Leading electroweak corrections at the TeV scale

P. Ciafaloni

Dipartimento di Fisica & INFN, via Arnesano, 73100 Lecce
E-mail: Paolo.Ciafaloni@le.infn.it

LC-TH-1999-024

Abstract

The planned next generation of linear colliders (NLCs) will be able to probe the infrared structure of Standard Model electroweak interactions, that determines the behavior of electroweak radiative corrections at TeV scale energies. I present results of a recent calculation at the leading log level, and discuss my view on open issues and possible future developments of this new and interesting subject.

1 Introduction

There has been recently an outburst of interest in the TeV scale behavior of Standard Model electroweak corrections, triggered by the observation that such behavior is dominated by the infrared (IR) structure of the theory [1]. The main motivation is of course the possibility of having, in a hopefully nearby future, linear colliders operating at such very high energies and with high luminosities [2]. As has been pointed out in [1, 3], at the TeV scale leading log (LL) one loop electroweak corrections have a typical magnitude of 10 % relative to the Born level. The reason for this* is that when the c.m. energy $\sqrt{s}$ is much bigger than the electroweak scale, the W and Z masses act as effective cutoffs for infrared divergences. One loop corrections then grow like a *double* logarithm of $\sqrt{s}$, i.e. like $\log^2 \frac{\sqrt{s}}{M}$ where $M(\approx M_W \approx M_Z)$ is the electroweak scale. This in contrast with the corrections related to the ultraviolet behavior of the theory, described by the usual RGE equations and growing like a *single* log, being therefore subdominant at very high energies. Moreover, besides leading logs also subleading effects of infrared origin are, generically speaking, numerically relevant [3]. Since big SM effects could mask possible effects of New Physics at NLCs, an accurate calculation of electroweak SM corrections is necessary, which in turn implies addressing higher order calculations. In addition, as I will try to show in the following, the IR structure of a broken theory like the electroweak sector of the Standard Model is interesting *in itself*, both from a theoretical and a phenomenological point of view.

Three different calculations [4, 5, 6] of LL electroweak corrections for processes relevant at NLCs have recently been done. These calculations give different results; this seems to indicate that maybe some issues are still to be understood. In any case, electroweak interactions have a distinctive feature that differentiates them from both QED and QCD: symmetry breaking. The importance of this feature and its relationship with the infrared structure of electroweak interactions are still to be clarified.

The main result of [5], and also its main motivation, is that, at a c.m. energy close to the TeV scale, soft QED effects† cannot be accounted for separately as has been done in the LEP-era [7]. What happens instead, is that there is a separation of scales such that below the electroweak scale M only QED LL effects are present, while above M the contributions of all gauge bosons γ, W, Z to leading log effects have to be taken into account together‡. While the precise meaning of this sentence will be specified in next

*at LEP electroweak corrections have a much smaller typical magnitude of $\frac{\alpha(M_Z)}{4\sin^2\theta_w \pi} \approx 2.7 \times 10^{-3}$

†here by "soft QED effects" I mean all photon contributions giving LL effects

‡as has also been noticed in [6], in this region considering separately only the W, Z contributions would violate gauge invariance

section, I would like to notice that the above implies that the approach to radiative electroweak corrections at NLCs must be substantially changed from the one which was customary at LEP, nicely described in [7].

Of course, since as I have said three different calculations give different results, the issues commented above are currently under debate. Eventually, a full, exact two loop calculation without any *apriori* assumption could discriminate between the different possibilities. In this case, I believe that one should consider a physical process as simple as possible; the one we examine in [5] is a good example.

2 The Z' electroweak form factor

We have computed electroweak LL corrections at TeV scale with the following approach in mind:

- *Complete* electroweak virtual corrections are calculated, taking into account also the photon contribution. The photon is given a mass λ to regularize IR divergences.
- soft photon emission is calculated, with photons having energy less than the experimental resolution ΔE

The effect of soft photon emission is basically that of substituting the photon mass λ, which is an IR regulator, with the experimental resolution ΔE. Therefore in the final result we give to λ the physical meaning of an energy-angle experimental resolution parameter. We are thinking about experimental resolutions of the order of $\lambda \approx 10$ GeV, much lower than the W and Z bosons mass so that a process with W- or Z- bremsstrahlung is experimentally resolved. In other words, we are inclusive with respect to emitted soft photons, but exclusive with respect to W, Z emission.

