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Predictions for $Z \to \mu\tau$ and Related Reactions [†]

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Abstract

We discuss predictions for the lepton-flavour changing decays $Z \to e\mu, \mu\tau, e\tau$ which may be searched for at the Giga–Z option of the Tesla Linear Collider project with Z resonance production rates as high as 10^9. We try to be as model-independent as possible and consider both the Dirac and Majorana mass cases. The Standard Model, if being minimally extended by the inclusion of light neutrino masses with some mixings as observed in neutrino oscillation search experiments, predicts completely negligible rates. With a more general neutrino content quite interesting expectations may be derived.

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1 Introduction

With the Giga–Z option of the Tesla project one may expect the production of about 10^9 Z bosons at resonance [1]. This huge rate, about a factor 100 higher than rates at LEP 1, allows one to study a number of problems with unprecedented precision. Among them is the search for lepton-flavour changes in Z decays:

$$Z \to e\mu, \mu\tau, e\tau. \tag{1.1}$$

Non-zero rates are expected if neutrinos are massive and mix [2, 3, 4]. Often one considers the branching ratio for the production of the following sum of charged states:

$$\mathrm{BR}(Z \to l_1^{\mp} l_2^{\pm}) = \frac{\Gamma(Z \to \bar{l}_1 l_2 + l_1 \bar{l}_2)}{\Gamma_Z}. \tag{1.2}$$

First predictions for flavour-changing Z decays in the framework of the Standard Model [5, 6, 7], using techniques developed in [8], were given in [9, 10, 11, 12]. The best direct limits are obtained by searches at LEP 1 (95% c.l.) [13]:

$$\begin{aligned}
\mathrm{BR}(Z \to e^{\mp}\mu^{\pm}) &< 1.7 \times 10^{-6} \;\; [14], && (1.3)\\
\mathrm{BR}(Z \to e^{\mp}\tau^{\pm}) &< 9.8 \times 10^{-6} \;\; [15,\; 14], && (1.4)\\
\mathrm{BR}(Z \to \mu^{\mp}\tau^{\pm}) &< 1.2 \times 10^{-5} \;\; [14,\; 16]. && (1.5)
\end{aligned}$$

A careful analysis shows, taking into account realistic conditions at future experiments, that the sensitivities for the branching ratios could be improved considerably at the Giga–Z [17], namely down to:

$$\begin{aligned}
\mathrm{BR}(Z \to e^{\mp}\mu^{\pm}) &< 2 \times 10^{-9}, && (1.6)\\
\mathrm{BR}(Z \to e^{\mp}\tau^{\pm}) &< f \times 6.5 \times 10^{-8}, && (1.7)\\
\mathrm{BR}(Z \to \mu^{\mp}\tau^{\pm}) &< f \times 2.2 \times 10^{-8}, && (1.8)
\end{aligned}$$

with $f = 0.2 \div 1.0$.

These numbers may be confronted with expectations derived from the signals for $\nu_\mu - \nu_\tau$ oscillations in atmospheric neutrino experiments [18, 19, 20, 21, 22, 23, 24]. They are at the 90% c.l. compatible with the following parameter set [25]:

$$\begin{aligned}
\Delta m^2_{\nu_\mu\nu_\tau} &\simeq (2 \div 8) \times 10^{-3}\mathrm{eV}^2, && (1.9)\\
\sin^2(2\vartheta_{\mu\tau}) &\simeq 0.8 \div 1. && (1.10)
\end{aligned}$$

There is also evidence for $\nu_e - \nu_\mu$ oscillations from solar neutrino experiments [26, 27, 28, 29, 30, 31], being compatible with:

$$\Delta m^2_{\nu_e\nu_\mu} \simeq 10^{-10} \div 10^{-5}\mathrm{eV}^2. \tag{1.11}$$

From reactor searches, there are no hints of $\nu_e - \nu_\tau$ oscillations [32, 33]. For more details see e.g. the review [25] and references therein.

The good news from the evidences for neutrino oscillations is that they suggest non-vanishing rates for reaction (1.1). The bad news is, that these rates are, if derived with (1.9)–(1.11) in the minimally extended Standard Model (νSM), extremely small[1]:

$$\begin{aligned} \mathrm{BR}(Z \to \mu^\mp \tau^\pm) &\sim 10^{-54}, &(1.12)\\ \mathrm{BR}(Z \to e^\mp \mu^\pm) \sim \mathrm{BR}(Z \to e^\mp \tau^\pm) &\lesssim 4 \times 10^{-60}. &(1.13) \end{aligned}$$

This is derived in Section 2 and is also in accordance with older calculations [9, 10, 35].

How do these small numbers arise? In Born approximation, the lepton-flavour changing Z decay into two charged leptons is forbidden in the νSM due to the GIM mechanism [36]. However, it may take place if n types of neutrinos have masses m_i and mix with each other (with mixing matrix $\mathbf{V}_{ij}$), i.e. if symmetry eigenstates and mass eigenstates are different in the lepton sector. Then, the virtual exchange of these neutrinos produces an effective lepton-flavour changing vertex and the corresponding branching ratio has the following structure:

$$\mathrm{BR}(Z \to l_1^\mp l_2^\pm) = \frac{\alpha^3}{192\pi^2 s_W^6 c_W^2} \frac{M_Z}{\Gamma_Z} |\mathcal{V}(M_Z^2)|^2 \approx 10^{-6} |\mathcal{V}(M_Z^2)|^2. \qquad (1.14)$$

The form factor $\mathcal{V}(Q^2)$ depends on the details of the interaction:

$$\mathcal{V}(Q^2) = \sum_{i=1}^{n} \mathbf{V}_{l_1 i} \mathbf{V}^*_{l_2 i} \; V(m_i^2/M_W^2). \qquad (1.15)$$

The vertex function V was calculated in 1982/83 independently by three groups [9, 10, 11, 12] for sequential Dirac particles, and in a more general context later [37, 38].

The function V depends quadratically on the mass *both* in the small neutrino mass limit [39, 9] *and* in the large neutrino mass limit [40, 10, 11, 12]:

$$\begin{aligned} V(\lambda_i \ll 1) - V(0) &\approx (2.56 - 2.30 \times i)\; \lambda_i + \mathcal{O}\left(\lambda_i^2 \ln \lambda_i\right), &(1.16)\\ V(\lambda_i \gg 1) - V(0) &\approx \frac{1}{2}\; \lambda_i + \mathcal{O}(\ln \lambda_i), &(1.17) \end{aligned}$$

with

$$\lambda_i = \frac{m_i^2}{M_W^2}. \qquad (1.18)$$

The constant terms are not shown in (1.16), (1.17) since they drop out in (1.15) due to the unitarity of the mixing matrix, and the branching ratio becomes proportional to the fourth power of the neutrino massses. It is this behaviour which makes the expected rates so extremely small for the experimentally evidenced tiny neutrino masses. For values of m_i of the order of M_W, the vertex V is of the order one, and could become large if the m_i would be much bigger than M_W.

[1] Our estimate is in clear distinction to Eqn. (6) of [34], where from the data a limit was derived which corresponds to $\mathrm{BR}(Z \to \mu^\mp \tau^\pm) \approx \mathcal{O}(10^{-8} \div 10^{-5})$.

The evidence of tiny neutrino masses may also be indicative for a mechanism which produces at the same time very large masses. Heavy neutrinos are expected by some GUTs [41] and string-inspired models [42, 43, 44], and are suggested by the seesaw mechanism [45, 46, 47]. Therefore, the above observations motivate us to have a closer look at the prospects of observing lepton-flavour changes with the Tesla Linear Collider. For the Giga–Z option, it might not be sufficient to apply the well-known and simple approximations for large masses m_i, but also the medium- or even small-mass cases may be of experimental interest.

To be concrete, we will explore the following scenarios:

(i) The νSM. We treat the known light neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ as *massive Dirac* particles. Individual lepton numbers L_e, L_μ, L_τ are not conserved any more. The lepton sector is then in exact analogy to the quark sector. As a by-product, the Z decay amplitude into two quarks of different flavours can be read off from our general expressions.

(ii) The νSM *sequentially extended* with *one* heavy *ordinary Dirac* neutrino. This case implies the existence of a heavy charged lepton as well[2]. It is not a very favoured scenario but we consider it as a simple application of the expressions of case (i) for heavier neutrinos. Again, total lepton number L is conserved.

(iii) The νSM *extended* with *two* heavy *right-handed singlet Majorana* neutrinos. Not only individual, but also total L is, in general, not conserved since the presence of Majorana mass terms involves mixing of neutrinos and their charge-conjugate partners (antineutrinos), with opposite fermion-number. For two equal and heavy masses this case reduces to the addition of one heavy *singlet Dirac* neutrino [38]. In this latter case L is recovered [48].

With our numerical estimates we will be as model-independent as possible and will assume no constraints on neutrino masses or mixings, except for the ones imposed by the unitarity of the leptonic and neutrino mixing matrices, by the present bounds on lepton universality, CKM unitarity, and the measured Z boson invisible width [49, 50, 51, 52, 53, 54], and by oscillation experiments (see [25] for a review).

In the following sections, we will discuss the predictions of the three scenarios for the lepton-number changing Z decay. In Appendices, the generalization of Lagrangian and Feynman rules of the Standard Model to the general case of Dirac and Majorana masses is explained, experimental limits on neutrino mixings and masses are quoted, and the calculation of the vertex function is sketched.

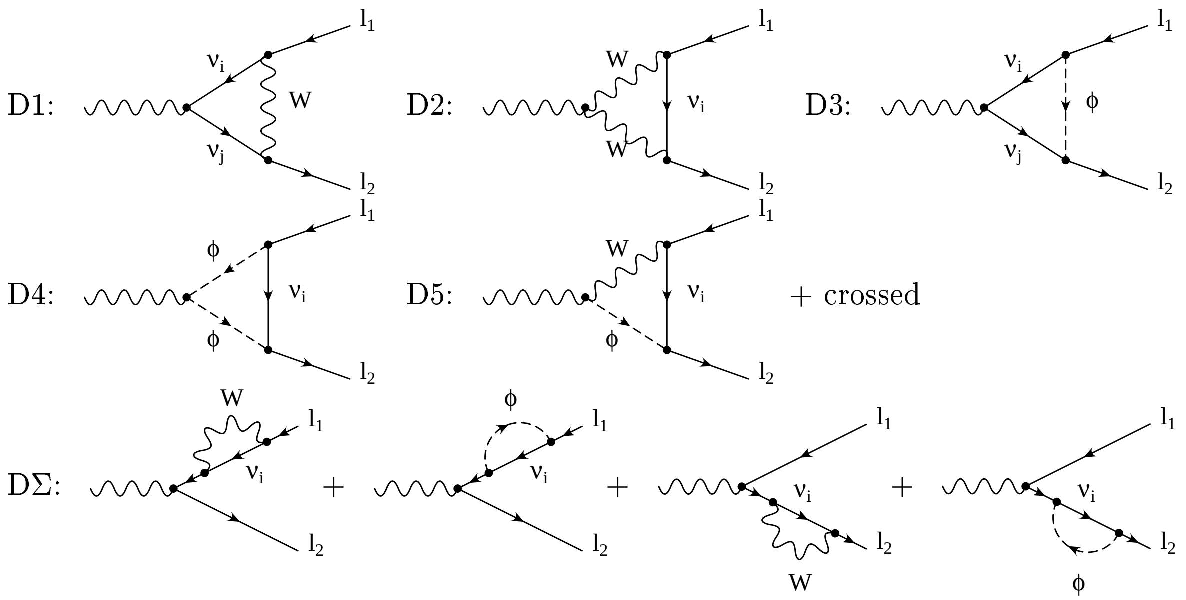

Figure 1: *Feynman diagrams for the lepton-flavour changing Z decay. In the case of virtual, ordinary Dirac neutrinos, the $Z\nu_i\nu_j$ vertices in D1 and D3 are diagonal and the analogous quark-flavour-changing process can be obtained by replacing l_k by down-quarks and ν_i by up-quarks.*

