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The Gain to Polarimetry and A_{LR} Measurements from a Polarised Positron Beam in TESLA

Gideon Alexander and Erez Reinherz-Aronis

School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel-Aviv University, 69978 Tel-Aviv, Israel

Abstract

A thorough analysis is presented on the benefits gained for the left-right asymmetry A_{LR} and beam polarisation measurements by adding a longitudinal polarised positron beam to the e^+e^- linear collider TESLA. Two centre of mass energies were considered, 91 and 500 GeV, where it is assumed that data was collected with a total luminosity of 30 fb^{-1} and 500 fb^{-1} respectively. In the analysis we consider 4-bunch spin-spin configurations setups according to the Blondel scheme and an alternative scheme. The results of these operating modes are compared to the case where the beam polarisations are measured by polarimeters which have a precision of 0.5% and to the situation where only the electron beam is polarised. Whereas a dramatic improvement in the statistical error of A_{LR} is achieved at 91 GeV by the application of the alternative Blondel four-spin configuration scheme, the gain at 500 GeV is rather moderate and is in the range 50 to 70 % for A_{LR}=0.15 and about 300% for A_{LR}=0.5.

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1 Introduction

In e^+e^- colliders the presence of at least one longitudinal polarised beam enhances significantly the spectrum of physics questions that can be studied within the Standard Model and beyond it. These topics were discussed in many publications in the past, among them, the CERN Yellow report 'Polarisation at LEP [1] and more recently in the TDR of TESLA [2], the planned e^+e^- 500 GeV linear collider.

In storage ring e^+e^- colliders, like LEP, the effort needed to produce a longitudinal beam is essentially the same for the electron and positron bunches. The reason is that the polarisation build-up starts after the beams are accelerated to the desired energy level. At first a transverse polarisation builds up through the mechanism described by Sokolov and Ternov [3] and the final desired longitudinal polarisation can be achieved by rotating the spin direction. In LEP the transverse polarisation step has been achieved and has been extensively used in calibrating the energy of the beams which was crucial in the efforts to reduce the error on the Z^0 mass measurement. The longitudinal polarisation phase however has never been realised and instead the option to increase the CM energy of LEP from 91 GeV to above 200 GeV has correctly been taken.

The situation in a linear collider is very different from that of a storage ring collider. In a linear collider one uses a longitudinal polarised electron source prior to its linear acceleration which is supposed not to destroy its spin state. The creation of the positron beam however follows a completely different path and is achieved in steps. In one proposed scheme at first the electron beam is forced to radiate circular polarised photons which subsequently hit a target to produce e^+e^- pairs which carry the longitudinal polarisation with a division between them proportional to their relative momentum. Thus by utilising only those pairs where the positron carries most of the momentum one obtains a positron beam with a high degree of longitudinal polarisation. Here it should be noted that in the accelerator where only the electron beam is polarised the collider is nevertheless able to tackle many of the central physics questions as has been demonstrated by the SLC, the Stanford linear collider, which operated on the Z^0 mass, where solely the electron beam was longitudinal polarised.

In the deliberations on a future linear collider like TESLA the advantage of having a polarised positron beam has been dealt with by quite a few authors. Their arguments for a polarised positron beam will eventually have to be weighted against the effort and costs of such a facility. These arguments can roughly be divided into four classes.

1) The first deals with the upgrading of the power and precision of the physics measurements which is also coupled to the reduction of the background when a longitudinal polarised positron

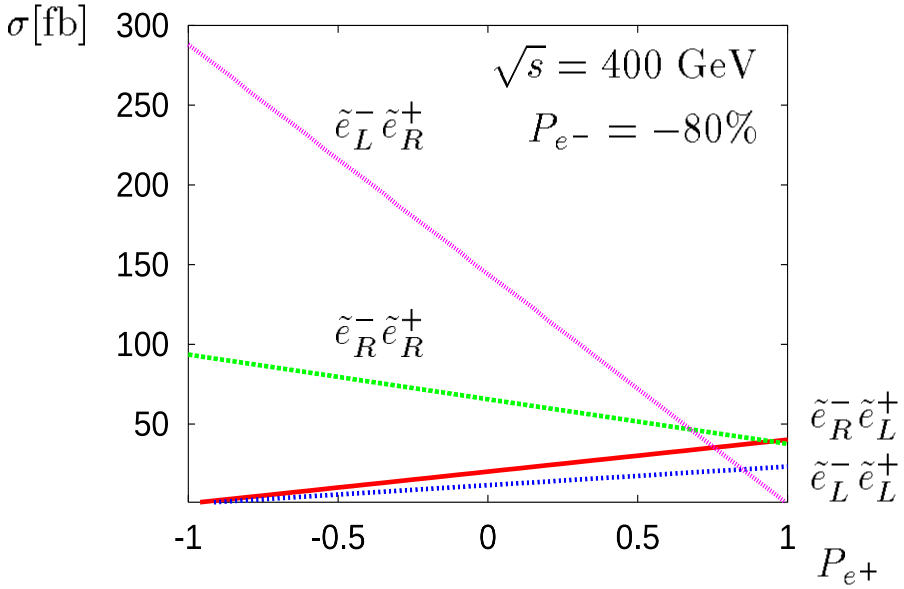

Figure 1: An example for the benefit that can be gained from the addition of a polarised positron beam taken from Ref. [4]. Shown are the cross sections for $e^+e^- \rightarrow \tilde{e}^+_{L,R}\tilde{e}^-_{L,R}$ for the fixed value of $P(e^-) = -0.8$ as a function of the $P(e^+)$. Observation of the reaction at different $P(e^+)$ can serve as a way to determine the quantum numbers of the selectrons.

beam is introduced to the linear collider.

Many papers deal with this category, which is beyond the scope of this note, and we only wish to point out that a major part of the described physics goals can also be achieved without a polarised positron beam provided sufficient luminosity is available. An example of the benefit gained from the positron polarisation is shown in Fig. 1, taken from Ref. [4], which illustrates the positron polarisation effect on the production of selectrons.

