LC-TH-1999-018
March 2000

The electroweak chiral Lagrangian revisited

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Abstract

Using a manifestly gauge-invariant approach we show that the set of low-energy constants in the electroweak chiral Lagrangian currently used in the literature is redundant. In particular, by employing the equations of motion for the gauge fields, one can choose to remove two low-energy constants which contribute to the self-energies of the gauge bosons. The relation of this result to the experimentally determined values for the oblique parameters S, T and U is discussed. We then evaluate the matching relation between gauge-invariant Green's functions in the full and the effective theory for the case of the Standard Model with a heavy Higgs boson and compare the results for the independent low-energy constants with those for a simple Technicolor model. Since the pattern of the low-energy constants is very different in these two models it may be misleading to mimic any strongly interacting symmetry breaking sector by a heavy Higgs boson. From our investigation we conclude that current electroweak precision data do not really rule out such strongly interacting models.

1 Introduction

In Ref. [1] we introduced a manifestly gauge-invariant approach to the bosonic sector of the Standard Model which deals with gauge-invariant Green's functions and provides a method to evaluate the corresponding generating functional without fixing a gauge. In Ref. [2] we employed this new approach to reanalyze the electroweak chiral Lagrangian [3, 4] in order to investigate two issues related to gauge invariance. We present here the main results of our investigation and refer to Refs. [1, 2] for a discussion of the problems encountered with gauge invariance when studying these issues in the usual approach to gauge theories, for the presentation of our solution to these problems and for all the details of the derivation. A more complete overview of the literature can be found in Refs. [1, 2] as well.

The first issue concerns the determination of the number of independent low-energy constants in the effective Lagrangian by employing the equations of motion to eliminate redundant terms. This is important for any general analysis of the data that will be measured at future colliders like the LHC or a linear collider, if the underlying dynamics of electroweak symmetry breaking is strongly interacting. See Ref. [5] for a recent overview. The second subject is the matching of a full and an effective field theory at low energies in order to evaluate the effective Lagrangian in gauge theories. We compare the results of the independent low-energy constants for the Standard Model with a heavy Higgs boson, obtained by matching gauge-invariant Green's functions [2], with earlier evaluations that used different methods [3, 6] and with those for a simple Technicolor model [7]. We end with some comments on the phenomenological implications of our investigation for strongly interacting models of electroweak symmetry breaking.

2 The electroweak chiral Lagrangian

Assuming a strongly interacting dynamics behind the breaking of the electroweak symmetry, the physics at low energies can be described by an effective Lagrangian, the electroweak chiral Lagrangian [4] built in analogy to the chiral Lagrangian for low-energy QCD [8, 9]. It is given by a series in powers of momenta and masses, $\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$, where $\mathcal{L}_k$ is of order p^k and has the general form $\mathcal{L}_k = \sum_i l_i^{(k)} \mathcal{O}_i^{(k)}$.

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The coefficients $l_i^{(k)}$ represent the low-energy constants of the effective theory and count as order p^0. The operators $\mathcal{O}_i^{(k)}$ involve the light fields in such a way that they respect the $SU(2)_L \times U(1)_Y$ gauge symmetry. We assume that $p^2, M_W^2, M_Z^2 \ll M^2$, where p is a typical momentum and M is the mass scale of the heavy particles in the underlying theory.

We first consider only the bosonic sector, i.e. the low-energy dynamics of the W- and Z-boson and the photon. At lowest order, the electroweak chiral Lagrangian [4] can be written in the following form [2]

$$\begin{aligned}\mathcal{L}_2 &= \frac{v^2}{2}\left(\mathcal{W}_\mu^+\mathcal{W}_\mu^- + \bar\rho\frac{1}{4}\mathcal{Z}_\mu\mathcal{Z}_\mu\right) + \frac{1}{4g^2}\mathcal{W}_{\mu\nu}^a\mathcal{W}_{\mu\nu}^a + \frac{1}{4g'^2}B_{\mu\nu}B_{\mu\nu} \\ &\quad -\frac{1}{2}K_{\mu\nu}B_{\mu\nu} + 2v^2(J_\mu^+\varphi^+\mathcal{W}_\mu^- + J_\mu^-\varphi^-\mathcal{W}_\mu^+) + v^2 J_\mu^{\mathcal{Z}}\mathcal{Z}_\mu + 4c_{\mathcal{W}}v^2 J_\mu^+J_\mu^- + c_{\mathcal{Z}}v^2 J_\mu^{\mathcal{Z}}J_\mu^{\mathcal{Z}} \ , \end{aligned} \tag{1}$$

with

$$\mathcal{W}_\mu^+ = i\widetilde{U}^\dagger D_\mu U \ , \ \mathcal{W}_\mu^- = i U^\dagger D_\mu \widetilde{U} \ , \ \mathcal{Z}_\mu = -2i U^\dagger D_\mu U \ , \tag{2}$$

$$\mathcal{W}_{\mu\nu}^a = \partial_\mu\mathcal{W}_\nu^a - \partial_\nu\mathcal{W}_\mu^a + \varepsilon^{abc}\mathcal{W}_\mu^b\mathcal{W}_\nu^c \ , \ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \ , \tag{3}$$

$$D_\mu U = \left(\partial_\mu - i\frac{\tau^a}{2}W_\mu^a - i\frac{1}{2}B_\mu\right)U \ . \tag{4}$$

The $SU(2)_L$ doublet U, confined to the sphere $U^\dagger U = 1$, describes the degrees of freedom of the Goldstone bosons. The Y-charge conjugate doublet is defined by $\widetilde{U} = i\tau_2 U^*$. Since we will employ a functional approach we have coupled external sources $K_{\mu\nu}$, $J_\mu^\pm$ and $J_\mu^{\mathcal{Z}}$ to gauge-invariant fields in Eq. (1). The phase factor $\varphi^\pm$, which appears in the source terms for the charged fields $\mathcal{W}_\mu^\pm$ in Eq. (1), is defined by

$$\varphi^\pm(x) = \exp\left(\mp i\int d^d y\,\mathcal{G}_0(x-y)\,\partial_\rho B_\rho(y)\right) \quad , \quad \mathcal{G}_0(x-y) = \langle x|\frac{1}{-\Box}|y\rangle \ , \tag{5}$$

and ensures gauge invariance under the full group $SU(2)_L \times U(1)_Y$. We will use Euclidean space notation throughout. At tree-level we have the relations

$$M_W^2 = \frac{v^2e^2}{4s^2} \ , \ M_Z^2 = \bar\rho\frac{v^2e^2}{4s^2c^2} \ , \ c^2 \equiv \cos^2\theta_W = \frac{g^2}{g^2+g'^2} \ , \ s^2 = 1-c^2 \ , \ e^2 = \frac{g^2g'^2}{g^2+g'^2} \ , \tag{6}$$

where $\bar\rho = c^2M_Z^2/M_W^2$ is the inverse of the usual ρ-parameter.

