PRECISION MEASUREMENT OF THE W MASS WITH A POLARISED THRESHOLD SCAN AT A LINEAR COLLIDER

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Threshold scans potentially offer the highest precision in the determination of the masses and widths of known and as yet undiscovered particles at linear colliders. Concentrating on the definite example of the WW threshold for determining the W mass (M_{W}), it is shown that the currently envisaged high luminosities and longitudinal polarisation for electrons **and positrons** allow M_{W} to be determined with an error of 6 MeV with an integrated luminosity of 100 fb^{-1} (One 10^7 s year with TESLA). The method using polarised beams is statistically powerful and experimentally robust; the efficiencies, backgrounds and luminosity normalisation may if needed be determined from the data. The uncertainties on the beam energy, the beamstrahlung sprectrum and the polarisation measurement are potentially large; required precisions are evaluated and methods to achieve them discussed.

This writeup is my proceedings contribution from LCWS1999 (Sitges) augmented at the end by a brief update on addressing some of the open questions of that writeup. Apologies for not having had time to write a coherent up to date document with more expansive documentation.

Present measurements of M_{W} to 40 MeV from the Tevatron and LEP2 combined [1] play an important role in testing the electro-weak theory through radiative corrections from the Higgs mass and top mass. The present measurement precision and projected errors from LEP2, TeV2 and LHC however indicate that an eventual precision below 20 MeV will be hard. With expected large improvements in the top mass from a linear collider (LC) and perhaps a precisely measured Higgs mass, the M_{W} precision would be uncomfortably poor on the scale of the M_{Z} error (2 MeV).

The LC offers the opportunity to measure M_{W} to better than 5 MeV in a single experiment which would vastly improve our knowledge of M_{W} and test the consistency of the observed particle spectrum with theoretical predictions [2].

1 Methods to Measure M_{W} in $e^+ e^-$ Collisions

As discussed in [3], two methods are used at LEP2 to measure M_{W}. Firstly with 10 pb^{-1} per experiment, a threshold cross-section measurement was made. This can be viewed as a special case (with one scan point) of a "classic" BES-like threshold scan for the tau mass. A proper threshold scan for measuring M_{W} was also discussed [4]. Secondly, direct kinematic reconstruction (DKR) of M_{W} using the kinematic constraints of energy and momentum conservation. Both methods are valid and for the integrated luminosities accumulable at LEP2 (2 fb^{-1} combined), DKR is preferred, leading to somewhat lower statistical errors per pb^{-1} and expected systematic errors commensurate with the statistics.

At a LC, where integrated luminosities of 100 fb^{-1} around $\sqrt{s} = 161$ GeV and 2000 fb^{-1} at 800 GeV may be achievable in 1 and 4 10^7s years running respectively

(TESLA design), there is clearly **statistical** scope for vast improvements in the measurement of M_W. However to take advantage one needs also to control **systematics** to unprecedented precision.

Two features of LCs look particularly challenging for precision measurements of M_W. Firstly the ability to control the absolute average beam energy (E_{beam}). Secondly, the beam-strahlung distortion of the luminosity spectrum resulting in an additional uncertainty on top of initial state radiation (ISR) on the actual $\sqrt{s}$ of the collision and the net momentum. The E_{beam} uncertainty will increase substantially with energy as all methods are likely to calibrate in some way to the Z. Beam-strahlung introduces an effective energy spread which grows with E^2_{beam}.

Let's proceed assuming that these can be dealt with and discuss them in more detail once it is established that other issues are potentially controllable. However the scaling of these effects with E_{beam} may favour a **threshold scan**.

