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Pair production of charged Higgs bosons in e^+e^- collisions at CLIC

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Abstract

In this paper, we consider the pair production of charged Higgs bosons in e^+e^- collisions at 3 TeV. High energy beam-beam effects are taken into account. With an integrated luminosity of 3000 fb^{-1}, the charged Higgs bosons are likely to be discovered at CLIC if m_H remains smaller than 1.2 TeV. The mass and the signal rate can be measured with a χ^2-analysis: the accuracy on m_H is better than 1%, and $\sigma \times \mathrm{Br}^2(H \to tb)$ can be determined with a precision of a few percent. This latter measurement allows a decent determination of the MSSM parameter $tan\beta$ in the region $3 \leq tan\beta \leq 10$. A measurement of the charged Higgs boson production cross section at various centre-of-mass energies may allow to compute $tan\beta$ in the complementary region.

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1 Introduction

One of the most important physics issues to be covered by a future multi-TeV e^+e^- linear collider is the exploration of the Higgs sector. In the Standard Model, one complex scalar doublet is responsible for the electroweak gauge symmetry breaking, and there is only one physical Higgs boson h^0. In several extensions of the Standard Model, the Higgs sector consists of two complex scalar doublets and there are five Higgs bosons, two of which are charged (H^+ and H^-).

The analysis of LEP data excludes the existence of charged Higgs bosons with masses lower than 78.5 GeV at a 95% confidence level, independently of the branching ratios of the charged Higgs boson [1]. A study of $e^+e^- \rightarrow H^+H^-$ at a 800 GeV linear collider showed that the sensitivity to the pair production of charged Higgs bosons is guaranteed up to 350 GeV, and that statistical accuracies of 1 GeV on m_H and of 10% on the production cross section can be obtained [2]. In this paper, we focus on the search for pair produced heavy charged Higgs bosons in e^+e^- collisions at $\sqrt{s} = 3$ TeV.

In Section 2, the cross section for $e^+e^- \rightarrow H^+H^-$ is computed at $\sqrt{s} = 3$ TeV, i.e. the nominal centre-of-mass energy of the Compact Linear Collider CLIC [3]. In Section 3, we focus on the reconstruction of the charged Higgs bosons and on the influence of the background from $\gamma\gamma \rightarrow$ hadrons. In Section 4, the discovery potential for pair produced charged Higgs bosons at CLIC is discussed. Then, in Section 5, we estimate the accuracy with which the charged Higgs bosons properties and the production cross section for $e^+e^- \rightarrow H^+H^-$ can be measured at CLIC and, in Section 6, we discuss the derivation of the MSSM parameter $tan\beta$ from these measurements. Finally, a summary and some conclusions are given in Section 7.

2 Production cross section for charged Higgs bosons at CLIC

In e^+e^- collisions, charged Higgs bosons are produced in pairs via an intermediate photon or Z boson. At tree level, the production cross section for $e^+e^- \rightarrow H^+H^-$ only depends on m_H and on the centre-of-mass energy $\sqrt{s}$ [4]:

$$\sigma = \frac{e^4}{48\pi s}\left(1 - \frac{4m_H^2}{s}\right)^{3/2}\left(1 + \frac{2c_V^\prime c_V}{1 - m_Z^2/s} + \frac{{c_V^\prime}^2(c_V^2 + c_A^2)}{(1 - m_Z^2/s)^2}\right)$$

with $c_V = \dfrac{-1 + 4sin^2\theta_W}{4sin\theta_W cos\theta_W}$, $c_A = \dfrac{-1}{4sin\theta_W cos\theta_W}$, $c_V^\prime = \dfrac{-1 + 2sin^2\theta_W}{2sin\theta_W cos\theta_W}$.

At CLIC, in addition to the initial state radiation of photons (ISR), one must take into account high energy beam-beam effects. Because of the very small transverse dimensions of the beams at the interaction point, electrons and positrons experience very strong transverse electromagnetic fields. The bending of their trajectories induces emission of photons (beamstrahlung). In a strong external field, a photon can turn into an e^+e^- pair, and the number of additional particles due to this coherent pair creation is not negligible, even compared to the number of beam particles (4×10^9 per bunch), so they also contribute to the beam fields. In the following, the nominal centre-of-mass energy is 3 TeV and the luminosity is 10^{35} cm^{-2}.s^{-1}. The average energy loss of a beam particle due to beamstrahlung is then 31%, and the number of coherent pairs per bunch crossing is 6.6×10^8 [5]. Figure 1 shows the resulting luminosity spectrum calculated with GUINEA-PIG [6] and then implemented into the event generator.

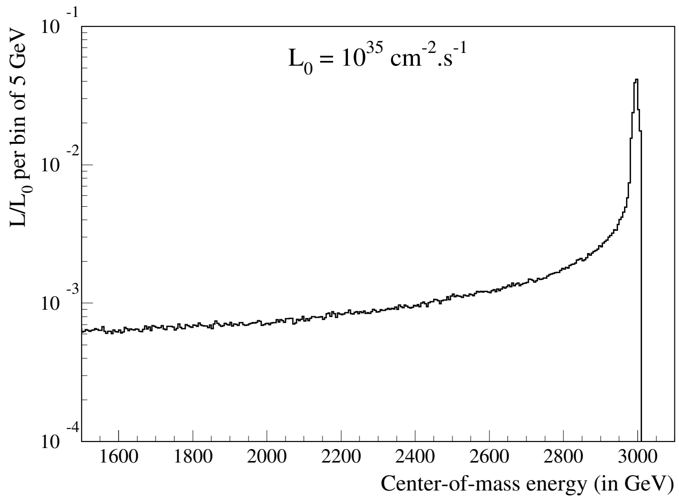

Figure 1: Luminosity spectrum at CLIC for a centre-of-mass energy of 3 TeV.

The cross section for $e^+e^- \to H^+H^-$ and the branching ratios of the various $H^\pm$ decay modes were computed with PYTHIA 6.1 [7]. The process 301 was used, and all s-particles were assumed to be heavier than the charged Higgs boson, so only the standard decays of $H^\pm$ were considered.

