \;=\; \Gamma_{\mathrm{top}} B(\mathrm{t} \to q\mathrm{W}) \tag{12}$$

and thus

$$\lvert V_{\mathrm{t}q}\rvert^2 \;=\; B(\mathrm{t} \to \mathrm{qW})\;\frac{\Gamma_{\mathrm{top}}}{1.42\ \mathrm{GeV}}\ . \tag{13}$$

Whereas the determination of the branching ratios is straight-forward and will be detailed in the next paragraphs, the measurement of the decay width of the top quark to high precision is less obvious. This is not possible via secondary vertex tagging because of the expected lifetime of $\mathcal{O}(10^{-25}$ sec). Instead other means have been suggested [29], the most promising one seeming to be single top production $\mathrm{e}^+\mathrm{e}^- \to \mathrm{e}\nu_{\mathrm{e}}\mathrm{tb}$ [30]. For such a measurement the background from top pair production has to be rejected. Scaling the precision estimated in [30] to the luminosities foreseen at TESLA, Γ_{top} should be measurable to $\sim 1\%$.

Assuming unitarity and that the top quark couples only to down, strange and bottom quarks, the Γ_{top} should agree with the theoretical expectation and thus

$$\lvert V_{\mathrm{t}q}\rvert^2 = B(\mathrm{t} \to q\mathrm{W}) \tag{14}$$

In general the determination of the branching ratios follows the procedure for the W bosons.

- Identification of the W boson decay products and consideration of the remaining jet in a top decay. This is rather straight-forward for leptonic W decays.
- Application of the tagging algorithm to this jet. The light flavour tag should have a very high purity even at the cost of a lower efficiency because of the overwhelming

	$\pi^\pm$	$K^\pm$	$p(\overline{p})$	K^0_S	$\Lambda(\overline{\Lambda})$	bottom tag
Number of tags	1000	620	125	90	50	499180

Table 10: Numbers of expected top quark decays tagged by a light quark tag combined with a high x_p particle or a heavy quark tag.

	Branching ratios	CKM matrix elements
$t \to d$	$(8\pm52) \times 10^{-5}$	$0.0060 \pm 0.026 \pm 0.00003$
$t \to s$	0.0015 ± 0.0005	$0.0400 \pm 0.006 \pm 0.0002$
$t \to b$	0.99840 ± 0.00028	$0.999200 \pm 0.000008 \pm 0.005$

Table 11: Expected precision on the CKM matrix elements from top decays assuming top only decays into d, s, and b quarks. The second error of the CKM matrix elements assumes $\Gamma_{\rm top}$ to be known to 1%.

fraction of top decays into bottom quarks. In the following we assume $\epsilon_{\rm uds} = 0.5$ and $\epsilon_{\rm b} = 0.01$. In the absence of FCNC decays, no charm quarks are directly produced. This allows one to reject bottom quarks by a very tight selection against secondary vertices, helping to achieve the preferred high purity of light flavours.

- Different from the situation with W decays, the mass of the colour neutral system in which hadronisation takes place depends on the details of the decays of both the top and anti-top quarks in the same event. This makes the assignment of the η_q^i somewhat more complicated. Whereas the Z^0 and the W bosons are colourless objects and therefore hadronisation proceeds in their respective rest frames, the top quark itself is coloured. Its colour is neutralised by the recoiling anti-top quark. In general the hadronisation should evolve in the $q\bar{q}'$ system given by the decays $(t \to qW^+, \bar{t} \to \bar{q}'W^-)$. This requires that the mass $M_{q\bar{q}'}$ of the $q\bar{q}'$ system be reconstructed. QCD scaling violations between the η_q^i at the Z^0 and those at $M_{q\bar{q}'}$ have to be taken into account. A JETSET simulation for $M_{q\bar{q}'}$ is shown in Figure 4 and the sizes of the necessary QCD corrections are given in Figure 1.

  If at least one of the top quarks decays fully hadronically, distortions may arise from colour reconnection between quarks from the W decay and those directly associated to the top. As mentioned previously, they are expected to be of no importance for our method. The experimental task, however, is complicated for hadronic W decays by the need to resolve the two jets that directly couple to the top quarks. This may require further selections.

- Solution of the equation system (Eq. 10).