On the theoretical side, **DKR** of M_W in the 4q channel is dogged at present by errors from final-state interactions resulting in "cross-talk" between decay products of different W's. The 4q channel may not contribute significantly to a precise measurement of M_W given that this source currently contributes a 58 MeV error at LEP2. If the 4q channel is unusable for DKR, the statistical sensitivity of a threshold scan is essentially identical to DKR at LEP2 energies per unit luminosity.[a]

The **threshold scan** relies on a theoretical understanding of the cross-section dependence on M_W. Fortunately most of this is simply kinematics and QED. The Coulomb correction at threshold of around 6% is known to all orders, see[5]. Nevertheless it will be necesary for theoretical calculations to be reviewed and extended substantially to reach the 0.06% precision which is needed to keep the theoretical error on the M_W interpretation of a threshold scan below 1 MeV.

2 Study of a M_W measurement from a polarised threshold scan

Having argued plausibly that a **threshold scan** may provide the better means of achieving precisions of order 5 MeV, we elaborate on how such a measurement might be made based on the following calculations.

Calculations. The unpolarised WW cross-section is calculated using the GENTLE2.0 program[6] for values of $\sqrt{s}$ ranging from 139.8 GeV to 170.3 GeV in steps of 0.25 GeV. The calculated cross-section is "CC03-only" for WW production ignoring effects of interfering contributions from the general 4-fermion production. This is carried out for 3 values of M_W : 80.31, 80.39 and 80.47 GeV corresponding to the $\pm 2\sigma$ range of the current preliminary world average. The cross-section is also calculated at each of these M_W values for 3 different values of α_S (0.06, 0.12, 0.24) to simulate the effect of a non-SM width. The resulting cross-section data is then interpolated linearly between 0.25 GeV "theoretical scan-points" and convolved with the expected beam-strahlung spectrum at TESLA evaluated for $\sqrt{s} = 170$ GeV using CIRCEV6[7] to give the expected measured cross-sections including beam-strahlung at values of nominal centre-of-mass energy $\sqrt{s}$ ranging from 150 GeV to 170 GeV in steps of 0.2 GeV. The calculation does not at present include beam energy spread

[a] For $\sqrt{s}$=500 GeV, the WW cross-section is much lower leading to less statistical power

Table 1: Multiplicative factors on the unpolarised cross-section used for each helicity combination (HC). P^- and P^+ are defined to be positive and are nominally 0.8 and 0.6 respectively. The background factors are calculated setting A_{LR} to zero (in general A_{LR} will be non-zero - addressed in the update.)

HC index (k)	e^+	e^-	σ_{WW}	$\sigma_{\rm bkgd}$
1	R	L	$(1+P^-)(1+P^+)$	$(1+P^-P^+)$
2	L	R	$(1-P^-)(1-P^+)$	$(1+P^-P^+)$
3	0	0	1	1
4	0	L	$1+P^-$	1
5	R	0	$1+P^+$	1
6	0	R	$1-P^-$	1
7	L	0	$1-P^+$	1

from the linac which is below about 0.1% for the TESLA design and is therefore small with respect to beamstrahlung.

Beam Polarisation. Near the WW threshold, the WW cross-section is dominated by the terms of the matrix element squared including the t-channel ν_e exchange amplitude. Due to the V-A nature of the charged-current interaction, Given that the neutrino is left-handed and the W is "maximally parity-violating", this amplitude only couples to the $e_R{}^+e_L{}^-$ helicity combination (HC). So for unpolarised beams, only 1 out of 4 e^+e^- HCs can make WW via the t-channel ν_e exchange amplitude. To a good approximation, this means that if one could have 100% polarised electron and positron beams of switchable polarity then one could either choose:

- $e_R{}^+e_L{}^-$. **Quadrupling** the WW cross-section and doubling the background [b]
- OR, $e_L{}^+e_R{}^-$. Switching off WW production, while doubling the background. Ideal for measuring the background **at** the scan-point in question

The signal approximation has been investigated using the WOPPER generator [8] and has been found to be excellent. The $e_L{}^+e_R{}^-$ cross-section is about 0.4% of the $e_R{}^+e_L{}^-$ cross-section at threshold and never exceeds 1% from 158 GeV to 200 GeV. For the purposes of the study, where we assume achievable electron beam polarisations of 80% and positron beam polarisations of 60%, we are justified in neglecting the WW production from the $e_L{}^+e_R{}^-$ HC. For calculation of cross-sections with polarised beams, the calculated unpolarised cross-sections for WW production and background are then multiplied by the factors given in Table 1.