Figure 2 shows that the production cross section computed with PYTHIA for a fixed centre-of-mass energy of 3 TeV agrees very well with the theoretical prediction at tree level (open crosses versus dashed line). When the ISR and beamstrahlung effects are taken into account, one has to make to a convolution of the tree level cross section with the luminosity spectrum. For the low values of m_H, since one integrates over almost the whole energy spectrum (from 1.5 TeV down to m_H), the effective cross section becomes larger than the tree level cross section. On the other hand, if m_H is close to the threshold, only a small fraction of the energy spectrum is available, so the effective cross section is smaller than the tree level cross section.

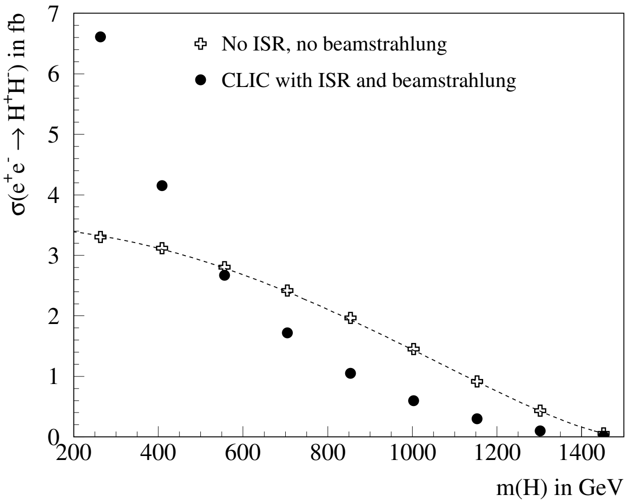

Figure 2: Cross section for $e^+e^- \rightarrow H^+H^-$. The calculations were performed with the PYTHIA event generator, in the theoretical framework of the MSSM. The open crosses and the dashed line show the cross section calculated at tree level, while the full circles show the cross section obtained after taking ISR and beamstrahlung effects into account.

The theoretical framework of our study is the minimal supersymmetric extension of the Standard Model (MSSM). At tree level, the masses and the couplings are fixed by only two parameters, $tan\beta$ and m_A. In the MSSM, the charged Higgs bosons decay into fermions pairs, i.e. tb or $\tau\nu_\tau$. If kinematically allowed, the $H \rightarrow Wh^0$ decay may also occur. However, it has a substantial branching ratio only when m_H is below the tb threshold and when $tan\beta$ is small: it will thus be neglected in the following of this study. If m_H is above the tb threshold, the main decays are $H \rightarrow tb$ and $H \rightarrow \tau\nu_\tau$. At first order, their branching ratios are [8]:

- $\mathrm{Br}(H \rightarrow tb) = \dfrac{3(m_b^2 tan^2\beta + m_t^2 cot^2\beta)}{3(m_b^2 tan^2\beta + m_t^2 cot^2\beta) + m_\tau^2 tan^2\beta}$,
- $\mathrm{Br}(H \rightarrow \tau\nu_\tau) = \dfrac{m_\tau^2 tan^2\beta}{3(m_b^2 tan^2\beta + m_t^2 cot^2\beta) + m_\tau^2 tan^2\beta}$.

Since $H \rightarrow tb$ is generally the dominant decay mode, we will only focus on $e^+e^- \rightarrow H^+H^- \rightarrow tbtb$, which leads to two W bosons and four b quarks in the final state.

3 Reconstruction of the charged Higgs bosons

3.1 Simulation of the detector response

Following the generation of $e^+e^- \rightarrow H^+H^-$ events, a fast detector simulation is performed by using the SIMDET code [9] with its default parametrizations for CLIC. This program applies a gaussian smearing to the momenta of the charged particles and to the energy measured in the electromagnetic and hadronic calorimeters. It takes into account the tracking efficiencies, the charge misidentification and the threshold energies for detection.

At CLIC, high energy $\gamma\gamma$ collisions will lead to the production of additional hadrons. If the nominal centre-of-mass energy is 3 TeV, one expects about 4 of these hadronic events per bunch crossing, with a centre-of-mass energy $\sqrt{s_{\gamma\gamma}}$ larger than 5 GeV [5]. The hadrons produced in the $\gamma\gamma$ collisions have a rather small transverse energy: most of them remain very close to the beam axis and, when they are found in the central part of the detector, their energy is very low. This is illustrated in Figure 3. At CLIC, bunch crossings will occur every 0.67 ns, which is much smaller than the time response of any existing or foreseen detector. One should thus integrate the contribution of the hadronic background over N_{bc} bunch crossings. About 8000 $\gamma\gamma \rightarrow$ hadrons events are stored in a database and can be added to the $e^+e^- \rightarrow H^+H^- \rightarrow tbtb$ events generated with PYTHIA.

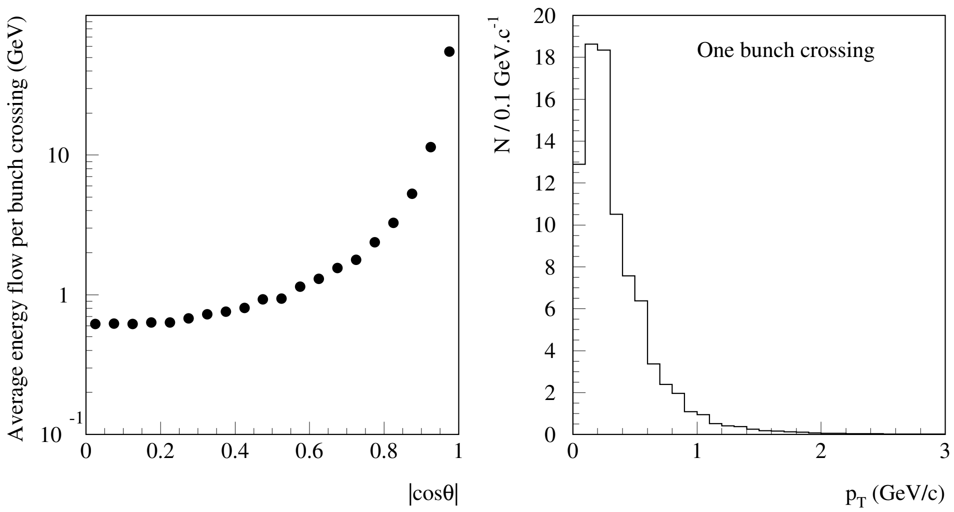

Figure 3: Average energy flow due to the hadronic background from $\gamma\gamma$ collisions per bunch crossing as a function of the angle with respect to the beam axis (left-hand side) and p_T distribution of the hadrons produced in the $\gamma\gamma$ collisions (right-hand side).