2 Predictions from the νSM

The amplitude for the decay of a Z boson into two charged leptons with different flavour, l_1 and l_2, is given in a self-explanatory notation by:

$$\mathcal{M} = -\frac{ig\alpha_W}{16\pi c_W}\ \mathcal{V}(Q^2)\ \varepsilon_Z^\mu \bar{u}_{l_2}(p_2)\gamma_\mu(1-\gamma_5)u_{l_1}(-p_1), \tag{2.1}$$

where

$$\alpha_W \equiv \frac{\alpha}{s_W^2}, \tag{2.2}$$

and the form factor $\mathcal{V}$ depends on $Q^2 = (p_2 - p_1)^2$ and can be written as:

$$\begin{aligned} \mathcal{V}(Q^2) &= \sum_{i=1}^{3} \mathbf{V}_{l_1 i}\mathbf{V}^*_{l_2 i}\ V(\lambda_i), & (2.3)\\ V(\lambda_i) &= \left[v_W(i) + v_{WW}(i) + v_\phi(i) + v_{\phi\phi}(i) + v_{W\phi}(i) + v_\Sigma(i)\right], & (2.4)\end{aligned}$$

with $\mathbf{V}_{ij}$ being the leptonic CKM mixing matrix. In general, there are besides the vector and axial-vector couplings f_V and f_A in (2.1) also contributions of the f_S, f_P, f_M, f_E types, but for the production of on-shell fermions (with their masses being neglected) they vanish here. Further, it is $f_V = f_A = \mathcal{V}(Q^2)$ due to the presence of W bosons coupling only to left-handed fermions. The contributions from the one-loop diagrams

[2] A fourth generation of quarks is also needed to keep the theory anomaly free.

of Figure 1 depend on λ_i and additionally on

$$\lambda_Q = \frac{Q^2}{M_W^2}, \tag{2.5}$$

which on the Z boson mass shell becomes

$$\lambda_Z = \frac{M_Z^2}{M_W^2} = \frac{1}{c_W^2} \approx 1.286. \tag{2.6}$$

In terms of the usual vector and axial-vector couplings,

$$v_i = I_3^{i_L} - 2Q_i s_W^2 = I_3^{i_L}(1 - 4s_W^2|Q_i|), \tag{2.7}$$
$$a_i = I_3^{i_L}, \tag{2.8}$$

the individual contributions are:

- From vertex diagrams:

$$\begin{aligned} \text{D1}: \quad v_W(i) &= -(v_i + a_i)\ [\lambda_Q\ (C_0 + C_{11} + C_{12} + C_{23}) - 2C_{24} + 1] \\ &\quad -(v_i - a_i)\ \lambda_i\ C_0, \end{aligned} \tag{2.9}$$

$$\text{D2}: v_{WW}(i) = 2c_W^2\ (2I_3^{i_L})\left[\lambda_Q\ (\bar{C}_{11} + \bar{C}_{12} + \bar{C}_{23}) - 6\bar{C}_{24} + 1\right], \tag{2.10}$$

$$\begin{aligned} \text{D3}: \quad v_\phi(i) &= -(v_i + a_i)\ \frac{\lambda_i^2}{2}\ C_0 \\ &\quad -(v_i - a_i)\ \frac{\lambda_i}{2}\ \left[\lambda_Q\ C_{23} - 2C_{24} + \frac{1}{2}\right], \end{aligned} \tag{2.11}$$

$$\text{D4}: \quad v_{\phi\phi}(i) = -(1 - 2s_W^2)\ (2I_3^{i_L})\ \lambda_i\ \bar{C}_{24}, \tag{2.12}$$

$$\text{D5}: \quad v_{W\phi}(i) = -2s_W^2\ (2I_3^{i_L})\ \lambda_i\ \bar{C}_0; \tag{2.13}$$

- From self-energy corrections to the external fermion lines:

$$\text{D}\Sigma: \ v_\Sigma(i) = \frac{1}{2}(v_i + a_i - 4c_W^2 a_i)\left[(2 + \lambda_i)B_1 + 1\right]. \tag{2.14}$$

The one-loop tensor integrals C_0, $\bar{C}_0$, C_{ij}, $\bar{C}_{ij}$, and B_1 are defined in Appendix C. With our numerical results for the Dirac case, we rely on two calculations, an old one [9, 10] and also this new, completely independent one. In the latter, the numerical evaluation of the tensor integrals is performed with the help of the computer program package `LoopTools` [55, 56].

Numerical results are shown in Figure 2. The quantity presented is related to a branching ratio definition often used in the literature:

$$B_Z \equiv \frac{\Gamma(Z \to f_1^{\mp} f_2^{\pm})}{2 \times \Gamma(Z \to \bar{\nu}_l \nu_l)} = \left(\frac{\alpha}{\pi}\right)^2 \frac{N_c}{16 s_W^4}\ |\mathcal{V}(M_Z^2)|^2, \tag{2.15}$$

with N_c as colour factor. Using

$$\Gamma(Z \to \bar{\nu}_l \nu_l) = \frac{\alpha_W}{24 c_W^2} M_Z, \tag{2.16}$$

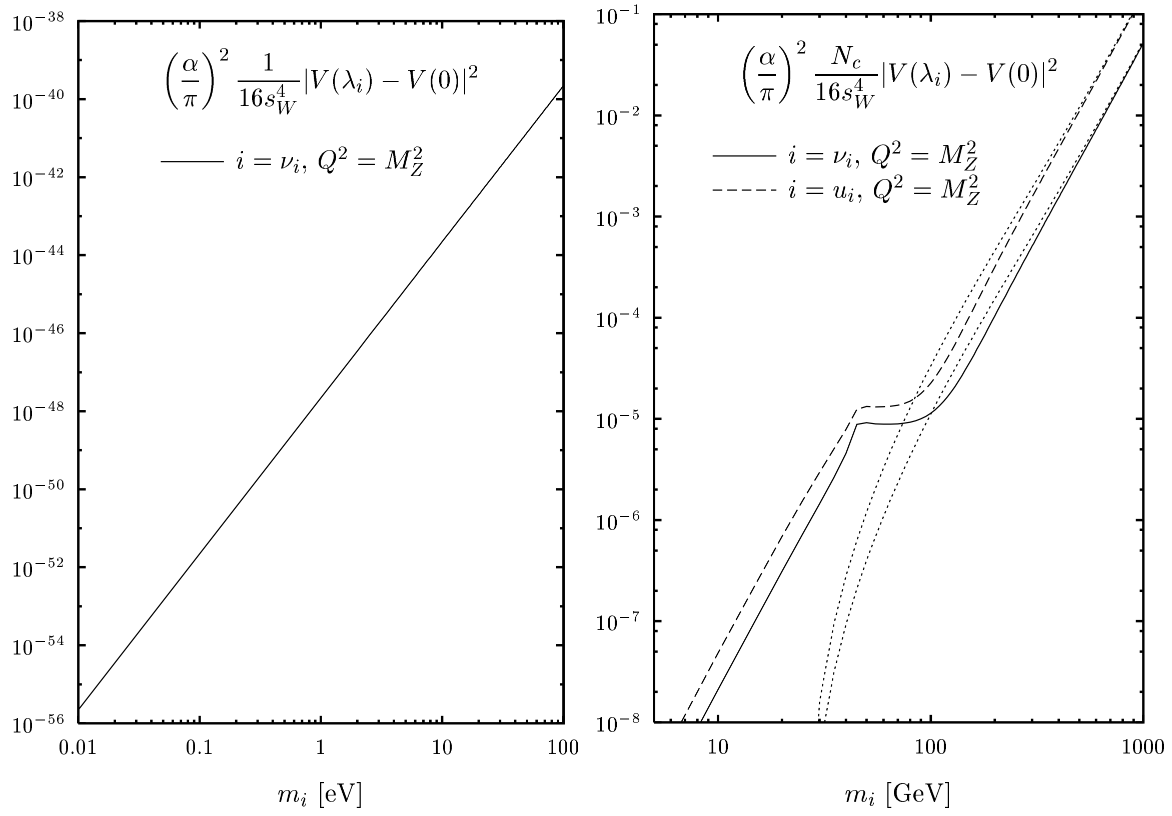

Figure 2: *Contribution of one neutrino generation to the ratio B_Z of Eqn. (2.15) in the small and large mass regions for virtual, ordinary Dirac neutrinos and the analogous quark case. We set $\mathbf{V}_{l_1 i}\mathbf{V}^*_{l_2 i} = 1$ here; dotted lines correspond to $Q^2 = 0$.*

we get a useful relation to the branching ratio introduced in (1.2) and (1.14):

$$\mathrm{BR}(Z \to f_1^{\mp} f_2^{\pm}) \;=\; \frac{2 \times \Gamma(Z \to \bar{\nu}_l \nu_l)}{\Gamma_Z}\, B_Z = 0.1333\; B_Z. \tag{2.17}$$

Figure 2 shows the contribution from one neutrino generation to the branching ratio as a function of the neutrino mass *without* the influence of the mixing matrix elements. We choose two interesting mass regions, namely that corresponding to the findings of neutrino oscillation searches (Figure 2(a)) and also that with potential predictions in the reach of the Giga–Z (Figure 2(b)). The latter figure nicely agrees with the earlier calculations [10, 11, 12], and the small mass limit with [9]. The dotted lines in Figure 2(b) correspond to the approximation $\lambda_Z = 0$: the large mass limit is reproduced quite well, while the small mass limit differs from the correct result. This is discussed in Appendix D.3.

We will now estimate the branching ratio under the assumption that there are three generations of light neutrino flavours with a unitary mixing matrix $\mathbf{V}$ as evidenced by experiment. The general form of this matrix may be chosen to be [3]:

$$
\begin{aligned}
\mathbf{V}_{l\nu} &= \begin{pmatrix} \mathbf{V}_{e1} & \mathbf{V}_{e2} & \mathbf{V}_{e3} \\ \mathbf{V}_{\mu 1} & \mathbf{V}_{\mu 2} & \mathbf{V}_{\mu 3} \\ \mathbf{V}_{\tau 1} & \mathbf{V}_{\tau 2} & \mathbf{V}_{\tau 3} \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\delta} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\delta} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\delta} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\delta} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}
\tag{2.18}
$$

Here, we have three mixing angles and one CP-violating phase as in the quark CKM case, plus two CP-violating phases α, β if neutrinos are Majorana particles (they are *strictly* neutral so that less phase factors may be 'eaten' by redefining complex fermion fields). Current data suggest the following form of this matrix:

$$
\mathbf{V}_{l\nu} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{2.19}
$$

where we have assumed [57] $\alpha = \beta = \delta = 0$, extracted from [32, 33] the $s_{13} = 0$, and further assumed $s_{23} = 1/\sqrt{2}$ (corresponding to maximal mixing) and left the θ_{12} free.

