LC-PHSM-2001-016
20 February 2001

Determination of the Strong Coupling Constant at Giga-Z

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Abstract

The strong coupling constant may be extracted from the production of hadronic (and leptonic) final states in e^+e^- annihilation at Giga-Z using the same Z^0 line-shape observables as at LEP-1. An attempt is made to show how the two orders of magnitude higher integrated luminosity and the unprecedented detector performances foreseen at the Linear Collider would allow to improve the present precision on the strong coupling constant. The study hints to the fact that an accuracy substantially better than $\pm$ 0.001 is achievable, since most of the systematic uncertainties of the LEP measurements are heavily influenced by the statistical power of their estimates.

1 Introduction

The determination of the strong coupling constant (α_s) may remain for long among the most essential Standard Model parameters to determine experimentally with the best possible accuracy. Its actual magnitude is not predicted by Quantum Chromodynamics (QCD) despite its importance for theoretical predictions . Top-pair production in e^+e^- collisions is an example where an accurate value of α_s is required for a precise determination of the top-quark mass from a scan of the $t\bar{t}$ kinematical threshold. An accurate measurement of α_s is also needed to test the hypothesis underlying Grand Unified Theories, which predict that all three gauge interaction constants would merge at some high energy scale. Furthermore, the comparison between values of α_s determined at different energy scales, which allows to test the non-abelian structure of the strong gauge group (SU(3)), calls for accurate measurements in order to cope with the slow (logarithmic) evolution of α_s in the perturbative regime. Finally, the objective of testing the Standard Model with increasing refinement can only be realised with an accurate knowledge of the QCD corrections entering the theoretical prediction of numerous electroweak observables.

The extraction of α_s from Z^0 line-shape observables at LEP-1 [1], [2], [3], [4] has resulted in a deep understanding of the sensitive aspects of the measurement and has shown that the latter would benefit substantially from larger statistical precision since most of the systematic uncertainties reflect the limited statistical power (based on real data) of their estimates. Furthermore, the accuracy of physics generators will improve in the coming years and the detector foreseen at the Linear Collider will achieve substantially higher performances than any LEP experiment in any specific physics area. All this should translate in a sizeable reduction of the total uncertainty on $\alpha_s(M_Z)$, as shown in this paper. The latter is organised as follows. The observables of the Z^0 line-shape sensitive to α_s are recalled in Section 2, which also summarises the advantages they provide in comparison with other measurements of α_s. The sensitivity of these observables are compared to each other in Section 3. Section 4 gives an insight of the experimental aspects underlying the determination of the relevant observables at LEP-1, and provides the ground to estimate how the LEP measurements can be transposed to Giga-Z. The systematic uncertainties achievable at Giga-Z for each relevant final state are discussed in Section 5 and the precision expected on $\alpha_s(M_Z)$ is exposed in Section 6. Section 7 provides a summary of the study.

2 Z^0 observables sensitive to α_s

The cross-section for the process $e^+e^- \to$ hadrons, combined with that of lepton-pair production, allows to extract the experimental value of four observables sensitive to α_s: the ratio (R_l) of the hadronic to the leptonic cross-section, the total width (Γ_Z) of the Z^0 resonance and the leptonic and hadronic Born cross-sections (σ_0^l and σ_0^h) at the resonance peak.

The advantage of using these observables is that their measurements are based on the production of quark-antiquark pairs in e^+e^- annihilation, a process among the few which can be analysed rigorously in the framework of QCD [5]: the measurement is inclusive, there is no phenomenological dependence on the details of the hadronic final state and the non-perturbative corrections are suppressed by $1/s^2$. The magnitude of the latter can actually be calculated in terms of matrix elements of gauge invariant local operators. The first non-perturbative correction to R_l depends on the vacuum expectation value of the square of the gluon field strength tensor, which is of order Λ^4_{QCD}/s^2, i.e. $\sim 10^{-10} - 10^{-11}$ at Giga-Z.

The extraction of $\alpha_s(M_Z)$ from the measurements of R_l, Γ_Z, σ_0^l and σ_0^h relies on the value of these observables predicted by the Standard Model, which depends upon the coupling of the quarks and leptons to the Z^0. The precision is such that higher order electroweak corrections to these couplings must be included. There are in particular theoretical uncertainties arising from the values of the top-quark and Higgs-boson masses which enter these radiative corrections. Hence, while this extraction method has small theoretical uncertainties from QCD itself, it relies sensitively on the electroweak couplings of the Z^0 to quarks and leptons and on the ability of the Standard Model to predict these correctly. These corrections are actually quite different for the cross-sections σ_0^l and σ_0^h and for R_l and Γ_Z: ignoring for instance the Higgs mass introduces less uncertainty on σ_0^l or σ_0^h than on R_l.

The presence of new physics affecting these couplings via electroweak radiative corrections would of course invalidate the value of $\alpha_s(M_Z)$ extracted.

3 Sensitivity of Z^0 observables to $\alpha_s(M_z)$

The sensitivity of R_l, Γ_Z, σ_0^l and σ_0^h to $\alpha_s(M_Z)$ is actually coming from the α_s dependence of the hadronic partial width (Γ_h), which can be approximated by the expression:

$$\Gamma_h \;=\; \Gamma_h^0 \cdot (1+\delta_{QCD})$$

where Γ_h^0 stands for the hadronic partial width without its QCD correction, δ_{QCD}. The α_s dependence of the latter can be expressed in the following approximated way [6]:

$$\delta_{QCD} \;\approx\; 1.06\,\frac{\alpha_s}{\pi} + 0.9\left(\frac{\alpha_s}{\pi}\right)^2 - 15\left(\frac{\alpha_s}{\pi}\right)^3 + \ldots$$

The first term of this expression (i.e. $1.06\ \alpha_s(M_Z)/\pi$) dominates δ_{QCD} in magnitude, and amounts typically to 4 % for $\alpha_s(M_Z) \sim 0.12$.