3.2 Jet reconstruction

A cluster analysis method based on particle momenta was used to reconstruct the jets. Their multiplicity strongly depends on the distance scale d_{join} above which two clusters may not be joined in the PYCLUS subroutine of PYTHIA. Let us assume that there is no hadronic background from the high energy $\gamma\gamma$ collisions and let us only consider the $e^+e^- \rightarrow H^+H^- \rightarrow tbtb$ decay chains that lead to four b quarks and four quarks from the first and second generations in the final state. The optimal value of d_{join} is the one for which the fraction of events with exactly eight reconstructed jets is maximal. It has been estimated for various values of m_H, as shown in Figure 4. In the following, only the events with at least eight reconstructed jets, including four b tagged jets are considered (the b tagging efficiency is set to 90%).

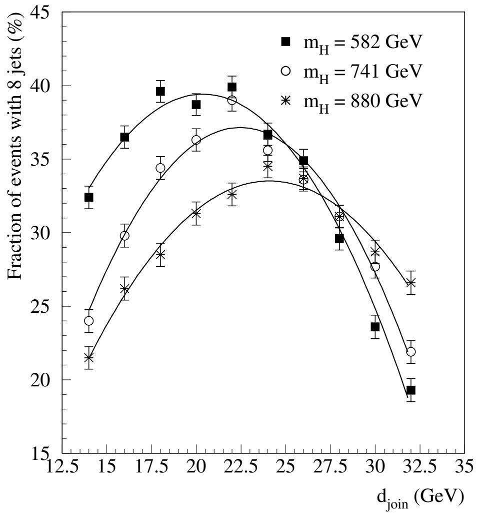

Figure 4: Fraction of events with eight jets as a function of the distance scale over which two clusters are not joined. Only the events with $H \rightarrow tb \rightarrow qqbb$ are selected and no hadronic background is added.

3.3 Jet assignment and mass reconstruction

In order to distinguish the pair production of charged Higgs bosons from the associated background processes and in order to accurately measure m_H, it is important to obtain a clean mass distribution of the multi-jet final states. In the following, the two W bosons in the final state can decay either into a quark-antiquark pair or into a charged lepton and a neutrino.

In a first step, the presence of two W bosons decaying hadronically is tested. If there was no hadronic background, the assignment of the four leading non-b tagged jets to their correct WW pair would be done by choosing the combination which minimizes $(m(j_a j_b) - m_W)^2 + (m(j_c j_d) - m_W)^2$. However, since high energy $\gamma\gamma$ collisions produce some additional hadrons which are merged into the jets coming from the decay of the charged Higgs bosons, the reconstructed masses of the intermediate states are higher than what they would be without the hadronic background. Therefore, if one wants to keep a good efficiency for the jet assignment, one should replace m_W by the expected mass peak value of the W candidates. In order to estimate $m_{rec}(W)$, one must run the event generator and the fast detector simulation a first time and assign the four leading non-b tagged jets to their correct WW pair by choosing the combination for which $m(j_a j_b)$ and $m(j_c j_d)$ are the closest. In a second step, each t quark is reconstructed from a W candidate paired with one of the four b tagged jets, such that the invariant mass of the (Wb) system is as close as possible to the expected mass peak value for the t candidates (here, $m_{rec}(t)$ is estimated with the same method as for the mass peak value of the W candidates). The mass spectra obtained for the intermediate W and t candidates at this stage of the reconstruction procedure are shown in Figure 5.

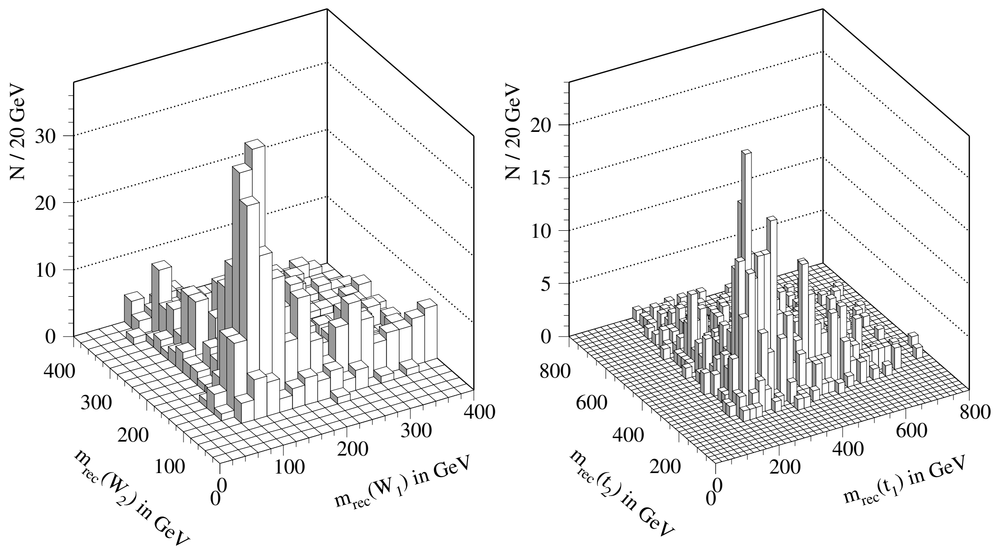

Figure 5: Mass reconstruction of the intermediate W and t candidates (left and right, respectively). The integrated luminosity is 3000 fb^{-1} and the production of additional hadrons through $\gamma\gamma$ collisions is taken into account, with $N_{bc} = 15$.