2) The second class of arguments addresses physics goals which essentially cannot be dealt with unless the two collider beams are polarised. In other words the introduction of a polarised positron beam constitutes a qualitative and not just a quantitative upgrade. To this category belong, for example, the production of s-channel scalars like Higgs bosons (see e.g. Ref. [5]) or the study of longitudinal polarised W bosons where both the electron and the positron beams should be transverse polarised [6].

3) The third point concerns the possibility to determine the polarisation level of the beams just by the introduction of a polarised positron beam in addition to the polarised electron beam. This possibility can be achieved by using electron-positron spin-spin configurations known as the 'Blondel scheme' [7] which further inspired also alternative schemes like the one described in Refs. [8, 9].

4) The last point which is related to the former one is connected to the possibility to study physics quantities, like the left-right asymmetry just from measurements of cross sections without referring to the polarisation values of the beams.

This short note addresses mainly the third and fourth points of the above list where we will try to reexamine in a rigorous way the errors associated with the left-right asymmetry A_{LR} measurement and the beams' polarisation estimations in the event that the original Blondel scheme is used and in the case that the alternative scheme, discussed in Refs. [8, 9], is chosen. Furthermore, we will also consider the situation where there is a need to collect multi-hadron events which are expected to represent a sample produced in non-polarised e^+e^- collisions. This possible need has in general been overlooked in former studies pertaining to polarised electron beam collisions. However such a sample may be of a considerable value in various studies and in particular when one would like to compare certain physics features as a function of the e^+e^- CM energy where at lower energies, like LEP, the collider beams were unpolarised.

In the various papers dealing with the polarised beams and collisions of electron and positrons one finds at least two notations. In order to avoid confusion we will throughout this note define a positive longitudinal polarisation as lying in the the lepton (antilepton) flight direction.

2 A_{LR} measurement with only one polarised beam

To appreciate the advantages gained by the introduction of a polarised positron beam let us just briefly recapitulate the measurement of the left-right asymmetry A_{LR} with only one polarised (electron) beam. To this end one measures the two e^+e^- annihilation cross sections where a vector particle (γ and/or Z^0) are exchanged in the s-channel. One with the electron spin parallel to the velocity direction, σ_+ and the other in the opposite direction, σ_-. These cross sections are equal to

$$\sigma_+ = \sigma_u(1 - PA_{LR}); \quad \text{and} \quad \sigma_- = \sigma_u(1 + PA_{LR}) \; ; \tag{1}$$

where P is the longitudinal polarisation of the electron beam and A_{LR} is the left-right asymmetry which is is equal to

$$A_{LR} \; = \; \frac{1}{P}\frac{\sigma_- - \sigma_+}{\sigma_- + \sigma_+} \; = \; \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2} \; = \; \frac{2av}{a^2 + v^2} \; ; \tag{2}$$

Here g_L and g_R are the left and right weak coupling constants and v and a are the vector and axial-vector couplings. The unpolarised annihilation cross section σ_u is equal to

$$\sigma_u = \frac{1}{2}(\sigma_+ + \sigma_-) \; .$$

The error squared of A_{LR} in term of the errors of the cross sections σ_+ and σ_- and the electron beam polarisation P, assuming them to be independent, is given by

$$(\Delta A_{LR})^2 = \frac{4}{P^2}\frac{\sigma_+^2(\Delta\sigma_-)^2}{(\sigma_- + \sigma_+)^4} + \frac{4}{P^2}\frac{\sigma_-^2(\Delta\sigma_+)^2}{(\sigma_- + \sigma_+)^4} + \frac{(\sigma_- - \sigma_+)^2}{(\sigma_- + \sigma_+)^2}\frac{(\Delta P)^2}{P^4} \ . \tag{3}$$

Thus at very high statistics the error on A_{LR} depends mainly on the quality of the polarimeter which in the case of a Compton polarimeter may reach a precision as low as $\Delta P/P = 0.5\%$. The Møller scattering polarimeters are somewhat less precise and they are estimated [10] to reach at high energy electron beams a precision of about 1.5%. As for the contribution to the A_{LR} error from the cross sections measurements, they will of course depend on the size of the data sample. At sufficiently high statistics the systematic errors are in general expected to dominate.

3 The incorporation of a polarised e$^+$ beam into TESLA

In the design of the TESLA linear collider, where the crossing angle of the beams is 180^o, the task to install a polarimeter at or near the beams interaction point (IP) has not been so far satisfactory solved due to major technical obstacles. These difficulties arise mainly from the fact that the beams have to be dumped right after they pass the IP so that very little space is left for the accommodation of a polarimeter device. Thus, notwithstanding the physics motivation for the incorporation of a polarised positron beam, there exists the additional interest to make use of the so called "Blondel" or similar scheme when both the electron and positron beams are polarised. These schemes may partly substitute the need for an absolute polarisation measurement of the beams and will require only a relative measurement of the polarisations. In the following subsections we will explore the benefits of the addition of a polarised positron beam to the linear collider to improve the measurements of A_{LR} and for a precision measurement of the beams polarisations.

3.1 The concept of the effective polarisation and its properties

The presence of two longitudinal polarised beam in an e^+e^- collider allows four spin-spin configurations and their corresponding cross sections. The two cross section which contribute to annihilation processes where a vector particle is produced in the s-channel (see section 3.3) are

$$\sigma_{-+} \ = \ \sigma_u[1 + P^{\text{`}}P - A_{LR}(P^{\text{`}} + P)] \tag{4}$$

$$\sigma_{+-} \ = \ \sigma_u[1 + P^{\text{`}}P + A_{LR}(P^{\text{`}} + P)] \tag{5}$$

where the first and second subscripts denote respectively the spin direction of the positron and electron. Here P and $P^{\text{`}}$ are respectively the polarisation of the electron and positron beams. Here it is important to note that throughout this work we will denote by $P$ and $P^{\text{`}}$ the absolute value of the polarisations while their relative direction with respect to their movement is given explicitly by the signs in the formulae. By dividing and multiplying these last two equations by $1 + P^{‘}P$ one obtains

$$\sigma_{-+} = \sigma_u(1 + P^{‘}P)[1 - P_{eff}A_{LR}] \tag{6}$$

$$\sigma_{+-} = \sigma_u(1 + P^{‘}P)[1 + P_{eff}A_{LR}] \tag{7}$$

where the *effective polarisation*, P_{eff}, is defined as

$$P_{eff} = \frac{P^{‘} + P}{1 + P^{‘}P} . \tag{8}$$

Thus the corresponding A_{LR} value is given by

$$A_{LR} = \frac{1}{P_{eff}} \times \frac{\sigma_{+-} - \sigma_{-+}}{\sigma_{+-} + \sigma_{-+}} . \tag{9}$$

A comparison between Eq. (9) and Eq. (2) shows that P_{eff} has replaced P, the polarisation of the electron beam, the addition of a polarised positron beam has increased the analysing power of the collider. In the case that the electron beam polarisation level is 0.8, the addition of a positron beam polarisation with a level of 0.6 reaches an effective polarisation value of 0.946.

