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Upper and lower bounds of the lightest CP-even Higgs boson mass in the two-Higgs-doublet model [1]

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By imposing validity of the perturbation and stability of vacuum up to an energy scale Λ ($\leq 10^{19}$ GeV), we evaluate mass bounds of the lightest CP-even Higgs-boson mass (m_h) in the two-Higgs-doublet model (2HDM) with a softly-broken discrete symmetry [1]. In the standard model (SM), both the upper and the lower bounds have been analyzed from these kinds of requirement as a function of Λ [2, 3]. There have already been several works on the Higgs mass bounds in the 2HDM without the soft-breaking term [4, 5]. Our analysis is a generalization of these works to the case with the soft-breaking term. Because the introduction of the soft-breaking scale changes property of the 2HDM, it is very interesting to see what happens for the mass bounds in this case. Our results are qualitatively different from the previous works in the region of the large soft-breaking mass, where only one neutral Higgs boson becomes light. We find that, while the upper bound is almost the same as in the SM, the lower bound is significantly reduced. In the decoupling regime where the model behaves like the SM at low energy, the lower bound is given, for example, by about 100 GeV for $\Lambda = 10^{19}$ GeV and $m_t = 175$ GeV, which is smaller by about 40 GeV than the corresponding lower bound in the SM. In general case, the m_h is no longer bounded from below by these conditions. If we consider the experimental $b \to s\gamma$ constraint, small m_h are excluded in Model II of the 2HDM.

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The Higgs potential of the 2HDM is given for both Model I and Model II as [1]

$$\begin{aligned} V_{\rm 2HDM} &= m_1^2 \left|\varphi_1\right|^2 + m_2^2 \left|\varphi_2\right|^2 - m_3^2 \left(\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1\right) + \frac{\lambda_1}{2}\left|\varphi_1\right|^4 + \frac{\lambda_2}{2}\left|\varphi_2\right|^4 \\ &\quad + \lambda_3 \left|\varphi_1\right|^2 \left|\varphi_2\right|^2 + \lambda_4 \left|\varphi_1^\dagger \varphi_2\right|^2 + \frac{\lambda_5}{2}\left\{\left(\varphi_1^\dagger \varphi_2\right)^2 + \left(\varphi_2^\dagger \varphi_1\right)^2\right\}. \end{aligned} \tag{1}$$

We here take all the self-coupling constants and the mass parameters in (1) to be real. In Model II, φ_1 has couplings with down-type quarks and leptons and φ_2 with up-type quarks. Only φ_2 has couplings with fermions in Model I.

The masses of the charged Higgs bosons ($\chi^\pm$) and CP-odd Higgs boson (χ_2) are expressed as $m_{\chi^\pm}^2 = M^2 - (\lambda_4 + \lambda_5)v^2/2$, and $m_{\chi_2}^2 = M^2 - \lambda_5 v^2$, respectively, where $M = m_3/\sqrt{\cos\beta\sin\beta}$, $\tan\beta = \langle\varphi_2\rangle/\langle\varphi_1\rangle$ and $v = \sqrt{2}\sqrt{\langle\varphi_1\rangle^2 + \langle\varphi_2^2\rangle} \sim 246$ GeV. The two CP-even Higgs boson masses are obtained by diagonalizing the 2×2 matrix, where each component is given by $M_{11}^2 = v^2\left(\lambda_1\cos^4\beta + \lambda_2\sin^4\beta + \frac{\lambda}{2}\sin^2 2\beta\right)$, $M_{12}^2 = M_{21}^2 = v^2\sin 2\beta\left(-\lambda_1\cos^2\beta + \lambda_2\sin^2\beta + \lambda\cos 2\beta\right)/2$ and $M_{22}^2 = v^2\left(\lambda_1+\lambda_2-2\lambda\right)\sin^2\beta\cos^2\beta + M^2$, where $\lambda \equiv \lambda_3+\lambda_4+\lambda_5$. The mass of the lighter (heavier) CP-even Higgs boson h (H) is then given by $m_{h,H}^2 = \left\{M_{11}^2 + M_{22}^2 \mp \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4M_{12}^4}\right\}/2$. For the case of $v^2 \ll M^2$, they can be expressed by

$$m_h^2 = v^2\left(\lambda_1\cos^4\beta + \lambda_2\sin^4\beta + \frac{\lambda}{2}\sin^2 2\beta\right) + \mathcal{O}(\frac{v^4}{M^2}), \tag{2}$$

$$m_H^2 = M^2 + v^2\left(\lambda_1+\lambda_2-2\lambda\right)\sin^2\beta\cos^2\beta + \mathcal{O}(\frac{v^4}{M^2}). \tag{3}$$