In order to have a well-defined effective field theory a consistent chiral momentum counting is required. Following the counting rules of chiral perturbation theory [8, 9], we treat the Goldstone boson field U and the phase factor $\varphi^\pm$ as quantities of order p^0. The momenta, the covariant derivative D_μ, the gauge fields $\mathcal{W}_\mu^\pm, \mathcal{Z}_\mu, B_\mu$, and the masses M_W and M_Z are counted as order p. The consistency of these rules requires that the coupling constants g, g' and e count as order p. This is similar to the counting of e as order p, if virtual photons are included in chiral perturbation theory [10]. The above rules imply that c and s are of order p^0. Furthermore, we count $\bar\rho - 1$ in Eq. (1) as order p^0 in the low-energy expansion. Since $\bar\rho - 1$ is experimentally known to be very small [11], the custodial symmetry breaking term $(\bar\rho-1)\mathcal{Z}_\mu Z_\mu$ is sometimes [6] only included at order p^4 in the effective Lagrangian. Finally, we count the external source $K_{\mu\nu}$ as order p^0 and $J_\mu^\pm, J_\mu^{\mathcal{Z}}$ as order p.

The most general effective Lagrangian at order p^4 is given by $\mathcal{L}_4 = \mathcal{L}_4^0 + \mathcal{L}_4^s$, where the CP-even terms without external sources can be written in the form $\mathcal{L}_4^0 = \sum_{i=1}^{18} l_i\mathcal{O}_i$. The operators $\mathcal{O}_i$ are given by

$$\begin{aligned}
\mathcal{O}_1 &= (\mathcal{W}_\mu^+\mathcal{W}_\mu^-)(\mathcal{W}_\nu^+\mathcal{W}_\nu^-) \ , & \mathcal{O}_{10} &= i\mathcal{Z}_\nu(d_\mu\mathcal{W}_\mu^+\mathcal{W}_\nu^- - d_\mu\mathcal{W}_\mu^-\mathcal{W}_\nu^+) \ , \\
\mathcal{O}_2 &= (\mathcal{W}_\mu^+\mathcal{W}_\nu^-)(\mathcal{W}_\mu^+\mathcal{W}_\nu^-) \ , & \mathcal{O}_{11} &= \mathcal{Z}_{\mu\nu}\mathcal{Z}_{\mu\nu} \ , \\
\mathcal{O}_3 &= (\mathcal{Z}_\mu\mathcal{Z}_\mu)(\mathcal{W}_\nu^+\mathcal{W}_\nu^-) \ , & \mathcal{O}_{12} &= B_{\mu\nu}\mathcal{Z}_{\mu\nu} \ , \\
\mathcal{O}_4 &= (\mathcal{Z}_\mu\mathcal{Z}_\nu)(\mathcal{W}_\mu^+\mathcal{W}_\nu^-) \ , & \mathcal{O}_{13} &= (d_\mu\mathcal{W}_\mu^+)(d_\nu\mathcal{W}_\nu^-) \ , \\
\mathcal{O}_5 &= (\mathcal{Z}_\mu\mathcal{Z}_\mu)(\mathcal{Z}_\nu\mathcal{Z}_\nu) \ , & \mathcal{O}_{14} &= (\partial_\mu\mathcal{Z}_\mu)(\partial_\nu\mathcal{Z}_\nu) \ , \\
\mathcal{O}_6 &= \epsilon_{\mu\nu\rho\sigma}\mathcal{Z}_\sigma(\mathcal{W}_\rho^-\mathcal{W}_{\mu\nu}^+ + \mathcal{W}_\rho^+\mathcal{W}_{\mu\nu}^-) \ , & \mathcal{O}_{15} &= M_W^2\left(\mathcal{W}_\mu^+\mathcal{W}_\mu^- + \tfrac{1}{4}\mathcal{Z}_\mu\mathcal{Z}_\mu\right) \ , \\
\mathcal{O}_7 &= i\mathcal{Z}_{\mu\nu}(\mathcal{W}_\mu^+\mathcal{W}_\nu^- - \mathcal{W}_\nu^+\mathcal{W}_\mu^-) \ , & \mathcal{O}_{16} &= M_Z^2\mathcal{Z}_\mu\mathcal{Z}_\mu \ , \\
\mathcal{O}_8 &= iB_{\mu\nu}(\mathcal{W}_\mu^+\mathcal{W}_\nu^- - \mathcal{W}_\nu^+\mathcal{W}_\mu^-) \ , & \mathcal{O}_{17} &= \mathcal{W}_{\mu\nu}^a\mathcal{W}_{\mu\nu}^a \ , \\
\mathcal{O}_9 &= i\mathcal{Z}_\mu(d_\mu\mathcal{W}_\nu^+\mathcal{W}_\nu^- - d_\mu\mathcal{W}_\nu^-\mathcal{W}_\nu^+) \ , & \mathcal{O}_{18} &= B_{\mu\nu}B_{\mu\nu} \ ,
\end{aligned} \tag{7}$$

where

$$d_\mu \mathcal{W}_\nu^\pm = (\partial_\mu \mp iB_\mu)\mathcal{W}_\nu^\pm \ , \ \mathcal{W}_{\mu\nu}^\pm = d_\mu \mathcal{W}_\nu^\pm - d_\nu \mathcal{W}_\mu^\pm \ , \ \mathcal{Z}_{\mu\nu} = \partial_\mu \mathcal{Z}_\nu - \partial_\nu \mathcal{Z}_\mu \ . \tag{8}$$

One can easily identify those operators in Eq. (7) that contribute to the self-energies of the gauge bosons and to anomalous triple and quartic gauge couplings, respectively. We have employed partial integrations to remove other allowed terms, like $d_\mu \mathcal{W}_\nu^+ d_\nu \mathcal{W}_\mu^-$. As shown in Ref. [2] the operators $\mathcal{O}_1, \ldots, \mathcal{O}_{14}$ can be expressed through the operators $L_1, \ldots, L_{14}$ in the usually employed basis [4]. The operator $\mathcal{O}_{16}$ corresponds to the operator L_0. Due to the factor M_Z^2 this terms counts as order p^4. The CP-even source terms can be written in the form $\mathcal{L}_4^s = \sum_{i=1}^{76} l_i^s \mathcal{O}_i^s$. The list of operators $\mathcal{O}_i^s$ can be found in Ref. [2].