In the spirit of choosing the simplest possible case for studying LL electroweak interactions at the TeV scale, we consider in [5] the two fermions decay rate of a Z' gauge boson unmixed with the usual Z boson and belonging to a group that commutes with the SM group. Since the Z' is neutral under the Standard Model SU(2)$\otimes$U(1) gauge group, the relevant LL electroweak corrections act only upon the two fermion external legs. Moreover, in the massless fermion limit we consider, chirality is conserved and one can consider separately the cases of left and right final fermions. Even though this is one of the simplest cases of phenomenological interest one can imagine, the basic formalism we set up and the general considerations we make about the IR structure of electroweak SM interactions (i.e. factorization, exponentiation and so on) are relevant for a more general class of processes of interest at NLC energies.

In order to compute the leading radiative corrections in the infrared region $\sqrt{s} \gg w \gg M$, where w is the virtual boson energy and M its mass, we use the method of soft insertions formulae, which are widely used in QED [8] and are known to provide in QCD [9, 10] the leading IR singularities at double log level. This method consists in factorizing the softest virtual momentum $k_1 = (w_1, \mathbf{k})$ by computing external line insertions only. The left-over diagram is then evaluated by setting $k_1 = 0$ (or, equivalently, the diagram is evaluated on-shell). This procedure is iterated and the final integration is performed in the region of phase space giving rise to the LL electroweak corrections we are interested in, i.e. the strongly ordered region: $w_1 \ll w_2 \ll ..... \ll w_n$ where 1 labels the outermost boson and n the innermost one (see fig. 1).

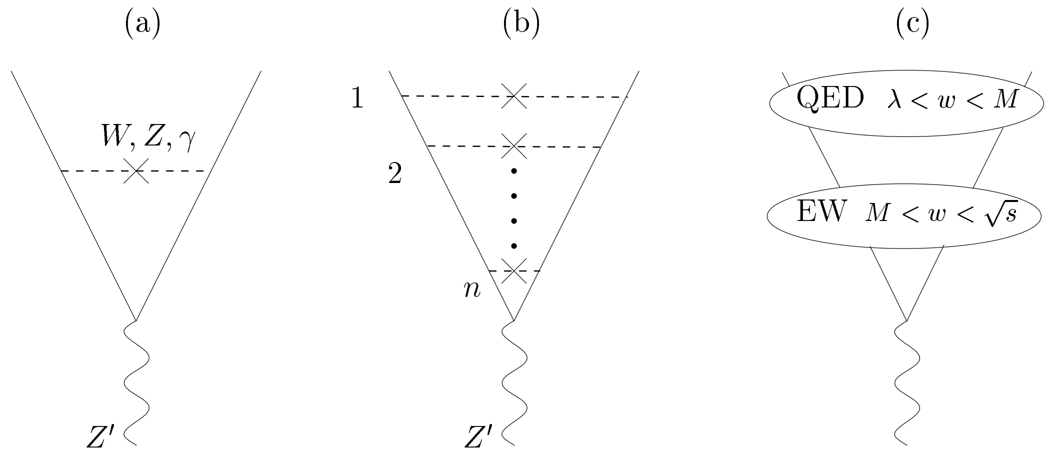

Figure 1: (a)-(b) :diagrams for soft boson insertion at 1 and n loops. Continuous lines are fermion lines, and dashed lines are W, Z, γ gauge bosons. Crosses indicate that gauge bosons are close to mass-shell, and energies are such that $w_1 \ll w_2 \ll ..... \ll w_n$ (see text). (c): Pictorial representation of eq. (1)

Referring to [5] for computational details and using here the same notations, we find that the matrix element at the LL level is given by ($x \propto \log w$ is used in place of w; M_0 is the Born level amplitude):

$$\mathbf{M}^{LL} = \exp[-e^2 q_f^2 \frac{l^2}{2}] \sum_{n=0}^{\infty} M_n = \exp[-e^2 q_f^2 \frac{l^2}{2}] \langle f | P_x \exp[-\int_0^L dx H_{EW}(x)] | f \rangle M_0 \quad (1)$$

$$H_{EW}(x) = (g'^2 Y \tilde{Y} + g^2 \bar{T} \cdot \bar{\tilde{T}}) x + e^2 Q \tilde{Q} l \qquad l = \log \frac{M}{\lambda} \quad L = \log \frac{\sqrt{s}}{M} \quad x_i = \log \frac{w_i}{M} \quad (2)$$