Each of the charged Higgs bosons is then reconstructed from a t candidate paired with one of the two remaining b tagged jets. Since one does not know the mass of the charged Higgs boson, the combination which minimizes the mass difference between the two (tb) systems is selected. At this stage, the events for which the jet combinations are not compatible with a W boson and a t quark, within the observed resolution, are discarded. A first estimation of m_H can be performed by simply calculating the average value of the invariant masses of the two H candidates, see Figure 6.

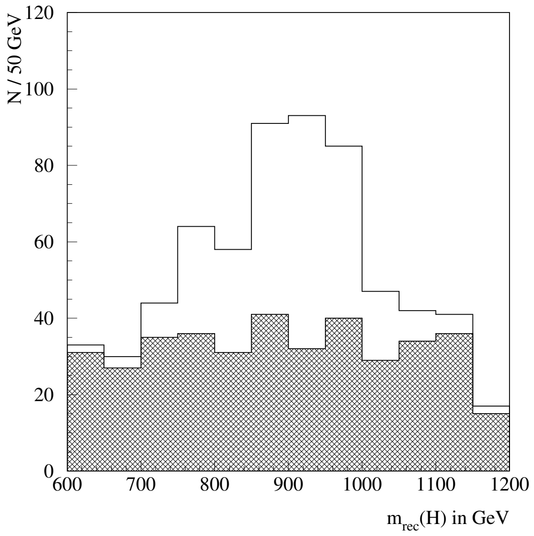

Figure 6: Mass reconstruction of a pair of 880 GeV charged Higgs bosons. Here, the integrated luminosity is 3000 fb^{-1} and the production of additional hadrons through $\gamma\gamma$ collisions is taken into account, with $N_{bc} = 15$. The shaded histogram shows the contribution of the $tbtb$ background.

After having assigned the jets to their W and t candidates, one can further improve the reconstruction of the charged Higgs bosons by using a kinematical fit with the following constraints:

- the mass of the intermediate states are equal to m_W or m_t,
- the two H candidates have the same mass,
- the system consisting of the four b tagged jets and the four leading non-b tagged jets has p_x and p_y equal to zero,
- the momentum and energy conservation at $\sqrt{s} = 3$ TeV (with neither ISR nor beamstrahlung losses) is applied to an imaginary system consisting of the four b tagged jets, the four leading non-b tagged jets and a neutrino with a zero transverse momentum.

This time, since a first estimation of the charged Higgs boson mass has already been performed, the assignment of the t candidates and the two remaining b

tagged jets is done such that the invariant mass of the selected (tb) combinations is as close to $m_{rec}(H)$ as possible. At this stage, the events for which the kinematical fit does not give a satisfying result for the intermediate W and t candidates are rejected. With this new reconstruction procedure, the signal efficiency becomes between two and three times larger, the mass resolution is slightly better and the signal-to-background ratio under the peak is improved by nearly 40%, see Figure 7.

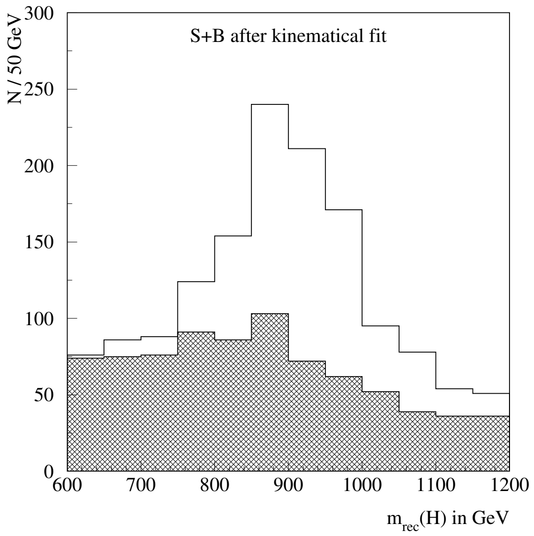

Figure 7: Mass reconstruction of a pair of 880 GeV charged Higgs bosons with a kinematical fit. Here, the integrated luminosity is 3000 fb^{-1} and the production of additional hadrons through $\gamma\gamma$ collisions is taken into account, with $N_{bc} = 15$. The shaded histogram shows the contribution of the $tbtb$ background.

4 Discovery potential for charged Higgs bosons

In order to estimate the discovery limits for pair produced charged Higgs bosons at CLIC, it is necessary to know the physics background first. A large number of processes may lead to hadronic final states. For many of them, the cross section is several orders of magnitude larger than for $e^+e^- \rightarrow H^+H^-$. However, the processes which have the largest cross sections, such as W boson or t quark pair production, lead only to four or six fermions, with two or less b quarks in the final state. Therefore, these backgrounds can be efficiently suppressed by requiring at least eight jets, including four b tagged jets. In the reconstruction algorithm, the requirements on the masses of the intermediate states (i.e. the W bosons and the t quarks) also reduce much further the event rates of the background processes in which the parton configuration differs from the signal.

Therefore, the most significant background processes are those which lead to genuine $tbtb$ final states. Their production cross section was calculated with COMPHEP [10]. For this purpose, the ISR was included but the beamstrahlung effect was not. Hence, the cross section that we found (1.51 fb) is likely to be slightly different from what it would be with beamstrahlung.

The signal and background event rates have been estimated in three different cases. In the first one, there is no hadronic background. In the second one, we integrate the hadronic background over one bunch crossing. Finally, in the third one, we choose $N_{bc} = 15$.

The following cuts are applied in order to select the good W and t candidates at the end of the first reconstruction procedure (no kinematical fit is applied):

- if there is no hadronic background, both W candidates have a reconstructed mass between 60 and 150 GeV, and both t candidates have a reconstructed mass between 160 and 250 GeV.
- if $N_{bc} = 1$, both W candidates have a reconstructed mass between 70 and 170 GeV, and both t candidates have a reconstructed mass between 170 and 270 GeV.
- if $N_{bc} = 15$, both W candidates have a reconstructed mass between 80 and 220 GeV, and both t candidates have a reconstructed mass between 180 and 320 GeV.

When using the second reconstruction procedure, one simply requires that the mass of the intermediate states indeed be equal to m_W or m_t, as requested in the kinematical fit.