The factorisation entering the expression of Γ_h is not exact since QCD corrections also affect the Z^0 self energies and the $Z \to b\bar{b}$ vertex, giving rise to α_s dependent terms in Γ_h^0. Furthermore, Γ_h is the sum of a vectorial and an axio-vectorial current components, which both receive different QCD corrections, hence forbidding factorisation in principle. However, the arguments developed in the present paper are not influenced by the non-factorisable contributions to Γ_h, which account actually for very small effects [7].

Since $R_l = \Gamma_h/\Gamma_l$, where Γ_l represents the leptonic partial width of the Z^0, the QCD correction to R_l is also about 4 %. The QCD corrections entering the Z^0 self-energy are neglected here since their absolute size is minute and furthermore, their effect is almost identical for the numerator (Γ_h^0) and the denominator (Γ_l) of the ratio, translating into a first order cancellation.

The total width (Γ_Z), is related to Γ_h^0 through the relation:

$$\Gamma_Z \;=\; \Gamma_L + \Gamma_h \;=\; \Gamma_L + \Gamma_h^0 \cdot (1+\delta_{QCD})$$

where Γ_L stands for the total leptonic width of the Z^0 (including charged and neutral leptons). It amounts to $\sim$ 30 % of Γ_Z, which allows to write:

$$\Gamma_Z \;=\; \Gamma_Z^0 + \Gamma_h^0 \cdot \delta_{QCD} \;\approx\; \Gamma_Z^0 \cdot (1 + 0.7 \cdot \delta_{QCD})$$

where $\Gamma_Z^0 \;=\; \Gamma_L + \Gamma_h^0$. One sees that the QCD correction to Γ_Z amounts to $\sim$ 2.8 %.

The sensitivity of Γ_Z to α_s is best exploited in the expression of the Born leptonic cross-section (σ_0^l), which varies like the square of Γ_Z:

$$\sigma_0^l \;=\; \frac{12\pi\Gamma_l^2}{M_Z^2\Gamma_Z^2} \;\approx\; \frac{12\pi\Gamma_l^2}{M_Z^2{\Gamma_Z^0}^2}\cdot\frac{1}{(1+0.7\cdot\delta_{QCD})^2} \;\approx\; {\sigma_0^l}^0 \cdot (1 - 1.4\cdot\delta_{QCD})$$

The QCD correction to σ_0^l is therefore in the range 5.5–6 %, i.e. the largest of the four considered here. Depending on the knowledge of the integrated luminosity w.r.t. to the level of understanding of the hadronic final states, the extraction of α_s from σ_0^l may come out more precisely than from R_l.

The situation of σ_0^h is much less favorable since the correction δ_{QCD} enters its numerator and denominator, i.e.:

$$\sigma_0^h = \frac{12\pi\Gamma_l\Gamma_h}{M_Z^2\Gamma_Z^2} \approx \frac{12\pi\Gamma_l\Gamma_h^0}{M_Z^2\Gamma_Z^{0\,2}} \cdot \frac{1+\delta_{QCD}}{(1+0.7\cdot\delta_{QCD})^2} \approx \sigma_0^{h0}\cdot(1-0.4\cdot\delta_{QCD})$$

The QCD correction to σ_0^h comes out to be ~ 1.5 %, which makes σ_0^h the least sensitive of the 4 observables.

In summary, the relation between the absolute precision on $\alpha_s(M_Z)$ and the relative experimental accuracy on the sensitive observables may be approximated by the following numerical expressions:

$$\begin{array}{lrcl}
R_l: & \Delta\alpha_s(M_Z) & \approx & 3.1\cdot\Delta R_l/R_l \\
\sigma_0^l: & \Delta\alpha_s(M_Z) & \approx & 2.2\cdot\Delta\sigma_0^l/\sigma_0^l \\
\sigma_0^h: & \Delta\alpha_s(M_Z) & \approx & 7.4\cdot\Delta\sigma_0^h/\sigma_0^h \\
\Gamma_Z: & \Delta\alpha_s(M_Z) & \approx & 4.4\cdot\Delta\Gamma_Z/\Gamma_Z
\end{array}$$

where the second digit of the numerical coefficients may vary slightly, depending on the approximations made in the calculus.

The goal of a future measurement of $\alpha_s(M_Z)$ is to achieve a precision better than 0.001, and possibly 0.0005. Table 1 compares the size of the QCD correction to each of the four observables with the precision of their measurements at LEP. The effect of a variation of $\alpha_s(M_Z)$ by 0.001 on R_l, Γ_Z, σ_0^l and σ_0^h is also indicated.

observable	R_l	Γ_Z (MeV)	σ_0^l (nb)	σ_0^h (nb)
Standard Model	20.740	2496.2		41.480
A.D.L.O.	20.767 ± 0.025	2495.2 ± 2.3	2.0003 ± 0.0027	41.540 ± 0.037
QCD correction	~ 0.80	~ 70	~ 0.11	~ 0.65
$\delta\alpha_s = 0.001$	~ 0.007	~ 0.55	~ 0.001	~ 0.005

Table 1: The first line shows the Standard Model prediction for R_l, Γ_Z and σ_0^h. The second line (A.D.L.O.) provides the combined values of the four LEP experiments found for each variable. The third line exhibits the size of the QCD correction for each observable and the bottom line displays the numerical variation of each observable consecutive to a change of α_s by 0.001.

With these values of R_l, Γ_Z, σ_0^l and σ_0^h, a global fit to the LEP observables [8] lead to a value of the strong coupling constant at the energy scale of the Z^0 mass of:

$$\alpha_s(M_Z) = 0.120 \pm 0.003$$

where the range allocated to the Standard Model Higgs mass in the fit contributes approximately to $\pm$ 0.001.

At Giga-Z, one expects an intrinsic luminosity of about 3 10^{33} cm^{-2} s^{-1} or more [9]. Since the Z^0 peak cross-section for muon- and tau-pair production amounts to $\sim$ 1.5 nb and that for hadronic final states production is close to 30 nb, a year of running (10^7 s) would produce about 9 10^8 hadronic, 4.5 10^7 muon and 4.5 10^7 tau decays of the Z^0.