With

$$
\begin{aligned}
M_W &= 80.41 \text{ GeV}, &(2.20)\\
M_Z &= 91.187 \text{ GeV}, &(2.21)\\
\Gamma_Z = \Gamma(Z \to \text{all}) &= 2.49 \text{ GeV}, &(2.22)
\end{aligned}
$$

we get after trivial calculations:

$$
\begin{aligned}
\mathrm{BR}(Z \to l_1^{\mp} l_2^{\pm}) = \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z} \Big| & \mathbf{V}_{l_1 1}\mathbf{V}_{l_2 1}^*[V(\lambda_1) - V(0)] \\
& + \mathbf{V}_{l_1 2}\mathbf{V}_{l_2 2}^*[V(\lambda_2) - V(0)] + \mathbf{V}_{l_1 3}\mathbf{V}_{l_2 3}^*[V(\lambda_3) - V(0)]\Big|^2,
\end{aligned}
\tag{2.23}
$$

with

$$
\frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z} = 1.127 \times 10^{-6}. \tag{2.24}
$$

For small neutrino masses, we show in Section D.2:

$$
V(\lambda_i) - V(0) = a_1(\lambda_Z)\,\lambda_i. \tag{2.25}
$$

The resulting branchings are, without approximations yet, but using the information from three-generation unitarity:

$$
\mathrm{BR}(Z \to l_1^{\mp} l_2^{\pm}) = \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z}\, |a_1|^2\, \left|\mathbf{V}_{l_1 1}\mathbf{V}_{l_2 1}^*\lambda_{12} - \mathbf{V}_{l_1 3}\mathbf{V}_{l_2 3}^*\lambda_{23}\right|^2, \tag{2.26}
$$

with

$$\lambda_{ij} \quad = \quad |\lambda_i - \lambda_j| \tag{2.27}$$

and the a_1 is given in (D.67):

$$|a_1|^2 = 11.832. \tag{2.28}$$

There are two different cases to be considered when using now the specific mixing matrix (2.19):

$$\mathrm{BR}(Z \to e^{\mp}\mu^{\pm}) \approx \mathrm{BR}(Z \to e^{\mp}\tau^{\pm}) \quad \approx \quad 1.333 \times 10^{-5}\, \frac{c_{12}^2 s_{12}^2}{2}\, \lambda_{12}^2, \tag{2.29}$$

and

$$\mathrm{BR}(Z \to \mu^{\mp}\tau^{\pm}) \quad = \quad 1.333 \times 10^{-5}\, \frac{1}{4}\, \left|s_{12}^2\lambda_{12} - \lambda_{23}\right|^2. \tag{2.30}$$

From the mass estimate (1.9) we get as additional input from atmospheric neutrino studies:

$$\lambda_{23} \approx \frac{(2 \div 8) \times 10^{-3}\mathrm{eV}^2}{M_W^2} = (3 \div 12) \times 10^{-25}, \tag{2.31}$$

and from solar neutrino searches, (1.11):

$$\lambda_{12} \approx 1.5 \times \left(10^{-32} \div 10^{-27}\right). \tag{2.32}$$

It is easy now to see that the expected rates for the lepton-flavour changing Z decays are limited to:

$$\mathrm{BR}(Z \to e^{\mp}\mu^{\pm}) \approx \mathrm{BR}(Z \to e^{\mp}\tau^{\pm}) \quad < \quad 3.75 \times 10^{-60}, \tag{2.33}$$

$$\mathrm{BR}(Z \to \mu^{\mp}\tau^{\pm}) \quad \approx \quad (3 \div 48) \times 10^{-55}. \tag{2.34}$$

In (2.33), we assumed arbitrarily a maximal mixing $s_{12} = 1/\sqrt{2}$. These rates are extremely small. In fact, we will neglect the effects of the light neutrino sector in the next sections, where we extend the νSM to accomodate heavy neutrinos, taking massless the known ones.

3 Predictions from the νSM plus One Heavy Dirac Neutrino

Here, the only effective difference to the case before is the existence of a fourth generation with a sequential Dirac neutrino of mass m_N. In this case, the branching ratio gets:

$$\mathrm{BR}(Z \to l_1^{\mp} l_2^{\pm}) \quad = \quad \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z}\, \left|\mathbf{V}_{l_1 N}\mathbf{V}_{l_2 N}^*\right|^2\, \left|V(\lambda_N) - V(0)\right|^2, \tag{3.1}$$

with $V(\lambda_N)$ given in (2.4). The numerical results depend crucially on the mixing between the light and heavy leptons. An optimistic assumption would be maximal mixing:

$$\left|\mathbf{V}_{l_1 N}\mathbf{V}^*_{l_2 N}\right|^2 = \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4}. \tag{3.2}$$

This is of course unrealistic. Stringent (though indirect) limits may be derived from the analysis of flavour-diagonal reactions as advocated e.g. in [50, 51, 52] as well as from the lepton flavour-changing process $\mu \to e\gamma$ [58, 59]. A short summary may be found in Appendix B. There the matrix $\mathbf{B}$ is, for this particular case of heavy Dirac neutrinos, $\mathbf{B} = \mathbf{V}$.

In order to be definite, we show in Figure 3a, solid line, the predictions for BR($Z \to \mu^\pm\tau^\mp$) and assume the upper limit of the mixings allowed from (B.9)–(B.11) and (B.12):

$$|\mathbf{V}_{\mu N}\mathbf{V}^*_{\tau N}|^2 < 1.5 \times 10^{-4}. \tag{3.3}$$

For the other two lepton flavour-changing Z decay channels, the corresponding graphs scale simply in accordance with the ratios of the mixing matrix elements:

$$|\mathbf{V}_{eN}\mathbf{V}^*_{\tau N}|^2 \;<\; 1.9 \times 10^{-4}, \tag{3.4}$$
$$\left|\mathbf{V}_{eN}\mathbf{V}^*_{\mu N}\right|^2 \;<\; 1.2 \times 10^{-4}. \tag{3.5}$$

These coupling factors are to be compared to (3.2): we observe a suppression of the expected branching ratio (for a given mass m_N) by more than three orders of magnitude.

As mentioned in Appendix B, these bounds on light-heavy mixings from flavour-diagonal processes are improved by flavour-changing processes involving the first two lepton families. In fact, from [60]

$$\mathrm{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}, \tag{3.6}$$

one obtains, for heavy neutrinos, the following (nearly) mass-independent limit [58, 59]:

$$\left|\mathbf{V}_{eN}\mathbf{V}^*_{\mu N}\right|^2 \;<\; 1.4 \times 10^{-8}, \tag{3.7}$$

much more stringent than (3.5).

The Giga–Z discovery range is indicated in Figure 3a, using the maximum values of the mixings allowed by (3.3). The range will be limited to the large neutrino mass limit if we believe in the relevance of the above mixing bounds. Then, the approximations of (D.43) apply. Numerically, this means for $\lambda_Z = 1.286$:

$$|V(\lambda_N) - V(0)|^2 \;=\; \frac{1}{4}\,\lambda_N^2 + 1.44\;\lambda_N \ln\lambda_N - 3.49\;\lambda_N + 2.07\;\ln^2\lambda_N + \mathcal{O}(\ln\lambda_N). \tag{3.8}$$

If neutrinos with a mass of several hundred GeV or more would exist, there is a good chance to observe some effect of them from the Z decays under study. Indeed one cannot constrain heavy neutrino masses from these processes since they also depend on the mixings, and vice versa. The fact that in Figure 3a the discovery reach of LEP or Giga–Z cuts the curves means: if no event is observed, mixings must be smaller than the ones employed (we took present upper bounds) for neutrino masses above the intersection point; or if some effect was observed then the plot would provide a lower bound for the heavy neutrino mass. In fact, it is roughly only the product $m_N^4 \left|\mathbf{V}_{l_1 N}\mathbf{V}^*_{l_2 N}\right|^2$ which can be constrained, assuming only one heavy Dirac neutrino.

4 Predictions from the νSM plus Two Heavy Right-Handed Singlet Majorana Neutrinos

Some basic features of Lagrangians with Majorana mass terms and their relations to the simpler Dirac case are summarized in Appendix A. For a derivation of Feynman rules with Majorana particles [61] we refer to [62, 63, 64]. The couplings of the virtual neutrinos to the W bosons are left-handed, $v_i + a_i = 1$, $v_i - a_i = 0$, and $2I_3^{i_L} = 1$, while those to the Z boson and to the Higgs particles are non-diagonal and contain right-handed admixtures. This may be seen from (A.44)–(A.46). The amplitude for the decay (1.1) is again given by Eqn. (2.1), but this time the form factor $\mathcal{V}_M$ is non-diagonal not only in the external charged leptons, but also in the virtual neutrinos due to the $Z\nu_i\nu_j$ coupling, see (A.45):

$$\begin{aligned} \mathcal{V}_M(Q^2) &= \sum_{i,j=1}^{n_G+n_R} \mathbf{B}_{l_1 i}\mathbf{B}^*_{l_2 j}\; V(i,j), \qquad (4.1)\\ V(i,j) &\equiv V(\lambda_i,\lambda_j,\mathbf{C}_{ij}) \\ &= \left[v_W(i,j) + \delta_{ij}v_{WW}(i) + v_\phi(i,j) + \delta_{ij}v_{\phi\phi}(i) + \delta_{ij}v_{W\phi}(i) + \delta_{ij}v_\Sigma(i)\right]. \qquad (4.2) \end{aligned}$$

The new non-diagonal terms arise in diagrams D1 and D3 of Figure 1:

$$\begin{aligned} \mathrm{D1}: v_W(i,j) &= -\mathbf{C}_{ij}\;\left[\lambda_Q\;(C_0 + C_{11} + C_{12} + C_{23}) - 2C_{24} + 1\right] \\ &\quad + \mathbf{C}^*_{ij}\;\sqrt{\lambda_i\lambda_j}\;C_0, \qquad (4.3)\\ \mathrm{D3}:\; v_\phi(i,j) &= -\mathbf{C}_{ij}\;\frac{\lambda_i\lambda_j}{2}\;C_0 + \mathbf{C}^*_{ij}\frac{\sqrt{\lambda_i\lambda_j}}{2}\;\left[\lambda_Q\;C_{23} - 2C_{24} + \frac{1}{2}\right]. \qquad (4.4) \end{aligned}$$

The matrices $\mathbf{B}$ and $\mathbf{C}$ are introduced in Appendix A.3. Both contributions are quite similar to (2.9) and (2.11) when there the right-handed couplings are retained. The other contributions are the same as in the ordinary Dirac case. The vertex reads:

$$\mathcal{V}_M(Q^2) = \sum_{i,j=1}^{n_G+n_R} \mathbf{B}_{l_1 i}\mathbf{B}^*_{l_2 j}\;\left[\delta_{ij}F(\lambda_i) + \mathbf{C}_{ij}G(\lambda_i,\lambda_j) + \mathbf{C}^*_{ij}\sqrt{\lambda_i\lambda_j}H(\lambda_i,\lambda_j)\right], \qquad (4.5)$$

to be compared to the case of n_H heavy sequential Dirac neutrinos ($\mathbf{B} = \mathbf{V}$):

$$\mathcal{V}(Q^2) = \sum_{i=1}^{n_G+n_H} \mathbf{B}_{l_1 i}\mathbf{B}^*_{l_2 i}\;V(\lambda_i). \qquad (4.6)$$

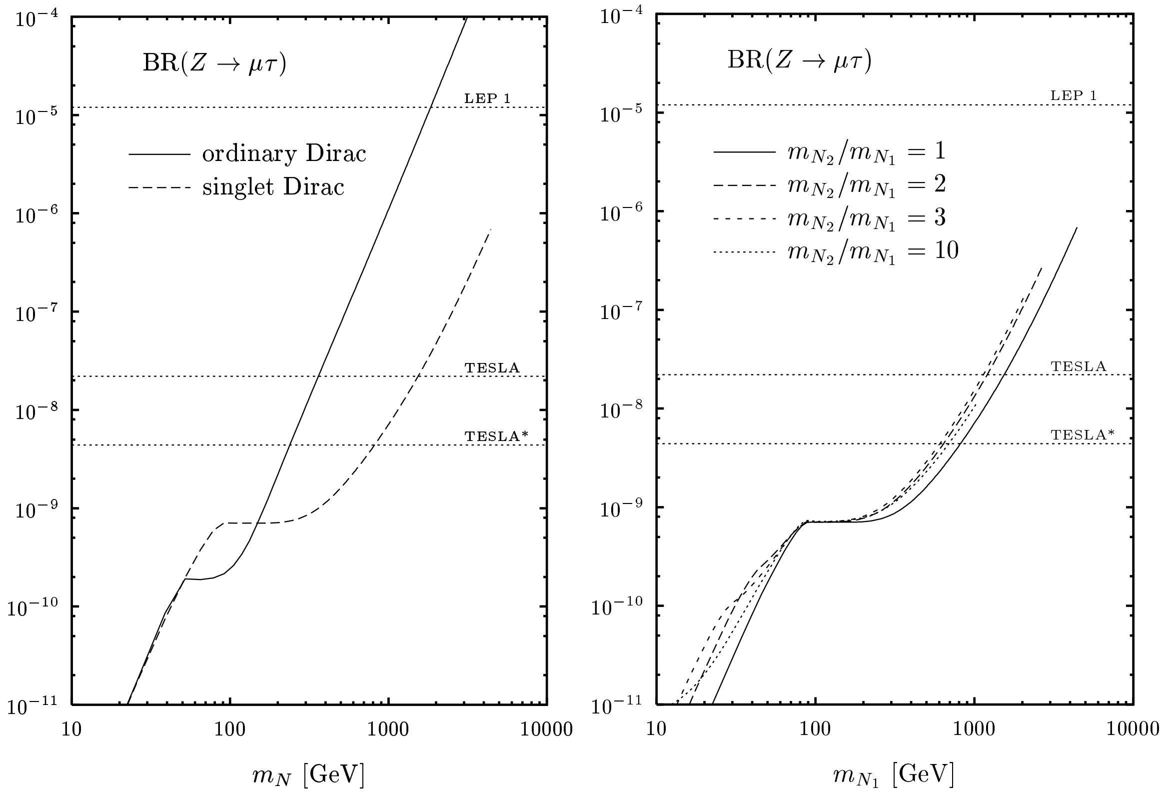

Figure 3: *Predictions for the branching fraction* BR($Z \to \bar{\mu}\tau + \mu\bar{\tau}$) *as defined in the text. Indirect upper limits on the mixing angles from flavour-diagonal observables, obtained in the models used, are taken into account.*