The numbers of expected tags are listed in Table 10. The expected precision of the CKM matrix elements involving top quarks are listed in Table 11. Their correlation matrix is given in Table 12.

	$\lvert V_{\rm td}\rvert$	$\lvert V_{\rm ts}\rvert$	$\lvert V_{\rm tb}\rvert$
$\lvert V_{\rm td}\rvert$	1.	-0.832	0.728
$\lvert V_{\rm ts}\rvert$		1.	-0.986
$\lvert V_{\rm tb}\rvert$			1.

Table 12: Correlation matrix between the top CKM matrix elements.

These results imply that one million top decays allow one to determine the branching ratio $B(\mathrm{t}\rightarrow \mathrm{bW})$ to be different from unity with high significance. This implies that the sum of $B(\mathrm{t}\rightarrow \mathrm{sW})$ and $B(\mathrm{t}\rightarrow \mathrm{dW})$ is larger than zero. The light flavour tagging efficiencies even allow one even to distinguish down and strange quark contributions. As a result the CKM matrix element $|V_{\rm ts}|$ can be determined directly and with reasonable precision. Despite low statistics and still formidable bottom background in the light quark samples, a first upper limit from direct measurements on the $\mathrm{t}\rightarrow\mathrm{d}$ transition can be derived. At 95% confidence,

$$B(\mathrm{t}\rightarrow \mathrm{dW}) \quad < \quad 8\times 10^{-4}.$$

To translate these into reasonably precise $|V_{\mathrm{t}q}|$ elements requires that $\Gamma_{\rm top}$ be measured with higher precision than what has been suggested so far. If this can be achieved one can set a limit of

$$|V_{\rm td}| \quad < \quad 0.029\ .$$

7 Conclusions

The large samples of W bosons and top quarks at a future $\mathrm{e}^+\mathrm{e}^-$ collider of high energy and luminosity provides the means to directly determine the fundamental CKM matrix elements. The current values are based on hadron decays and deep inelastic scattering results and invoke QCD symmetries. We propose a complementary method which is free of any assumptions on QCD modelling at a low mass scale. In addition the resulting precision is at least competitive to the ones from hadronic decays. In this paper we have only discussed the experimental feasibility. The potential experimental precision may have to be complemented by further theoretical scrutiny of higher order effects.

Compared to the current results for individual measurements as summarised in [22], there are improvements on all elements in the charm sector. The precision is better than what is anticipated from other measurements in the future. Even measurements of $|V_{\rm cb}|$ at B factories hardly reach the potential TESLA precision because of the anticipated theoretical uncertainty. The results for transitions involving the up quark are a factor three to five worse than those from hadron decays. However, even for these elements the measurement in W boson decays may become an interesting complement.

The direct determination of CKM matrix elements involving top quarks may be pioneered at TESLA. So far only $|V_{\rm tb}|$ has been determined directly using top decays

	current uncertainty	projected other	TESLA
$\lvert V_{\mathrm{ud}}\rvert$	$\pm$ 0.0008		±0.0028
$\lvert V_{\mathrm{us}}\rvert$	±0.0023		±0.0124
$\lvert V_{\mathrm{ub}}\rvert$	$\sim\pm0.008$	±0.0004 [31]	±0.011
$\lvert V_{\mathrm{cd}}\rvert$	±0.016		±0.0072
$\lvert V_{\mathrm{cs}}\rvert$	±0.16		±0.0017
$\lvert V_{\mathrm{cb}}\rvert$	±0.0019	±0.0012 [31]	±0.0011
$\lvert V_{\mathrm{td}}\rvert$	$\lvert V_{\mathrm{td}}\rvert/\lvert V_{\mathrm{ts}}\rvert$ <0.24	<0.016	±0.026 ±0.00003
$\lvert V_{\mathrm{ts}}\rvert$			±0.006 ±0.0002
$\lvert V_{\mathrm{tb}}\rvert$	(+0.29,-0.12) [26]	$\sim\pm0.05$ [27]	±0.000008 ±0.005

Table 13: Current (mostly from [22]) and expected precision of measurements of the CKM matrix elements. Only direct measurements and those that do not rely on the knowledge of other CKM matrix elements are used. Prospective measurements are listed for TESLA, e^+e^- B factories [31] and the LHC [27]. For the TESLA limits on $\lvert V_{tq}\rvert$ the second error is due to an uncertainty of Γ_{top} of 1%.