Notice that the soft-breaking parameter M characterizes the model. In the case of $M^2 \gg \lambda_i v^2$, these heavy Higgs bosons but the lightest decouple from the low-energy observable, and below the scale M the effective theory is the SM with one Higgs doublet. On the other hand, if $M^2 \sim \lambda_i v^2$, the masses are controlled by the self-coupling constants, and thus the heavy Higgs bosons do not decouple and the lightest CP-even Higgs boson can have a different property from the SM Higgs boson [6].

As the condition of validity of perturbation theory, we here require that the running coupling constants of the Higgs self-couplings and the Yukawa couplings do not blow up below a certain energy scale Λ: this leads the constraints on the coupling constants;

$$\forall\,\lambda_i(\mu) < 8\pi,\ y_t^2(\mu) < 4\pi\,,\ (\mu < \Lambda). \tag{4}$$

Next, from the condition of the vacuum stability we obtain constraints;

$$\begin{aligned} &\lambda_1(\mu) > 0,\quad \lambda_2(\mu) > 0, \\ &\sqrt{\lambda_1(\mu)\lambda_2(\mu)} + \lambda_3(\mu) + \min\left[0, \lambda_4(\mu)+\lambda_5(\mu), \lambda_4(\mu)-\lambda_5(\mu)\right] > 0\,,\ (\mu < \Lambda). \end{aligned} \tag{5}$$

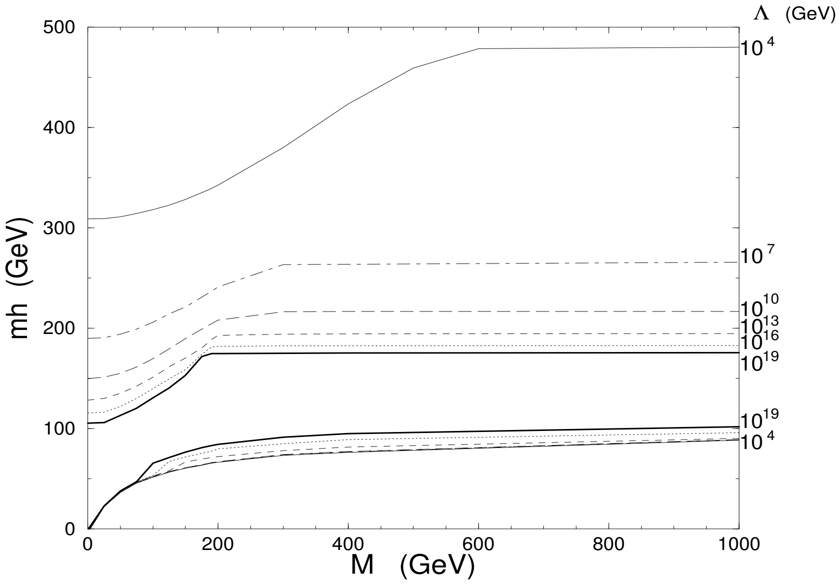

Figure 1: The mass bounds of the lightest CP even Higgs boson mass as a function of M for various Λ in the Model I 2HDM at $m_t = 175$ GeV.

We assume that the tree-level Higgs potential at the weak scale does not have any global minimum except for the one we consider: there is no CP nor charge breaking at the minimum [1]. The conditions (4) and (5) constrain low-energy coupling constants through renormalization group equations (RGE's). Thus the mass bounds of the Higgs boson are obtained.

In the decoupling regime ($M^2 \gg \lambda_i v^2$), the 2HDM effectively becomes the SM with one Higgs doublet below M. In order to include this effect, we use the one-loop SM RGE below M, and the one-loop 2HDM RGE [7] above M. They are connected at M by identifying the lightest CP-even Higgs boson in the 2HDM as the SM one in the mass formulas in both the models. We here use this procedure for the case $M^2 \sim \lambda_i v^2$ too, because the correction from the SM RGE is numerically very small in this case, although this procedure is not really justified there.

The 2HDM receives rather strong experimental constraints from the low energy precision data, especially on the ρ parameter. The extra contribution of the 2HDM to the ρ parameter should satisfy $\Delta\rho_{\rm 2HDM} = -0.0020 - 0.00049\frac{m_t - 175{\rm GeV}}{5{\rm GeV}} \pm 0.0027$ [1]. Another important experimental constraint comes from the $b \to s\gamma$ measurement [8]: there is a strong constraint on the charged-Higgs boson mass from below by this process in Model II, while Model I is not strongly constrained. We examine the general mass bounds of h as a function of Λ varing all the free parameters under these experimental constraints.