Since the selection efficiency for hadronic final states is almost 100 % and that for muon- and tau-pairs is about 85 %, with a very modest residual background, the statistical uncertainties associated to the production rates will be close to 0.003 % for the multihadrons and 0.015 % for each leptonic final state. These figures provide the scale at which one should aim to reduce the systematic uncertainties of the measurements. They also set the ultimate precision one could hope for if systematic uncertainties were negligible. For instance, the statistical uncertainties associated to the hadronic final states and to the muon- and tau-pairs would turn into a remarkable precision on α_s better than 0.0004 from R_l alone. The question is whether it will be possible to squeeze the systematic uncertainties to a similar magnitude. An attempt is made in the coming sections to provide an answer based on the precision achieved at LEP.

4 Achievement of LEP experiments and gains expected from Giga-Z

The measurements performed at LEP act as a very useful guide-line for the estimates of the accuracy which may be achieved at Giga-Z. Most of the speculations are obviously on the systematic accuracy one can expect when increasing the LEP statistics by two orders of magnitude and doing the measurements with a detector providing substantially better performances. One can also count on more accurate physics generators.

LEP collaborations have mainly extracted the experimental values of R_l, Γ_Z, σ_0^l and σ_0^h from the hadronic, muon-pair and tau-pair final states, taking advantage of the high precision achieved on the integrated luminosity. The systematic accuracy of the measurements performed by each of the four collaborations varies substantially from one to another. This is illustrated in Table 2, where the range of the total systematic uncertainties on the selection efficiency and the residual background subtraction is shown for each of the four relevant final states.

$\frac{\Delta\epsilon_h}{\epsilon_h} \oplus \frac{\Delta bg_h}{bg_h}$	$\frac{\Delta\epsilon_\mu}{\epsilon_\mu} \oplus \frac{\Delta bg_\mu}{bg_\mu}$	$\frac{\Delta\epsilon_\tau}{\epsilon_\tau} \oplus \frac{\Delta bg_\tau}{bg_\tau}$	$\frac{\Delta L^{exp}_{syst}}{L}$	$\frac{\Delta L^{theo}_{syst}}{L}$
0.04 - 0.10 %	0.09 - 0.31 %	0.18 - 0.65 %	0.033 - 0.09 %	0.054 %

Table 2: Range of systematic uncertainties obtained at LEP on the selection efficiency and the residual background subtraction for the hadronic, muon-pair and tau-pair final states, as well as on the integrated luminosity (experimental and theoretical components separately).

The most accurate selection of hadronic final states was obtained by the L3 collaboration [1], the most precise muon- and tau-pair selections were performed in ALEPH [2], and the luminosity determination of the OPAL collaboration came out with the smallest systematic uncertainty [15].

The purpose of this section is to present the main characteristics of the most precise out of the four measurements concerned, with an emphasis on the estimate of the different systematic uncertainties and, consequently, on their extrapolation to Giga-Z. The latter are performed taking into account that systematic uncertainties at LEP reflecting only the statistical power of the comparison between real and simulated distributions used to estimate them, may not only scale according to statistics since some discrepancy between the two samples may occur at Giga-Z. It was assumed that if this discrepancy would become sizeable, it would be understood and the Monte-Carlo would be corrected accordingly. For this reason, most of the systematic uncertainties which were of pure statistical origin were assumed to decrease by a factor of only 5 instead of the factor of 10 one would observe if only the statistical power of the comparison would govern the evolution of the uncertainty.

4.1 Quark-pair selection of L3

Multihadronic final states were selected in order to derive the cross-section for quark-pair production restricted to e^+e^- annihilations taking place at an effective energy ($\sqrt{s'}$) of at least 10 % of the collision energy ($\sqrt{s}$). This was to avoid being sensitive to the theoretical description of events radiated back to the vector-meson resonances, i.e. lighter than ~ 10 GeV. The acceptance above the lower bound in $\sqrt{s'}$ was found to be $\sim$ 99.3–99.4 %. Its computation was based on version 7.4 of the JETSET programme [10].

Table 3 summarises the systematic errors on the hadronic cross-section, as extracted by the L3 collaboration from the data collected in 1994 [1].

source of uncertainty	relative uncertainty [%]
Acceptance	0.021
Selection cuts	0.030
Trigger efficiency	0.012
Non-resonant background	0.010
Monte-Carlo statistics	0.004
Total	0.040

Table 3: Sources of systematic uncertainties on the multihadronic final state cross-section measured in 1994 by the L3 collaboration. Values are expressed as a percentage of the cross-section.

Projections on how uncertainties may evolve when repeating the measurements at Giga-Z are discussed below, based on the difficulties encountered at LEP-1[11]:

- The uncertainty on the acceptance was among the most difficult to determine. Its components were estimated by comparing the acceptance determinations obtained with the JETSET (version 7.4) and the HERWIG (version 5.7) [12] generators, and by taking half of the difference between the two values as systematic uncertainty. Since more than 99 % of the events were accepted, the issue was actually to estimate the precision on the number of rejected events. This question had several aspects. A first one was of geometrical origin: most of the detection inefficiency originated from quark-pairs produced in the very forward region; for instance, when quarks were produced with a polar angle corresponding to $0.99 < \cos\theta < 1$, the chance to accept the event was only ~ 60 %. Since the detector at the Linear Collider should have good tracking efficiency down to $cos\theta \sim 0.995$, and calorimetry down to $cos\theta > 0.999$, the selection efficiency should exceed 99.90 %, translating in a substantial reduction of the number of rejected events and hence of the uncertainty on this rejection. The second aspect concerns the control of fragmentation: the rejected hadronic events were at the edge of distributions entering the event selection, such as the charged multiplicity. This is typical of a quark-pair fragmenting into very few hadrons produced at very small polar angle. The probability to detect such final states must be calculated reliably by Monte-Carlo. Likewise the sharing between neutral and charged hadrons, the sharing between electromagnetic and hadronic energy, etc. All these aspects may improve with better detector hermiticity and larger data samples. One can in particular expect that the tails of the sensitive distributions will be much better understood. One should also reconsider the definition of the signal (e.g. $\sqrt{s'} > 0.10 \sqrt{s}$). A more precise measurement would probably rely on a more inclusive definition of the signal, including the description of the ISR to the low energy resonances. Overall, substantial progresses seem achievable, and a reduction of the systematic uncertainty on the acceptance by a factor of at least 3 to 5 is therefore not unrealistic;