at the Tevatron [26]. The measurement of the branching ratio $B(\mathrm{t}\rightarrow\mathrm{bW})$ may be improved at the Tevatron Run 2 and particularly at the LHC. For example, [27] assumes a 0.2% statistical error for one year of low luminosity running at LHC (10 fb^{-1}). The systematic error is not yet evaluated. Translating this into $\lvert V_{\mathrm{tb}}\rvert$ is more complicated and a direct measurement via single top quark production is deemed "challenging". No direct measurements of $\lvert V_{\mathrm{td}}\rvert$ and $\lvert V_{\mathrm{ts}}\rvert$ are possible at the LHC. At TESLA not only the dominant decay branching ratio $B(\mathrm{t}\rightarrow\mathrm{bW})$ can be measured with similar or better precision compared to the LHC, but also the top transition to light quarks can be probed. The element $\lvert V_{\mathrm{ts}}\rvert$ can be determined with a significance of about six standard deviations and, if Γ_{top} is known to sufficient precision, a significant upper limit on $\lvert V_{\mathrm{td}}\rvert$ can be set.

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A Formulae for Single and Double Tags in W Decays

Equations 3 and 4 show only the general structure of the equation system to solve for the branching ratios of the W bosons. As pointed out, these equations become more involved if experimental distortions are taken into account, such as kinematical and geometrical correlations ρ_{W} between the two jets, and the probability $\Pi_{\mathrm{W}\leftrightarrow\mathrm{jet}}$ of correctly assigning a jet to a W boson.

For $N_{\mathrm{W}}^{\mathrm{had}}$ denoting the number of accepted hadronically decaying W bosons, the number of double tags of types i and j with a correct assignment of a jet to a W boson is given by

$$N_{ij}^{\mathrm{correct}} = N_{\mathrm{W}}^{\mathrm{had}} \cdot \Pi_{\mathrm{W}\leftrightarrow\mathrm{jet}} \cdot (1 - 0.5\delta_{ij})\rho_{\mathrm{W}} \sum_{qq'} (\eta_q^i \eta_{q'}^j + \eta_q^j \eta_{q'}^i) R_{qq'} \tag{15}$$

where the sum ranges over all six quark pairs possibly produced in W decays: $qq' = (\mathrm{u\bar{d}}$, $\mathrm{u\bar{s}}$, $\mathrm{u\bar{b}}$, $\mathrm{c\bar{d}}$, $\mathrm{c\bar{s}}$, $\mathrm{c\bar{b}})$. In W pair events where the two W bosons decay like $\mathrm{W}_1 \to q_1\tilde{q}_1$ and $\mathrm{W}_2 \to q_2\tilde{q}_2$, those with misassigned jets can be approximated by

$$N_{ij}^{\mathrm{misassigned}} = N_{\mathrm{W}}^{\mathrm{had}}(1 - \Pi_{\mathrm{W}\leftrightarrow\mathrm{jet}})(0.5 - 0.25\delta_{ij}) \left[\sum_{q_1\tilde{q}_1} (\eta_{q_1}^i + \eta_{\tilde{q}_1}^i) R_{q_1\tilde{q}_1}\right] \left[\sum_{q_2\tilde{q}_2} (\eta_{q_2}^j + \eta_{\tilde{q}_2}^j) R_{q_2\tilde{q}_2}\right] \tag{16}$$

In this case an incoherent sum over the branching ratios is assumed. Correlations between wrong combinations slightly modify the equation. As an example assume a true $\mathrm{W^+W^-} \to (\mathrm{c\bar{s}})(\mathrm{\bar{u}d})$ event. If, for example, the c quark is combined with the d quark, the other combination is fixed as being $\mathrm{\bar{s}\bar{u}}$. Such a correlation is not accounted for in the above equation. However, it is rather straight-forward to include it. Such a modification is important only if all four quarks are tagged, which is rather unlikely.

In addition one has to estimate the double tags from background, mainly from continuum QCD events with two hard gluons and from Z^0 pair events. The amount and particle content of these events must be estimated from simulation.

