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Tests of the left-right electroweak model at linear collider

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Abstract

The left-right model is a gauge theory of electroweak interactions based on the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The main motivations for this model are that it gives an explanation for the parity violation of weak interactions, provides a mechanism (see-saw) for generating neutrino masses, and has $B-L$ as a gauge symmetry. The quark-lepton symmetry in weak interactions is also maintained in this theory. The model has many predictions one can directly test at a TeV-scale linear collider. We will consider here two processes ($e^-e^- \to qq\bar{q}\bar{q}$ and $e^-e^- \to \mu\nu q\bar{q}$) testing the lepton flavour violation predicted by the model. We will also discuss constraints on supersymmetric versions of the model.

1 Introduction

The left-right symmetric model (LRM) has the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which is spontaneously broken to the Standard Model (SM) symmetry $SU(2)_L \times U(1)_Y$ at low energies [1]. The Lagrangian of the model is parity conserving in the symmetric limit. The parity violation of weak interactions observed at low-energy domain is generated dynamically via the spontaneous breaking of the gauge symmetry. This is in contrast with the SM which is parity violating by definition, and it was originally the main motivation for the LRM.

The LRM differs from the Standard Model (SM) also in another essential respect. Yukawa couplings between neutrinos and the fundamental scalars give rise to the see-saw mechanism [2], which provides the simplest explanation for the lightness of neutrinos, if neutrinos do have a mass. In the SM neutrinos are exactly massless by construction. The recent observations on the atmospheric neutrino fluxes by the Super-Kamiokande [3] seem to verify that neutrinos indeed have a mass. Massive neutrinos are also indicated by the observed deficit of solar neutrinos [4] and the observation of $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions in the LSND experiment [5].

So far there has been, however, no evidence of left-right symmetry in weak interactions but everything seems to be well described in terms of the ordinary $V-A$ currents. This fact can be used to set constraints on the parameters of the LRM, such as the masses of the new gauge bosons $W_R^\pm$ and Z_R, associated with the gauge symmetry $SU(2)_R$, and their mixings with the $SU(2)_L$ bosons $W_L^\pm$ and Z_L [6, 7, 8]. Such constraints depend quite crucially on the assumptions one makes. This is true, for example, for the often quoted mass bound $M_{W_R} \gtrsim 1.6$ TeV from the $K_S - K_L$ mass difference [9], as well as for the limit $M_{W_R} \gtrsim 1.1$ TeV from the double beta decay [10].

In general, the new weak gauge bosons W_R and Z_R mix with their SM counterparts, but the observed $V-A$ structure of the weak force indicates that this mixing is quite small. Let us denote the heavier mass eigenstate charged boson as the superposition $W_2 = \sin\zeta W_L + \cos\zeta W_R$ and identify the orthogonal state $W_1 = \cos\zeta W_L - \sin\zeta W_R$ with the ordinary W boson. In the case of the manifest left-right symmetry semileptonic-decay data can be used to derive an upper limit of 0.005 on the mixing angle ζ [11]. From neutral current data one can derive the lower bound $M_{Z_2} \gtrsim 400$ GeV for the mass of the new Z boson and the upper bound of 0.008 for the Z_1, Z_2 mixing angle [7], where Z_1 denotes the ordinary neutral weak boson.

At the Tevatron direct searches have been made for W_2 ($\simeq W_R$) in the channels $pp \to W_2 \to eN$ or μN, where N is a neutrino. In the LRM N should be identified with the heavy right-handed neutrino, since the coupling of the ordinary light neutrino to W_2 is strongly suppressed. The most stringent bound announced is $M_{W_2} \gtrsim 720$ GeV [12]. It should be emphasized that this bound is based on several assumptions. It is assumed that the quark-W_2 coupling has the SM strength, the CKM matrices $V_L^{\rm CKM}$ and $V_R^{\rm CKM}$ are equal, and the right-handed neutrino does not decay in the detector but appears as missing E_T. It has been argued that if one relaxes the first two assumptions, the mass bound will be degraded considerably [13].

The Tevatron mass limit for the new neutral intermediate boson from the dimuon and dielectron decay channels is $M_{Z_2} \gtrsim 620$ GeV [14].