Assuming that the e^+ and e^- beam polarisations can be measured with the same relative precision x, the relative error of the effective polarisation (adding errors in quadrature) becomes [9]:

$$\frac{1}{x}\frac{\Delta P_{eff}}{P_{eff}} = \frac{\sqrt{(1 - P^{‘2})^2P^2 + (1 - P^2)^2P^{‘2}}}{(P^{‘} + P)(1 + P^{‘}P)} . \tag{10}$$

For $P = 0.8$ and $P^{‘} = 0.6$ it gives a reduction factor of 0.27 in the relative polarisation error as compared to a single polarised beam. If on the other hand the relative errors of the e^+ and e^- are fully correlated, like depolarisation effects from Bremsstrahlung, then the errors can be linearly added and one gets:

$$\frac{1}{x}\frac{\Delta P_{eff}}{P_{eff}} = \frac{1 - P^{‘}P}{1 + P^{‘}P} . \tag{11}$$

This corresponds to a reduction factor in the relative polarisation error of 0.35 again for $P = 0.8$ and $P^{‘} = 0.6$. In the case that both beams are polarised and equipped with a precision polarimeter having the smallest believed attainable relative error measurement of 0.5% then $\Delta P_{eff}/P_{eff}$ reaches the value 0.135 % for the completely independent errors and 0.175 % in the case the individual measurement are completely correlated.

Whereas the value of A_{LR} does depends only on the P_{eff} value, independent of the P and $P^{‘}$ values out of which it is constructed, its resulting relative error does depend on the specific choice of the two beam polarisations components. This is clearly illustrated in Fig. 2 where $\Delta P_{eff}/P_{eff}$ is shown for a fixed P_{eff}=0.946 as a function of the positron polarisation value.

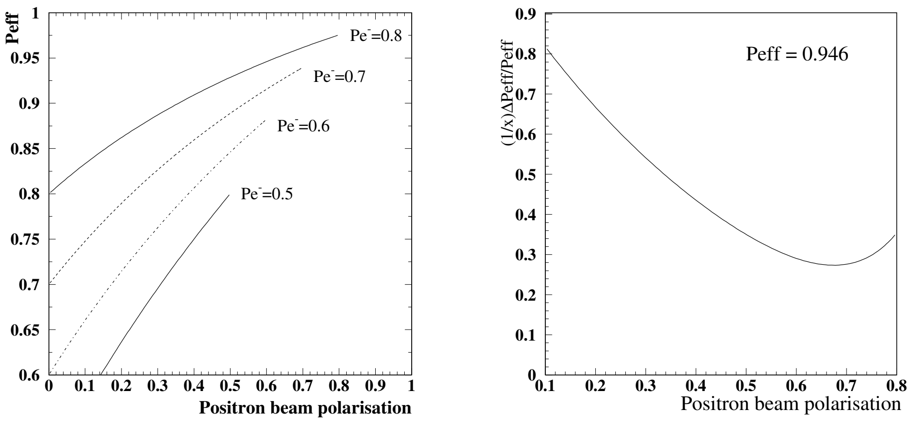

Figure 2: Left: The relation between P_{eff}, P(e^+) and P(e^-). Right: The relative error (1/x)$\Delta P_{eff}/P_{eff}$ as a function of the positron polarisation level at fixed P_{eff} value of 0.946.

The minimum error is found to be when $P = P^{`}$.

Finally to note is that in the expressions of the spin-spin configuration cross sections the replacement of P by P_{eff} is not the only change needed when a second beam is polarised. In fact, the unpolarised cross section σ_u is multiplied by the factor $1 + P^{`}P$ which in turn influences the error value associated with A_{LR}.

3.2 Two polarised beams in the Blondel scheme

The Blondel scheme has been studied in the past in connection with the plans, which have never been realised, to equip the large electron-positron storage ring collider LEP [1] with longitudinal polarised electron and positron beams. This scheme is of a particular interest when the e^+e^- collider energy is on the Z^0 mass where the vector bosons (Z^0 and $\gamma^\star$) are dominating in the s-channel annihilation. At LEP the longitudinal polarisation was planned to be achieved from the natural build up of a transverse polarisation in the storage rings which then could be rotated to a longitudinal one. Here to note is that in this planned experimental setup the possibility to flip the longitudinal polarisation from one direction to the opposite one could not be achieved as the direction of the longitudinal polarisation is fixed by the rotator configuration. Nevertheless, having 4 electron and 4 positron bunches in LEP one could form the Blondel scheme by the following longitudinal polarisation configurations:

electron bunches	1$\Leftarrow$	2	3	4$\Leftarrow$
positron bunches	1	2$\Rightarrow$	3	4$\Rightarrow$
cross sections	σ_1	σ_2	σ_3	σ_4
event numbers	N_1	N_2	N_3	N_4

In this scheme the electron bunches 2 and 3 and the positron bunches 1 and 3 are unpolarised, the others are either left or right-handed longitudinal polarised as indicated by the arrows.