We now have to go into a specific model in order to make definite predictions. For the case chosen, namely the Standard Model extended by two heavy Majorana singlets, the sums over virtual neutrinos involve n_G=3 light and n_R=2 heavy neutrinos. The model is described in Appendix A.3. For some important properties of the mixing matrices **B** and **C** see (A.49)–(A.52). The light neutrino sector is practically massless. Using the properties (A.49)–(A.52), one can then write the form factor $\mathcal{V}_M$ in terms of the heavy-neutrino sector only. Actually, it is:

$$V(\lambda_i) = F(\lambda_i) + G(\lambda_i, \lambda_i). \tag{4.7}$$

Further, the functions F, G, H are uniquely defined and from (A.49)–(A.52), and taking $\lambda_i = 0$ for $i = 1, \ldots, n_G$, it is straightforward to prove that:

$$\begin{aligned} \mathrm{BR}(Z \to l_1^{\mp} l_2^{\pm}) \;\; = \;\; & \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z} \Big| \sum_{i,j=1}^{n_R} \mathbf{B}_{l_1 N_i} \mathbf{B}^*_{l_2 N_j} \\ & \times \big\{ \delta_{N_i N_j} \; [F(\lambda_{N_i}) - F(0) + G(\lambda_{N_i}, 0) + G(0, \lambda_{N_i}) - 2G(0,0)] \\ & \;\; + \mathbf{C}_{N_i N_j} \; [G(\lambda_{N_i}, \lambda_{N_j}) - G(\lambda_{N_i}, 0) - G(0, \lambda_{N_j}) + G(0,0)] \\ & \;\; + \mathbf{C}^*_{N_i N_j} \; \sqrt{\lambda_{N_i} \lambda_{N_j}} \; H(\lambda_{N_i}, \lambda_{N_j}) \big\} \Big|^2 . \end{aligned} \tag{4.8}$$

It is possible to express the couplings $\mathbf{B}, \mathbf{C}$ on the right-hand side by the mass ratio $r = m_{N_2}^2/m_{N_1}^2$ plus the three light-heavy mixings $s_{\nu_e}, s_{\nu_\mu}, s_{\nu_\tau}$. The relations are explicitly given in (A.53)–(A.57). The upper limits for these mixings (given in Appendix B.1) have been used to obtain the graphs in Figure 3(b). The mass ratio r has been taken as a free parameter. Perturbative unitarity constrains the masses of the neutrinos so that they cannot be arbitrarily heavy. This is discussed in Appendix B.2. We see again from the figure that Giga–Z has a discovery potential, preferentially in the large neutrino mass region. Similar curves can be obtained for $Z \to e\mu$ and $Z \to e\tau$.

The distinguished case of two Majorana singlet neutrinos with *equal* masses, forming effectively a singlet Dirac particle [38], results for $r = 1$, see the solid line. This line has been taken over from Figure 3(a), there as the dashed line, in order to show that due to the different coupling structure the simple sequential Dirac neutrino case does not constitute a limiting case for large masses. The deviations are due to the terms from the non-diagonal elementary Z couplings proportional to $\mathbf{C}$ and $\mathbf{C}^*$ in (4.8). The $\mathbf{C}^*$ terms drop out for $r = 1$.

In contrast, predictions for Majorana and Dirac neutrinos approach each other in the limit of small masses. This may be seen from (4.8) using the unitarity relations (A.49)–(A.52) and the Taylor series expansion of the vertex function in powers of the neutrino mass (Appendix D.2). This phenomenon is just another example of what is called in the literature the "practical Dirac-Majorana confusion theorem" [65] (see also the recent discussion in [66, 67] and references therein).

At the end of this section, we would like to comment on the literature for the reaction (1.1). The early papers did not include the more "realistic" and physically most interesting models with Majorana masses. These cases were studied in detail by [38] and, in the context of left-right symmetric models, by [68]. While we reproduce the large mass limit given there, we obtain slight deviations in the medium mass case as given in Eqn. (B1) of [68]. Further, the Figures 7 and 8 in [38] are not exactly reproducible in the intermediate mass range. However, the large mass limit seems to be the only potentially relevant case.

5 Summary

From our study of the decays $Z \to e\mu, e\tau, \mu\tau$, in the context of the Giga–Z option of the Tesla linear collider project, we conclude:

- Neglecting the influence of the mixing angles, the expected branching ratios depend on the neutrino masses m_i and are of the order BR $\sim 10^{-5}(m_i/M_W)^4$ both in the small and large mass limits.

- If there exist only the known three generations of ultra-light neutrinos, $m_\nu \ll M_W$, there is absolutely no hope to see any effect.

- For heavier neutrinos with masses of the order of the weak scale, $m_\nu \sim M_W$, one has to calculate the form factors describing the vertex without any approximation. The necessary exact formulas have also been given.

- The light-heavy mixing angles are not so strongly restricted as with the minimal (one-family) seesaw mechanism when interfamily seesaw type models are considered. But, unfortunately, these mixings have already been constrained to be very small, so that the lepton flavour-violating reactions under study are beyond the discovery reach of the Giga–Z for $m_\nu \sim M_W$.

- Since, ignoring the light-heavy mixings, the branching ratios are proportional to the fourth power of the neutrino masses, there is a discovery potential in the large mass case, $m_\nu \gg M_W$. Nevertheless, they are limited in practice by potentially small mixing factors (bound independently of the heavy neutrino masses by other experiments) and upper neutrino mass bounds from unitarity considerations.

- In fact, there is an interplay between heavy masses and light-heavy mixings: the mixings must be small for very large neutrino masses, since otherwise the scattering matrix elements would grow above the unitarity limit.

Summarizing, the Giga–Z offers nearly three orders of magnitude gain of sensitivity compared to LEP 1. This opens quite interesting opportunities to search for lepton-number changing processes, if there exist heavy neutrinos sufficiently mixing with the light sector, within a quite broad allowed region according to present limits.

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A Lagrangians and Feynman Rules

We make extensive use of the notion of Majorana particles, since in general neutrinos may be of this type, and in fact in GUTs often exactly this happens. Majorana particles [61] are neutral fermions ψ, fulfilling

$$\psi^c = \psi, \tag{A.1}$$

where ψ^c is the charge-conjugate of ψ.

Some introduction on notations are given in Appendix A.1. We observe experimental evidences for ultra-light neutrinos in the known fermion families on the one hand, and on the other there are unifying theories with potentially ultra-heavy Majorana neutrinos. That both phenomena might be related will be made plausible by a toy example in Appendix A.2. A more general ansatz for Majorana mass terms is finally given in Appendix A.3.

A.1 Dirac neutrinos rewritten

The mass-term Lagrangian for Dirac neutrinos is:

$$-\mathcal{L}_D = m_D\left(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L\right) \equiv \frac{1}{2}\left(\overline{\chi^0}\right)\begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix}\left(\chi^0\right), \tag{A.2}$$

where we introduce (self-conjugate) Majorana fields (χ^0)

$$\left(\chi^0\right) = \begin{pmatrix} \nu_L + \nu_L^c \\ \nu_R + \nu_R^c \end{pmatrix}, \tag{A.3}$$

with

$$\nu^c \equiv C\bar{\nu}^T \tag{A.4}$$

and C being the charge-conjugation matrix. The mass matrix can be brought into a diagonal form in the basis of (χ):

$$\begin{aligned} -\mathcal{L}_D &= \frac{1}{2}\left(\overline{\chi}\right)\begin{pmatrix} -m_D & 0 \\ 0 & m_D \end{pmatrix}(\chi) = \frac{1}{2}m_D\left(-\bar{\chi}_1\chi_1 + \bar{\chi}_2\chi_2\right) \\ &= \frac{1}{2}\left(\overline{\xi}\right)\begin{pmatrix} m_D & 0 \\ 0 & m_D \end{pmatrix}(\xi) = \frac{1}{2}m_D\left(\bar{\xi}_1\xi_1 + \bar{\xi}_2\xi_2\right). \end{aligned} \tag{A.5}$$

In the last step, the field component ξ_1 is introduced as a chiral transform of χ_1 in order to make the positive mass eigenvalue explicit. The fields (χ) and (χ^0) are related by the unitary matrix $\mathbf{U}$, and ξ_1 additionally by γ_5:

$$\left(\chi^0\right) = \mathbf{U}\,(\chi); \quad \mathbf{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \tag{A.6}$$

$$\xi_1 = \gamma_5\chi_1 = \frac{1}{\sqrt{2}}\left(-\nu_L + \nu_L^c - \nu_R + \nu_R^c\right), \tag{A.7}$$

$$\xi_2 = \chi_2 = \frac{1}{\sqrt{2}}\left(\nu_L + \nu_L^c + \nu_R + \nu_R^c\right). \tag{A.8}$$

With the above chain of equations we have shown that one ordinary Dirac neutrino is equivalent to two Majorana neutrinos of equal mass and opposite CP parities. Evidently, if the mass matrix is not of type (A.2), then true Majorana particles are realized.

A.2 The seesaw mechanism

The seesaw mechanism [45, 46, 47] allows to understand the lightness of the known neutrinos by the introduction of heavier ones. However, it is not the only possible solution to this puzzle; see e.g. [59, 69] for a nice discussion.

In Appendix A.1 it was shown that a Dirac-neutrino mass term may be written as symmetric 2×2 matrix with only off-diagonal elements. Consider now the case where only *one* right-handed singlet neutrino is added to the ordinary left-handed doublets of the SM. Let's take one generation of left-handed light neutrinos for simplicity and with no loss of generality. The corresponding mass matrix is in general

$$\mathbf{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \tag{A.9}$$

and can be diagonalized by

$$\mathbf{U} = \begin{pmatrix} \cos\theta_\nu & \sin\theta_\nu \\ -\sin\theta_\nu & \cos\theta_\nu \end{pmatrix}, \tag{A.10}$$

with

$$\tan 2\theta_\nu = \frac{2m_D}{m_R - m_L}, \quad \cos 2\theta = \frac{m_R - m_L}{\sqrt{(m_R - m_L)^2 + 4m_D^2}} \tag{A.11}$$

yielding two eigenstates with different masses

$$m_\nu, m_N = \frac{1}{2}\left\{m_R + m_L \mp \left[(m_R - m_L)^2 + 4m_D^2\right]^{1/2}\right\}. \tag{A.12}$$

That is, the most general mass term of one four-component self-conjugate field describes two Majorana particles with different masses. There is a particular configuration that corresponds to one Dirac neutrino, as shown in Appendix A.1.

In the SM, $m_L = 0$ since the SM Higgs sector consists of a Higgs doublet (see Appendix A.3). Take now $m_R \gg m_D$. Then, the two physical states[3] are a light and a heavy neutrino with masses

$$m_\nu \simeq m_D^2/m_R, \tag{A.13}$$

$$m_N \simeq m_R, \tag{A.14}$$

and the light-heavy mixing angle is:

$$s_\nu \equiv \sin\theta_\nu \simeq m_D/m_R \simeq \sqrt{m_\nu/m_N}. \tag{A.15}$$

This is the minimal *seesaw* mechanism: the larger the heavy scale is, the smaller the light neutrino masses are, and, at the same time, the smaller the mixing becomes.