- Selection cuts originated the largest contribution to the systematic uncertainty. They were derived from variations of the cut values around their nominal ones. Some fraction of this uncertainty was of statistical nature and reflected the modest number of events around the nominal cut values. This uncertainty overlaps partially with the acceptance one, discussed above. The main contribution, however, was the precision to which one could estimate the contamination by physical and non-physical backgrounds. This relied on the Monte-Carlo modelling of signal and physics background and on the determination of components such as the beam related background, cosmic events, etc. The large statistics foreseen at Giga-Z should allow that most of these uncertainties will go down by a factor of 3 to 5. On the other hand, attention should be given to beam-related background and to cosmic events, which will certainly not be as harmless as at LEP-1;

- The trigger inefficiency was derived from the fraction of events with single to multiple trigger as a function of the polar angle. The resulting uncertainty was essentially given by the number of events triggered by only one trigger component, meaning that it would scale with the overall statistical uncertainty. Provided the trigger system is redundant enough at Giga-Z, the systematic uncertainty should go down by a factor of at least 5;

- The understanding of non-resonant background contributions demanded refined inspection: it was in particular difficult to control accurately the contribution from two-photon interactions, since they were not well described by the Monte-Carlo. The control of backgrounds from beam and detector noise entered also the uncertainty estimates. They were mainly determined from the data. Data collected at

LEP-2 and ongoing progress on the generators allows to foresee much better conditions at Giga-Z. The uncertainty on these backgrounds should therefore evolve quite similarly to the statistical one, i.e. a reduction factor of 5 can safely be assumed.

- The uncertainty due to Monte-Carlo statistics can easily be reduced below 0.001 %;

In conclusion, one observes that all systematic uncertainties are likely to decrease by a factor of at least 3 at Giga-Z.

4.2 Lepton-pair selection of ALEPH

The selection of lepton-pair final states [2] was performed with two different analyses: one (called *exclusive*) based on three independent selections where each flavour was isolated in a dedicated way, and another (called *global*) aimed to optimise the measurement of $\mathrm{R_l}$ by reducing the uncertainties and correlations arising from the flavour separation. The latter is however needed at a late stage of the global analysis in order to subtract the proportion of Bhabha events due to the t-channel exchange (including its interference with the s-channel diagrammes). For the sake of simplicity and in order to keep a rather modest optimism about the accuracy one may achieve on α_s at Giga-Z, only the flavour-dependent analysis of $\mu^+\mu^-$ and $\tau^+\tau^-$ final states was considered. A global analysis, including Bhabha events, would certainly allow to improve the precision forecasted in the coming sections, but the improvement would not change the overall picture.

Both analyses went first through a common preselection, where all lepton flavours were treated on a common footing.

4.2.1 Lepton-pair preselection

The selection was based on the KORALZ 4.0 generator [13], which includes initial and final state bremsstrahlung corrections up to $O(\alpha_\mathrm{s}^2)$ with exclusive exponentiation and $O(\alpha_\mathrm{s})$ electroweak corrections. QED initial-final bremsstrahlung interference in the presence of multiple QED hard bremsstrahlung was however not included. A correction for this effect was evaluated with the Monte-Carlo generator KK [14].

The main sources of systematic uncertainty associated with preselection and acceptance definition arise from tracking inefficiencies and from any systematic mismeasurement of the angles, viz. the production angle (θ^*) of the outgoing lepton w.r.t. the beam electron computed in the reaction center-of-mass and the acollinearity angle between the two leptons. The different components entering this estimate are summarised in Table 4.

The understanding of tracking inefficiencies was quantified by comparing simulated and real distributions of sensitive parameters. No discrepancy appeared but the statistical uncertainty on the comparison (which amounted to 0.03 %), mainly reflecting the real data sample size, was taken as a systematic uncertainty. This value is likely to evolve in a purely statistical way when extrapolating it to Giga-Z.

The position of the TPC end-plates were the dominant source of angular systematic uncertainties, which were estimated using toy Monte-Carlo techniques. This uncertainty (which amounts to 0.01 %) may only be reduced by a factor of 2 or 3 at Giga-Z.

The uncertainty on the signal versus background definition and separation resulted from the accuracy of the four fermion generators used for the signal extraction and from the ambiguity between lepton-pair and quark-pair signals in $l^+l^-q\overline{q}$ final states. These

source of relative uncertainty [%]	$\mu^+\mu^-$	$\tau^+\tau^-$
TPC tracking	0.03	0.03
$cos\theta^*$	0.01	0.01
ISR/FSR simulation	0.03	0.03
total acceptance	0.04	0.04
Monte-Carlo statistics	0.06	0.07

Table 4: Sources of systematic uncertainties on the $\mu^+\mu^-$ and $\tau^+\tau^-$ cross-sections due to preselection and acceptance requirements for the 1994 peak data of ALEPH, which corresponds to about 50 pb^{-1} integrated luminosity. Values are expressed as a percentage of the dilepton cross-section.

uncertainties (which add up to 0.03 %) are largely reflecting the experimental accuracy of the studies of four fermion production. It is therefore justified to consider that the uncertainty will substantially go down (typically by a factor of 3–5) with a larger four-fermion sample and time to improve the generators.