The window in which the signal and background event rates are calculated is $m_{fit}(H) \pm 2.5\sigma_{fit}(H)$, where $m_{fit}(H)$ and $\sigma_{fit}(H)$ are the peak mass and the r.m.s of the reconstructed charged Higgs boson, respectively. For a discovery, one requires the signal S to be at least 10 events and it must exceed 5 statistical fluctuations of the background B. In the following, the discovery limit for the charged Higgs bosons is estimated assuming an integrated luminosity of 3000 fb^{-1} and with $\mathrm{Br}(H \to tb) = 100\%$.

Table 1 shows that the discovery potential for pair produced charged Higgs bosons at CLIC does not significantly depend on the amount of hadronic background. It also indicates that the procedure with the kinematical fit allows to increase the discovery reach by 150-200 GeV, up to about 1.25 TeV, thanks to a larger value for both the reconstruction efficiency and the signal-to-background ratio.

N_{bc} for the hadronic background	0	1	15
Discovery limit without kinematical fit	1.10 TeV	1.07 TeV	1.11 TeV
Discovery limit with kinematical fit	1.28 TeV	1.27 TeV	1.25 TeV

Table 1: Discovery limit for the pair produced charged Higgs bosons at CLIC as a function of the number of bunch crossings over which the hadronic background is integrated. Here, we have considered an integrated luminosity of 3000 fb^{-1} and we have assumed that Br$(H \to tb)$ = 100%.

Note that the discovery limits of Table 1 were found using a b-tagging efficiency ϵ_b of 90% in the reconstruction procedure of both the signal and background events. For a given value of m_H, the significance $S/\sqrt{B}$ is proportional to ϵ_b^2. If the b-tagging efficiency was increased to 100%, then the discovery limit for the charged Higgs bosons at CLIC would be about 30 GeV higher. Similarly, if the b-tagging efficiency was 65% instead of 90%, then the discovery limit would be 1.15 TeV instead of 1.25 TeV.

When estimating the discovery limit for pair produced charged Higgs bosons, we have assumed a 100% branching ratio for the $H \to tb$ decay. However, the possible presence of other decay modes may result in a smaller value of Br$(H \to tb)$ and therefore a somewhat lower discovery limit in terms of m_H, as it is suggested by Figure 8.

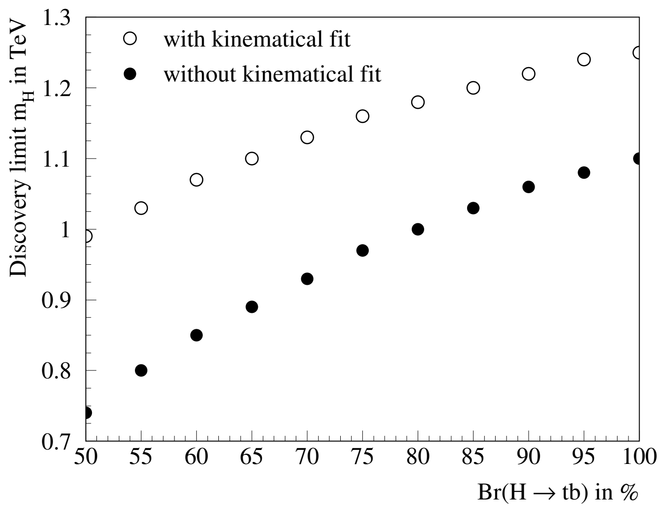

Figure 8: Discovery limit in terms of m_H as a function of Br$(H \to tb)$.

5 Measurement of the charged Higgs boson mass and production cross section

In this section, we focus on the measurement of the mass of the charged Higgs bosons, as well as on the determination of the cross section for $e^+e^- \to H^+H^-$. Let us assume that the charged Higgs boson only decays into tb and that its mass (respectively its width) is 880 GeV (respectively 14.5 GeV). This corresponds to the CMSSM benchmark point J of reference [11]. In this case, the production cross section was found to be 1.0 fb. In the following, we consider an integrated luminosity of 3000 fb^{-1} and we assume that $\mathrm{Br}(H \to tb) = 100\%$.

A sample of 3000 "real" $e^+e^- \to H^+H^- \to tbtb$ events was generated, together with the corresponding "real" $tbtb$ background. Various samples of "simulated" $e^+e^- \to H^+H^- \to tbtb$ events were also generated, with slightly different values for the mass and for the production cross section. A sample of "simulated" $tbtb$ background events was generated as well and then normalized to an integrated luminosity of 3000 fb^{-1}. Comparing "real" and "simulated" event samples allows to derive the parameters of interest. In this study, the hadronic background is taken into account ($N_{bc} = 15$) and the reconstruction procedure of Section 3.3 is used, with the kinematical fit.

For each sample of "simulated" events (signal + background), one calculates:

$$\chi^2 = \sum_i \frac{(N_r(i) - N_s(i))^2}{N_r(i)},$$

where $N_s(i)$ and $N_r(i)$ are respectively the number of "simulated" and "real" events in the bin i of the reconstructed mass histogram (here, the amount of "simulated" events is large enough to neglect the uncertainty on $N_s(i)$ in the estimation of χ^2). For Poisson-distributed data, minimizing χ^2 is equivalent to maximizing the likelihood function for the contents of the bins.

Let us choose several values of the cross section between 0.7 and 1.4 fb. For each of them, χ^2 is calculated as a function of the mass of the charged Higgs boson used in the "simulated" samples, and the minimal value of χ^2 is then derived from a parabolic fit. Knowing how $\mathrm{Min}(\chi^2)$ depends on the cross section, it is possible to derive the value of σ for which $\mathrm{Min}(\chi^2)$ is the smallest with a parabolic fit: we found $\sigma = 0.97 \pm 0.06$ fb. If this value is chosen for the cross section, then χ^2 becomes minimal when $m_H = 884.9 \pm 7.7$ GeV, see Figure 9. This analysis was also performed before using the kinematical fit. In that case, the minimisation of χ^2 leads to $\sigma = 0.99 \pm 0.09$ fb and $m_H = 881.5 \pm 10.3$ GeV: these results are thus somewhat worse than the ones obtained with the kinematical fit.