3 Number of independent parameters in $\mathcal{L}_{eff}$

The generating functional for the Green's functions of the gauge-invariant fields $\varphi^\mp \mathcal{W}_\mu^\pm, \mathcal{Z}_\mu$ and $B_{\mu\nu}$ is given by the path integral

$$e^{-W_{eff}[K_{\mu\nu}, J_\mu^\pm, J_\mu^z]} = \int \mathrm{d}\mu[U, W_\mu^a, B_\mu] \ e^{-\int d^dx \, \mathcal{L}_{eff}} \ . \tag{9}$$

As discussed in Ref. [2] it is possible to evaluate the path integral in Eq. (9) without fixing a gauge because we consider only Green's functions of gauge-invariant fields. In general, there are two different kinds of contributions to the generating functional in Eq. (9). On the one hand, one has tree-level contributions given by the integral $\int d^dx \mathcal{L}_{eff}$, which has to be evaluated at the stationary point, i.e., with the solutions of the equations of motion. On the other hand there are contributions from loops, which ensure unitarity. General power counting arguments show that n-loop corrections are suppressed by $2n$ powers of the momentum [8, 9]. For instance, at order p^2 there are only tree-level contributions from $\mathcal{L}_2$, whereas at order p^4 one has to consider tree-graphs from $\mathcal{L}_4$ and one-loop graphs with vertices from $\mathcal{L}_2$.

At order p^2, the generating functional of the effective field theory is given by the classical action

$$W_2[K_{\mu\nu}, J_\mu^\pm, J_\mu^z] = \int d^dx \ \mathcal{L}_2\left(\mathcal{W}_\mu^\pm, \mathcal{Z}_\mu, B_\mu; K_{\mu\nu}, J_\mu^\pm, J_\mu^z\right) \ , \tag{10}$$

where the gauge fields satisfy the equations of motion

$$\begin{aligned} -d_\mu \mathcal{W}_{\mu\nu}^\pm &= -M_W^2(\mathcal{W}_\nu^\pm + 4j_\nu^\pm) \pm i(\mathcal{Z}_{\mu\nu} + B_{\mu\nu})\mathcal{W}_\mu^\pm \mp i\mathcal{W}_{\mu\nu}^\pm \mathcal{Z}_\mu \mp i(\partial_\mu \mathcal{Z}_\mu)\mathcal{W}_\nu^\pm \pm i(\partial_\mu \mathcal{Z}_\nu)\mathcal{W}_\mu^\pm \\ &\quad \pm i\mathcal{Z}_\nu d_\mu \mathcal{W}_\mu^\pm \mp i\mathcal{Z}_\mu d_\mu \mathcal{W}_\nu^\pm - (\mathcal{Z}_\mu \mathcal{Z}_\mu)\mathcal{W}_\nu^\pm + (\mathcal{Z}_\mu \mathcal{Z}_\nu)\mathcal{W}_\mu^\pm \pm 2\mathcal{W}_\mu^\pm(\mathcal{W}_\mu^+ \mathcal{W}_\nu^- - \mathcal{W}_\nu^+ \mathcal{W}_\mu^-) \end{aligned} \tag{11}$$

$$\begin{aligned} -\partial_\mu(\mathcal{Z}_{\mu\nu} + B_{\mu\nu}) &= -c^2 M_Z^2(\mathcal{Z}_\nu + \frac{4}{\bar\rho}J_\nu^z) - 4\mathcal{Z}_\nu \mathcal{W}_\mu^+ \mathcal{W}_\mu^- + 2\mathcal{Z}_\mu(\mathcal{W}_\mu^+ \mathcal{W}_\nu^- + \mathcal{W}_\nu^+ \mathcal{W}_\mu^-) \\ &\quad +2i(\mathcal{W}_{\mu\nu}^+ \mathcal{W}_\mu^- - \mathcal{W}_{\mu\nu}^- \mathcal{W}_\mu^+) - 2i(d_\mu \mathcal{W}_\mu^+ \mathcal{W}_\nu^- - d_\mu \mathcal{W}_\mu^- \mathcal{W}_\nu^+ - d_\mu \mathcal{W}_\nu^+ \mathcal{W}_\mu^- + d_\mu \mathcal{W}_\nu^- \mathcal{W}_\mu^+) \end{aligned} \tag{12}$$

$$-\partial_\mu B_{\mu\nu} = s^2 M_Z^2 \mathrm{PT}_{\nu\mu}(\mathcal{Z}_\mu + \frac{4}{\bar\rho}J_\mu^z) - \frac{e^2}{c^2}\partial_\mu K_{\mu\nu} \ . \tag{13}$$

The equations of motion (constraints) for the Goldstone bosons are given by

$$d_\mu \mathcal{W}_\mu^\pm = -4d_\mu j_\mu^\pm \pm i\mathcal{Z}_\mu(\mathcal{W}_\mu^\pm + 4j_\mu^\pm) \mp i\bar\rho(\mathcal{Z}_\mu + \frac{4}{\bar\rho}J_\mu^z)\mathcal{W}_\mu^\pm \ , \tag{14}$$

$$\partial_\mu \mathcal{Z}_\mu = -\frac{4}{\bar\rho}\partial_\mu J_\mu^z + \frac{8i}{\bar\rho}(\mathcal{W}_\mu^+ j_\mu^- - \mathcal{W}_\mu^- j_\mu^+) \ , \tag{15}$$

where $j_\mu^\pm = \varphi^\pm J_\mu^\pm$ and $\mathrm{PT}_{\mu\nu} = \delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}$. As discussed in Refs. [1, 2] it is possible to solve the equations of motion (11) – (15) for the physical degrees of freedom without the need to fix a gauge.