The background approximation of $A_{LR} = 0$ is not very good since in fact A_{LR} is expected to be around 0.15, 0.33, 0.49 for the ll, lh and hh channels respectively. The quantitative aspects of this approximation are discussed in the update section.

Scan strategy and fit ansatz. The observables for the threshold scan are the observed number of selected events, $N^{ijk}_{\rm obs}$ at each $\sqrt{s}$ (index i), for each channel (index j) where by channel we mean fully leptonic ($\ell\ell$), mixed leptonic-hadronic (ℓh) and fully hadronic (hh), and for each HC with index k defined in Table 1. The fraction of the available luminosity spent at each $\sqrt{s}$ and HC can be optimised

[b] ignoring A_{LR} effects and assuming that the background is "Z-like"

Table 2: Assumed event selection performance and the fraction of WW produced in each channel.

Channel (j)	Efficiency (%)	Unpolarised σ_{bkgd} (fb)	WW fraction (%)
$\ell\ell$	75	20	10.5
ℓh	75	80	44.0
h h	67	400	45.5

according to the expected total error on M_{W}. In order to evaluate a reasonable strategy while taking into account likely systematic errors, we define a fit to the observed numbers of events seen at all ijk values with the following 11 parameters.

1 M_W, the W mass.

2-4 σ^j_{bkgd}, ($j = \ell\ell$, ℓh, hh), the unpolarised background cross-section per channel.

5 f_{lumi}, a scale factor for uncertainty on the absolute integrated luminosity.

6-8 ε_j. Correction factors to the estimated efficiency in each channel.

9-10 The actual polarisation of the electron and positron beams.

11 α_S. This is a convenient way to investigate sensitivity to Γ_W, the W width[c].

We will usually fix α_S to the measured value of 0.12 and assume that Γ_W follows the Standard Model (SM) dependence on M_{W}. The fits carried out assume that the absolute luminosity, channel efficiencies and beam polarisations are known with some accuracy. We assume that the absolute luminosity and channel efficiencies are measured with a relative precision of 0.25% [d] and that the beam polarisations are both measured[e] with an absolute uncertainty of 0.25% (best achievement to date at SLC is 0.5% [9]).

A χ^2 fit to the simulated data is carried out with parameters 5 to 10 constrained by χ^2 penalty functions to be consistent with their measured errors. No useful a priori knowledge of the background cross-sections is assumed and they are determined principally using the measured cross-section with HC2.

The assumptions for the event selection efficiency and background acceptance are given in Table 2 based on the Oxford97 workshop [10] where the achieved performance of LEP2 at 161 GeV was reviewed. These appear to be reasonable if perhaps conservative performance numbers for the LC detector. The study assumes for simplicity that the efficiencies and backgrounds are constant with $\sqrt{s}$ and that point-to-point systematics are negligible. Other systematics such as 4-fermion effects and the background spin model have not been investigated.

A number of scans have been investigated. Since the polarisation allows one to determine the background at the scan point in question, there appears to be no need to measure the cross-section well below threshold unless one is interested in measuring Γ_W. An example scan is the one given in Table 3 with 6 scan points of which

[c] Equivalent to fitting for Γ_W

[d] Challenging, but factors of 4 looser than achieved for the LEP Z line-shape scan

[e] The achievable precision on the polarisation at the LC has not yet been worked through in detail, it appears that 0.5% is possible, whereas 0.1% may well be unachievable

$\sqrt{s}$ (j)	Luminosity weight
160.4	0.2
161.0	1.0
161.2	1.0
161.4	1.0
162.0	0.2
170.0	1.2