[3]Actually, one of the mass eigenvalues in (A.12) is negative. This is not a problem since then the true mass eigenstate is a chiral transform of the original field which has a mass term of opposite sign as in (A.8) [70].

Actually, by taking $m_N \gtrsim M_Z$ and the present bounds on the light neutrino masses [13],

$$m_{\nu_e} \lesssim 2.5 \text{ eV}, \tag{A.16}$$
$$m_{\nu_\mu} \lesssim 160 \text{ keV}, \tag{A.17}$$
$$m_{\nu_\tau} \lesssim 15 \text{ MeV}, \tag{A.18}$$

we get from (A.11) the light-heavy mixing angles:

$$s^2_{\nu_e} \lesssim 3 \times 10^{-11}, \tag{A.19}$$
$$s^2_{\nu_\mu} \lesssim 2 \times 10^{-6}, \tag{A.20}$$
$$s^2_{\nu_\tau} \lesssim 2 \times 10^{-4}. \tag{A.21}$$

These mixing angles are too small to be constrained by the experimental LEP and low-energy limits (B.6)–(B.8) [50, 51].

Nevertheless, one might have $m_L \neq 0$, by introducing a Higgs triplet in the SM, [4] and hope for a conspiracy between m_L and m_R to get light-heavy mixings of order one. This does not seem very reasonable. Thus, the minimal seesaw apparently lacks of phenomenological interest.

A.3 Majorana neutrinos

In the *interfamily* seesaw-type models, several right-handed neutrinos are introduced and large light-heavy mixings can be obtained even for $m_L = 0$ [71, 72, 73].

Therefore following [38], we now introduce in our study *two* heavy right-handed singlet neutrinos and treat the light-heavy mixings s_{ν_l} and the heavy masses m_{N_1}, m_{N_2} as free phenomenological parameters.

Consider an extension of the SM that incorporates n_R right-handed neutrinos ν_R (singlets under SU(2)⊗U(1)) to the already present set of $n_G = 3$ generations of left-handed neutrino doublets. We keep the Higgs sector untouched.

It is convenient to arrange all the independent neutrino degrees of freedom into two vectors of *left-handed* fields:

$$(\nu^0) \equiv (\nu_e, \nu_\mu, \nu_\tau), \tag{A.22}$$
$$(N^0) \equiv (\nu_R^c)_i \quad i = 1, \ldots, n_R. \tag{A.23}$$

The charge-conjugates of right-handed neutrinos have been introduced:

$$\nu_R^c \equiv C\overline{\nu_R}^T. \tag{A.24}$$

The conjugate field has chirality and lepton/fermion numbers opposite to the original field (e.g. the ν_R^c's are left-handed).

[4] The introduction of a Higgs triplet affects the value of the parameter $\rho = M_W^2/(M_Z^2 c_W^2)$.

In the basis

$$(n^0) = (\nu^0, N^0) \tag{A.25}$$

the most general mass-term Lagrangian reads:

$$-\mathcal{L}_M = \frac{1}{2}(\overline{n^{0c}})\ \mathbf{M}\ (n^0) + \text{h.c.} = \frac{1}{2}(n^0)^T C\ \mathbf{M}\ (n^0) + \text{h.c.}, \tag{A.26}$$

where the Majorana fields

$$(\chi^0) \equiv (n^0) + (n^{0c}), \tag{A.27}$$

containing both chiralities, have been introduced.

The mass matrix $\mathbf{M}$ is symmetric and can be written in a block form:

$$\mathbf{M} = \begin{pmatrix} \mathbf{m}_L & \mathbf{m}_D^T \\ \mathbf{m}_D & \mathbf{m}_R \end{pmatrix}. \tag{A.28}$$

It has dimension $n_G + n_R$ and can be diagonalized by a unitary matrix $\mathbf{U}$ of the same dimension,

$$\begin{aligned} \hat{\mathbf{M}} = \mathbf{U}^T\mathbf{M}\mathbf{U} = (\mathbf{U}^*)^\dagger\mathbf{M}\mathbf{U} &= \mathbf{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, m_{N_1}, m_{N_2}) \\ &\simeq \mathbf{diag}(0, 0, 0, m_{N_1}, m_{N_2}), \end{aligned} \tag{A.29}$$

since we are mostly interested in a heavy neutrino sector consisting of two right-handed neutrinos. The interaction eigenstates, in terms of the left- and right-handed components of the mass eigenstates, are given by:

$$\text{Left-handed: } (n^0) \equiv \begin{pmatrix} \nu^0 \\ N^0 \end{pmatrix} = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \mathbf{U}\ P_L\ (\chi) \tag{A.30}$$

$$\text{Right-handed: } (n^{0c}) \equiv \begin{pmatrix} \nu^{0c} \\ N^{0c} \end{pmatrix} = \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} = \mathbf{U}^*\ P_R\ (\chi), \tag{A.31}$$

with

$$P_{R,L} \equiv \frac{1}{2}(1 \pm \gamma_5). \tag{A.32}$$

The diagonal blocks in $\mathbf{M}$, connecting states with opposite fermion number ($\mathbf{m}_L, \mathbf{m}_R$) are called "Majorana" mass terms, while the off-diagonal ones ($\mathbf{m}_D$) are "Dirac" mass terms that conserve fermion number.[5] The mass terms arise, after the spontaneous symmetry breaking (SSB) of $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{U}(1)_Q$, from the Yukawa coupling of the fermion fields to the neutral Higgs field. The $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ quantum numbers of a Higgs doublet and the fermion bilinears that can be constructed from the doublet and singlet neutrinos are

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (\mathbf{2}, 1) \tag{A.33}$$

$$\Delta L = \ \ 0: \quad \overline{\nu_R}\nu_L \sim (\mathbf{1}, 0) \otimes (\mathbf{2}, -1) = (\mathbf{2}, -1) \tag{A.34}$$

$$\Delta L = \pm 2: \quad \overline{\nu_L^c}\nu_L \sim (\mathbf{2}, -1) \otimes (\mathbf{2}, -1) = (\mathbf{1}, -2) \oplus (\mathbf{3}, -2) \tag{A.35}$$

$$\Delta L = \pm 2: \quad \overline{\nu_R^c}\nu_R \sim (\mathbf{1}, 0) \otimes (\mathbf{1}, 0) = (\mathbf{1}, 0) \quad \text{(neutral)}. \tag{A.36}$$

[5] Non-diagonal Dirac mass terms produce transitions between states with different individual lepton numbers but the total lepton number, i.e. the fermion number, is conserved.

Therefore in a model with only Higgs doublets, the entry $\mathbf{m}_L = \mathbf{0}$.

Our convention for the covariant derivative acting on a fermion doublet is

$$D_\mu = \partial_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu + ig'\frac{Y}{2}B_\mu \tag{A.37}$$

with

$$Q_f = I_3^{f_L} + Y/2. \tag{A.38}$$

After SSB one gets

$$\begin{aligned} e &= g s_W = g' c_W, && \text{(A.39)} \\ \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} &= \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}, && \text{(A.40)} \\ W^\mp_\mu &= \frac{1}{\sqrt{2}}\left(W^1_\mu \pm i W^2_\mu\right). && \text{(A.41)} \end{aligned}$$

The interaction Lagrangians of neutrinos with W, Z and charged Goldstone bosons $\phi^\pm$, in the weak basis[6]

$$\begin{aligned} \mathcal{L}_W &= -\frac{g}{\sqrt{2}} W^-_\mu \sum_{i,j=1}^{n_G} \bar{l}_i \gamma^\mu P_L \nu_{L_j} + \text{h.c.}, && \text{(A.42)} \\ \mathcal{L}_Z &= -\frac{g}{2c_W} Z_\mu \sum_{k=1}^{n_G} \left[\overline{\nu_{L_k}} \gamma^\mu P_L \nu_{L_k} - \overline{\nu^c_{L_k}} \gamma^\mu P_R \nu^c_{L_k}\right], && \text{(A.43)} \end{aligned}$$

become, in terms of physical neutrinos χ_j with masses m_j, $j = 1, ..., n_G + n_R$:

$$\begin{aligned} \mathcal{L}_W &= -\frac{g}{\sqrt{2}} W^-_\mu \sum_{i=1}^{n_G} \sum_{j=1}^{n_G+n_R} \mathbf{B}_{l_i j} \bar{l}_i \gamma^\mu P_L \chi_j + \text{h.c.}, && \text{(A.44)} \\ \mathcal{L}_Z &= -\frac{g}{2c_W} Z_\mu \sum_{i,j=1}^{n_G+n_R} \bar{\chi}_i \gamma^\mu (P_L \mathbf{C}_{ij} - P_R \mathbf{C}^*_{ij}) \chi_j, && \text{(A.45)} \\ \mathcal{L}_{\phi^\pm} &= -\frac{g}{\sqrt{2}} \phi^- \sum_{i=1}^{n_G} \sum_{j=1}^{n_G+n_R} \mathbf{B}_{l_i j} \bar{l}_i \left(\frac{m_{l_i}}{M_W} P_L - \frac{m_j}{M_W} P_R\right) \chi_j + \text{h.c.}, && \text{(A.46)} \end{aligned}$$

with

$$\begin{aligned} \mathbf{B}_{l_i j} &\equiv \sum_{k=1}^{n_G} \mathbf{V}^*_{l_i k} \mathbf{U}_{kj}, && \text{(A.47)} \\ \mathbf{C}_{ij} &\equiv \sum_{k=1}^{n_G} \mathbf{U}^*_{ki} \mathbf{U}_{kj}. && \text{(A.48)} \end{aligned}$$

[6]The interaction of the Z to two (right-handed) charge-conjugate neutrinos (non-existent in the ordinary SM) has been included in (A.43), making use of the bilinear transformation properties under charge-conjugation: $\bar{\psi}\gamma^\mu P_{L,R}\psi \to -\overline{\psi^c}\gamma^\mu P_{R,L}\psi^c$.

$\mathbf{V}_{l_i j}$ and $\mathbf{B}_{l_i j}$ are the leptonic CKM mixing matrices and its *generalized* version, respectively [62, 72]. For Dirac particles $\mathbf{B} = \mathbf{V}$, $\mathbf{C}_{ij} = \delta_{ij}$ and $\mathbf{C}^*_{ij} = 0$. For Majorana particles, in contrast, there are NC couplings of different flavours (FCNC) and with both left- and right-handed components.

The matrix $\mathbf{V}_{l_i j}$ is quadratic of dimension n_G and $\mathbf{B}_{l_i j}$ is rectangular $n_G \times (n_G + n_R)$ and incorporates lepton-flavour changing mixings. The matrix $\mathbf{C}_{ij}$ is quadratic, has dimension $(n_G + n_R)$, and causes flavour non-diagonal $Z\chi_i\chi_j$ interactions.

These interaction Lagrangians involve Majorana fermions that complicate the evaluation of S matrix elements, since extra Wick contractions survive in comparison to the case with only Dirac fermions. Following [74, 64], one can write Feynman rules resembling the ones of the Dirac fermions, based on the well defined *fermion flow*, rather than on the *fermion number flow* which is not preserved in the vertices with Majorana fermions: after fixing an arbitrary orientation (fermion flow) for a given diagram, the vertices can be read off from the Lagrangian as usual, *but* for every vertex $\bar{f}_1 \Gamma f_2$ one has to add the reversed one, $\overline{f_1^c}\Gamma' f_2^c$, with $\Gamma' = C\Gamma^T C^{-1}$. The fermion propagators are the usual ones. This effectively yields the same result for the vertex $W l_i \chi_j$ but a *factor two* larger for the vertex $Z\chi_i\chi_j$ in comparison to the case of Dirac neutrinos, since for two Majorana fermions $(\chi = \chi^c)$ $\Gamma = \Gamma'$.