Here $|f\rangle$ is a fermion belonging to a given representation of the SU(2)$\otimes$U(1) gauge group, coupling with gauge bosons through SU(2)$\otimes$U(1) generators T^a normalized to to $\mathrm{Tr}\{T^aT^b\} = \frac{1}{2}\delta^{ab}$. The charge operator is defined as $Q = T^3 + Y$[§]; tilded operators $\tilde{T}^a$ act on the antifermion flavor indices[¶] while untilded operators act on the fermion line. Furthermore, P_x denotes the x-ordered product; at the LL level we can identify $M_Z \approx M_W \equiv M$ with the weak scale.

Let us now discuss equation (1) in various regimes, by noting first that in this problem there are three relevant mass scales: the c.m. energy $\sqrt{s}$, the electroweak scale M and the parameter λ, which in the final result has the meaning of an experimental resolution, as mentioned above.

First, when $\sqrt{s} \sim M$, only QED soft effects are present, which amounts to saying that the "LEP-approach" is correct at LEP energies of course. However, for $\sqrt{s} \gg M \sim \lambda$ (or, equivalently, $l \ll L$) a completely different situation occurs and eqn. (1) then reduces to:

$$\mathbf{M}^{LL} = \exp[-(g'^2 y_f^2 + \frac{3}{4}g^2)\frac{1}{2}\log^2\frac{\sqrt{s}}{M}]M_0 \tag{3}$$

In other words, at very high energies what really factorizes and exponentiates is the whole SU(2)$\otimes$U(1) group contribution, and <u>not</u> the U$(1)_{em}$ component. This was to be expected, since the symmetry of relevance to the problem is related to the energy scale. In particular, at scales typical of electroweak symmetry breaking, one "sees" the pattern of breaking SU(2)$\otimes$U(1)$\rightarrow$ U$(1)_{em}$, but when the energy gets much bigger than this scale, the full gauge symmetry SU(2)$\otimes$U(1) is restored.

In general, and for arbitrary values of $\sqrt{s}$ above the electroweak scale, one sees that QED effects are not completely factorized. In fact if this were the case, then we would find a QED prefactor $\exp[-e^2q_f^2\frac{1}{2}\log^2\frac{\sqrt{s}}{\lambda}]$ in place of $\exp[-e^2q_f^2\frac{l^2}{2}] = \exp[-e^2q_f^2\frac{1}{2}\log^2\frac{M}{\lambda}]$ in (1). In other words, only photons with energies such that $\lambda < w_\gamma < M$ indeed factorize, as is depicted pictorially in fig. 1c. For energies higher than M the photon contribution is taken into account in a nontrivial way in H_{EW}; this is precisely what we mean by "nonfactorizable soft QED effects". As is shown in [5], if one insists in extrapolating to TeV scale energies the "LEP approach" in which soft QED corrections are computed separately, then the error one makes with respect to the correct result is at the LL level, i.e. an error precisely of the same order of the effect one is trying to calculate.

[§]Operators are in capital letters, c-numbers in small letters. Thus Q is an operator with values $q_e = -1, q_\nu = 0$

[¶]here and in the following, by "flavor" I mean SU(2)$\otimes$U(1) quantum numbers; we only consider a single generation of fermions