The four measured cross sections can be expressed via the polarisation of the electron bunches P and the positron $P^{‘}$ as

$$\sigma_1 = \sigma_u(1 - PA_{LR}) \tag{12}$$

$$\sigma_2 = \sigma_u(1 + P^{‘}A_{LR}) \tag{13}$$

$$\sigma_3 = \sigma_u \tag{14}$$

$$\sigma_4 = \sigma_u[1 - P^{‘}P + (P^{‘} - P)A_{LR}] \tag{15}$$

These constitute four equations and four unknown. Since the 3rd equation is $\sigma_3 = \sigma_u$ there are really only three equations and three unknown, namely: P, $P^{‘}$ and A_{LR}. From these we get:

$$P = \frac{\sigma_u - \sigma_1}{\sigma_u \sqrt{\frac{(-\sigma_2 + \sigma_u)(\sigma_1 - \sigma_u)}{\sigma_u(-\sigma_u - \sigma_4 + \sigma_2 + \sigma_1)}}}$$

$$P^{‘} = \frac{\sigma_2 - \sigma_u}{\sigma_u \sqrt{\frac{(-\sigma_2 + \sigma_u)(\sigma_1 - \sigma_u)}{\sigma_u(-\sigma_u - \sigma_4 + \sigma_2 + \sigma_1)}}}$$

For the ratio of the polarisations one simply has:

$$\frac{P}{P^{‘}} = \frac{\sigma_u - \sigma_1}{\sigma_2 - \sigma_u}$$

.

For the asymmetry A_{LR} one obtains

$$A_{LR} = \sqrt{\frac{(-\sigma_2 + \sigma_u)(\sigma_1 - \sigma_u)}{\sigma_u(-\sigma_u - \sigma_4 + \sigma_2 + \sigma_1)}} \, .$$

If the errors on the cross section measurements $\Delta\sigma_i$ are completely independent then one has for the relative error of A_{LR}

$$\frac{\Delta A_{LR}}{A_{LR}} = \frac{1}{A_{LR}} \sqrt{\sum_1^4 \left(\frac{\partial A_{LR}}{\partial \sigma_i}\right)^2 (\Delta\sigma_i)^2}$$

where

$$\frac{1}{A_{LR}} \frac{\partial A_{LR}}{\partial \sigma_1} = \frac{1}{2} \frac{\sigma_4 - \sigma_2}{(\sigma_1 - \sigma_u)(\sigma_u + \sigma_4 - \sigma_2 - \sigma_1)}$$

$$\frac{1}{A_{LR}} \frac{\partial A_{LR}}{\partial \sigma_2} = -\frac{1}{2} \frac{\sigma_4 - \sigma_1}{(-\sigma_2 + \sigma_u)(\sigma_u + \sigma_4 - \sigma_2 - \sigma_1)}$$

$$\frac{1}{A_{LR}}\frac{\partial A_{LR}}{\partial \sigma_4} = -\frac{1}{2}\frac{1}{\sigma_u + \sigma_4 - \sigma_2 - \sigma_1}$$

$$\frac{1}{A_{LR}}\frac{\partial A_{LR}}{\partial \sigma_u} = \frac{1}{2}\frac{{\sigma_1}^2\,\sigma_2 + ({\sigma_2}^2 - (2\,\sigma_u + \sigma_4)\,\sigma_2)\,\sigma_1 + \sigma_4\,{\sigma_u}^2}{\sigma_u\,(-\sigma_4 + \sigma_2 - \sigma_u + \sigma_1)\,(-\sigma_2 + \sigma_u)\,(\sigma_1 - \sigma_u)}$$

These formulae can serve to determine A_{LR} and estimate its statistical error directly from the measured cross sections without the need to rely on an intermediate evaluation of the polarisation levels of the two collider beams.

In addition to the fact that in the LEP Blondel scheme A_{LR} and its error are derived from the cross sections, it also provides a direct collection of a data sample produced from non-polarised e^+e^- annihilations. This sample allows the comparison of many physics quantities with the corresponding ones collected at other E_{CM} energies where no polarised beams existed. The fraction of the luminosity given to the unpolarised configuration L_u can be determined according to need and wish. A simple example of such a case is the comparison of the emitter size as obtained from Bose-Einstein correlations at energies of the planned collider and the corresponding values obtained at lower e^+e^- collisions (see e.g. reference [11]).

3.3 Two polarised beams in the alternative Blondel scheme

In the case of a linear collider like TESLA all polarisation states can in principle be reached for both the electron and positron beams. Specifically if we denote by $\sigma_{\pm\pm}$ the e^+e^- cross sections where the first subscript denotes the positron polarisation and the second the electron one, both defined with respect to their velocity direction, then one has [8,9]:

$$\sigma_{++} = \sigma_u[1 - P^{‘}P + A_{LR}(P^{‘} - P)]$$

$$\begin{aligned}
\sigma_{-+} &= \sigma_u[1 + P^{‘}P + A_{LR}(-P^{‘} - P)] \\
\sigma_{+-} &= \sigma_u[1 + P^{‘}P + A_{LR}(P^{‘} + P)] \\
\sigma_{--} &= \sigma_u[1 - P^{‘}P + A_{LR}(-P^{‘} + P)]
\end{aligned}$$

where σ_u is not a measured quantity but can be estimated from the relation

$$\sigma_u = \frac{1}{4}(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--}) \ .$$

To note is that up to $\mathrm{O}(m_e/E_e)$ the σ_{++} and σ_{--} cross sections cannot form a vector particle in the s-channel.