We have considered the linear collider [15] phenomenology of the left-right symmetric model, both with and without supersymmetry, in several previous publications [16], and we summarized the results of our studies in the foregoing issue of this series [17]. Here we shall report the studies we have carried out since that previous summary. In Section 2 we briefly recall the basic features of the LRM. In Section 3 we will consider various processes where one can test the lepton number violation predicted by the model. The supersymmetric left-right model (SLRM) is described in Section 4 and we discuss various theoretical and phenomenological constraints on it. Section 6 is a summary.

2 Description of the left-right symmetric model

In the left-right symmetric model quark and leptons are assigned to the doublets of the gauge groups $SU_L(2)$ and $SU_R(2)$ according to their chirality [18]:

$$\begin{aligned} \Psi_L &= \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L = (2,1,-1), \qquad \Psi_R = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_R = (1,2,-1), \\ Q_L &= \begin{pmatrix} u \\ d \end{pmatrix}_L = \left(2,1,\tfrac{1}{3}\right), \qquad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R = \left(1,2,\frac{1}{3}\right), \end{aligned} \tag{1}$$

and similarly for the other families. The minimal set of fundamental scalars, the theory to be symmetric under the $L \leftrightarrow R$ transformation, consists of the following Higgs multiplets:

$$\begin{aligned} \Phi &= \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} = (2,2,0), \\ \Delta_L &= \begin{pmatrix} \Delta_L^+ & \sqrt{2}\Delta_L^{++} \\ \sqrt{2}\Delta_L^0 & -\Delta_L^+ \end{pmatrix} = (3,1,2), \\ \Delta_R &= \begin{pmatrix} \Delta_R^+ & \sqrt{2}\Delta_R^{++} \\ \sqrt{2}\Delta_R^0 & -\Delta_R^+ \end{pmatrix} = (1,3,2). \end{aligned} \tag{2}$$

They transform according to $\Phi \to U_L \Phi U_R^\dagger$, $\Delta_L \to U_L \Delta_L U_L^\dagger$ and $\Delta_R \to U_R \Delta_R U_R^\dagger$, where $U_{L(R)}$ is an element of $SU(2)_{L(R)}$. The vacuum expectation value of the bidoublet Φ is given by

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}. \tag{3}$$

This breaks the Standard Model symmetry $SU(2)_L \times U(1)_Y$, and it generates masses to fermions through the Yukawa couplings

$$\mathcal{L}_\Phi^{\text{Yukawa}} = \bar{\Psi}_L^i (f_{ij} \Phi + \tilde{f}_{ij} \tilde{\Phi}) \Psi_R^j + \bar{Q}_L^i (f_{ij} \Phi + \tilde{f}_{ij} \tilde{\Phi}) Q_R^j + h.c., \tag{4}$$

where $\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2$.

The vacuum expectation values of the scalar triplets are denoted by

$$\langle \Delta_{L,R} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}. \tag{5}$$

The right-triplet Δ_R breaks the $SU(2)_R \times U(1)_{B-L}$ symmetry, and at the same time the discrete $L \leftrightarrow R$ symmetry, and it yields a Majorana mass to the right-handed neutrinos through the Yukawa coupling

$$\mathcal{L}_\Delta^{\text{Yukawa}} = i h_L \Psi_L^T C \sigma_2 \Delta_L \Psi_L + i h_R \Psi_R^T C \sigma_2 \Delta_R \Psi_R + h.c., \tag{6}$$

where $\Delta_{L,R} = \Delta_{L,R}^i \sigma_i$. The conservation of electric charge prevents the triplet Higgses from coupling to quarks. The Yukawa Lagrangian (6) is the origin of lepton number violating interactions, a novel feature of the LRM.

In order to have the tree-level value of the ρ parameter close to unity, $\rho = 0.9998 \pm 0.0008$ [19], one should assume $v_L \lesssim 9$ GeV.

It is argued in ref. [8] that to suppress the FCNC one must require the Higgs potential to be such that in the minimum $\kappa_1 \ll \kappa_2$ or $\kappa_1 \gg \kappa_2$. This requirement has the consequence that the W_L, W_R mixing angle ζ is necessarily small, since $\zeta \sim (g_L/g_R)^2 |\kappa_1 \kappa_2| / |v_R|^2$.