The single tag rate is given by:

$$\begin{aligned} N_i \;=\; & N_W\{ \\ & \eta_{\mathrm{u}}^i\, [\mathcal{K}_{\mathrm{d}} \cdot B(\mathrm{W} \to \mathrm{ud}) + \mathcal{K}_{\mathrm{s}} \cdot B(\mathrm{W} \to \mathrm{us}) + \mathcal{K}_{\mathrm{b}} \cdot B(\mathrm{W} \to \mathrm{ub})] \\ + \; & \mathcal{K}_{\mathrm{u}} [\eta_{\mathrm{d}}^i \cdot B(\mathrm{W} \to \mathrm{ud}) + \eta_{\mathrm{s}}^i \cdot B(\mathrm{W} \to \mathrm{us}) + \eta_{\mathrm{b}}^i \cdot B(\mathrm{W} \to \mathrm{ub})] \\ + \; & \eta_{\mathrm{c}}^i\, [\mathcal{K}_{\mathrm{d}} \cdot B(\mathrm{W} \to \mathrm{cd}) + \mathcal{K}_{\mathrm{s}} \cdot B(\mathrm{W} \to \mathrm{cs}) + \mathcal{K}_{\mathrm{b}} \cdot B(\mathrm{W} \to \mathrm{cb})] \\ + \; & \mathcal{K}_{\mathrm{c}} [\eta_{\mathrm{d}}^i \cdot B(\mathrm{W} \to \mathrm{cd}) + \eta_{\mathrm{s}}^i \cdot B(\mathrm{W} \to \mathrm{cs}) + \eta_{\mathrm{b}}^i \cdot B(\mathrm{W} \to \mathrm{cb})]\ \} \end{aligned} \tag{17}$$

where $\mathcal{K}_q$ is the probability that jet from quark q does *not* lead to any tag:

$$\mathcal{K}_q \;=\; [1 \;-\; \rho_{\mathrm{W}} \;+\; (1 + \rho_{\mathrm{W}}) \sum_q \eta_q^i]. \tag{18}$$

References

[1] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. **D2** (1970) 1285;
S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264;
A. Salam, *Elementary Particle Theory*, ed. N. Svartholm (Almquist and Wiksells, Stockholm, 1969) pp. 367.

[2] D. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343;
S. Weinberg Phys. Rev. Lett. **31** (1973) 494;
H. Fritzsch, M. Gell-Mann and H. Leutweyler, Phys. Lett. **47B** (1973) 365.

[3] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531;
M. Kobayashi and T. Maskawa, Prog. Thor. Phys **49** (1973) 652.

[4] ALEPH Collaboration, R. Barate *et al.*, Phys. Lett. **B484** (2000) 205;
DELPHI Collaboration, P. Abreu *et al.*, Phys. Lett. **B479** (2000) 89;
L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. **B496** (2000) 19;
OPAL Collaboration, G. Abbiendi *et al.*, Phys. Lett. **B493** (2000) 249.

[5] ALEPH Collaboration, R. Barate *et al.*, Phys. Lett. **B465** (1999) 349;
DELPHI Collaboration, P. Abreu *et al.*, Phys. Lett. **B439** (1998) 209.

[6] OPAL Collaboration, G. Abbiendi *et al.*, Phys. Lett. **B490** (2000) 71.

[7] OPAL Collaboration, R. Akers *et al.*, Z. Phys. **C60** (1993) 397;
DELPHI Collaboration, P. Abreu *et al.*, Eur. Phys. J. **C14** (2000) 613;
SLD Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **85** (2000) 5059.

[8] T. Sjöstrand, Comp. Phys. Comm. **39** (1986) 347;
T. Sjöstrand and M. Bengtsson, Comp. Phys. Comm. **43** (1987) 367.

[9] G. Marchesini *et al.*, Comp. Phys. Comm. **76** (1992) 464.

[10] J. Letts and P. Mättig, Z. Phys. **C73** (1997) 217.

[11] OPAL Collaboration, G. Abbiendi *et al.*, Eur. Phys. J. **C16** (2000) 407.

[12] R.P. Feynman and R.D. Field, Nucl. Phys. **B136** (1978) 1;
TASSO Collaboration, R. Brandelik *et al.*, Phys. Lett. **100B** (1981) 357.