The left-right asymmetry is given in this alternative Blondel scheme by:

$$A_{LR} = \sqrt{\frac{(\sigma_{++}+\sigma_{-+}-\sigma_{+-}-\sigma_{--})(-\sigma_{++}+\sigma_{-+}-\sigma_{+-}+\sigma_{--})}{(\sigma_{++}+\sigma_{-+}+\sigma_{+-}+\sigma_{--})(-\sigma_{++}+\sigma_{-+}+\sigma_{+-}-\sigma_{--})}}\,.$$

In this scheme the beam polarisations are given by:

$$P = \left[\frac{-\sigma_{++}+\sigma_{+-}-\sigma_{--}+\sigma_{-+}}{\sigma_{++}+\sigma_{+-}-\sigma_{--}-\sigma_{-+}}\right] A_{LR}$$

and

$$P^{'} = \left[\frac{\sigma_{++}+\sigma_{+-}-\sigma_{--}-\sigma_{-+}}{\sigma_{+-}+\sigma_{--}+\sigma_{++}+\sigma_{-+}}\right]\frac{1}{A_{LR}}\,.$$

Taking the individual errors on the cross sections $\Delta\sigma_{\pm\pm}$ to be completely independent then the relative error of A_{LR} is equal to

$$\frac{\Delta A_{LR}}{A_{LR}} = \frac{1}{A_{LR}}\sqrt{\sum\left(\frac{\partial A_{LR}}{\partial\sigma_{\pm\pm}}\right)^2(\Delta\sigma_{\pm\pm})^2}$$

where the individual contributions to the error are:

$$\frac{\partial A_{LR}}{\partial\sigma_{++}} = \frac{1}{8A_{LR}\sigma_u^2}\frac{(-{\sigma_{--}}^2+{\sigma_{++}}^2+{\sigma_{-+}}^2+{\sigma_{+-}}^2)\,\sigma_{--}-2\,\sigma_{-+}\,\sigma_{++}\,\sigma_{+-}}{(\sigma_{-+}+\sigma_{+-}-\sigma_{--}-\sigma_{++})^2}$$

$$\frac{\partial A_{LR}}{\partial\sigma_{--}} = \frac{1}{8A_{LR}\sigma_u^2}\frac{(-{\sigma_{++}}^2+{\sigma_{--}}^2+{\sigma_{-+}}^2+{\sigma_{+-}}^2)\,\sigma_{++}-2\,\sigma_{-+}\,\sigma_{--}\,\sigma_{+-}}{(\sigma_{-+}+\sigma_{+-}-\sigma_{--}-\sigma_{++})^2}$$

$$\frac{\partial A_{LR}}{\partial\sigma_{+-}} = \frac{1}{8A_{LR}\sigma_u^2}\frac{(-{\sigma_{-+}}^2+{\sigma_{--}}^2+{\sigma_{++}}^2+{\sigma_{+-}}^2)\,\sigma_{-+}-2\,\sigma_{--}\,\sigma_{+-}\,\sigma_{++}}{(\sigma_{-+}+\sigma_{+-}-\sigma_{--}-\sigma_{++})^2}$$

$$\frac{\partial A_{LR}}{\partial\sigma_{-+}} = \frac{1}{8A_{LR}\sigma_u^2}\frac{(-{\sigma_{+-}}^2+{\sigma_{-+}}^2+{\sigma_{++}}^2+{\sigma_{--}}^2)\,\sigma_{+-}-2\,\sigma_{--}\,\sigma_{-+}\,\sigma_{++}}{(\sigma_{-+}+\sigma_{+-}-\sigma_{--}-\sigma_{++})^2}$$

4 The precision reached for A_{LR}, P and $P^{'}$

In the present study on the expected precision of the left-right asymmetry and beam polarisations measurements the following remarks are in place.

1) The errors considered are only the statistical ones. No attempt has been made in this work to assess the systematic effects which at high statistics data are expected to be the dominant ones. In particular one should note that systematic errors strongly depend on the experimental setup of the collider and devices.

2) The errors quoted correspond at $\sqrt{s} = 91$ GeV to a total data of 10^9 events (the Giga Z^0 case) whereas at $\sqrt{s} = 500$ GeV, where a cross section of 2 pb is expected for the no radiative

return, a total luminosity of 500 fb^{-1} was assumed which corresponds to 10^6 events in an unpolarised colliding e^+e^- beams.

3) The left-right asymmetry at $\sqrt{s} = 91$ GeV has been measured [12] to be 0.15 which was used in our analysis. The outcome of the analyses at $\sqrt{s}$= 500 GeV is given for the two left-right asymmetry values of $A_{LR} = 0.15$ and 0.5.

4) Throughout our analysis we have assumed that the polarimeter measurement of the longitudinal beam polarisations reached the precision of $\Delta P/P$=0.005.

5) As for the luminosity division between the different spin polarisation configurations. In the Blondel scheme the results are given as a function of L_4 for several values of L_u, the luminosity fraction given to the unpolarised configuration, where the remaining luminosity is divided equally between L_1 and L_2. In the alternative scheme the results are presented as a function of $(L_{++} + L_{--})/L$ with the condition that $L_{++} = L_{--}$ and $L_{+-} = L_{-+}$.

4.1 Results from the Blondel scheme

The Blondel scheme was at first proposed for e^+e^- storage ring colliders and in particular for LEP. However it can of course also be applied to linear colliders like TESLA and it may well turn out also to be easier to implement than the alternative scheme. Furthermore, as mentioned before, in the Blondel scheme there is the freedom to choose the fraction of luminosity $L_3/L = L_u/L$ to be given to the non-polarised spin configuration which could be an additional argument to opt for this scheme. In Fig. 3 the dependence of $\Delta A_{LR}/A_{LR}$ and the error of the electron beam polarisation is shown as a function of $L_4/(L - L_u)$ at $\sqrt{s}$=91 and 500 GeV. As can be seen, the best precision obtained both for A_{LR} and the beams' polarisation is obtained for values around $L_4/(L - L_u) = 0.1$ irrespective of the chosen L_u/L value. Furthermore, at $\sqrt{s} = 90$ GeV one finds out that by adding a polarised positron beam and when about 10^9 events are collected, the precision of the left-right asymmetry is by far better than that which can be achieved by a single polarised beam and/or when the polarisations are measured by polarimeters with a precision of 0.5 %. This achievement can be traced to the very large data sample which suppresses the statistical error on the cross sections measurements to an insignificant amount. Specific values for the gain in the statistical precision are given in Table 2. At the centre of mass energy of 500 GeV the Blondel scheme is performing rather poor.