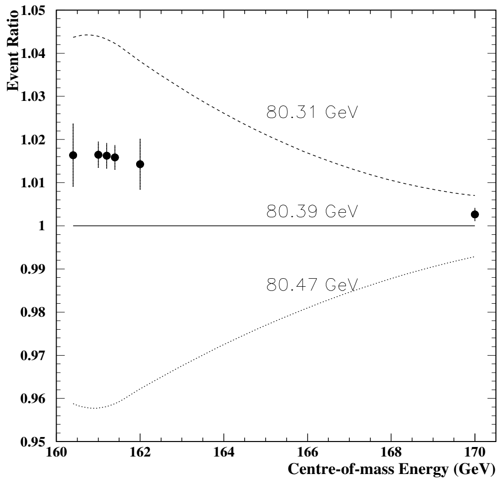

Figure 1: The expected value of the ratio of observed events to that predicted for $M_{\rm W} = 80.39$GeV is shown for an actual value of $M_{\rm W} = 80.36$GeV from the summed HC1 data in the proposed scan

4 have significant fractions of the luminosity. In order to constrain the polarisation with data,[f] for each scan point, 5% of the available luminosity is spent on HC3 and 2.5% each on helicity combinations 4 to 7 (total 15%). The remaining luminosity is split 85:15 between HC1 (WW production) and 2 (background measurement). So in total, one would have 6*3*7=126 measured numbers of selected events from which to measure $M_{\rm W}$ and test whether the fit ansatz actually describes the data. It is found useful to have at least one scan point at higher $\sqrt{s}$ presumably because this constrains the efficiency while being insensitive to $M_{\rm W}$.

The example scan in which a total number of 860,000 events are selected in 100 fb^{-1} leads to an error on $M_{\rm W}$ of 4.9 MeV with Γ_W fixed by the SM relationship. The scan is illustrated in figure 1. The sensitivity to the measurement errors on absolute luminosity, relative efficiency and polarisation is studied; the fits were repeated assuming that the error source in question was removed. The $M_{\rm W}$ error would reduce to 4.2 MeV, 4.5 MeV and 4.6 MeV respectively indicating that compared to the assumed 0.25% errors, the luminosity uncertainty is the most critical[g].

Alternative scans with scan points substantially below threshold offer the opportunity of optimising a Γ_W measurement. An inexhaustive study indicated that Γ_W errors of 3 MeV may be achievable but at considerable expense to the $M_{\rm W}$ uncertainty (degrades to 6 MeV). For the standard $M_{\rm W}$ optimised scan when fitting also for the width, the width error is 4 MeV while the mass error degrades to 14 MeV. Given that 3 MeV on the width is equivalent within the Standard Model to the knowledge that $\alpha_S = 0.12 \pm 0.03$, a Γ_W scan does not seem well motivated.

[f] The optimal amount of data dedicated to understanding the polarisation, or how to split among HCs has not been determined well due to convergence problems in the fits when the polarisation uncertainty is large.

[g] The efficiency and polarisation contributions would also be worrying if the uncertainties are substantially higher (like 0.5%)

LC experimental systematics In addition to the uncertainties discussed above we must return to the E_{beam} uncertainty. Two methods look very promising. Firstly the method proposed by the author[11] consisting of using Z γ events where the Z decays to $\mu\mu$ which can measure the average $\sqrt{s}$ of the $\mathrm{e^+e^-}$ luminosity, integrating over beamstrahlung effects. The statistics are sufficient at WW threshold that the statistical error for the muon channel of 3.4 MeV per 100 fb^{-1} polarised scan combined with the electron channel, would lead to a statistical error of around 2.5 MeV. Systematic errors will depend on controlling the detector absolute polar angle to 10^{-5} and controlling radiative corrections to Z γ production, both possibly achievable. Secondly, one could install a precise beam spectrometer whose absolute field calibration could be established by re-measuring M_{Z} from the line-shape and then extrapolating to the WW threshold. In each case M_{Z} from LEP1 (relative error of 2.3×10^{-5}) is used as input, so is an additional absolute error of 1.9 MeV on E_{beam} at threshold. This error cancels in the ratio $M_{\mathrm{W}}/M_{\mathrm{Z}}$ which can be argued is most relevant to testing the theory.