An additional uncertainty of 0.06 % (resp. 0.07 %) affected the muon-pair (resp. tau-pair) production cross-section due to limited Monte-Carlo statistics. These values would diminish below 0.01 % at Giga-Z.

4.2.2 $\mu^+\mu^-$ final states

The selection efficiency was found to be about 98 % within the acceptance, estimated to $\sim$ 85 %. The associated systematic uncertainty including selection efficiency, combined with the uncertainty reflecting residual background subtraction, amounted to $\pm$ 0.09 %, i.e. more than 5 times the statistical uncertainty expected at Giga-Z. By inspecting the different systematic uncertainty contributions of the ALEPH measurement, one sees that most of them are very likely to be reduced substantially. These contributions are summarised in Table 5.

The acceptance and the Monte-Carlo contributions come from Table 4. The calibration of the particle momenta was successfully cross-checked with a statistical accuracy of 0.006 % in 1994, which was considered as a systematic uncertainty. This uncertainty may diminish below 0.001 % at Giga-Z. A similar remark applies to the knowledge of the momentum resolution, for which the systematic uncertainty resulted from a variation of the cross-section consecutive to a momentum smearing and from the statistical precision on this variation.

About 2 % of the simulated events were selected as $\mu^+\mu^-\gamma$ final states with a photon detected in the electromagnetic calorimeter. The most critical cut for these events concerned the difference between the expected and the measured photon energy. The associated systematic uncertainty was estimated by making the simulated distribution of this energy difference reproduce the observed one. The consecutive variation of the cross-section was used to derive the associated uncertainty. This estimate would gain from higher statistics and better detector performances (calorimeter and tracking de-

source	$\Delta\sigma/\sigma$ [%]
acceptance	0.04–0.05
momentum calibration	0.006
momentum resolution	0.005
photon energy	0.05
radiative events	0.05
muon identification	$\approx$ 0.001
Monte-Carlo statistics	0.06

Table 5: Contributions to the systematic uncertainty on the $\mu^+\mu^-$ final state selection and residual background for the measurements performed by the ALEPH collaboration with the data collected in 1994. Values are expressed as a percentage of the dimuon cross-section.

vices) such as those expected at Giga-Z. One can therefore foresee an improvement by a factor of at least 3 on this uncertainty.

The simulation of radiative events was checked by recomputing the cross-section after loosening some crucial cuts. The systematic uncertainty coming out from this estimate was also influenced by the statistical power of the check and by the detector performance and simulation. It is therefore justified to expect a reduction of this uncertainty at Giga-Z by a factor of at least 3.

The muon identification efficiency was measured with real data, using two complementary identification methods. The statistical uncertainty associated to this measurement ($\approx$ 0.001 %) was taken as the systematic uncertainty associated to the identification. This value would probably go down by a factor of at least 5 at Giga-Z.

4.2.3 $\tau^+\tau^-$ final states

The $\tau^+\tau^-$ background fractions were estimated using real and simulated samples. The breakdown of the systematic uncertainties associated to the residual background subtractions is provided in Table 6.

The hadronic and the two-photon collision contaminations were measured on the data by fitting the distributions of the relevant Monte-Carlo observables; these were checked on enriched samples of real $q\bar{q}$ and two-photon collision events and the assigned uncertainties (resp. 15 % and 10 % of the amount subtracted) reflect the observed level of agreement between the real and the simulated distributions. A factor 100 increase of statistics and improved generators would certainly allow to reduce the corresponding systematic uncertainties by a factor of at least 5.

The contamination due to electron- and muon-pairs depended on the lepton identification probabilities, which were not perfectly reproduced by the Monte-Carlo. These were estimated with the real data, using selected samples of e^+e^- and $\mu^+\mu^-$ events. The uncertainties on these measurements are the main contributions to the systematic uncertainty

Source	$\Delta\sigma/\sigma$ [%]
e^+e^- final states	0.05
$\mu^+\mu^-$ final states	0.02
$q\overline{q}$ final states	0.04
two-photon collisions	0.04
Total	0.08

Table 6: Contributions to the systematic uncertainty on the residual background subtraction from the $\tau^+\tau^-$ final states applied by the ALEPH collaboration to the 1994 data. Values are expressed as a percentage of the dilepton cross-section.

associated to these backgrounds. A reduction factor of at least 3 on the systematic uncertainty may be expected from Giga-Z, consecutive to Monte-Carlo improvements w.r.t. LEP.

The selection efficiency was determined with *artificial* $\tau^+\tau^-$ events, constructed by associating two hemispheres of two different real $\tau^+\tau^-$ events. The two hemispheres were selected by applying severe τ selection criteria in their opposite hemispheres. The artificial event sample was therefore made of almost unbiased real $\tau^+\tau^-$ final states. A selection efficiency of $\sim$ 94 % inside an acceptance of about 82 % came out from this procedure. The latter introduced biases, which were estimated from Monte-Carlo samples. The corresponding uncertainty (i.e. 0.08 %) would presumably not decrease at Giga-Z as fast as the increase of the statistical power but a factor of at least 3 is expectable.

The breakdown of all systematic uncertainties associated to the $\tau^+\tau^-$ final states is shown in Table 7. The column labeled A contains uncertainties mainly due to discrepancies between real and simulated event samples. At Giga-Z, one can consider that these uncertainties would be at least 3 times smaller. The column labeled B is dominated by the statistics of the artificial event sample used to measure the efficiency. A reduction factor of 5 at Giga-Z may therefore be assumed.

4.3 Luminosity determination of OPAL

The luminosity was derived from the analysis of Bhabha events produced at small polar angle ($\sim$ 25–58 mrad) and intercepted with a SiW calorimeter those visible cross-section was about 79 nb.

The precision of the measurement was defined by the accuracy on the luminometer acceptance, which depended on the precision on the theoretical Bhabha cross-section and on the experimental acceptance corrections.