4.2 The expected precision for A_{LR}, P and $P^{`}$ from the alternative scheme

The expected precision for the A_{LR} and beam polarisation measurements in the alternative scheme are shown in Figs 4 and 5. These results were obtained from a sample of some 10^9

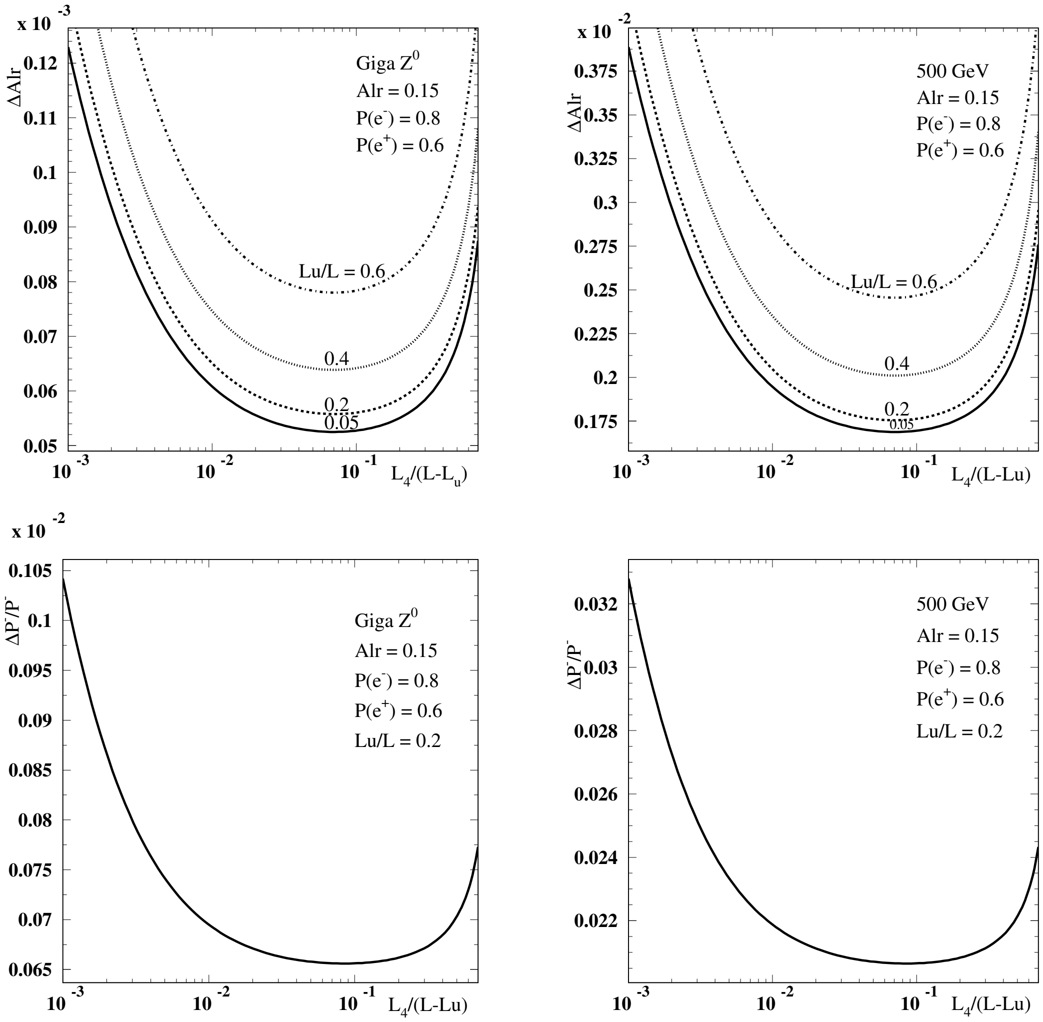

Figure 3: The ΔA_{LR} and the beam relative polarisation errors obtained in the Blondel scheme as a function of $L_4/(L-L_u)$ where L_4 is the amount of luminosity given to the 4th spin-spin configuration and L_u stands for the amount of luminosity given to the non-polarised colliding e^+e^- bunches (see text).

events at $\sqrt{s} = 91$ GeV and a sample of some 10^6 events at $\sqrt{s} = 500$ GeV. Our findings are given as a function of $(L_{++} + L_{--})/L$ tacitly assuming that $L_{++} = L--$. As expected, the precision increases with the polarisation level of the positron and in all cases the best precision is obtained in the vicinity of $(L_{++} + L_{--})/L$=0.1.

Even though the effective polarisation replaces in the evaluation of A_{LR} the polarisation of the single polarised beam it cannot be considered as the only variable which determines the error ΔA_{LR} as is illustrated in Fig. 6. In this figures are shown the minimum error of A_{LR} as a function of P_{eff} for three values of the electron polarisation level. As can be seen both at 91 GeV and at 500 GeV the error dependence on P_{eff} does also depend on the individual polarisation values of the electron and positron beams.

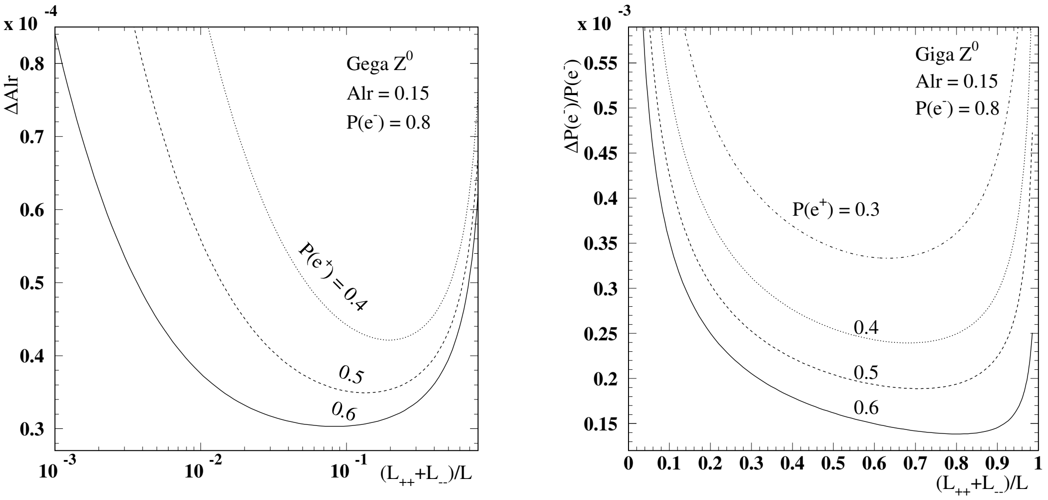

Figure 4: ΔA_{LR} and the relative errors on the electron beam polarisation, obtained in the alternative scheme, as a function of the fraction of luminosity given to the the $++$ and $--$ polarisation states of the e^+e^- system. The errors are calculated at $\sqrt{s} = 91$ GeV taking 10^9 Z^0 hadronic decays events using the measured value of A_{LR}=0.15 [12].