Important note: elsewhere in the text we refer to ν_i and N_i as the neutrino physical states, rather than χ_i, to simplify the presentation, but with no possible confusion, since we always work in the physical basis.

The matrices $\mathbf{B}$ and $\mathbf{C}$ obey a number of useful relations [37]:

$$\sum_{j=1}^{n_G+n_R} \mathbf{B}_{l_1 j}\mathbf{B}^*_{l_2 j} = \delta_{l_1 l_2}, \tag{A.49}$$

$$\sum_{k=1}^{n_G+n_R} \mathbf{C}_{ik}\mathbf{C}^*_{jk} = \sum_{k=1}^{n_G} \mathbf{B}_{l_k i}\mathbf{B}^*_{l_k j} = \mathbf{C}_{ij}, \tag{A.50}$$

$$\sum_{k=1}^{n_G+n_R} \mathbf{B}_{lk}\mathbf{C}_{kj} = \mathbf{B}_{lj}, \tag{A.51}$$

$$\sum_{k=1}^{n_G+n_R} m_k \mathbf{C}_{ik}\mathbf{C}_{jk} = \sum_{k=1}^{n_G+n_R} m_k \mathbf{B}_{lk}\mathbf{C}^*_{ki} = \sum_{k=1}^{n_G+n_R} m_k \mathbf{B}_{l_1 k}\mathbf{B}_{l_2 k} = 0. \tag{A.52}$$

In the case of $n_R = 2$ the matrix elements involving heavy neutrinos can be obtained from (A.50), (A.52) in terms of the light-heavy mixing angles (B.3) and the ratio of the two heavy masses squared $r \equiv m^2_{N_2}/m^2_{N_1}$, assuming that the light sector consists of massless neutrinos [38]:

$$\mathbf{B}_{l_k N_1} = \frac{r^{1/4}}{\sqrt{1 + r^{1/2}}}\, s_{\nu_k}, \tag{A.53}$$

$$\mathbf{B}_{l_k N_2} = \frac{i}{\sqrt{1 + r^{1/2}}}\, s_{\nu_k}, \tag{A.54}$$

and

$$\mathbf{C}_{N_1N_1} = \frac{r^{1/2}}{1+r^{1/2}} \sum_{k=1}^{n_G} s_{\nu_k}^2, \tag{A.55}$$

$$\mathbf{C}_{N_2N_2} = \frac{1}{1+r^{1/2}} \sum_{k=1}^{n_G} s_{\nu_k}^2, \tag{A.56}$$

$$\mathbf{C}_{N_1N_2} = -\mathbf{C}_{N_2N_1} = \frac{ir^{1/4}}{1+r^{1/2}} \sum_{k=1}^{n_G} s_{\nu_k}^2. \tag{A.57}$$

B Constraints on Heavy Neutrinos

B.1 Experimental bounds

Several kinds of constraints on heavy-neutrino masses and light-heavy mixings can be obtained from experiment.

Direct production searches establish the following limits on the neutrino masses at 95% c.l. [13]:

$$\text{Stable neutrinos: } m_N > 45.0\ (39.5)\ \text{GeV} \tag{B.1}$$

$$\text{Unstable neutrinos: } m_N > 69.0\ (58.2)\ \text{GeV}, \tag{B.2}$$

for Dirac (Majorana) particles, respectively.

A general formalism to describe light-heavy mixings was developed in [49, 58]. The mixing angles in our notation [38] correspond to

$$s_{\nu_k}^2 \equiv \sum_i |\mathbf{B}_{l_kN_i}|^2. \tag{B.3}$$

Indirect constraints on the masses and bounds on the mixings are provided by two categories of LEP and low energy experiments:

(i) Flavour-diagonal processes. They include mass-independent and model-independent light-heavy mixing constraints [49, 50, 51, 52] from tests of lepton universality and CKM unitarity and measurements of the Z boson invisible width, as well as other less sensitive studies like W mass measurements and low energy experiments like neutrino scattering, atomic parity violation, etc. Flavour-conserving leptonic decays $Z \to l^-l^+$ depend on masses and mixings through loop contributions and provide alternative constraints [53].

(ii) Flavour-changing processes. They include rare processes like $\mu \to e\gamma$, $\mu \to ee^+e^-$, $\mu \to e$ conversion in nuclei, $\tau \to l_a l_b^+ l_c^-$ and $Z \to l_a^- l_b^+$ [49, 59]. They are mass-dependent (except for $\mu \to e\gamma$ to a good approximation). One can get also less stringent light-heavy mixing constraints from oscillation experiments [58, 52].

The most stringent *present* bounds on the light-heavy mixings are provided by the flavour-diagonal processes. Exceptions are the ones involving the first two lepton families such as $\mu \to e\gamma$, $Z \to e\mu$ and $\mu \to ee^+e^-$ [59].

For illustration we show an example of the flavour-diagonal constraints. The effective muon decay constant G_μ is related to the coupling G_F of the standard model by [49, 50, 51]:

$$G_\mu = c_{\nu_e} c_{\nu_\mu} G_F. \tag{B.4}$$

The unitarity constraint for the first row of the CKM quark-mixing matrix implies [51]

$$\sum_{i=1}^{3} |V_{ui}|^2 \;=\; \left(\frac{c_{\nu_e} G_F}{G_\mu}\right)^2 = \frac{1}{c^2_{\nu_\mu}} = 0.9992 \pm 0.0014. \tag{B.5}$$

As a summary, from [49, 50, 51] (90% c.l.):

$$\begin{aligned} s^2_{\nu_e} &< 0.0071\ (0.005), && \text{(B.6)}\\ s^2_{\nu_\mu} &< 0.0014, && \text{(B.7)}\\ s^2_{\nu_\tau} &< 0.033\ (0.01), && \text{(B.8)} \end{aligned}$$

where the most conservative bounds are obtained assuming any kind of heavy neutrinos and the ones in brackets correspond to the case of SU(2) singlets.

From the most recent update[7] by [52], assuming only heavy singlets, one gets (90% c.l.):

$$\begin{aligned} s^2_{\nu_e} &< 0.012, && \text{(B.9)}\\ s^2_{\nu_\mu} &< 0.0096, && \text{(B.10)}\\ s^2_{\nu_\tau} &< 0.016. && \text{(B.11)} \end{aligned}$$

A final remark is in order here. Using the Schwartz inequalities [58],

$$|\sum_i \mathbf{B}_{l_a N_i} \mathbf{B}^*_{l_b N_i}|^2 < s^2_{\nu_a} s^2_{\nu_b}, \tag{B.12}$$

one can infer *indirect* upper limits on the off-diagonal mixings (relevant for the flavour-changing processes) from the previous flavour-diagonal constraints. Nevertheless, for our scenario (iii), as already mentioned, the mixings can be obtained exactly from the properties of **B** and **C** and such inequalities are not needed.

B.2 Decoupling and neutrino-mass upper limits from perturbative unitarity

The heavy-neutrino masses are restricted by the perturbative unitarity condition on the decay width of heavy neutrinos [75, 76, 77, 78, 79],

$$\Gamma_{N_i} \leq \frac{1}{2} m_{N_i}. \tag{B.13}$$

[7]These latest bounds are more conservative than the earlier ones in [51] due to the fact that present determinations of the elements of the first row of the CKM matrix are not compatible with unitarity, and hence this constraint is eliminated from the analysis.

The total decay width of a heavy Dirac neutrino (four d.o.f.) with mass $m_{N_i} \gg M_W, M_Z, M_H$ is [78, 79, 80, 72]:

$$\begin{aligned}
\Gamma_{N_i} &= \sum_{k=1}^{3} \Gamma(N_i \to l_k^- W^+) + \sum_{k=1}^{3} [\Gamma(N_i \to \nu_k Z) + \Gamma(N_i \to \nu_k H)] \\
&\simeq \frac{\alpha_W}{8M_W^2} m_{N_i}^3 \sum_{k=1}^{3} |\mathbf{B}_{l_k N_i}|^2, \qquad \text{(B.14)}
\end{aligned}$$

and a factor two larger for a heavy Majorana neutrino (two d.o.f.). Therefore, the perturbative unitarity bound expressed in (B.13) reads

$$m_{N_i}^2 \sum_{k=1}^{3} |\mathbf{B}_{l_k N_i}|^2 = m_{N_i}^2 \mathbf{C}_{N_i N_i} \leq \begin{cases} 4M_W^2/\alpha_W \text{ for a Dirac neutrino} \\ \\ 2M_W^2/\alpha_W \text{ for a Majorana neutrino,} \end{cases} \qquad \text{(B.15)}$$

(no summation over repeated indices is understood) which shows implicitly that *heavy neutrinos decouple* [78, 79], in accordance with the Appelquist-Carazzone theorem [81, 82]: the unacceptable large-mass behaviour of the amplitudes ($\sim m_N^2$) is actually cured when the light-heavy mixing ($\sim m_N^{-2}$) is taken into account [83, 84].

Taking the values of $\mathbf{B}_{l_k N_i}$ in terms of the light-heavy mixing angles (B.3) for one Dirac or for two heavy Majorana neutrinos (A.53), (A.54) one can get the following upper limits:

$$m_N^2 \leq \frac{4M_W^2}{\alpha_W} \left[\sum_{k=1}^{3} s_{\nu_k}^2 \right]^{-1} \qquad \text{(B.16)}$$

for a heavy Dirac neutrino, and

$$m_{N_1}^2 \equiv \frac{1}{r} m_{N_2}^2 \leq \frac{2M_W^2}{\alpha_W} \frac{1 + r^{1/2}}{r} \left[\sum_{k=1}^{3} s_{\nu_k}^2 \right]^{-1} \qquad \text{(B.17)}$$

for two heavy Majorana singlets. The latter bound is very stringent when m_{N_1} and m_{N_2} are very different. The upper mass limits on heavy neutrinos, from (B.16), (B.17) are then:

$$\begin{aligned}
\text{scenario (ii):} \quad m_N^2 &\lesssim (4.2, 4.4 \text{ TeV})^2 \qquad \text{(B.18)} \\
\text{scenario (iii):} \ m_{N_1}^2 \equiv \frac{1}{r} m_{N_2}^2 &\lesssim \frac{1 + r^{1/2}}{r} \times (3.0, 3.1 \text{ TeV})^2, \qquad \text{(B.19)}
\end{aligned}$$

using the bounds (B.6)–(B.8) and (B.9)–(B.11), respectively.

C The Vertex Function

We use the notations of [85]. The vertex function contains the following one-loop integrals in D dimensions [8, 86]:

$$\frac{i}{16\pi^2}A(m_0^2) = \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{1}{\mathcal{D}_0}, \tag{C.1}$$

$$\frac{i}{16\pi^2}\{B_0, B^\mu\}(p_1^2; m_0^2, m_1^2) = \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{\{1, q^\mu\}}{\mathcal{D}_0\mathcal{D}_1}, \tag{C.2}$$

$$\frac{i}{16\pi^2}\{C_0, C^\mu, C^{\mu\nu}\}(p_1^2, Q^2, p_2^2; m_0^2, m_1^2, m_2^2) = \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{\{1, q^\mu, q^\mu q^\nu\}}{\mathcal{D}_0\mathcal{D}_1\mathcal{D}_2}, \tag{C.3}$$

with

$$Q^2 = (p_2 - p_1)^2 \tag{C.4}$$

and

$$\mathcal{D}_0 = q^2 - m_0^2 + i\epsilon, \tag{C.5}$$

$$\mathcal{D}_1 = (q + p_1)^2 - m_1^2 + i\epsilon, \tag{C.6}$$

$$\mathcal{D}_2 = (q + p_2)^2 - m_2^2 + i\epsilon. \tag{C.7}$$

They are decomposed into tensor integrals according to their Lorentz structure:

$$B^\mu = p^\mu B_1, \tag{C.8}$$

$$C^\mu = p_1^\mu C_{11} + p_2^\mu C_{12}, \tag{C.9}$$

$$C^{\mu\nu} = p_1^\mu p_1^\nu C_{21} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu)C_{23} + g^{\mu\nu}C_{24}. \tag{C.10}$$

We employ the following dimensionless tensor integrals and their abbreviations:

$$\begin{aligned} B_1 &\equiv B_1(0; \lambda_i, 1) \\ &= B_1(0; m_i^2, M_W^2), \end{aligned} \tag{C.11}$$

$$\begin{aligned} C_{\{0,11,12,21,22,23\}} &\equiv C_{\{\dots\}}(0, \lambda_Q, 0; 1, \lambda_i, \lambda_j) \\ &= M_W^2 C_{\{\dots\}}(0, Q^2, 0; M_W^2, m_i^2, m_j^2), \end{aligned} \tag{C.12}$$

$$\begin{aligned} C_{24} &\equiv C_{24}(0, \lambda_Q, 0; 1, \lambda_i, \lambda_j) \\ &= C_{24}(0, Q^2, 0; M_W^2, m_i^2, m_j^2), \end{aligned} \tag{C.13}$$

$$\begin{aligned} \bar{C}_{\{0,11,12,21,22,23\}} &\equiv C_{\{\dots\}}(0, \lambda_Q, 0; \lambda_i, 1, 1) \\ &= M_W^2 C_{\{\dots\}}(0, Q^2, 0; m_i^2, M_W^2, M_W^2), \end{aligned} \tag{C.14}$$

$$\begin{aligned} \bar{C}_{24} &\equiv \bar{C}_{24}(0, \lambda_Q, 0; \lambda_i, 1, 1) \\ &= C_{24}(0, Q^2, 0; m_i^2, M_W^2, M_W^2). \end{aligned} \tag{C.15}$$

On the Z mass shell it is $\lambda_Q = \lambda_Z$. The $\lambda_Q, \lambda_Z, \lambda_i$ are introduced in (1.18), (2.5), and (2.6).