The computation of the theoretical cross-section relied on the BHLUMI V4.04 generator [16], based on exact $\mathcal{O}(\alpha^2)$ calculations. The corresponding systematic uncertainty on the Bhabha cross-section was estimated to be 0.061 % [17], which went down to 0.054 % once the contribution of soft pair production [18] was accounted for. Within the coming ten years, further developments of the cross-section calculations may occur, which would include higher order contributions. It is therefore not excluded that the theoretical

Source	A [%]	B [%]	total
Acceptance	0.03	0.04	0.05
Monte-Carlo statistics		0.07	0.07
preselection cuts	0.05	0.04	0.07
$q\overline{q}$ rejection cuts	0.06	0.09	0.11
e^+e^- rejection cuts	0.03	0.04	0.05
$\mu^+\mu^-$ rejection cuts	0.03	0.03	0.05
Total efficiency	0.08	0.11	0.14
Total (with backgrounds)	0.12	0.13	0.18

Table 7: Contributions to the systematic uncertainty on the $\tau^+\tau^-$ cross-section associated to the measurement performed by the ALEPH collaboration with the 1994 data. Column A displays the contributions related to the Monte-Carlo simulation. Column B contains the remaining component, dominated by the statistics of the sample of artificial events used to measure the efficiency.

systematic uncertainty on the luminosity will decrease by a factor of about 2, if no new uncertainties, like those related to the beamstrahlung, have to be accounted for.

The experimental acceptance corrections were divided in two categories: those evaluated using a parametrised detector simulation (ϵ_{sim}) and those excluded from the detector simulation, and applied externally (ϵ_{ext}). Table 8 displays a breakdown of the systematic uncertainties related to ϵ_{sim} and ϵ_{ext}.

Both components were of similar size, i.e. 0.023 % for the external component and 0.024 % for the simulation one, adding up to a total uncertainty of 0.033 %.

One can expect that the knowledge of the luminometer geometry at Giga-Z will be at least as good as at LEP. It will presumably be even better, due to the experience gained at LEP and to the large statistics accessible at Giga-Z. The uncertainty on the simulation results could certainly benefit from larger real event samples, and a reduction of the corresponding systematic uncertainty by a factor of 2 or 3 is not excluded.

The total systematic uncertainty affecting the luminosity of the OPAL collaboration may therefore translate to a value close to 0.02 % at Giga-Z, but the effect of the beamstrahlung may introduce new systematic uncertainties of roughly the same size.

5 Precision expected at Giga-Z

The advantage of measuring α_s at Giga-Z follows mainly from the unprecedented statistical power of the measurements. These will allow to reduce the statistical accuracy achieved at LEP by a factor of about 10, as well as the systematic uncertainties by a factor which will essentially depend on the level of accuracy of the Monte-Carlo simulations: if the simulated distributions of the parameters reflecting the measurement's accuracy reproduce the real ones well, the corresponding precision will come from the statistical

Source	uncertainty [%]
Radial metrology	0.014
Inner radial coordinate bias	0.014
Background	0.011
Others	0.006
Total external (ϵ_{ext})	0.023
Energy	0.018
Beam parameters	0.008
Clustering	0.010
Monte-Carlo statistics	0.008
Others	0.004
Total simulation (ϵ_{sim})	0.024
Total	0.033

Table 8: Contributions to the experimental systematic uncertainty on the integrated luminosity measured by the OPAL collaboration based on the 1994 data. They are subdivided into external and simulation components.

power of the comparison between both distributions. But if residual discrepancies appear, which are not to be solved, the corresponding uncertainty will come from the instability of the measurement reflecting this discrepancy. One may then consider two cases: either this discrepancy showed up at LEP, and then one may wonder if larger statistics will help to reduce it, or this discrepancy will appear in distributions where no feature was observed at LEP. Depending on the case, the systematic uncertainties obtained at LEP were reduced by a factor of 5 when their estimate was believed to follow mainly the increase of statistics, or by a factor of 3 (or eventually 2) when possible discrepancies between the real data and the Monte-carlo sample were suspected to dominate the final uncertainty.

In general, one should keep in mind that the analyses of the data collected at LEP showed that the modelling of the detector in the Monte-Carlo improved with increasing number of recorded events, more or less in proportion of the statistical accuracy.

5.1 Potential of a virtual LEP detector

When extrapolating from LEP to Giga-Z, it is justified to consider that the detector to be installed at the Linear Collider will at least be as well performing as a *virtual LEP detector* which combines the performances of L3 for multihadronic final states, those of ALEPH for the muon- and tau-pair production, and those of OPAL for the luminosity determination. This leads to the set of systematic uncertainties indicated in Table 9[1].

[1]The values are taken from Table 3, 5, 7 and 8.

uncertainty component	systematic [%]	statistical [%]
$\Delta q\overline{q}$	0.040	0.003
$\Delta\mu^+\mu^-$	0.090	0.015
$\Delta\tau^+\tau^-$	0.170	0.015
ΔL_{exp}	0.033	0.002
ΔL_{theo}	0.054	–

Table 9: Systematic and statistical uncertainties affecting the selected samples of multihadronic, tau-pair, muon-pair and luminosity events at Giga-Z if there were no improvements on the systematic precision w.r.t. LEP.

Table 10 shows which precision on $\alpha_s(M_Z)$ one may expect in this case after one year of data taking at Giga-Z. As predicted in [19], the precision on Γ_Z was assumed to be 1 MeV.

observable	relative uncertainty [%]	$\Delta\alpha_s(M_Z)$
R_l	0.09	0.0027
Γ_Z	0.04	< 0.002
σ_0^h	0.07	0.0055
σ_0^l	0.10	0.0022

Table 10: Total uncertainties on the four Z^0 observables sensitive to α_s and related precision on $\alpha_s(M_Z)$ foreseen at Giga-Z if the detector were to have the same performances as L3 for the multihadrons, as ALEPH for muon- and tau-pairs, and as OPAL for the integrated luminosity.