In the general case, the charged current Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{wk}^{CC} \;\simeq\; & \frac{g_L}{2\sqrt{2}} \Big[\left(\cos\zeta \, J_{L\,\mu}^{+} + \sin\zeta \, J_{R\mu}^{+} \right) W_1^{+\mu} \\ & + \left(\cos\zeta \, J_{R\,\mu}^{+} - \sin\zeta \, J_{L\,\mu}^{+} \right) W_2^{+\mu} + \text{h.c.} \Big] . \end{aligned} \tag{7}$$

where $J^+_{R/L} = \overline{e}\gamma(1 \pm \gamma_5)\nu_{R/L}$. In the limit $\zeta \to 0$, the couplings of $W_1 \simeq W_L$ are similar to those of the W boson in the SM. We have assumed that the gauge couplings g_L and g_R associated with the symmetries $SU(2)_L$ and $SU(2)_R$, respectively, are equal, as the manifest left-right symmetry would necessitate.

The neutral current Lagrangian is given in terms of mass eigenstate bosons by

$$\begin{aligned}\mathcal{L}^{NC}_{wk} &= eJ^{em}_\mu A^\mu + \frac{g_L}{\cos\theta_W}\left\{\left[J^Z_{L\,\mu} - \lambda\cos\theta_W\left(\sin^2\theta_W J^Z_{L\,\mu} + \cos^2\theta_W J^Z_{R\,\mu}\right)\right]Z^\mu_L\right. \\ &\quad \left. + \left(\cos^2\theta_W\right)^{-1/2}\left(\sin^2\theta_W J^Z_{L\,\mu} + \cos^2\theta_W J^Z_{R\,\mu}\right)Z^\mu_R\right\}\,, \end{aligned} \tag{8}$$

where $\lambda = (M_{W_L}/M_{W_R})^2$, and the weak neutral currents are given by $J^Z_{L/R} = J^3_{L/R} - Q\sin^2\theta_W\, J^{em}$ with $J_{L/R} = \overline{\psi}\gamma T^3_{L,R}\psi$ and $J^{em} = \overline{\psi}\gamma Q\psi$. The couplings of Z_L approach the SM Z couplings in the limit $\lambda \ll 1$.

From the left-handed and right-handed neutrino states one can form three types of Lorentz-invariant mass terms: Dirac term $\overline{\nu}_L\nu_R$, and Majorana terms $\overline{\nu}^c_L\nu_R$ and $\overline{\nu}^c_R\nu_L$, where the latter two terms break the lepton number by two units. All these terms are realized in the left-right symmetric model with the Yukawa coupling (6) and the vevs given in eqs. (3) and (5). The see-saw mass matrix of neutrinos is given by

$$M = \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix}. \tag{9}$$

The entries are 3×3 matrices given by $m_D = (f\kappa_1 + g\kappa_2)/\sqrt{2}$, $m_L = h_L v_L$ and $m_R = h_R v_R$. The mass of the charged lepton is given by $m_l = (f\kappa_2 + g\kappa_1)/\sqrt{2}$, and therefore if f and g are comparable, one has $m_D \simeq m_l$. Unless there is an extraordinary hierarchy among the couplings, one has $m_L \ll m_D \ll m_R$. In this case the approximate masses of the Majorana states that diagonalize the neutrino Lagrangian are given by $m_\nu \simeq m_D^T m_R^{-1} m_D$ and $m_N \simeq m_R$.

3 Some tests of the LRM in e^-e^--collisions.

The main prediction of left-right symmetric model to be tested at future experiments is the existence of the right-handed gauge bosons W_R and Z_R. Another essential prediction of the model are the Higgs triplets. In what follows we will show that the process $e^-e^- \to qq\bar{q}\bar{q}$ provides a good place to test the right-handed gauge sector below the W_R threshold, while the process $e^-e^- \to \mu\,\nu\,q\,\bar{q}$ is very promising for the search of the left-handed triplet Higgses Δ_L.