[13] "Conceptual Design of a 500 GeV e^+e^- Linear Collider with Integrated X-ray Laser Facility", eds. R. Brinkmann, G. Materlik, J. Rossbach and A. Wagner, DESY 1997-048, ECFA 1997-182.

[14] N. Walker, "Luminosity at Low Energies", talk at the ECFA/DESY Linear Collider Workshop, Frascati, 8-10 November 1998;
R. Brinkmann, "Progress with TESLA", talk at the ECFA/DESY Linear Collider Workshop, Orsay, 5-7 April 1999.

[15] See, for example,
http://www.desy.de/∼njwalker/ecfa-desy-wg4/parameter_list.html.

[16] R. Hawkings, "Vertex Detector and Flavour Tagging Studies for the TESLA Linear Collider", LC-PHSM-2000-021-TESLA.

[17] R. Hawkings, private communication.

[18] M. Hauschild, private communication.

[19] M. Hauschild *et al.*, Nucl. Instr. and Meth. **A379** (1996) 43.

[20] OPAL Collaboration, K. Ackerstaff *et al.*, Z. Phys. **C76** (1997) 387.

[21] ZFITTER:
D. Bardin *et al.*, CERN-TH 6443/92;
D. Bardin *et al.*, Phys. Lett. **B255** (1991) 290;
D. Bardin *et al.*, Nucl. Phys. **B351** (1991) 1;
D. Bardin *et al.*, Z. Phys. **C44** (1989) 493.

[22] D.E. Groom *et al.*, Eur. Phys. J. **C15** (2000) 1.

[23] M. Frank, P. Mättig, R. Settles and W. Zeuner, "Experimental Aspects of Gauge Boson Production in e^+e^- Collisions at $\sqrt{s}$ = 500 GeV", MPI-PHE-92-02, in *Proc. of the Workshop on a 500 GeV Linear* e^+e^- *Collider*, ed. P. Zerwas.

[24] T. Sjöstrand and V.A. Khoze, Z. Phys. **C62** (1994) 281;
T. Sjöstrand and V.A. Khoze, Phys. Rev. Lett **72** (1994) 28.

[25] See, for example:
A. Pich, "Tau Physics", hep-ph/9912294, in *Proc. of the XIX International Symposium on Lepton and Photon Interactions at High Energies*, Stanford, California, 9-14 August 1999, eds. J. A. Jaros and M. E. Peskin.

[26] CDF Collaboration, T. Affolder *et al.*, "First Measurement of the Ratio $B(\mathrm{b} \to \mathrm{Wb})/B(\mathrm{t} \to \mathrm{W}q)$ and Associated Limit on the CKM Element $|V_{\mathrm{tb}}|$", hep-ex/0012029.

[27] M. Beneke *et al.*, "Top Quark Physics", hep-ph/0003033, CERN-TH-2000-100, in *1999 CERN Workshop on Standard Model Physics (and more) at the LHC,* eds. G. Altarelli and M. L. Mangano, CERN-2000-004.

[28] I. Bigi *et al.*, Phys. Lett. **B181** (1986) 157.

[29] K. Fujii, T. Matsui and Y. Sumino, Phys. Rev. **D50** (1994) 4341;
V.A. Khoze, L.H. Orr and W.J. Stirling, Nucl. Phys. **B378** (1992) 413;
V.A. Khoze, J. Ohnemus and W.J. Stirling, Phys. Rev. **D49** (1994) 1237;
G.V. Jikia, Phys. Lett. **B257** (1991) 196.

[30] E. Boos *et al.*, Z. Phys. **C70** (1996) 255.

[31] The BaBar Collaboration, D. Boutigny *et al.*, *The BaBar Physics Book: Physics at an Asymmetric B Factory*, ed. P. F. Harrison, SLAC-R-0504.