The precision expected on Γ_Z would lead to a rather precise estimate of $\alpha_s(M_Z)$, provided no theoretical uncertainty (like the one associated to the hadronic contribution to the evolution of α_{EM}) would spoil the experimental accuracy. Combining the precisions displayed in Table 10 with the proper correlation between the components leads to a precision on $\alpha_s(M_Z)$ of about 0.0013 (see Section 3).

This result is already significantly more precise than the present value. The next section shows the gain expected from improvements on the systematic uncertainty.

5.2 Gain from reduced systematic uncertainties

As discussed in section 4, a large majority of systematic uncertainties affecting the LEP measurements are likely to shrink by a factor of 3–5 at Giga-Z. The corresponding improvements expected on $\alpha_s(M_Z)$ are discussed below.

5.2.1 Quark-pair final states

The reduction of the systematic uncertainties affecting the hadronic final states produced at Giga-Z is mainly based on the assumption that the detector will be more hermetic and that the larger statistics will translate in a more accurate Monte-Carlo simulation and a smaller statistical uncertainty associated to cut variations. Furthermore, the generators used to determine the signal selection efficiency and the residual backgrounds will progressively become more realistic.

The corresponding reduction on the systematic uncertainties obtained by the L3 collaboration and discussed in Section 4.1 is summarised in Table 11.

source of uncertainty	relative uncertainty [%]
Monte-Carlo statistics	0.001
Acceptance	0.004–0.007
Selection cuts	0.006–0.010
Trigger efficiency	0.002
Non-resonant background	0.002
Total	0.008–0.013

Table 11: Sources and magnitude of systematic uncertainties on the multihadronic final state cross-section foreseen at Giga-Z.

5.2.2 Lepton-pair selection

Lepton-pair selection will benefit from better controlled tracking efficiency, consecutive to larger statistics but also to better track finding and reconstruction performances, based on a lighter and more hermetic detector. The simulation of ISR-FSR interferences will improve both because of their more accurate study at the Linear Collider and because of the natural improvements of the generators with time. Thank's to the large statistics, to the good hermeticity and to the high performance tracking, the acceptance to lepton-pairs will be known more precisely than at LEP. The generation and simulation of radiative events ($\mu^+\mu^-\gamma$ and $\tau^+\tau^-\gamma$) will also be more accurate, due to more complete and rigorous generators and to reduced material (originating reinteractions which confuse the event reconstruction) which will be simulated more accurately. The control of radiative events will also benefit from the high performance of the electromagnetic calorimeter, combined with that of the tracking system. Finally, it is assumed that the cosmic background, which will be much higher than at LEP, will not contribute significantly to the measurement's uncertainties.

The uncertainties associated to Monte-Carlo statistics and to acceptance knowledge expected at Giga-Z, introduced in Section 4.2.1, are summarised in Table 12.

source of uncertainty	relative uncertainty [%]
TPC tracking	0.003
$cos\theta^*$	0.003–0.005
ISR/FSR simulation	0.005–0.01
total acceptance	0.007–0.012
Monte-Carlo statistics	0.005–0.01

Table 12: Expected sources of systematic uncertainties on the $\mu^+\mu^-$ and $\tau^+\tau^-$ cross-sections due to preselection and acceptance requirements at Giga-Z. The size of the uncertainties are expressed as a percentage of the dilepton cross-section.

5.2.3 $\mu^+\mu^-$ final states

The uncertainty components associated to the selection of $\mu^+\mu^-$ final states, introduced in Section 4.2.2, are summarised in Table 13.

source	$\Delta\sigma/\sigma$ [%]
acceptance	0.007–0.012
momentum calibration	0.002–0.003
momentum resolution	$\leq$ 0.0025
photon energy	0.010–0.015
radiative events	0.010–0.015
muon identification	$\approx$ 0.0002
Monte-Carlo statistics	0.005–0.01
Total	0.015–0.025

Table 13: Expected contributions to the systematic uncertainty on the $\mu^+\mu^-$ final state selection and residual background for the measurements foreseen at Giga-Z.

In total, the systematic uncertainty may be in the range 0.015–0.025 %, which is quite close to the statistical uncertainty, of the order of 0.015 %. The total uncertainty may therefore be in the range 0.02–0.03 %.

5.2.4 $\tau^+\tau^-$ final states

The uncertainties on the residual background to the $\tau^+\tau^-$ final states and on the selection efficiency foreseen at Giga-Z are summarised in Table 14. The meaning of the different components is explained in Section 4.2.3.

Source	$\Delta\sigma/\sigma$ [%]
e^+e^- and $\mu^+\mu^-$ final states	0.02
two-photon collisions and $q\bar{q}$ final states	0.01
acceptance	0.01
Monte-Carlo statistics	0.005–0.01
total efficiency (A)	0.025
total efficiency (B)	0.02
Total	0.04

Table 14: Expected contributions to the systematic uncertainty on the $\tau^+\tau^-$ final state selection and residual background for the measurements foreseen at Giga-Z.

The total systematic uncertainty ($\sim$ 0.04 %) will be combined with about 0.015 % statistical uncertainty, translating into a 0.045 % total uncertainty on the $\tau^+\tau^-$ final states.

5.2.5 Luminosity determination

As at LEP, the determination of the integrated luminosity will be based on the observed number of Bhabha events within a given time intervalle inside a well controlled geometrical acceptance. There will however be a major new difficulty, originated by the strong focussing of the beams before the interaction region, which translates into intense beamstrahlung. This process degrades the energies of the beams: the mean energy loss amounts to $\sim$ 1 % for beam energies of 250 GeV, and the energy spread of each beam increases substantially. Reconstructing the acollinearity of the Bhabha events should allow to evaluate the consequences of beamstrahlung on the visible Bhabha cross-section. This method may encounter difficulties and the luminosity determination may therefore not be more accurate at Giga-Z than at LEP-1. The scenarios considered in the present study assume that either there will be no gain in precision at Giga-Z w.r.t. to LEP or that the improvement will amount to a factor of 2–3 on the systematic uncertainty.