The Lagrangian $\mathcal{L}_4$ only contributes at tree-level if we go up to order p^4 in the low-energy expansion of the generating functional in Eq. (9). Therefore we can use the equations of motion for the gauge fields as well as the constraints for the Goldstone bosons to eliminate redundant terms in the list of operators in Eq. (7). Note that in our gauge-invariant approach no gauge artefacts can enter through this procedure. Using the constraints (14) and (15) one can remove the following three operators

$$\begin{aligned}
\mathcal{O}_{10} &= -2(1-\bar{\rho})\mathcal{O}_4 + 4\mathcal{O}_4^s - 4\mathcal{O}_6^s - 4\mathcal{O}_{46}^s \ , &(16)\\
\mathcal{O}_{13} &= (1-\bar{\rho})^2\mathcal{O}_4 - 4(1-\bar{\rho})\mathcal{O}_4^s + 4(1-\bar{\rho})\mathcal{O}_6^s + 16\mathcal{O}_{14}^s - 16\mathcal{O}_{17}^s \\
&\quad + 16\mathcal{O}_{19}^s + 4(1-\bar{\rho})\mathcal{O}_{46}^s - 16\mathcal{O}_{51}^s + 16\mathcal{O}_{53}^s + 16\mathcal{O}_{74}^s \ , &(17)\\
\mathcal{O}_{14} &= (64/\bar{\rho}^2)(2\mathcal{O}_{10}^s - \mathcal{O}_{12}^s) + (64/\bar{\rho}^2)(\mathcal{O}_{49}^s - \mathcal{O}_{52}^s) + (16/\bar{\rho}^2)\mathcal{O}_{76}^s \ . &(18)
\end{aligned}$$

This corresponds to removing the operators L_{11}, L_{12}, L_{13} in the usual basis, as was done in Ref. [4]. Using the equations of motions for the gauge fields, Eqs. (11) – (13), one can remove the two operators

$$\begin{aligned}
\mathcal{O}_{11} &= -8\mathcal{O}_1 + 8\mathcal{O}_2 - 16\mathcal{O}_3 + 16\bar{\rho}\mathcal{O}_4 + 8\mathcal{O}_7 - 8\mathcal{O}_9 - 8\mathcal{O}_{15} - 2c^2(2-1/\bar{\rho})\mathcal{O}_{16} - \mathcal{O}_{17} + \mathcal{O}_{18} \\
&\quad +32\mathcal{O}_4^s - 32\mathcal{O}_6^s - 32\mathcal{O}_{46}^s - 16\mathcal{O}_{64}^s - 16(c^2/\bar{\rho})\mathcal{O}_{66}^s \ , &(19)\\
\mathcal{O}_{12} &= 8\mathcal{O}_1 - 8\mathcal{O}_2 + 8\mathcal{O}_3 - 8\bar{\rho}\mathcal{O}_4 - 4\mathcal{O}_7 + 4\mathcal{O}_9 + 8\mathcal{O}_{15} + 2c^2(1-1/\bar{\rho})\mathcal{O}_{16} + \mathcal{O}_{17} - \mathcal{O}_{18} \\
&\quad -16\mathcal{O}_4^s + 16\mathcal{O}_6^s + 16\mathcal{O}_{46}^s + 16\mathcal{O}_{64}^s + 8(c^2/\bar{\rho})\mathcal{O}_{66}^s \ . &(20)
\end{aligned}$$

This step, which corresponds to removing L_1 and L_8 in the usual basis, has not been taken before in the literature. The operators $\mathcal{O}_{11}$ and $\mathcal{O}_{12}$ contribute to the self-energies of the gauge bosons which are not directly observable. The relations between the operators L_i in the usual basis which follow from Eqs. (16) – (20) can be found in Ref. [2]. A similar procedure can be applied to eliminate four of the source terms [2].

Furthermore, the operators $\mathcal{O}_{15}, \ldots, \mathcal{O}_{18}$ in Eq. (7), as well as several source terms in $\mathcal{L}_4^s$, are proportional to corresponding terms in $\mathcal{L}_2$. One can remove these terms from the basis by a renormalization of the parameters, low-energy constants and sources in $\mathcal{L}_2$. For instance we get

$$\frac{v^2}{2}\left(\mathcal{W}_\mu^+\mathcal{W}_\mu^- + \frac{1}{4}\mathcal{Z}_\mu\mathcal{Z}_\mu\right) + l_{15}M_W^2\left(\mathcal{W}_\mu^+\mathcal{W}_\mu^- + \frac{1}{4}\mathcal{Z}_\mu\mathcal{Z}_\mu\right) \quad\to\quad \frac{v_{eff}^2}{2}\left(\mathcal{W}_\mu^+\mathcal{W}_\mu^- + \frac{1}{4}\mathcal{Z}_\mu\mathcal{Z}_\mu\right) \ . \qquad (21)$$

The complete list of redefinitions can be found in Ref. [2].

At the end we find [2] that there are 9 independent low-energy constants $l_1', \ldots, l_9'$ in $\mathcal{L}_4^0$. Furthermore, there are 63 independent low-energy constants $l_i^{s\prime}$ which appear in the source terms $\mathcal{L}_4^s$. Since the use of the relations (16) – (20) modifies the low-energy constants of the remaining terms, we have denoted the independent constants by l_i' and $l_i^{s\prime}$. We can cover the conventions used in Ref. [6], where a custodial symmetry breaking term proportional to $Z_\mu Z_\mu$ is included only in $\mathcal{L}_4$, by setting $\bar{\rho} = 1$ and including the operator $\mathcal{O}_0 \doteq M_Z^2\mathcal{Z}_\mu Z_\mu \equiv \mathcal{O}_{16}$ into the basis with the low-energy constant l_0'. In a purely bosonic effective field theory the source terms will not contribute to physical quantities (on-shell gauge bosons) [2]. Therefore, apart from the universal parameters v^2, g and g' in $\mathcal{L}_2$, there are only 10 physically relevant low-energy constants up to order p^4, namely $l_1', \ldots, l_9'$ and $\bar{\rho} - 1$, or l_0', depending on the momentum counting, to distinguish different underlying theories.