The explicit expressions for the loop functions are:

$$
\begin{aligned}
C_{\{0,11,23\}} &= \int_0^1 dx \int_0^x \frac{dy}{\mathcal{D}}\{-1, y, -(1-x)y\}, && \text{(C.16)}\\
C_{12}(\lambda_i, \lambda_j) &= C_{11}(\lambda_j, \lambda_i), && \text{(C.17)}\\
C_{24} &= -\frac{1}{2(D-4)} - \frac{1}{2}\int_0^1 dx \int_0^x dy \ln \mathcal{D}, && \text{(C.18)}
\end{aligned}
$$

with

$$
\begin{aligned}
\mathcal{D} &= D_{ijk}\\
&= \lambda_Q xy + (\lambda_k - \lambda_j)x + (-\lambda_Q + \lambda_i - \lambda_k)y + \lambda_j - i\epsilon, && \text{(C.19)}
\end{aligned}
$$

and the correspondences are: for the non-abelian diagrams (with elementary ZWW, $ZW\phi$, $Z\phi\phi$ vertices), (i,j) are virtual W, ϕ bosons and k a neutrino, while for the abelian diagrams k is the W, ϕ boson and (i,j) are neutrinos. On the Z boson mass shell:

$$
\begin{aligned}
\mathcal{D} &\equiv D_{ijW} = \lambda_Z xy + (1-\lambda_j)x + [-\lambda_Z + (\lambda_i - 1)]y + \lambda_j - i\epsilon, && \text{(C.20)}\\
\bar{\mathcal{D}} &\equiv D_{WWi} = \lambda_Z xy - (1-\lambda_i)x + [-\lambda_Z - (\lambda_i - 1)]y + 1 - i\epsilon. && \text{(C.21)}
\end{aligned}
$$

In the Dirac case, it is $\lambda_i = \lambda_j$. Further,

$$
\begin{aligned}
B_1 &= \frac{1}{D-4} + \int_0^1 x dx \ln[x + \lambda_i(1-x) - i\epsilon]\\
&= \frac{1}{D-4} + \frac{\lambda_i}{2(1-\lambda_i)}\left(1 + \frac{\lambda_i \ln \lambda_i}{1-\lambda_i}\right) - \frac{1}{4}. && \text{(C.22)}
\end{aligned}
$$

As may be seen, the tensor integrals B_1, C_{24} and $\bar{C}_{24}$ are ultraviolet–divergent in D dimensions. We mention that the functions are defined here with Minkowskian metric, so that all the functions introduced, except for $B_0, B_1, C_{24}, \bar{C}_{24}$, have different sign from those used in e.g. [10] (Euclidean metric). In principle, the divergent parts depend on the regularization scheme and could differ by a finite, universal term yet. Recalling the UV-behaviour of the divergent integrals B_1, C_{24} and $\bar{C}_{24}$ (see (C.22) and (C.18)), we get divergent, mass-dependent contributions from individual diagrams to the vertex V.

For the Dirac case:

$$
\begin{aligned}
v_\phi(i) &\sim -(v_i + a_i) \times [\text{finite}] - (v_i - a_i) \times \left[\frac{1}{2}\frac{\lambda_i}{D-4} + \text{finite}\right], && \text{(C.23)}\\
v_{\phi\phi}(i) &\sim -(2I_3^{i_L})\,(1 - 2s_W^2) \times \left[-\frac{1}{2}\frac{\lambda_i}{D-4} + \text{finite}\right], && \text{(C.24)}\\
v_\Sigma(i) &\sim \frac{1}{2}(v_i + a_i - 4c_W^2 a_i)\left[\frac{2}{D-4} + \frac{\lambda_i}{D-4} + \text{finite}\right]. && \text{(C.25)}
\end{aligned}
$$

The sum of the terms proportional to $\lambda_i/(D-4)$ vanishes. The constant divergent terms (not shown here) sum up for individual vertices $V(\lambda_i)$, but vanish due to the unitarity of the mixing matrix for the complete vertex function (1.15):

$$
\mathcal{V} \sim (2I_3^{i_L})\, 4c_W^2\, \frac{1}{D-4}\, \delta_{l_1 l_2} = 0. \qquad \text{(C.26)}
$$

For the Majorana case ($2I_3^{i_L} = 1$, $v_i = a_i = 1/2$):

$$v_\phi(i,j) \sim -\mathbf{C}_{ij} \times [\text{finite}] + \mathbf{C}^*_{ij} \times \left[\frac{1}{2} \frac{\sqrt{\lambda_i \lambda_j}}{D-4} + \text{finite} \right], \tag{C.27}$$

$$v_{\phi\phi}(i,j) \sim -\delta_{ij}\ (1 - 2s_W^2) \times \left[-\frac{1}{2} \frac{\lambda_i}{D-4} + \text{finite} \right], \tag{C.28}$$

$$v_\Sigma(i,j) \sim \delta_{ij}\ \frac{1}{2}(-1 + 2s_W^2) \left[\frac{2}{D-4} + \frac{\lambda_i}{D-4} + \text{finite} \right]. \tag{C.29}$$

The mass-dependent divergent terms in (C.28) and (C.29) cancel each other, and the one proportional to $\mathbf{C}^*_{ij}$ in (C.27) drops out due to the unitarity condition (A.52) when the sum over virtual neutrinos is performed in (4.1). The constant divergent terms vanish again due to the unitarity relations (A.49)–(A.52):

$$\mathcal{V}_M \sim 4c_W^2\ \frac{1}{D-4}\ \delta_{l_1 l_2} = 0 \tag{C.30}$$

D The Vertex Function for Large and Small Neutrino Masses

D.1 The vertex for large neutrino mass, $\lambda_i \gg 1$

We now consider the vertex function in the limit of large Dirac-neutrino masses ($\lambda_i = \lambda_j$). The leading terms of the one-loop functions for $\lambda_Q = \lambda_Z$ are:[8]

$$
\begin{aligned}
C_0 &= -\frac{1}{\lambda_i} + \frac{\ln\lambda_i}{\lambda_i^2} - (12+\lambda_Z)\frac{1}{12\lambda_i^2} + \dots, && \text{(D.31)}\\
C_{11} &= C_{12} = \frac{1}{4\lambda_i} + \dots, && \text{(D.32)}\\
C_{23} &= -\frac{1}{18\lambda_i} + \dots, && \text{(D.33)}\\
C_{24} &= -\frac{1}{2(D-4)} - \frac{1}{4}\ln\lambda_i + \frac{1}{8} + (-9+\lambda_Z)\frac{1}{36\lambda_i} + \dots, && \text{(D.34)}\\
B_1 &= \frac{1}{(D-4)} + \frac{1}{2}\ln\lambda_i - \frac{3}{4} + \frac{\ln\lambda_i}{\lambda_i} - \frac{1}{2\lambda_i} + \dots, && \text{(D.35)}\\
\bar{C}_0 &= -\frac{\ln\lambda_i}{\lambda_i} - [1-4a(y)]\frac{1}{\lambda_i} + \dots, && \text{(D.36)}\\
\bar{C}_{11} &= \bar{C}_{12} = \frac{1}{2}\frac{\ln\lambda_i}{\lambda_i} + \dots, && \text{(D.37)}\\
\bar{C}_{23} &= -\frac{1}{6}\frac{\ln\lambda_i}{\lambda_i} + \dots, && \text{(D.38)}\\
\bar{C}_{24} &= -\frac{1}{2(D-4)} - \frac{1}{4}\ln\lambda_i + \frac{3}{8} + (-6+\lambda_Z)\frac{\ln\lambda_i}{12\lambda_i}\\
&\quad + [-30+5\lambda_Z+24(4-\lambda_Z)a(y)]\frac{1}{72\lambda_i} + \dots, && \text{(D.39)}
\end{aligned}
$$

with

$$
\begin{aligned}
y &= \sqrt{1/\lambda_Z - 1/4}, && \text{(D.40)}\\
a(y) &= y\arctan(1/2y). && \text{(D.41)}
\end{aligned}
$$

The large Dirac-neutrino mass limit of the vertex function is:

$$
\begin{aligned}
V(\lambda_i) = I_3^{\nu_L}\Bigg[&\lambda_i + \left(3 - \frac{\lambda_Z}{6}(1-2s_W^2)\right)\ln\lambda_i\\
&+ \frac{1}{18}\left(-66-\lambda_Z+96s_W^2+5s_W^2\lambda_Z\right) && \text{(D.42)}\\
&+ \frac{1}{3}\left(-8+2\lambda_Z-32s_W^2-4s_W^2\lambda_Z\right)a(y) + \frac{8c_W^2}{D-4}\Bigg] + \mathcal{O}\left(\frac{\ln\lambda_i}{\lambda_i}\right).
\end{aligned}
$$

[8] The functions $\bar{C}_{11}$, $\bar{C}_{12}$, $\bar{C}_{23}$ do not contribute to the large mass limit of the vertex, and we reproduce only their first leading terms, of order $\ln\lambda/\lambda$. If these functions are of relevance for an application, one should determine also the terms of order $1/\lambda$.

Numerically, this means for the value $\lambda_Z = 1.286$:

$$V(\lambda_N) - V(0) \;=\; \frac{1}{2}\left[\lambda_4 + 2.88 \ln \lambda_4 - 4.47 - (2.52 + 2.11 \times i)\right]. \tag{D.43}$$

Here, we subtracted from the large mass limit of the vertex function its value at zero mass (D.66) in order to obtain (3.8).

D.1.1 A relation to the $Zb\bar{b}$ vertex

The loop contributions to the vertex function $V(\lambda)$ are gauge-invariant. We calculate them in the 't Hooft-Feynman gauge. They describe not only the flavour-changing $Zf\bar{f}'$ vertex, but also the mass dependent terms of the flavour-diagonal vertex. There is one case where the effect is quite visible, namely the $Zb\bar{b}$ vertex with virtual t quark exchanges. The exact one-loop expression for the t quark mass dependent part $\mathcal{W}$ of the $Zb\bar{b}$ vertex correction was first given in Eqn. (22) of [87] (form factor $\delta\kappa$, calculated in the unitary gauge). The large t quark mass limit is (Eqn. (2.4.30) of [88]):

$$\mathcal{M}(Z \to \bar{b}b) \;\sim\; \epsilon^\mu \bar{u}\left[\gamma_\mu v_b - \gamma_\mu\gamma_5 a_b + \gamma_\mu(1-\gamma_5)\mathcal{W}(\lambda_t, \lambda_Z)\right] u, \tag{D.44}$$

$$\frac{\mathcal{W}(\lambda_t, \lambda_Z)}{a_b} \;=\; \frac{\alpha}{\pi}\frac{1}{4 s_W^2}|P_{tb}|^2 V_t, \tag{D.45}$$

$$V_t \;=\; \frac{1}{2}\left[\lambda_t + \left(\frac{8}{3} + \frac{\lambda_Z}{6}\right)\ln \lambda_t + \mathcal{O}(1)\right]. \tag{D.46}$$

This agrees with the leading terms in (D.42).