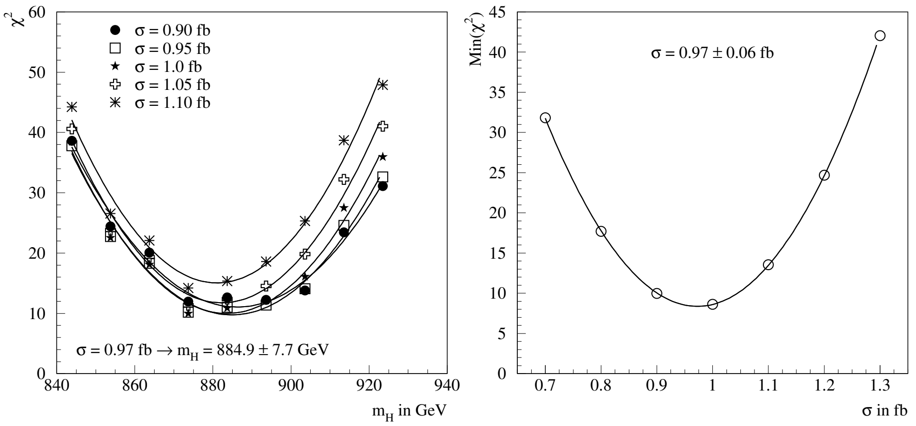

Figure 9: Determination of the statistical errors for m_H and the cross section for $e^+e^- \to H^+H^- \to tbtb$. The variation of χ^2 with m_H is shown on the left, for various values of the cross section. The minimal value of χ^2 is derived from a parabolic fit. The variation of Min(χ^2) with the cross section is shown on the right. Here, the integrated luminosity is 3000 fb^{-1} and Br($H \to tb$) = 100%.

Two other benchmark points of reference [11] were considered as well, with lower values for the charged Higgs boson mass. A similar analysis was performed in order to estimate the best relative accuracy for the determination of the mass and of the signal rate. The results that we obtained are summarized in Table 2.

Benchmark point	C	D	J
m_H (GeV)	582	741	880
$tan\beta$	10	10	35
$\delta\sigma/\sigma$	3.6%	4.9%	5.6%
$\delta m/m$	0.4%	0.7%	0.9%

Table 2: Accuracy for the determination of the signal rate and of the charged Higgs boson mass in three different configurations: we consider an integrated luminosity of 3000 fb^{-1} and we assume that Br($H \to tb$) = 100%.

The presence of other decay modes may result in a value of Br($H \to tb$) which is smaller than 100%. In the following, we assume that the charged Higgs boson does not decay into supersymmetric particles. Therefore, only the $H \to \tau\nu_\tau$ channel can compete with $H \to tb$. In that case, the branching ratio Br($H \to tb$) is always larger than 86% and the relative accuracy on m_H or on the signal rate does not strongly depend on Br($H \to tb$), as shown in Figure 10.

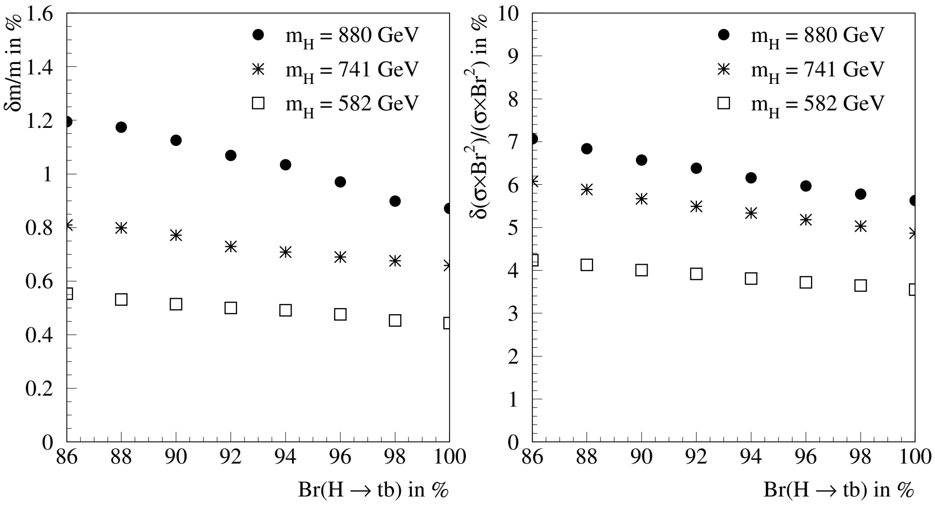

Figure 10: Relative accuracy on the mass and signal rate measurement as a function of $\mathrm{Br}(H \to tb)$, for various values of the charged Higgs boson mass.

6 Determination of $tan\beta$

The ratio $tan\beta$ of the vacuum expectation values of the two Higgs doublets is a very important parameter in the MSSM. Various methods have been proposed in order to accurately measure it at linear e^+e^- colliders [12].

6.1 From the measurement of the signal rate

At first order, the total production cross section for the charged Higgs bosons in e^+e^- collisions does not depend on the MSSM parameter $tan\beta$. It is indeed fully determined by m_H, $\sqrt{s}$ and the luminosity spectrum. Any uncertainty on the measured charged Higgs boson mass m_H thus induces errors when estimating the total production cross section σ for $e^+e^- \to H^+H^-$.