4 Inclusion of fermions and the oblique parameters S, T, U

In Ref. [4] the equations of motion for the gauge fields, Eqs. (11) – (13), have not been employed to find relations between operators in $\mathcal{L}_4^0$. Therefore the set of 12 low-energy constants in $\mathcal{L}_4$ found in these references is redundant. Before any conclusions can be drawn, however, one has to study the inclusion of fermions into the effective field theory [2]. At order p^2 the fermionic part of the effective Lagrangian contains several terms

$$\mathcal{L}_2^f = \mathcal{L}_2^{f,kin} + \mathcal{L}_2^{f,Yuk} + \mathcal{L}_2^{f,CC} + \mathcal{L}_2^{f,NC} + \mathcal{L}_2^{f,4F} + \mathcal{L}_2^{f,s} \ . \qquad (22)$$

They denote the kinetic terms, the Yukawa couplings, the coupling to charged and neutral currents, four-fermion interactions and source terms, respectively. In the following we will concentrate on the charged current interactions, a more complete discussion can be found in Ref. [2]. These interactions can be expressed as follows

$$\mathcal{L}_2^{f,CC} = \sum_{ij} c_{CC}^{ij,L} \left(\mathcal{W}_\mu^+ j_\mu^{L,ij-} + \mathcal{W}_\mu^- j_\mu^{L,ij+} \right) + c_{CC}^{ij,R} \left(\mathcal{W}_\mu^+ j_\mu^{R,ij-} + \mathcal{W}_\mu^- j_\mu^{R,ij+} \right) , \tag{23}$$

$$j_\mu^{L/R,ij+} = \bar{d}_{L/R}^i \gamma_\mu u_{L/R}^j \quad , \quad j_\mu^{L/R,ij-} = \bar{u}_{L/R}^i \gamma_\mu d_{L/R}^j , \tag{24}$$

where the fermionic currents are written in terms of $SU(2)_L$ gauge-invariant fields

$$u_L^k = \tilde{U}^\dagger \Psi_L^k \quad , \quad d_L^k = U^\dagger \Psi_L^k . \tag{25}$$

Here, Ψ^k denotes the usual fermion doublet for flavor number k. The fields u_R^k and d_R^k in Eq. (24) are the usual $SU(2)_L$ singlets. As can be seen from Eq. (23), the effective Lagrangian contains a host of a priori undetermined low-energy constants c^{ij} and more appear in $\mathcal{L}_2^f$ in Eq. (22) and at higher orders in $\mathcal{L}_3^f, \mathcal{L}_4^f, \ldots$. Of course, there exist strong bounds on couplings which deviate from the corresponding values in the Standard Model. In a general effective Lagrangian analysis one has to include, however, such terms and, in fact, they are expected to be small if they are of higher order in the low-energy expansion.

Using the equations of motion for the Goldstone bosons (constraints) one can still remove the terms $\mathcal{O}_{10}, \mathcal{O}_{13}$ and $\mathcal{O}_{14}$. For instance, in the relation (17) we get a contribution $(d_\mu j_\mu^+)(d_\nu j_\nu^-)$ from the source terms which now contain the fermionic currents. Employing the equations of motion for the fermions one observes that $(d_\mu j_\mu^\pm)$ is proportional to the fermion masses which are very small for light external fermions, cf. Ref. [4]. On the other hand, if we employ the equations of motion for the gauge fields we obtain, cf. Eq. (19),

$$\mathcal{O}_{11} = \ldots - 16 M_W^2 (\mathcal{W}_\mu^+ j_\mu^- + \mathcal{W}_\mu^- j_\mu^+) . \tag{26}$$

The last term is proportional to a term in $\mathcal{L}_2^{f,CC}$ and therefore leads only to a non-observable renormalization of the low-energy couplings c^{ij} in Eq. (23), similarly to the redefinition in Eq. (21). Therefore, the operator $\mathcal{O}_{11}$ can again be removed in the framework of a general effective field theory approach. The same statement applies to $\mathcal{O}_{12}$.

In contrast, in Ref. [4] the assumption was made that all couplings of the fermions to the gauge bosons are identical to their values in the Standard Model. In this case the use of the equations of motion for the gauge fields does not reduce the number of parameters, but simply shifts some low-energy constants from the bosonic to the fermionic sector. The assumptions made in Ref. [4] might, however, be too stringent.

We have *chosen* to remove the operators $\mathcal{O}_{11}$ and $\mathcal{O}_{12}$ from the basis, which correspond to the operators L_1 and L_8 in the usual basis. Since these operators contribute to the self-energies of the gauge bosons, the low-energy constants a_1 and a_8 are sometimes identified with the oblique correction parameters S and U [13][1]. As discussed in Ref. [2], we believe that such an identification is not really possible. The parameters S, T and U, as used nowadays by the Particle Data Group [11], describe the effects of heavy new physics *beyond* the Standard Model on the self-energies of the gauge bosons. In particular, they depend on the Higgs boson mass M_H, whereas the effective Lagrangian approach also works if there exists no Higgs boson at all. In order to make contact between the two descriptions one might do the following.

The low-energy constants at order p^k have the general form $l_i^{(k)} = \delta_i^{(k)} \Lambda_\varepsilon + l_i^{(k),r}(\mu)$. The coefficients $\delta_i^{(k)}$ of the pole term, $\Lambda_\varepsilon \doteq \frac{\mu^{d-4}}{16\pi^2}\left(\frac{1}{d-4} - \frac{1}{2}(\ln 4\pi + \Gamma'(1) + 1)\right)$, are universal, i.e. independent of the underlying theory. Following the conventions of chiral perturbation theory [8, 9], we denote the finite, scale dependent parts, $l_i^{(k),r}(\mu)$, as renormalized low-energy constants. One could now try to mimic any strongly interacting symmetry breaking sector by studying the large Higgs mass limit. One cannot completely remove the Higgs particle from the theory in this way, however, due to non-decoupling effects for $M_H \to \infty$. We now assume that the renormalized low-energy constants can be decomposed as follows

$$l_i^{(k),r}(\mu) = l_i^{(k),SM}(\mu) + l_i^{(k),new}(\mu) , \tag{27}$$

[1] The parameter T is often identified with the low-energy constant a_0 which corresponds to $\bar{\rho} - 1$ or l_0', depending on the momentum counting.

where the first term describes the contributions for the Standard Model with a heavy Higgs boson (see the next Section). The definition of S, T and U now amounts to setting $l_i^{(k),new}(\mu) = 0$ for all i, k except for $k = 4$ and $i = 0, 11, 12$. This introduces three finite parameters, independent of each other, to describe new physics effects. Again, the use of the equations of motion for the gauge fields does not reduce the number of parameters in this case, but simply shifts some low-energy constants from the bosonic to the fermionic sector. The assumptions made in Ref. [13], i.e. that new physics effects predominantly show up in oblique corrections and that a decomposition according to Eq. (27) is possible in the absence of a Higgs boson, might, however, again be too restrictive.