D.2 The vertex for small neutrino mass, $\lambda_i \ll 1$

The limits of the C functions for $\lambda_i = \lambda_j$ and $\lambda_Q = \lambda_Z \gg \lambda_i$ are:

$$C_0 \;=\; -c_0 + \frac{2}{\lambda_Z}\lambda_i \ln \lambda_i - \frac{2}{\lambda_Z(1+\lambda_Z)}(1 + \lambda_Z + \ln \lambda_Z - i\pi)\lambda_i + \ldots, \tag{D.47}$$

$$\begin{aligned}
\lambda_Z C_{11} \;&=\; \lambda_Z C_{12} \\
&=\; -c_0 + 1 - \ln \lambda_Z + i\pi - \lambda_i \ln \lambda_i + \left[c_0 + \frac{2}{1+\lambda_Z}(\ln \lambda_Z - i\pi)\right]\lambda_i \\
&\quad + \ldots,
\end{aligned} \tag{D.48}$$

$$\begin{aligned}
\lambda_Z^2 C_{23} \;&=\; -(\lambda_Z + 2)c_0 + \frac{\lambda_Z}{2} + 2 - 2(\ln \lambda_Z - i\pi) \\
&\quad - \left[-4(c_0 - 1) - 2\frac{2+\lambda_Z}{1+\lambda_Z}(\ln \lambda_Z - i\pi)\right]\lambda_i + \ldots,
\end{aligned} \tag{D.49}$$

$$\begin{aligned}
4\lambda_Z C_{24} \;&=\; -\frac{2\lambda_Z}{D-4} - 2(1+\lambda_Z)c_0 + 3\lambda_Z + 2 - (\lambda_Z + 2)(\ln \lambda_Z - i\pi) \\
&\quad + 4\left[c_0 - 1 + \ln \lambda_Z - i\pi\right]\lambda_i + \ldots,
\end{aligned} \tag{D.50}$$

$$
\begin{aligned}
B_1 &= \frac{1}{D-4} - \frac{1}{4} + \frac{1}{2}\lambda_i + \ldots\,, & \text{(D.51)}\\
\bar{C}_0 &= -\bar{c}_0 - \lambda_i \ln \lambda_i - (B-1)\lambda_i + \ldots\,, & \text{(D.52)}\\
\lambda_Z \bar{C}_{11} &= \lambda_Z \bar{C}_{12} = -(\bar{c}_0 - B + 1)(\lambda_i - 1) + \ldots\,, & \text{(D.53)}\\
\lambda_Z^2 \bar{C}_{23} &= -2(\bar{c}_0 - B + 1) + \frac{\lambda_Z}{2} - \;\left[\lambda_Z \bar{c}_0 - 4(\bar{c}_0 - B + 1)\right]\lambda_i + \ldots\,, & \text{(D.54)}\\
2\lambda_Z \bar{C}_{24} &= -\frac{\lambda_Z}{D-4} - (\bar{c}_0 - B + 1) + \frac{3\lambda_Z}{2} - \pi\lambda_Z y + 2\lambda_Z y \arctan(2y) & \\
&\quad + \left[2(\bar{c}_0 - B + 1) - \lambda_Z \bar{c}_0\right]\lambda_i + \ldots\,, & \text{(D.55)}
\end{aligned}
$$

with

$$
B = 2y\left[\arctan(2y) + \arctan\left(\frac{\lambda_Z - 1}{3 - \lambda_Z} 2y\right)\right] \approx 1.75, \qquad \text{(D.56)}
$$

for $\lambda_Z = 1.286$, $y \approx 0.73$, and the values of $-C_0$ and $-\bar{C}_0$ at $\lambda_i = 0$ [10]:

$$
\begin{aligned}
\lambda_Z\; c_0 &= \frac{\pi^2}{6} - \mathrm{Li}_2\left(\frac{1}{1+\lambda_Z}\right) - \frac{1}{2}\ln^2(1+\lambda_Z) + \pi \ln(1+\lambda_Z) \times i, & \text{(D.57)}\\
\lambda_Z\; \bar{c}_0 &= \frac{\pi^2}{6} - \mathrm{Li}_2(1-\lambda_Z) + 2\Re e\mathrm{Li}_2\left[(\lambda_Z - 1)\left(\frac{\lambda_Z}{2} - 1 + \lambda_Z y \times i\right)\right] & \\
&\quad - 2\Re e\mathrm{Li}_2\left(1 - \frac{\lambda_Z}{2} - \lambda_Z y \times i\right) & \text{(D.58)}
\end{aligned}
$$

Only the functions $\bar{C}$ develop imaginary parts, and only for

$$
\lambda_Z > 4\lambda_i. \qquad \text{(D.59)}
$$

The functions $C_0, C_{11}, \bar{C}_0$ contain terms of the order $\lambda_Z \ln \lambda_Z$ at $\lambda_i \ll \lambda_Z$, but these terms cancel in the form factor V. They survive in the case $\lambda_Z \to 0$ at constant λ_i.

A further note: the Euler dilogarithm is badly converging on the unit circle. In fact, for one of the Li_2 above it occurs $|1 - \lambda_Z/2 \pm i \times \lambda_Z y| = 1$. In that case, one may use:

$$
\Re e\mathrm{Li}_2\left(e^{i\phi}\right) = -\frac{1}{2}\mathrm{Li}_2(1) + \frac{1}{4}(\phi \pm \pi)^2, \qquad \text{(D.60)}
$$

taking the value of ϕ fulfilling the condition:

$$
(\phi \pm \pi) \in (-\pi, \pi). \qquad \text{(D.61)}
$$

The resulting small mass limit of the vertex is given in [39, 9] for the case of sequential Dirac neutrinos:

$$
V(\lambda_i \ll 1, \lambda_Z) = I_3^{i_L} \frac{8c_W^2}{D-4} + a_0 + a_L\,(\lambda_i \ln \lambda_i) + a_1 \lambda_i + \ldots \qquad \text{(D.62)}
$$

The divergent constant was left out in the introduction, Eqn. (1.16).

We want to stress that in this limit,

$$a_L = 0. \tag{D.63}$$

The ansatz in Eqn. (1) of [12] allowing for $a_L \neq 0$ is too general for the case of small neutrino masses in this respect. Such a term occurs for $\lambda_Q = 0$, see section D.3.

A numerical value of a_0 was given in [39]. The analytical expression is:

$$\begin{aligned} a_0 &= \frac{(1+\lambda_Z)^2}{\lambda_Z} c_0 + \frac{2}{\lambda_Z^2}(1+2\lambda_Z)(\bar{c}_0 - B) + \frac{6}{\lambda_Z}[\pi y - 2y \arctan(2y)] \\ &+ \frac{2+3\lambda_Z}{2\lambda_Z}(\ln \lambda_Z - \pi \times i) - \frac{1}{4\lambda_Z^2}(7\lambda_Z^2 + 14\lambda_Z - 8). \end{aligned} \tag{D.64}$$

The general expression for a_1 is [9]:

$$\begin{aligned} a_1(\lambda_Z) &= -\frac{2}{\lambda_Z}(1+\lambda_Z)c_0 + \frac{1}{2\lambda_Z^2}(4\lambda_Z^2 - 5\lambda_Z - 6)\bar{c}_0 - \frac{2}{\lambda_Z}(\ln \lambda_Z - \pi \times i) \\ &+ \frac{1}{8\lambda_Z^2}(25\lambda_Z^2 - 38\lambda_Z - 24) + \frac{1}{2\lambda_Z}(2-\lambda_Z)\pi y \\ &+ \frac{1}{\lambda_Z^2}(\lambda_Z^2 + 7\lambda_Z + 6) y \arctan(2y) \\ &+ \frac{3}{\lambda_Z^2}(3\lambda_Z + 2) y \arctan\left(\frac{\lambda_Z - 1}{3 - \lambda_Z} 2y\right), \end{aligned} \tag{D.65}$$

with y and B from (D.40) and (D.56). Further, we used c_0 and $\bar{c}_0$ of (D.57) and (D.58). With the inputs $M_W = 80.410$ GeV and $M_Z = 91.187$ GeV, these formulae yield:

$$a_0 = 1.2584 + 1.0524 \times i, \tag{D.66}$$

$$a_1 = 2.5623 - 2.2950 \times i. \tag{D.67}$$

D.3 The vertex for $\lambda_Q = 0$

The first calculation of the non-diagonal Zf_1f_2 vertex seems to be in [89], where the approach was simplified considerably by the approximation $Q^2 = 0$. We should mention that this limit is of physical relevance only in the large neutrino mass limit, since only then it is $\lambda_i \gg \lambda_Z \approx \lambda_Q = 0$. If we are interested in applications where $\lambda_Q > 4\lambda_i$, the amplitude gets essentially complex valued. The limit of [89], however, implies automatically the relation $\lambda_i > \lambda_Q$, even for $\lambda_i \ll 1$, and the amplitude is essentially real. So one cannot expect a continuous behaviour for the small or medium neutrino mass range. The virtual fermions are considered Dirac particles ($\lambda_i = \lambda_j$).

The vertex may be easily derived from the formulae given in [10]:

$$V(\lambda_i, \lambda_Q = 0) = I_3^{i_L}\left[\lambda_i(\lambda_i - 10)\mathcal{I}_1 + 8\mathcal{L} + 6 + \frac{8c_W^2}{D-4}\right], \tag{D.68}$$

with

$$\mathcal{I}_1 = \int_0^1 dx \frac{x}{(1-\lambda_i)x+\lambda_i} = \frac{\lambda_i \ln \lambda_i}{(1-\lambda_i)^2} + \frac{1}{1-\lambda_i}, \tag{D.69}$$

$$\mathcal{L} = \int_0^1 dx x \ln\left[(1-\lambda_i)x+\lambda_i\right] = \frac{\lambda_i^2 \ln \lambda_i}{2(1-\lambda_i)^2} + \frac{1}{2(1-\lambda_i)} - \frac{1}{4}. \tag{D.70}$$

The divergent part is independent of λ_i and of λ_Q.

Explicitly:

$$V(\lambda_i, \lambda_Q = 0) = I_3^{i_L}\left(3\frac{\lambda_i^2 \ln \lambda_i}{(1-\lambda_i)^2} + 2\frac{\lambda_i \ln \lambda_i}{(1-\lambda_i)^2} - \frac{\lambda_i^2}{1-\lambda_i} + 6\frac{\lambda_i}{1-\lambda_i} + \frac{8c_W^2}{D-4}\right), \tag{D.71}$$

with $I_3^{i_L}$ being the weak isospin of the virtual neutrinos. The constant finite terms of the vertex vanish for small λ_i:

$$V(\lambda_i \ll 1, \lambda_Q = 0) = I_3^{i_L}\left(2\lambda_i \ln \lambda_i + 6\lambda_i + \frac{8c_W^2}{D-4}\right). \tag{D.72}$$

This has to be compared to (D.62) and (D.63).

The large mass limit is:

$$V(\lambda_i \gg 1, \lambda_Q = 0) = I_3^{i_L}\left(\lambda_i + 3\ln \lambda_i - 5 + \frac{8c_W^2}{D-4}\right). \tag{D.73}$$

This has to be compared to (D.42) and (D.43).

Finally, for the sake of completeness, the value of the vertex at the weak scale:

$$V(\lambda_i = 1, \lambda_Q = 0) = I_3^{i_L}\left(\frac{3}{2} + \frac{8c_W^2}{D-4}\right). \tag{D.74}$$

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