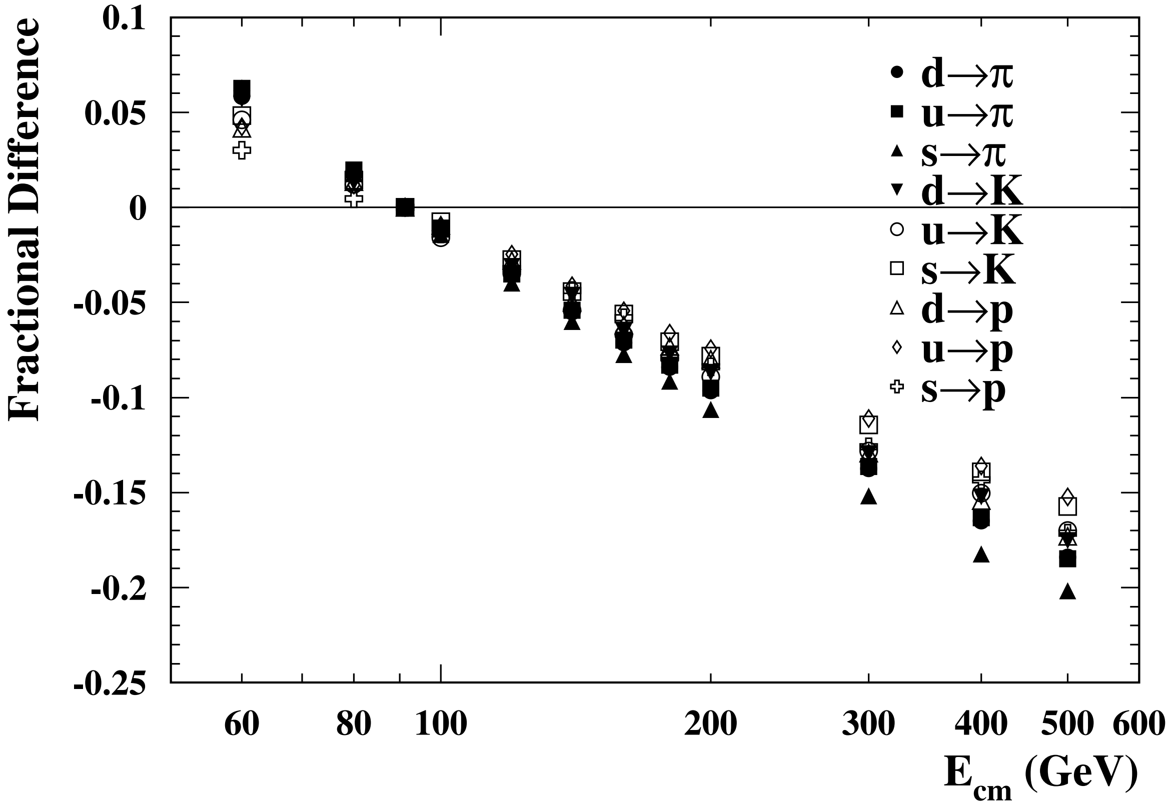

Figure 1: Fractional difference $(\eta_q^i(E_{\mathrm{cm}}) - \eta_q^i(M_{\mathrm{Z}^0}))/\eta_q^i(M_{\mathrm{Z}^0})$ for various η_q^i as a function of the centre-of-mass energy, E_{cm}, from the JETSET Monte Carlo.