As for the branching ratio $\mathrm{Br}(H \to tb)$, it does vary with $tan\beta$. Indeed, if one only considers the decays into tb and $\tau\nu_\tau$, which is reasonable for heavy charged Higgs bosons, then:

$$\mathrm{Br}(H \to tb) = \frac{3(m_b^2 tan^2\beta + m_t^2 cot^2\beta)}{3(m_b^2 tan^2\beta + m_t^2 cot^2\beta) + m_\tau^2 tan^2\beta}.$$

At large $tan\beta$, the branching ratio $\mathrm{Br}(H \to tb)$ remains constant, and it is thus impossible to derive $tan\beta$ from the measured signal rate. Several values of $tan\beta$ were considered in the 1-20 range. For each of them, $\mathrm{Br}(H \to tb)$ was estimated with the HDECAY program [13], which takes into account the running quark masses at the scale of the charged Higgs boson mass, and the corresponding value of the relative statistical error on the signal rate $\sigma \times \mathrm{Br}^2$ was derived from a χ^2-analysis. The absolute uncertainty $\delta\mathrm{Br}$ on the branching ratio into tb was then estimated, taking into account the uncertainty in the determination of σ that is due to the statistical error on the measured value of m_H:

$$\mathrm{Br} = \sqrt{\frac{\sigma \times \mathrm{Br}^2}{\sigma}} \Longrightarrow \delta\mathrm{Br} = \frac{\mathrm{Br}}{2}\sqrt{\left(\frac{\delta(\sigma \times \mathrm{Br}^2)}{\sigma \times \mathrm{Br}^2}\right)^2 + \left(\frac{\delta\sigma}{\sigma}\right)^2}$$

The uncertainty on $\mathrm{Br}(H \to tb)$ mainly comes from the statistical error in the determination of the signal rate and to a lower extent on $\delta\sigma/\sigma$, thanks to an accurate measurement of m_H. Note however that we have not taken into account possible systematic errors on the energy and/or the luminosity spectrum, which may also affect the determination of the total production cross section σ.

Knowing how $\mathrm{Br}(H \to tb)$ depends on $tan\beta$, one can estimate the absolute error $\delta tan\beta$, as illustrated in the left-hand side plot of Figure 11. The results obtained with this method are shown in the right-hand side plot of Figure 11: it seems very difficult to properly derive the MSSM parameter $tan\beta$ from the signal rate measurement, except in a narrow window centred around $tan\beta \simeq 6.5$, where the relative error on $tan\beta$ is likely to remain less than 50%.

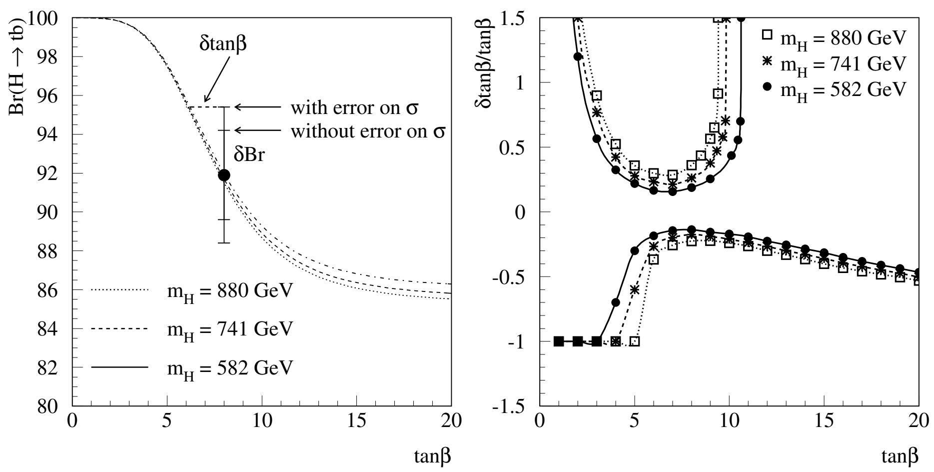

Figure 11: Variation with $tan\beta$ of the branching ratio $\mathrm{Br}(H \to tb)$ and of the relative statistical uncertainty in the determination of $tan\beta$, for various values of the charged Higgs boson mass.

6.2 From the measurement of the width

The MSSM parameter $tan\beta$ also controls the partial widths of the charged Higgs boson. In particular, one has:

$$\Gamma(H \rightarrow tb) \propto m_b^2 tan^2\beta + m_t^2 cot^2\beta$$

At large $tan\beta$, this equation leads to:

$$\frac{\delta tan\beta}{tan\beta} \simeq \frac{1}{2} \times \frac{\delta\Gamma}{\Gamma}.$$

Calculations with HDECAY show that the charged Higgs boson natural width is always much smaller than the experimental mass resolution, as it is shown in Table 3. An accurate measurement of Γ_H and the subsequent determination of $tan\beta$ are therefore extremely difficult.

Benchmark point	C	D	J
m_H in GeV	582	741	880
$tan\beta$	10	10	35
$\Gamma(H \rightarrow tb)$ in GeV	1.1	1.4	14.5
Experimental fwhm in GeV	126.7	152.2	166.9

Table 3: Comparison of the partial width for the $H \rightarrow tb$ decay channel with the experimental mass resolution (fwhm = full width at half of the maximum amplitude), as obtained after having forced Γ_H to be zero in PYTHIA.

6.3 From the measurement of the cross section at various centre-of-mass energies

According to the theoretical study performed in reference [14], $tan\beta$ may also be derived from the measurement of the production cross section for charged Higgs bosons at various energies.

In the framework of the MSSM, let σ_{tree} and $\sigma_{tree+1loop}$ be the production cross section for $e^+e^- \rightarrow H^+H^-$ at tree level and after having included SUSY one-loop terms, respectively. In order to account for these effects at a given centre-of-mass energy $\sqrt{s}$, one defines the ratio:

$$\epsilon(s) = \frac{\sigma_{tree+1loop}(s) - \sigma_{tree}(s)}{\sigma_{tree}(s)}.$$

Let us consider a set of N independent measurements at $\sqrt{s_1}$, $\sqrt{s_2}$, ..., $\sqrt{s_N}$ and let $\delta_i = \epsilon(s_i) - \epsilon(s_1)$ be the difference of each measurement with respect to the one at lowest energy. Under certain conditions, which are detailed in [14], δ_i can be written as:

$$\delta_i = F(tan\beta)\ln\frac{s_i}{s_1},$$

where $F(tan\beta) = -\frac{3\alpha}{4\pi m_W^2 sin^2\theta_W}(m_b^2 tan^2\beta + m_t^2 cot^2\beta)$, with $\alpha = \frac{e^2}{4\pi}$.

Then, a χ^2-analysis allows to derive the best estimate for $tan\beta$ from this set of measurements, and the standard deviation, which is fixed by the condition $\Delta\chi^2 = 1$, is given by:

$$\delta tan\beta = 2\left(\sum_i \left(\frac{F'(tan\beta)\ln(s_{i+1}/s_1)}{\delta\epsilon_i}\right)^2\right)^{-1/2},$$

where $\delta\epsilon_i$ is the experimental error on $\epsilon(s_i)$.