Concretely, two predictions on the total systematic uncertainty affecting the luminosity will be considered for the purpose of the precision achievable on α_s at Giga-Z: one where the theoretical component of the systematic uncertainty amounts to 0.05 % and the experimental component to 0.03%, and one where they amount respectively to 0.03 % and 0.02 %. The total systematic uncertainty was considered accordingly to amount to 0.06 % or to 0.035 %.

The estimate of the statistical uncertainty was based on the assumption that the visible Bhabha cross-section used for the integrated luminosity determination would at least amount to 50 nb. The corresponding statistical uncertainty would be $\sim$ 0.0025 %, a negligible quantity when compared to the systematic uncertainty.

6 Results

The precision range which can be obtained on $\alpha_s(M_Z)$ from each line-shape observable was estimated by varying the assumptions on the systematic uncertainties in order to explore the range of scenarios discussed in previous sections. The values obtained were compared to those of Table 10.

The results associated to the measurement of R_l are shown in Table 15.

$\Delta^{q\bar{q}}_{stat}$ [%]	$\Delta^{q\bar{q}}_{syst}$ [%]	$\Delta^{l^+l^-}_{syst}$ [%]	$\Delta^{l^+l^-}_{stat}$ [%]	$\Delta R_l/R_l$ [%]	$\Delta\alpha_s(M_Z)$
0.003	0.04	0.08	0.011	0.09	0.0027
0.003	0.013	0.02	0.011	0.03	0.0008
0.003	0.009	0.015	0.011	0.02	0.0006

Table 15: Various scenarios in terms of systematic uncertainties on the ingredients to the measurement of R_l and associated precision on $\alpha_s(M_Z)$.

One observes that the precision achievable on $\alpha_s(M_Z)$ from R_l alone lies within the range 0.0006–0.0008, i.e. significantly below 0.001.

The results associated to the measurement of σ^l_0 are shown in Table 16.

$\Delta^{l^+l^-}_{syst}$ [%]	$\Delta^{l^+l^-}_{stat}$ [%]	ΔL^{exp}_{syst} [%]	ΔL^{theo}_{syst} [%]	$\Delta\sigma^l_0/\sigma^l_0$ [%]	$\Delta\alpha_s(M_Z)$
0.08	0.011	0.033	0.054	0.10	0.0022
0.03	0.011	0.03	0.05	0.066	0.0014
0.02	0.011	0.02	0.03	0.043	0.0009

Table 16: Various scenarios in terms of the systematic uncertainties on the ingredients to the measurement of σ^l_0 and associated precision on $\alpha_s(M_Z)$.

The precision achievable on $\alpha_s(M_Z)$ with σ^l_0 is less appealing than with R_l, contrary to what was observed at LEP-1. This reflects the accuracy assumed on the luminosity. A more optimistic approach may translate in a larger influence of σ^l_0 and a better combined precision on $\alpha_s(M_Z)$. Furthermore, enriching the lepton-pair sample with e^+e^- final states and using a flavour independent analysis would translate in a more precise determination of σ^l_0, as well as of R_l, though presumably not striking.

As shown in Table 17, the contribution of σ^h_0 is clearly less important, due to its smaller sensitivity to α_s.

The last contribution comes from Γ_Z, already shown in Table 10. Table 18 summarises the precision achievable on the average value of $\alpha_s(M_Z)$ from R_l, σ^l_0, σ^h_0 and Γ_Z, for the explored range of systematic uncertainties.

One observes that the combined value may have an accuracy of ~ 0.0005.

$\Delta^{q\bar{q}}_{stat}$ [%]	$\Delta^{q\bar{q}}_{syst}$ [%]	ΔL^{exp}_{syst} [%]	ΔL^{theo}_{syst} [%]	$\Delta\sigma^h_0/\sigma^h_0$ [%]	$\Delta\alpha_s(M_Z)$
0.003	0.04	0.033	0.054	0.075	0.0055
0.003	0.013	0.03	0.05	0.059	0.0044
0.003	0.009	0.02	0.03	0.037	0.0027

Table 17: Various scenarios in terms of the systematic uncertainties on the ingredients to the measurement of σ^h_0 and associated precision on $\alpha_s(M_Z)$.

variable	$\Delta\alpha_s(M_Z)$
R_l	0.0006–0.0008
σ^l_0	0.0009–0.0014
σ^h_0	0.0027–0.0044
Γ_Z	$\sim$ 0.002
combined	0.0005–0.0007

Table 18: Total uncertainties on $\alpha_s(M_Z)$ from each Z^0 observable foreseen at Giga-Z, embracing the range of assumptions made on the systematic uncertainties. The bottom line shows the combined value.

7 Summary and conclusion

By examining the analyses applied to the LEP-1 data to extract the strong coupling constant from the Z^0 observables, an attempt was made to predict the precision achievable on $\alpha_s(M_Z)$ at Giga-Z. An absolute precision of $\pm$ 0.0005–0.0007, depending on the assumptions made on the systematic uncertainties governing the measurements, came out from this study. Non-perturbative QCD corrections and theoretical uncertainties on the Z^0 observables (such as those due to the top and Higgs masses) should have negligible influence on the measurements.

The factor of $\sim$ 4–6 improvement w.r.t. the LEP average originates mainly from the 100 times larger data sample, combined with better detector performances, improved physical process generators and more accurate Standard Model predictions. As a consequence of this bonus, one can expect more precise detector simulation and alignment, which would also benefit to the precision on $\alpha_s(M_Z)$.

The latter was estimated assuming a modest improvement on the luminosity determination as compared to the progress assumed for the lepton-pair and quark-pair final states. As a consequence, R_l would be the observable most sensitive to α_s and the additional accuracy from Γ_Z, σ^h_0 and σ^l_0 would amount to less than 20 % in total. If the improvement of the luminosity would follow that on quark- and lepton-pair final states, the role of σ^l_0 would become more significant and a precision better than $\pm$ 0.0005 would not be out of scope.

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