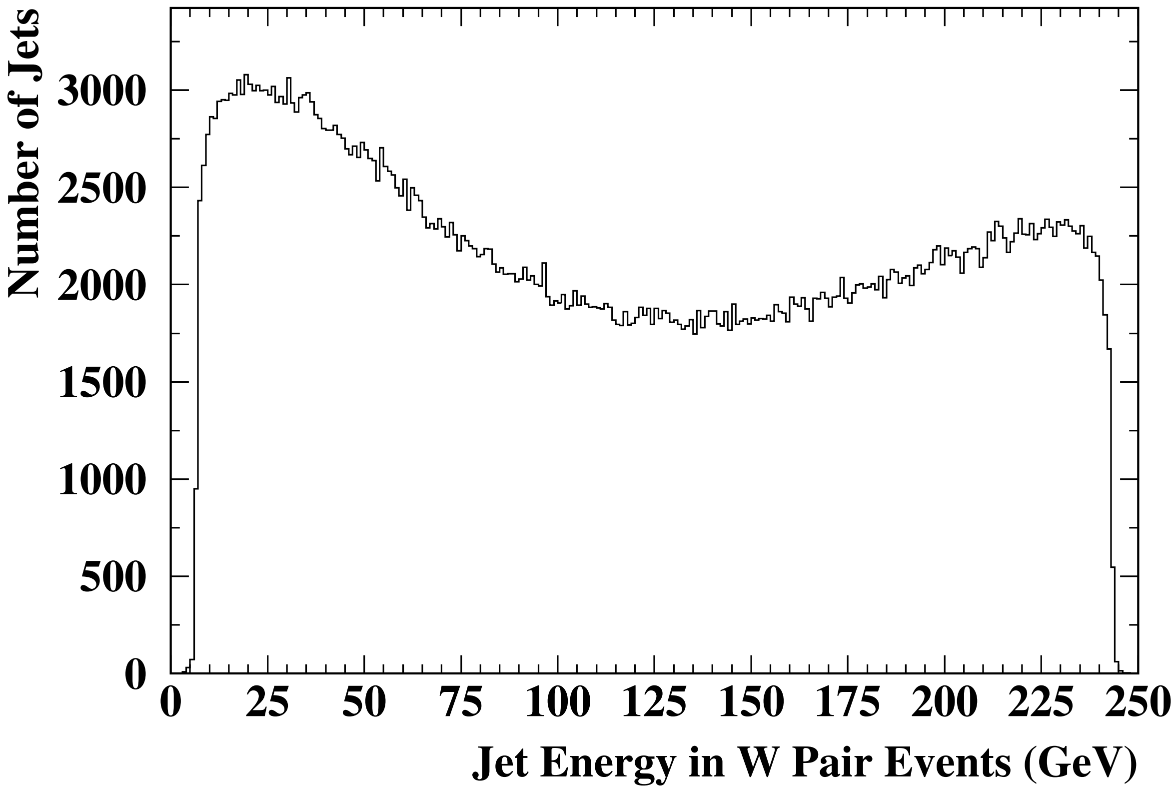

Figure 2: Energies of jets in W pair events generated with the PYTHIA Monte Carlo at a centre-of-mass energy of 500 GeV.