5 Low-energy constants for a heavy Higgs boson and for a simple Technicolor model

The low-energy constants in the effective Lagrangian are determined by the underlying theory of electroweak symmetry breaking and can, in principle, be calculated by matching the full and the effective theory at low energies. In the case of a heavy Higgs boson, which we consider here as one illustrative example of a strongly interacting symmetry breaking sector, the matching condition can be evaluated using perturbative methods. One has to make sure, however, that no gauge artefacts enter in this procedure, if one matches, for instance, gauge-dependent Green's functions of gauge bosons [3, 6]. In Ref. [2] we have redone the calculation within our manifestly gauge-invariant approach by matching the generating functionals of our gauge-invariant Green's functions, i.e. by requiring $W_{eff} = W_{SM}$. The matching condition was evaluated up to one-loop order in the Standard Model (bosonic sector) and up to order p^4 in the low-energy expansion. At the level of the bare effective Lagrangian we reproduce the results found in Ref. [6]. The complete result for the effective Lagrangian $\mathcal{L}_2$ after renormalization can be found in Ref. [2]. The independent low-energy constants of the non-source terms in the bosonic sector at order p^4 are given by

$$\begin{aligned}
l_0' &= s_p^2\left(\frac{3}{4}\Lambda_\varepsilon + \frac{3}{8}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,p}^2}{\mu^2}\right) + \frac{1}{16}\frac{1}{16\pi^2}\right) , \\
l_1' &= -\frac{1}{3}\Lambda_\varepsilon - \frac{2s_p^2 M_{W,p}^2}{e_{\rm res}^2 M_{H,p}^2} - \frac{1}{6}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,p}^2}{\mu^2}\right) + \frac{176-27\sqrt{3}\pi}{36\cdot 16\pi^2} , \\
l_2' &= -\frac{2}{3}\Lambda_\varepsilon - \frac{1}{3}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,p}^2}{\mu^2}\right) - \frac{1}{18}\frac{1}{16\pi^2} , \\
l_3' &= \frac{1}{6}\Lambda_\varepsilon - \frac{s_p^2 M_{W,p}^2}{e_{\rm res}^2 M_{H,p}^2} + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,p}^2}{\mu^2}\right) + \frac{178-27\sqrt{3}\pi}{72\cdot 16\pi^2} , \\
l_4' &= -\frac{2}{3}\Lambda_\varepsilon - \frac{1}{3}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,p}^2}{\mu^2}\right) - \frac{1}{18}\frac{1}{16\pi^2} , \\
l_5' &= -\frac{1}{16}\Lambda_\varepsilon - \frac{s_p^2 M_{W,p}^2}{8e_{\rm res}^2 M_{H,p}^2} - \frac{1}{32}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,p}^2}{\mu^2}\right) + \frac{58-9\sqrt{3}\pi}{192\cdot 16\pi^2} , \\
l_6' &= 0 , \\
l_7' &= -\frac{1}{6}\Lambda_\varepsilon - \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,p}^2}{\mu^2}\right) - \frac{13}{72}\frac{1}{16\pi^2} , \\
l_8' &= -\frac{1}{6}\frac{1}{16\pi^2} , \\
l_9' &= \frac{1}{6}\Lambda_\varepsilon + \frac{1}{12}\frac{1}{16\pi^2}\ln\left(\frac{M_{H,p}^2}{\mu^2}\right) + \frac{13}{72}\frac{1}{16\pi^2} .
\end{aligned} \tag{28}$$

In the Standard Model the custodial symmetry breaking effects in $\Delta\rho$ are proportional to g'^2. Therefore we have $\bar{\rho} = 1$ in $\mathcal{L}_2$ and there are 10 low-energy constants in $\mathcal{L}_4$. As physical input parameters we have chosen the masses of the Higgs, and the W- and Z-boson and the electric charge. The masses are defined by the pole position of gauge-invariant two-point functions [1] and are denoted by $M_{H,p}$, $M_{W,p}$ and $M_{Z,p}$, respectively. The electric charge, denoted by e^2_{res}, is defined as the residue at the photon-pole of the two-point function $\langle 0|TB_{\mu\nu}B_{\rho\sigma}|0\rangle$. The value agrees [1] with the usual result for the electric charge in the Thompson limit. Furthermore, we have introduced in Eq. (28) the notations $c_p^2 \doteq M^2_{W,p}/M^2_{Z,p}$, $s_p^2 = 1 - c_p^2$. The results for the low-energy constants are identical to those obtained in the ungauged $O(4)$-linear sigma model, in all cases where such a comparison is possible. This can easily be understood from our counting rules where $g^2, g'^2 = \mathcal{O}(p^2)$. Gauge boson loops lead either to corrections of order p^6 or to terms in $\mathcal{L}_4$ which are proportional to corresponding terms in $\mathcal{L}_2$ that are therefore not observable.

For illustration, we have listed in Table 1 the values of the renormalized low-energy constants $l_i'^r(\mu)$, $i = 0, 2, 3, 7$, for different scales μ and for several values of the Higgs boson mass $M_{H,p}$. The low-energy constant $l_0'^r$ is related to $-\Delta\rho$. The low-energy constants $l_2'^r$ and $l_3'^r$ represent anomalous quartic couplings of the gauge bosons, whereas $l_7'^r$ contributes to anomalous triple gauge couplings. Note that $l_4'^r = l_2'^r$ and $l_9'^r = -l_7'^r$. Furthermore, $l_6'^r$ vanishes and $l_8'^r$ is independent of $M_{H,p}$ and μ. The low-energy constants $l_1'^r, l_3'^r$ and $l_5'^r$ receive tree-level contributions which dominate numerically for Higgs masses below 1 TeV. Thus we have in this region $l_1'^r \approx 2l_3'^r \approx 16 l_5'^r$. For Higgs masses above 1 TeV the application of perturbation theory is certainly questionable. Therefore, the values of the low-energy constants quoted for $M_{H,p} = 2\ TeV$ are only given as illustration. On the other hand, if the Higgs mass is too small, e.g. below 250 GeV, the low-energy expansion will break down.

Table 1: Values of some of the renormalized low-energy constants $l_i'^r(\mu)$ for the Standard Model at different scales and for several values of the mass of the heavy Higgs boson.