As mentioned earlier, there might be cases where there is an interest to accumulate hadronic data sample identical to that collected in unpolarised electron-positron collisions. These cases may occur for example when one would like to compare the TESLA results at e^+e^- collisions at 500 GeV with those reported at lower energies, such as at the LEP energies, where no longitudinal polarisation beams existed. The obvious question is then if a reliable unpolarised data sample can be extracted from the one accumulated with the longitudinal polarised beams.

Energy and $\mathcal{L}umi$ configuration	A_{LR}	$\Delta P_{eff}/P_{eff}$	
		$(\mathcal{L}_{++}+\mathcal{L}_{--})/\mathcal{L}$=0.1	$(\mathcal{L}_{++}+\mathcal{L}_{--})/\mathcal{L}$=0.5
Giga Z^0 $\mathcal{L}$= 30 fb^{-1} $\sigma = 33 \times 10^{-9} b$	0.15	5.32×10^{-5}	2.47×10^{-5}
	0.50	2.26×10^{-5}	1.10×10^{-5}
$\sqrt{s} = 500$ GeV $\mathcal{L}$=500 fb^{-1} $\sigma = 2 \times 10^{-12} b$	0.15	1.67×10^{-3}	7.76×10^{-4}
	0.50	7.11×10^{-4}	3.46×10^{-4}

Table 1: The relative error $\Delta P_{eff}/P_{eff}$ obtained in the alternative Blondel scheme at two e^+e^- centre of mass energies, two luminosity partitions, and two A_{LR} values.

In the alternative Blondel scheme, unpolarised bunch-bunch collisions do not exist. If all four

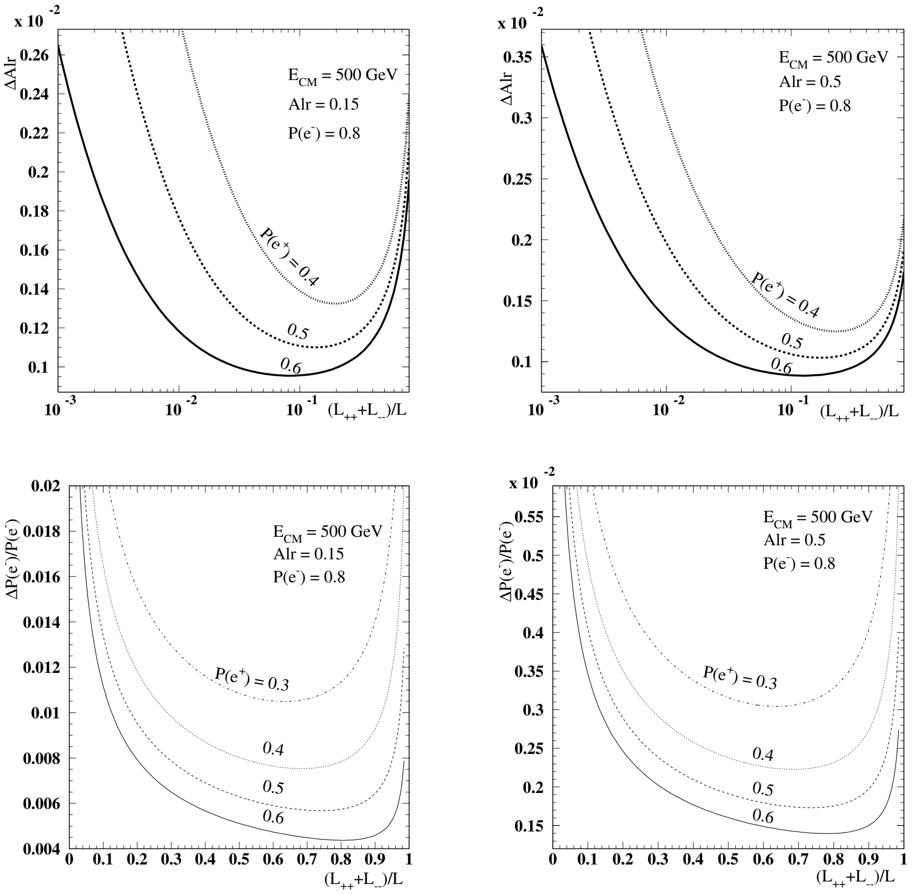

Figure 5: The alternative scheme at $\sqrt{s}$=500 GeV. Upper figures: The errors on A_{LR} as a function of the luminosity ratio $(L_{++} + L_{--})/L$ for two different A_{LR} values where the electron polarisation is $P(e^-) = 0.8$ and the positron polarisation assumes three different values. Lower figures: The relative error on the positron beam polarisation as a function of $(L_{++} + L_{--})/L$.

spin configurations receive the same amount of luminosity then the whole combined data sample is obviously equivalent to a sample obtained from non-polarised beams and no loss of data does occur. However, as shown in the former sections, the best measurement of A_{LR} is achieved when $(L_{++} + L_{--})/L \simeq 0.1$. In this case the unbiased unpolarised data sample that can safely be formed is by adding from each spin state sample only an equivalent of the lowest given luminosity L_{min} which in the outcome of the analysis given here is 0.05. Thus the total representative sample from a collision of unpolarised beams is $0.05\times$ 4 from which follows that it will correspond to only 20 % of the total sample. Whereas in the very high statistics Giga Z^0 case this does not pose any problem whereas it may introduce a hindrance in the high energy region of $\sqrt{s} = 500$ GeV where the data statistics is by far smaller.