By looking at the RGE's the qualitative result may be understood. In decoupling

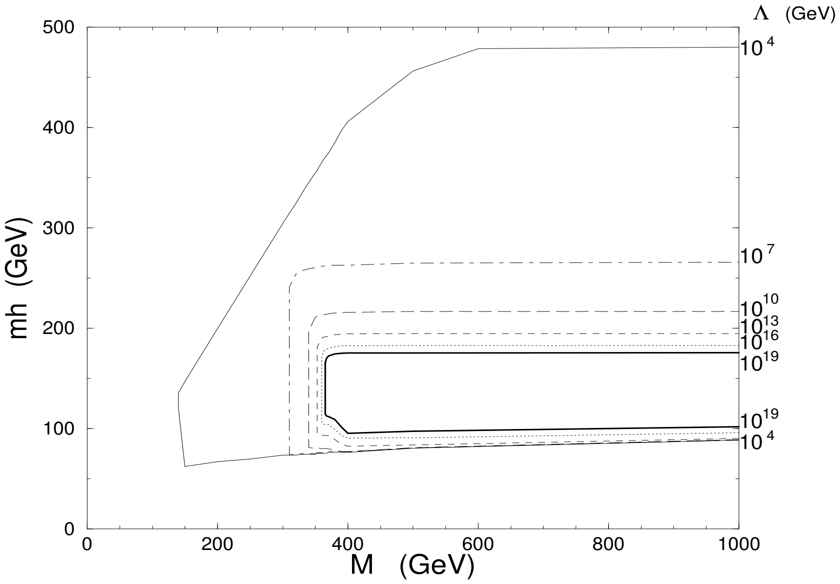

Figure 2: The mass bounds of the lightest CP even Higgs boson mass as a function of M for various Λ in the Model II 2HDM at $m_t = 175$ GeV. Small M region is excluded by the $b \to s\gamma$ results.

regime, from Eq. (2) we have $m_h^2 \sim \lambda_2 v^2$ for $tan\beta \gg 1$. The RGE for λ_2 is given by

$$16\pi^2 \mu \frac{d\lambda_2}{d\mu} = 12\lambda_2^2 - 3\lambda_2(3g^2 + g'^2) + \frac{3}{2}g^4 + \frac{3}{4}(g^2 + g'^2)^2 + 12\lambda_2 y_t^2 - 12y_t^4 + A, \quad (6)$$

where $A = 2\lambda_3^2 + 2(\lambda_3 + \lambda_4)^2 + 2\lambda_5^2 > 0$. The SM RGE for $\lambda_{SM}(\equiv m_H^{SM^2}/v^2)$ takes the same form as Eq. (6) substituting λ_{SM} and y_t^{SM} to λ_2 and y_t and neglecting the A term in the RHS. Hence the only difference from the SM RGE is the existence of the positive A term, which works to keep the stability of vacuum. Thus the lower bound is expected to be reduced in the 2HDM in comparison with the SM results.

In Fig. 1 and 2, the upper and lower bounds of the m_h are shown as a function of M for various cut-off Λ for Model I and II, respectively. In Fig. 1, the allowed region of m_h lies around $m_h \sim M$ for $M^2 \ll \lambda_2 v^2$, where the m_h comes from $M_{22} \sim M$ and the heavier Higgs boson mass m_H has the mass of $M_{11} \sim \sqrt{\lambda_2}v$. At $M = 0$, though there are the upper bounds of m_h for each Λ, m_h is not bounded from below by our condition. Our results at $M = 0$ are consistent with Ref. [5]. On the other hand, in the decoupling regime ($M^2 \gg \lambda_2 v^2$), the situation is reversed; $m_h \sim M_{11} \sim \sqrt{\lambda_2}v$, and the bounds no longer depend on M. If we take account of the experimental result of $b \to s\gamma$, m_h is bounded from below in the Model II as seen in Fig. 2, because the small M region necessarily corresponds to small $m_{\chi^\pm}$ and this is excluded by the $b \to s\gamma$ constraint.