	$10^3 \times l_i'^r(\mu = M_Z)$				$10^3 \times l_i'^r(\mu = 1\ TeV)$				$10^3 \times l_i'^r(\mu = 2\ TeV)$			
$M_{H,p}[GeV]$	250	500	1000	2000	250	500	1000	2000	250	500	1000	2000
$l_0'^r$	1.15	1.89	2.62	3.35	-1.38	-0.64	0.088	0.82	-2.11	-1.38	-0.64	0.088
$l_2'^r$	-4.61	-7.54	-10.5	-13.4	5.50	2.57	-0.35	-3.28	8.43	5.50	2.57	-0.35
$l_3'^r$	-247	-58.2	-10.4	2.07	-250	-60.7	-12.9	-0.46	-250	-61.5	-13.7	-1.19
$l_7'^r$	-2.21	-2.94	-3.67	-4.40	0.32	-0.41	-1.14	-1.87	1.05	0.32	-0.41	-1.14

For comparison, in a two-flavor QCD-like Technicolor model [7] one obtains the following estimates for the independent low-energy constants[2]

$$\begin{aligned}
l_1'^r(\mu = M_{\rho_{TC}}) &= \frac{3}{4}\frac{v^2}{M^2_{\rho_{TC}}}\,, \quad l_2'^r(\mu = M_{\rho_{TC}}) = -\frac{3}{4}\frac{v^2}{M^2_{\rho_{TC}}}\,, \quad l_3'^r(\mu = M_{\rho_{TC}}) = \frac{3}{4}\frac{v^2}{M^2_{\rho_{TC}}}\,, \\
l_4'^r(\mu = M_{\rho_{TC}}) &= -\frac{3}{4}\frac{v^2}{M^2_{\rho_{TC}}}\,, \quad l_7'^r(\mu = M_{\rho_{TC}}) = -\frac{1}{4}\frac{v^2}{M^2_{\rho_{TC}}}\,, \quad l_8'^r(\mu = M_{\rho_{TC}}) = \frac{1}{8}\frac{v^2}{M^2_{\rho_{TC}}}\,, \\
l_9'^r(\mu = M_{\rho_{TC}}) &= \frac{1}{4}\frac{v^2}{M^2_{\rho_{TC}}}\,, \quad l_i'^r(\mu = M_{\rho_{TC}}) = 0, \quad i = 0, 5, 6,
\end{aligned} \tag{29}$$

where $M_{\rho_{TC}}$ denotes the mass of the Technirho meson. We note that these expressions have not been obtained through a matching calculation. Instead, they are taken over from the corresponding estimates for the low-energy constants in the ordinary chiral Lagrangian, obtained by assuming that the exchange of the lowest lying resonances dominates the numerical values of the renormalized low-energy constants in the resonance region, i.e. at the scale of the Technirho mass. This assumption works reasonably well

[2] In the usually employed basis one obtains the following results for the non-vanishing renormalized low-energy constants at the scale $\mu = M_{\rho_{TC}}$: $-(2/3)a_1^r = -a_2^r = a_3^r = 2a_4^r = -2a_5^r = v^2/(4M^2_{\rho_{TC}})$. After eliminating the operators L_1 and L_8 one gets: $-a_2'^r = -2a_3'^r = 2a_4'^r = -2a_5'^r = v^2/(4M^2_{\rho_{TC}})$ at the scale $\mu = M_{\rho_{TC}}$.

for the coefficients in the chiral Lagrangian for QCD [9, 14] and can be justified using large-N_c arguments and constraints from sum-rules [15]. By taking over these estimates from QCD we assume that the electroweak interactions (gauge-boson loops) will not drastically change the corresponding values of the low-energy constants in the electroweak chiral Lagrangian. In contrast to the matching calculation for the Standard Model with a heavy Higgs boson this cannot be shown explicitly in the Technicolor model, because of the strongly interacting dynamics. In any case, we will use the values given in Eq. (29) only for illustration, since QCD-like Technicolor models are phenomenologically ruled out due to large flavor-changing neutral-currents.

In Table 2 we have collected the values of all renormalized low-energy constants $l_i'^r(\mu)$ for the Standard Model with a heavy Higgs boson, for $M_{H,p} = 250\ GeV$ and $1000\ GeV$, and for the Technicolor model. We have chosen two representative scales, the mass of the Z-boson and the mass of the Technirho meson $M_{\rho_{TC}}$. Simply scaling up from QCD we used $M_{\rho_{TC}} = 2\ TeV$ and $v = 246\ GeV$.

Table 2: Values of the renormalized low-energy constants $l_i'^r(\mu)$ for the Standard Model and the two-flavor Technicolor (TC) model at the scale of the Z-boson mass and at the mass-scale of the Technirho. For the Standard Model case two values for the mass of the heavy Higgs boson have been chosen.

	$10^3 \times l_i'^r(\mu = M_Z)$			$10^3 \times l_i'^r(\mu = M_{\rho_{TC}})$		
	$M_{H,p} = 250\ GeV$	$M_{H,p} = 1000\ GeV$	TC	$M_{H,p} = 250\ GeV$	$M_{H,p} = 1000\ GeV$	TC
l_0'	1.15	2.62	3.26	-2.11	-0.64	0.0
l_1'	-498.8	-31.3	4.83	-492.3	-24.8	11.3
l_2'	-4.61	-10.5	-24.4	8.43	2.57	-11.3
l_3'	-247.1	-10.4	14.6	-250.4	-13.7	11.3
l_4'	-4.61	-10.5	-24.4	8.43	2.57	-11.3
l_5'	-31.5	-2.61	-1.22	-30.2	-1.39	0.0
l_6'	0.0	0.0	0.0	0.0	0.0	0.0
l_7'	-2.21	-3.67	-7.04	1.05	-0.41	-3.78
l_8'	-1.06	-1.06	1.89	-1.06	-1.06	1.89
l_9'	2.21	3.67	7.04	-1.05	0.41	3.78

One has to evolve the estimates for the low-energy constants for the Technicolor model in Eq. (29) from the resonance region down to the scale of the Z-boson in order to compare with LEP I / SLC physics. In this way the constant $l_0'^r(\mu = M_Z)$ turns out to be nonzero, even though the Technicolor model preserves the custodial symmetry. We observe that at both scales the pattern of the low-energy constants for the Standard Model with a heavy Higgs boson and for the Technicolor model is very different. Therefore, it may be misleading to mimic any strongly interacting symmetry breaking sector by a heavy Higgs boson, as is done, for instance, when fitting values for the oblique parameters S, T and U in order to get bounds on Technicolor models [11, 12].