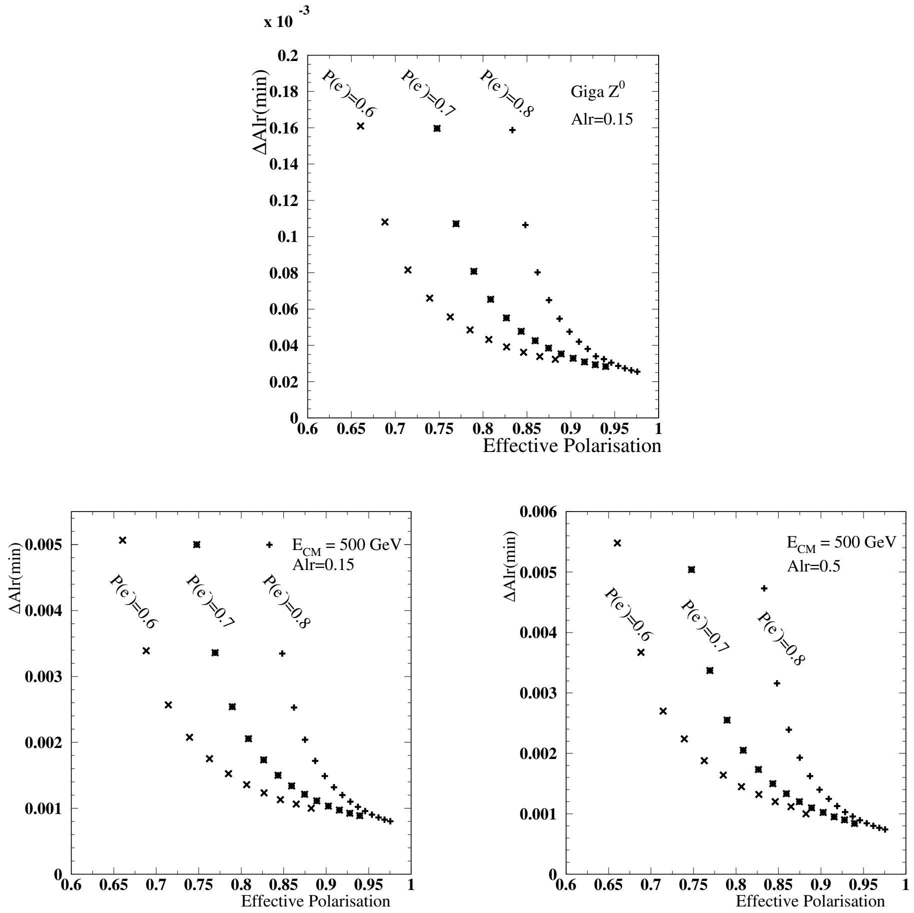

Figure 6: The minimum ΔA_{LR} value as function of the effective polarisation P_{eff} for three different electron beam polarisation values.

5 Summary and conclusions

In trying to assess the improvement obtained by adding a longitudinal polarised positron beam to a linear collider one should again stress the fact that no attempt has been made to try and evaluate the systematic errors. Neither did we incorporate the uncertainty due to the relative polarisation measurements which seems to be needed even when the collider operates in the Blondel or alternative schemes.

An overview of the outcome of the present analysis is given in Tables 1 and 2. In the first table the precision obtained for the effective polarisation for several operating configurations is presented. In Table 2 the gain in ΔA_{LR} is shown when one adds a longitudinal positron beam to that of the electron beam. The results are given in the case that two polarimeters are in use, in the case that the Blondel scheme is applied and in the case where the alternative scheme is chosen.

In the event that the polarisation of the beams is determined by polarimeters then the error

$\sqrt{s}$ (GeV)	A_{LR}	$\frac{\Delta P/P}{\Delta P_{eff}/P_{eff}}$	$\frac{\Delta A_{LR}(P_{e^-}=0.8;P_{e^+}=0.0)}{\Delta A_{LR}(P_{e^-}=0.8;P_{e^+}=0.6)}$		
			Two Polarimeters	Blondel Scheme	Alternative Blondel Scheme
91	0.15	3.73	3.73	13.5	25
“	0.50	”	“	40	87
500	0.15	”	1.65	0.82	1.5
“	0.50	”	2.7	1.38	3.1

Table 2: The factor gained for the ΔA_{LR} precision at A_{LR} values of 0.15 and 0.50 by the addition of a polarised positron beam to the collider. Two CM energies are considered the first at 91 GeV with 10^9 events and the second at 500 GeV with 2 pb$\times$ 500 fb^{-1} events. The gain factor is defined as the ratio of ΔA_{LR} obtained with only one polarised beam measured with a precision of 0.5 % and the ΔA_{LR} value expected from two polarised beams considering two cases. In the first the two beams polarisations are measured to a precision of 0.5% by polarimeters and the second by using the alternative Blondel scheme. These results are compared to the precision of the P_{eff} as compared to a collider where only the electron beam is polarised. The listed ΔA_{LR} values are the best ones which can be achieved in the given configuration. For the Blondel scheme we have chosen $L_3/L = L_u/L = 0.2$.

on A_{LR} depends first of all on the precision of these polarimeters which in Table 2 is taken to be $\Delta P/P = 0.005$. In the Blondel and its alternative schemes these statistical errors are above all a reflection of the size of the data sample. Hence the precision obtained for the left-right asymmetry A_{LR} in the Giga Z^0 case is impressive in both spin configuration schemes, where the alternative model is the winner.

At the CM energy of 500 GeV the precision gained in the Blondel scheme is clearly poor and below that achieved by the application of two polarimeters. The precision gained when the alternative scheme is employed is still below that obtained with two polarimeters but it consists an improvement over the Blondel scheme. Finally it should be noted that the addition of a polarised positron beam at the TESLA energy of $\sqrt{s} = 500$ GeV is providing a precision improvement of not more that 65% over the one that can be reached by having only one polarised beam when A_{LR} value lies in the vicinity of 0.15. However for a value of A_{LR}=0.5, which is expected at high energy as long as the Standard Model is still the correct description of particles and fields, a precision factor of 3.1 is gained in the alternative scheme by the addition of a longitudinal polarised positron beam.

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