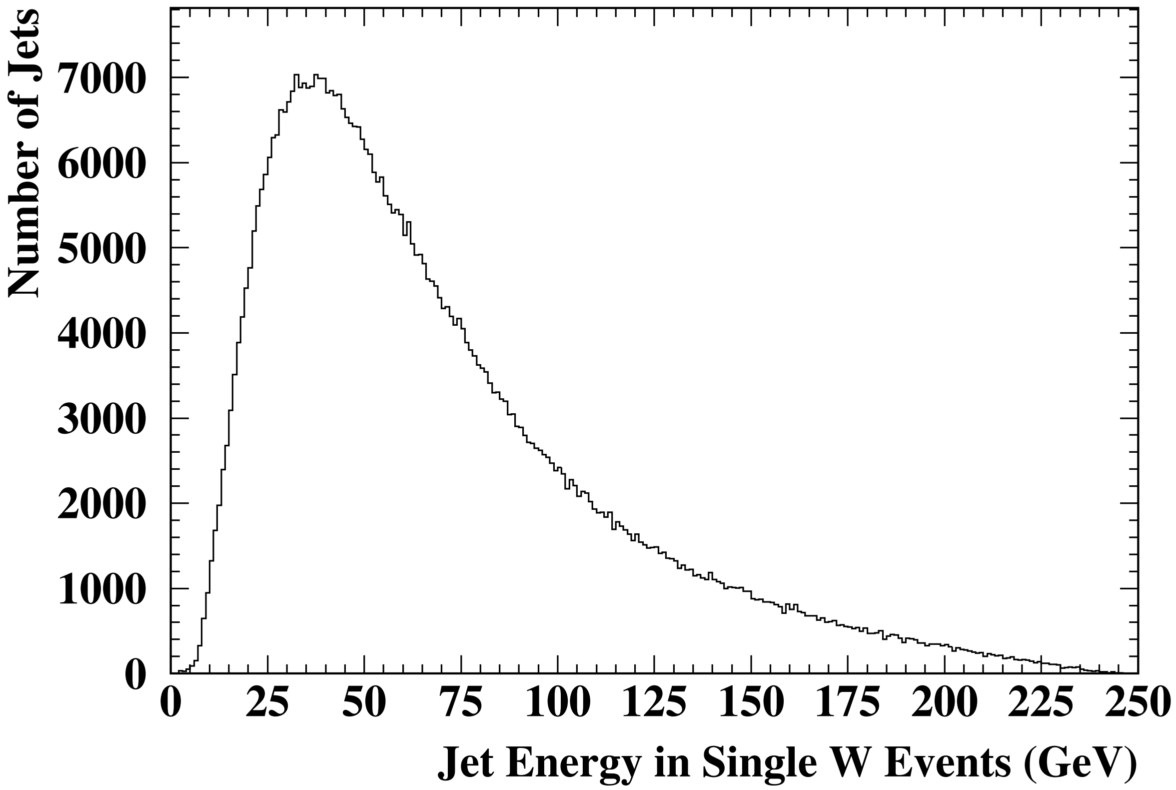

Figure 3: Energies of jets in single W events generated with the PYTHIA Monte Carlo at a centre-of-mass energy of 500 GeV.

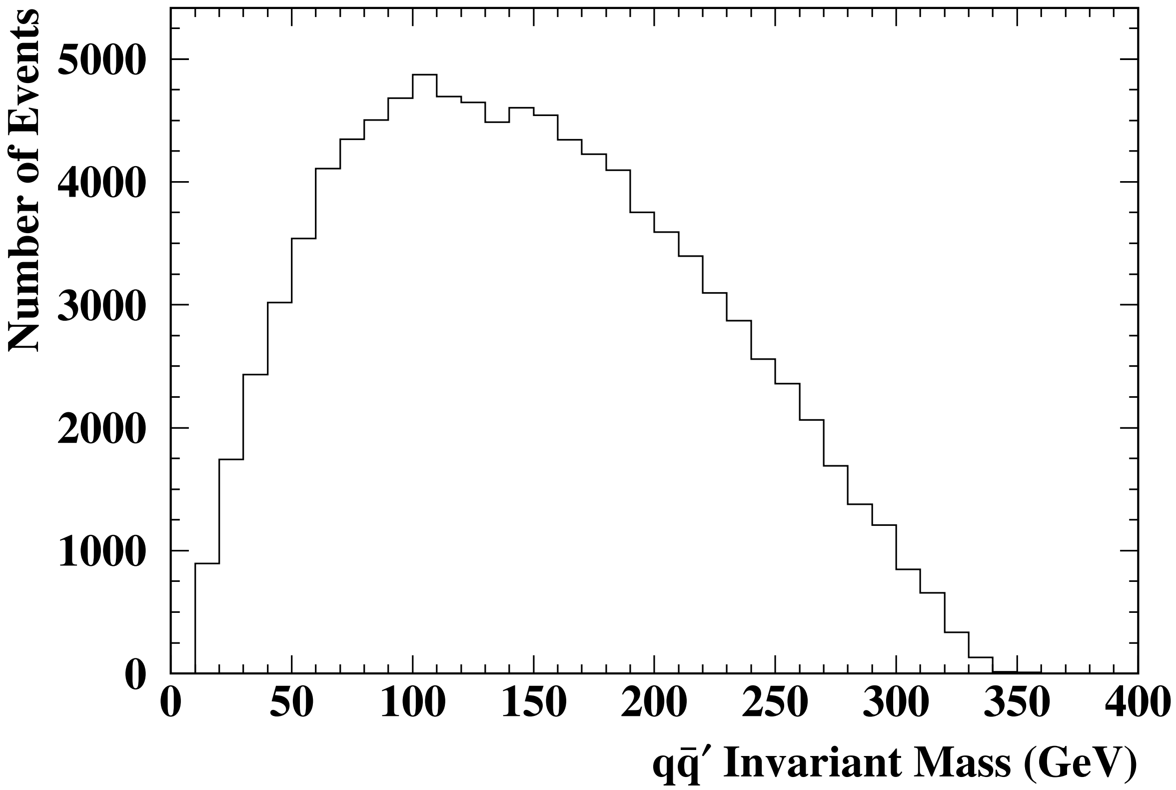

Figure 4: Invariant mass of the $q\bar{q}'$ system from the decays $\mathrm{t} \rightarrow q\mathrm{W}^+$ and $\overline{\mathrm{t}} \rightarrow \overline{q}'\mathrm{W}^-$ generated with the JETSET Monte Carlo at a centre-of-mass energy of 500 GeV.