We cannot directly compare the values for the low-energy constants quoted in Table 2 with experimental data, since we consider here only the bosonic part of the effective Lagrangian. Nevertheless, one might be tempted to make the identification $\widehat{\alpha}(M_Z)T = -\widehat{l}_0'^r(\mu = M_Z)$, cf. Eq. (6). Quantities with a hat are defined in the $\overline{MS}$-scheme. A recent fit of the parameters S, T and U yields $T = 0.00 \pm 0.15$ [12]. Using the relation $l_0'^r(\mu) = \widehat{l}_0'^r(\mu) + (3s_p^2)/(8 \cdot 16\pi^2)$, this would translate to the bounds $-1.8 < 10^3 \times l_0'^r(\mu = M_Z) < 2.9$ at the 2σ-level. In Ref. [12] and in the approach adopted by the Particle Data Group [11] the parameter T is equivalent to the parameter ρ_0, using $\widehat{\alpha}(M_Z)T \equiv 1 - \rho_0^{-1}$, where $\rho_0 \equiv M_W^2/(M_Z^2 \widehat{c}_Z^2 \widehat{\rho}(m_t, M_H))$. Fitting the electroweak data with ρ_0 as a free parameter leads to $\rho_0 = 0.9996^{+0.0031}_{-0.0013}$ at the 2σ-level [12]. From this we would obtain the bounds $-2.2 < 10^3 \times l_0'^r(\mu = M_Z) < 2.2$. Note, however, that these fits for ρ_0 or T are usually performed by including radiative corrections within the context of the Standard Model [11, 12]. These quantities depend therefore in a particular way on the masses of the top quark and of the Higgs boson. These dependencies may change drastically, however, in other models where

$\rho \neq 1$ at tree level, if the full radiative corrections of that model are taken into account, see Ref. [16]. Furthermore, the quantities ρ_0 or T are defined in Refs. [11, 12] to parametrize only new physics effects *beyond* the Standard Model. Therefore, a reference value for the Higgs boson mass has to be chosen for the fit of S, T and U, e.g. for the value for T quoted above, $M_H = 600\ GeV$ has been used to mimic Technicolor models. In contrast, the low-energy constant $l_0^{\prime r}(\mu = M_Z) \equiv -s_p^2 a_0^{\prime r}(\mu = M_Z)$ corresponds to the parameter $-\Delta\rho$, see Ref. [18] for the exact relation. Within the Standard Model with a heavy Higgs boson we obtain from $l_0^{\prime r}$ in Eq. (28) the known Higgs contribution to $\Delta\rho$. In other models, however, the low-energy constant $l_0^{\prime r}$ may receive contributions that are quite different from the Standard Model or from the Technicolor model. Therefore we conclude that there exist no strong, model-independent bounds on the low-energy constant $l_0^{\prime r}$ and thus on models for a strongly interacting symmetry breaking sector using current electroweak precision data. A similar conclusion has been drawn recently in Ref. [17] where it was argued that the electroweak precision data cannot rule out a strongly interacting symmetry breaking sector with a scale of new physics as high as $3\ TeV$.

The strongest bounds for anomalous triple gauge couplings from LEP II [19] translate at the 2σ-level to $10^3 \times |l_i^{\prime r}| \lesssim 100, i = 7, 8, 9$. Therefore they do not really constrain at the moment models for a strongly interacting symmetry breaking sector where the size of the renormalized low-energy constants is expected to be of the order of $\frac{1}{16\pi^2}$ [20]. All low-energy constants in Table 2 are of this order of magnitude, except for $l_i^{\prime r}, i = 1, 3, 5$, which receive large tree-level contributions in the Standard Model.

6 Conclusions

We have reported here on our recent investigation [2] of the electroweak chiral Lagrangian, which is based on the gauge-invariant approach to the Standard Model introduced in Ref. [1]. For a purely bosonic effective field theory we have shown, by using the equations of motion and by redefining the universal parameters in the lowest order Lagrangian $\mathcal{L}_2$, that there are only 10 independent low-energy constants in the effective Lagrangian up to order p^4. In particular, it is possible to remove two operators that contribute to the self-energies of the gauge bosons. Hence, there is only one low-energy constant in the effective Lagrangian, related to $\Delta\rho$, that parametrizes oblique corrections. This result persists in the presence of fermions in a general effective field theory analysis, where, however, many more new low-energy constants arise in the fermionic sector. We briefly discussed the relation of this result to those found in the usual approach to the electroweak chiral Lagrangian in Ref. [4] where the couplings of the fermions are assumed to have their Standard Model values. Furthermore we pointed out that the oblique parameters S, T and U [13], as determined by the fits in Ref. [11, 12], cannot directly be identified with the low-energy constants in the effective Lagrangian. They parametrize new physics effects *beyond* the Standard Model and depend, in particular, on the Higgs boson mass, whereas the effective Lagrangian approach also works if there exists no Higgs boson at all.

We then discussed the evaluation of the effective Lagrangian by matching gauge-invariant Green's function at low energies. For the case of the Standard Model with a heavy Higgs boson the matching relation can be evaluated in perturbation theory. With our approach we reproduce the results of earlier calculations [3, 6]. This can be understood from the momentum counting rule $g^2, g'^2 = \mathcal{O}(p^2)$. We then compared the values of the independent low-energy constants with those for a simple Technicolor model. We find that the pattern of the low-energy constants is very different in these two models. Therefore it may be misleading to mimic any strongly interacting symmetry breaking sector with a heavy Higgs boson, e.g. when fitting the oblique parameters S, T and U [11, 12]. Note that there are non-decoupling effects, if one tries to send the Higgs mass to infinity. Since, in addition, all these fits are based on the inclusion of radiative corrections within the context of the Standard Model, which may be quite misleading, cf. Ref. [16], we believe that the current electroweak precision data do not really rule out models for a strongly interacting symmetry breaking sector, see also Ref. [17].

If the electroweak symmetry breaking is indeed governed by a strongly interacting underlying dynamics it will be one of the tasks to be performed at a linear collider to determine these low-energy constants in the effective Lagrangian experimentally as precisely as possible [5], especially, if the collider is running at energies below the resonance region.

Acknowledgements

I would like to thank A. Schenk for the pleasant collaboration on the topics presented here. Furthermore, I am indebted to J. Gasser, F. Jegerlehner, M. Knecht, H. Leutwyler, E. de Rafael, V. Ravindran. T. Riemann, R. Sommer, J. Stern, and A. Vicini for useful discussions. This work was supported in part by Schweizerischer Nationalfonds.

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