In second place, (1,2) are responsible for the fact that electroweak LL effects do not exponentiate in a trivial way. In fact one finds the one and 2 loop results:

$$M_1 = -(a_f \frac{L^2}{2} + b_f l L) M_0 \qquad M_2 = \left\{ \frac{1}{2}(a_f \frac{L^2}{2} + b_f l L)^2 - \frac{1}{3} e^2 g^2 l L^3 y_f t_f^3 \right\} M_0 \qquad (4)$$

$$a_f = g^2 \frac{3}{4} + g'^2 y_f^2 \qquad\qquad b_f = e^2 q_f^2 \qquad (5)$$

and in the 2 loop result a "spurious" term $\propto e^2 g^2 t^3$ is present. It is easy to show why there is no exponentiation in our approach. In fact, suppose that one sets the $Q\tilde{Q}$ term in (2) to 0. Then, what is left is the SU(2)⊗U(1) Casimir, which is a c-number. The x ordered exponential then simply produces the regular exponential of the abovementioned Casimir. However of course, the $Q\tilde{Q}$ term can not be set to 0 in our approach: this is a noncommuting operator and determines the terms that break exponentiation, that are proportional to l as one can see from (4). It is interesting to notice that these effects come about because even for $w > M$, the Z and γ bosons are still distinguished by their masses, the latter acting as collinear cutoffs. The "exponentiation breaking" term turns out to be proportional to $l = \log \frac{M}{\lambda}$. Therefore without symmetry breaking, that causes mixing in the neutral sector and gives rise to different mass scales for the gauge bosons, we would have exponentiation. This is what happens for instance in QCD which shares with the electroweak sector the property of being a nonabelian theory, but in which effects analogous to the ones we are studying do in fact exponentiate.

3 Conclusions

The bulk of radiative corrections at energies much higher than the electroweak scale is determined by the infrared structure of the theory; this makes the subject a very interesting (and experimentally testable!) one. The requirement of controlling Standard Model contributions at a level comparable to the experimental precision at NLCs implies addressing higher order calculations. The fact that the first calculations of this kind of effects are (at least partially) in disagreement, might indicate that there is a peculiar feature that differentiates the infrared structure of a broken theory like the Standard Model of electroweak interactions from the one of an unbroken theory like QCD for instance. The main conclusion that emerges from our work [5] is that the "LEP-approach" to electroweak corrections must be substantially changed, or improved, when considering electroweak corrections at TeV scale energies. From a theoretical point of view, in this problem the interplay between symmetries of the theory and energy scales at which the theory is tested is an important issue and in my opinion, a not yet fully understood one.

References

[1] P. Ciafaloni and D. Comelli, Phys. Lett. **B446** (1999) 278

[2] E. Accomando *et al*, Phys. Rept. **299** (1998) 1
Y. Sugimoto, KEK-PROCEEDINGS-97-2.
See also the web page http://lcwws.physics.yale.edu/lc/

[3] M. Beccaria, P. Ciafaloni, D. Comelli, F.M. Renard and C. Verzegnassi, hep-ph/9906319.
P. Ciafaloni, hep-ph/9906354
M. Beccaria, P. Ciafaloni, D. Comelli, F.M. Renard and C. Verzegnassi, hep-ph/9907389.

[4] J.H. Kuhn and A.A. Penin, hep-ph/9906545.

[5] P. Ciafaloni and D. Comelli, hep-ph/9910278.

[6] V.S. Fadin, L.N. Lipatov, A.D. Martin and M. Melles, hep-ph/9910338.

[7] For reviews of the approach to electroweak corrections at LEP, see for instance:
G. Montagna, O. Nicrosini, F. Piccinini, "Precision physics at LEP", Riv. Nuovo Cim. **21** (1998) No. 9
G. Altarelli, R. Kleiss, C. Verzegnassi, "Z physics at LEP 1", CERN 89-08, Vol. 1
G. Altarelli, T. Sjostrand and F. Zwirner, "Physics at LEP 2", CERN 96-01, Vol. 1

[8] V. V. Sudakov, Sov. Phys. JETP 3, 65 (1956)
F. Low, Phys. Rev. **110** (1958) 974
D.R. Yennie, S.C. Frautschi, H. Suura, Annals of Phys. **13** (1961) 379

[9] J. M. Cornwall and G. Tiktopoulos, Phys. Rev. **D13** (1976) 3370
T. Kinoshita and A. Ukawa, Phys.Rev. **D15** (1977) 1596; Phys.Rev. **D16** (1977) 332
A. Sen, Phys. Rev. **D24** (1981) 3281; Phys. Rev. **D28** (1983) 860

[10] A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rept. **100** (1983) 201
S.Catani and M.Ciafaloni, Nucl.Phys. **B249** (1985) 301