In the following, we consider that $\delta\epsilon_i$ does not depend on the centre-of-mass energy. With this assumption, the previous equation becomes:

$$\frac{\delta tan\beta}{tan\beta} = \frac{\delta\epsilon}{\sqrt{\sum_i \ln^2(s_{i+1}/s_1)}} \times \frac{1}{\frac{3\alpha}{4\pi m_W^2 sin^2\theta_W}|m_b^2 tan^2\beta - m_t^2 cot^2\beta|}$$

Let us now consider 10 equally spaced centre-of-mass energies between 0.8 TeV and 3.5 TeV. In that configuration, the relative uncertainty on $tan\beta$ was derived for various values of the error $\delta\epsilon$ between 0.5% and 5%. Our results are shown in Figure 12. If one manages to keep $\delta\epsilon$ at the percent level, then a decent accuracy can be obtained for the determination of $tan\beta$, at either very small or large values of $tan\beta$, i.e. in the region that is not accessible through the measurement of the signal rate at $\sqrt{s} = 3$ TeV (see Figure 11).

The relative uncertainty on $tan\beta$ is directly proportional to the experimental error $\delta\epsilon$. Therefore, it is crucial to keep this parameter as small as possible, thanks to a precise knowledge of the quantities that are used in the calculation of the cross section for $e^+e^- \to H^+H^-$, in particular m_H, the centre-of-mass energies, the corresponding luminosity spectra, as well as the SUSY parameters used in the one-loop calculations. Nevertheless, $\delta tan\beta/tan\beta$ also strongly depends on how the set of measurements is chosen. In particular, one should have as many centre-of-mass energies $\sqrt{s_i}$ as possible, with a large spacing between them. This requires the use of a very flexible facility that is able to scan a wide range of energies, which should be the case for CLIC.

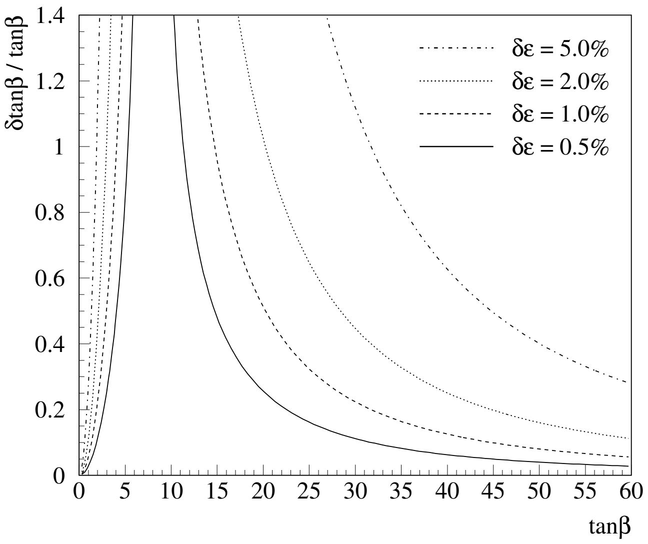

Figure 12: Relative uncertainty on $tan\beta$, as obtained from the measurement of the charged Higgs boson production cross section at 10 different centre-of-mass energies, as a function of $tan\beta$ and for various values of the experimental error $\delta\epsilon$ (see text for details).

7 Conclusion and outlooks

In this paper, we have studied the pair production of charged Higgs bosons in e^+e^- collisions at $\sqrt{s} = 3$ TeV. Only the dominant decay channel (i.e. $H \to tb$) was considered, since it has a much larger branching ratio than $H \to \tau\nu_\tau$ and other decay modes. The final state of interest for this study thus consists of two W bosons and four b quarks. Here, we only considered the hadronic decays of the W bosons, which leads to highly distinctive, yet challenging, multi-jet final states with four b tagged jets. In the reconstruction procedure, special care was taken in order to make our results robust in the presence of high energy beam-beam effects, such as beamstrahlung and additional $\gamma\gamma \to$ hadrons events.

With an integrated luminosity of 3000 fb^{-1}, we showed that the charged Higgs bosons can be discovered at CLIC if m_H is smaller than 1.2 TeV. At first order, this result does not depend on the MSSM parameter $tan\beta$. This is not the case at the Large Hadron Collider (LHC), where only the regions with low or high values of $tan\beta$ are accessible [15]. Then, the charged Higgs boson mass and the signal rate were measured with a χ^2-analysis: the accuracy that we obtained for m_H is better than 1%, and we showed that $\sigma \times \text{Br}^2(H \to tb)$ can be determined with a precision of a few percent. This latter measurement allows determination of $tan\beta$ in the region $3 \leq tan\beta \leq 10$, i.e. where LHC can not observe the charged Higgs bosons. Also, we showed that the measurement of the charged Higgs boson production cross section at various centre-of-mass energies may allow to compute $tan\beta$ in the complementary region.

For a complete study of the charged Higgs bosons at CLIC, one should also take into account the $H \rightarrow \tau\nu_\tau$ decay mode. For instance, the final states with a $\tau\nu_\tau$ pair and a tb pair could be used to measure the ratio between Br($H \rightarrow tb$) and Br($H \rightarrow \tau\nu_\tau$), which may then allow improvement of the $tan\beta$ determination in the region $3 \leq tan\beta \leq 10$. Also, if one manages to reconstruct efficiently τ leptons and to measure their charge, then the analysis of final states with $\tau\nu_\tau$ and tb pairs could also be used to investigate whether there is a difference in the decay rates of the charged Higgs boson into $t\bar{b}$ and $\bar{t}b$, which would be an indication for CP-violation in the MSSM with complex parameters, as suggested in reference [16]. However, for the final states with $\tau\nu_\tau$ and tb, the number of b tagged jets is two only, and the associated background processes are therefore likely to have a much larger cross section. This imposes a very good identification and reconstruction of the τ jets in the CLIC environment, which is not yet implemented in the SIMDET program.

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