Beamstrahlung reduces the WW threshold cross-section by up to 7% and distorts the shape, but the effective sensitivity to M_{W} is unchanged provided the beamstrahlung spectrum is known. For a contribution to the M_{W} uncertainty below 1 MeV, one would need to know the distortion absolutely to 0.06% (1% relative precision). In reference[12], it was shown that the differential luminosity spectrum induced by beam-strahlung and energy spread can be controlled with good precision by measuring the acolinearity angle distribution of Bhabha scattering events at forward angles. In fact the event rates and expected angular resolution are excellent[13] allowing one to measure the average energy loss to below 0.1% of its value far exceeding the requirement. However this is not the full story; one needs to also control the shape (eg the rms) of the luminosity spectrum with good precision. So beamstrahlung may not be such a big issue, but it is still a somewhat open one.

Conclusions. Threshold scans will be done at the LC. The top threshold scan is an already established one. Threshold scans are ideal for measuring new fermions eg. charginos[14]. Maybe a threshold scan is best suited to the Higgs mass determination. Here, a threshold scan for measuring M_{W} has been demonstrated to be feasible with a total error including E_{beam} error of 6 MeV potentially achievable within a reasonable time frame. Given the physics interest in precise mass measurements, we should therefore plan now to do threshold scans **well**. This may have serious implications on the machine design. We should strive to achieve the best possible precision on the measurements of E_{beam}, beamstrahlung and polarisation while not neglecting the consequent need for the highest possible luminosity at each (even unknown) threshold and highly polarised beams.

Update on further work since Sitges

A number of issues have been investigated more thoroughly during 2000 and the most important ones are discussed in [15].

Having been misled by literature that indicated that A_{LR} is "small" for fermion pairs, the sensitivity of the W mass measurement was re-evaluated for realistic A_{LR} background values of 0.15, 0.33 and 0.49 for the ll, lh and hh channels respectively.

In the Sitges writeup, the sensitivity to polarisation systematics was noticable. It has been demonstrated in [15] that the beam polarisation can be measured "in situ" using a modest fraction of the luminosity (7%) to collect data with all possible helicity combinations. This uses the radiative return process (Z (γ) production) at centre-of-mass energies of around 161 GeV with a cross-section of 100 pb to measure the polarisation of each beam for each sign of the helicity and for the apparently zero polarisation data. Basically one has 9 measurements to determine 8 variables : the 6 polarisations, the radiative return left-right asymmetry and the radiative return cross-section. So, basically the need for precision absolute polarisation measurements is reposed as a need to do a good relative measurement of radiative return events. This appears quite tractable.

In total, the quoted 4.9 MeV beam energy independent error on the W mass from experimental effects quoted in the Sitges proceedings becomes 5.2 MeV after refining the polarisation and accounting for a more correct background spin model.

The error coming from an uncertainty on the background spin model has also been investigated. If the A_{LR} of all the background was 0.50 and one knew this with an error of 0.01, this would induce an error on the W mass of 4 MeV. The best strategy to optimise the measurement facing this systematic is probably to improve the purity of the fully hadronic selection over that achieved at LEP2 and may argue in favour of an event selection more targeted at the l nu l nu , qqenu and qqmunu channels.

The excellent detector performance characteristics of the proposed linear collider detector offer substantial potential to improve further the performance and test with control samples aspects crucial to this measurement.

In summary, a number of additional issues have been investigated in more detail and the studies have demonstrated that the sensitivity to the W mass is very much as initially evaluated. **I reiterate that to realise fully this potential a precise beam energy measurement is essential.**

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