Finally, we combine the results in the SM and the 2HDM (Model I and II) (Fig. 3). We

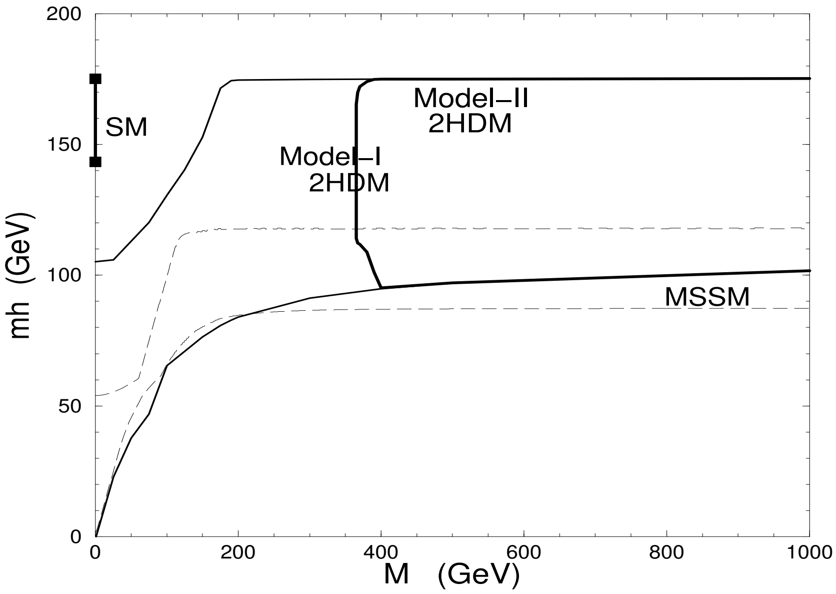

Figure 3: The mass bounds of the lightest CP even Higgs boson in the Model I and II 2HDM as well as of the SM Higgs boson for $\Lambda = 10^{19}$ GeV. As a reference, the bounds of the lightest Higgs boson mass are also shown in the MSSM at the 1 TeV stop mass, in which M corresponds to CP-odd Higgs boson mass.

here choose, as an example, $\Lambda = 10^{19}$ GeV for comparison of the results in the SM and the 2HDM at $m_t = 175$ GeV. For a reference, the bounds of the lightest CP-even Higgs mass in the MSSM are also given for the 1 TeV stop mass [10]. (In the MSSM, M corresponds to the CP-odd Higgs boson mass exactly.) From Fig. 3 it is easy to observe the difference of the bounds among the SM, the 2HDM(I) and the 2HDM(II). While the upper bounds are all around 175 GeV in these models, the lower bounds are completely different as we expect; about 145 GeV in the SM, about 100 GeV in the Model II (with respect to $b \to s\gamma$ constraints[2]) and no lower bound in Model I. Although we have shown figures in which $m_t = 175$ GeV is taken, the top mass dependence cannot be neglected especially for the lower bounds [1]. For example, the lower line for $\Lambda = 10^{19}$ GeV in the 2HDM shown in Fig.3 shifts to lower (upper) by 9 GeV for $m_t = 170$ (180) GeV at $M = 1000$ GeV.

In the SM, the next-to-leading order analysis of the effective potential shows that the lower bound reduces by about 10 GeV ($\Lambda = 10^{19}$ GeV) [3]. It may be then expected that a similar reduction of the lower bound would occur in the 2HDM by such higher order

[2] If we use more conservative way to add theoretical uncertainties for the $b \to s\gamma$ evaluation, the bound on the charged Higgs boson or on the M in Model II becomes rather smaller [9]. The lower bound of m_h due to the $b \to s\gamma$ constraint is then reduced for a few GeV according to the changed allowed region of M.

analysis.

In the decoupling regime, the properties of the lightest Higgs boson such as the production cross section and the decay branching ratios are almost the same as the SM Higgs boson. We have not explicitly considered constraint from the Higgs boson search at LEP II [9], but if the Higgs boson is discovered with the mass around 100 GeV at LEP II or Tevatron experiment in near future and its property is quite similar to the SM Higgs boson, the 2HDM with very high cut-off scale is another candidate of models which predict such light Higgs boson along with the MSSM [10, 11] and its extensions.

In summary, we have discussed the mass bounds of the h as a function of a cutoff Λ by the requirement of validity of perturbation theory and vacuum stability in the non-SUSY 2HDM with the softly-broken discrete symmetry. The upper bounds are almost the same as the SM results, while the lower bounds are significantly reduced even for the decoupling regime. In general case, the mass is no longer bounded from below. If we consider the experimental $b \to s\gamma$ constraint, the very light h is excluded.

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