3.1 Process $e^-e^- \to q\,q\,\bar{q}\,\bar{q}$ as a test of the LRM below W_R threshold.

Let us first give the arguments that make us to consider the reaction $e^-e^- \to q\,q\,\bar{q}\,\bar{q}$ particularly suitable for testing the LRM. First of all, the final state particles are all light, so that there is no kinematical suppression for the process, in contrast with, e.g., the W_R pair production. Consequently, one may expect to detect evidence of the LRM through this reaction well below the W_R threshold. Reactions with ordinary neutrinos in the final state are not very useful as invisibility of neutrinos makes them not easy to distinguish from the background processes. Also, reactions with final state electrons are not that good because of the possible mix-up of the initial and final state particles.

One could consider, of course, for the leptonic final states, for example for the reaction $e^-e^- \to \mu^-\ \mu^-\ \mu^-\ \mu^+$. The reactions like $e^-e^- \to b\,b\,\bar{t}\,\bar{t}$ that involve charged currents but not neutral currents offer however a more unambiguous test of the LRM than the leptonic processes. We prefer final states with b-quarks as the b-jets are relatively easy to identify in experiment [20]. From this point of view, the best process for a study would be $e^-e^- \to b\,b\,\bar{t}\,\bar{t}$. However, as will be seen from our numerical results, it will possible to measure the cross section also for the 4-jet reactions with no b-jets, as well as for the reactions with a single b-jet.

We have derived the squared matrix elements for $e^-e^- \to b\,b\,\bar{t}\,\bar{t}$ and computed the ensuing cross sections at the collision energies $\sqrt{s} = 1$ TeV and $\sqrt{s} = 1.5$ TeV by means of CompHEP [21].

In Fig. 1 we show the energy dependence of the total cross section of the process $e^-e^- \to b\,b\,\bar{t}\,\bar{t}$ for various values of masses of the right-handed triplet Higgs Δ_R^{--} and the right-handed neutrino ν_2. In all the cases the right-handed boson mass is taken to be $M_{W_R} = 700$ GeV.

In Fig. 2 we present the cross section and sensitivity contours for $\sqrt{s} = 1.5$ TeV with the masses of the right-handed neutrinos equal to 1.5 TeV for the abovementioned process and for the process with 1 b-jet (or only with light quarks) in the final state. The

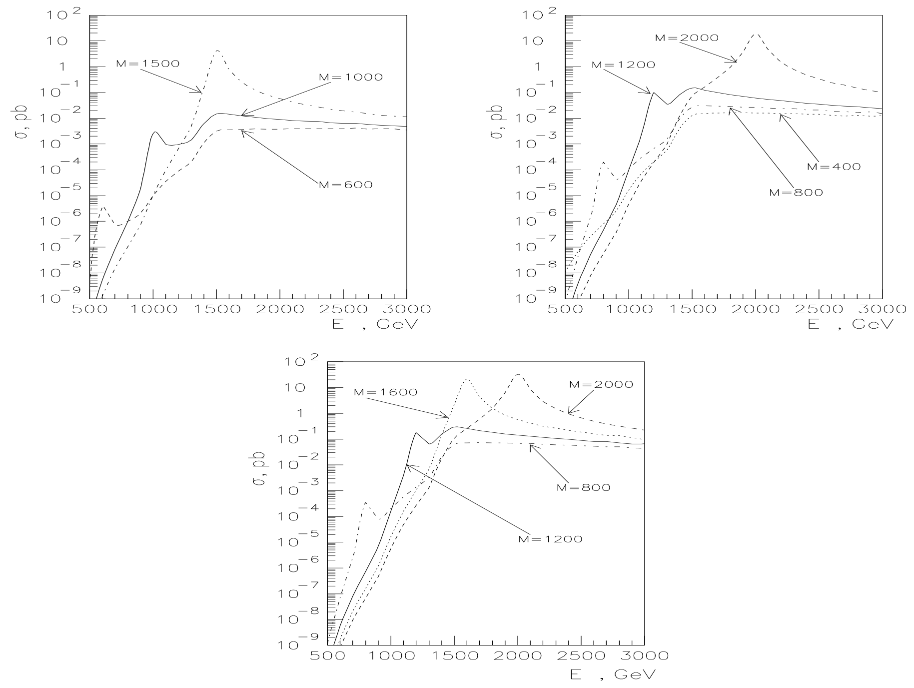


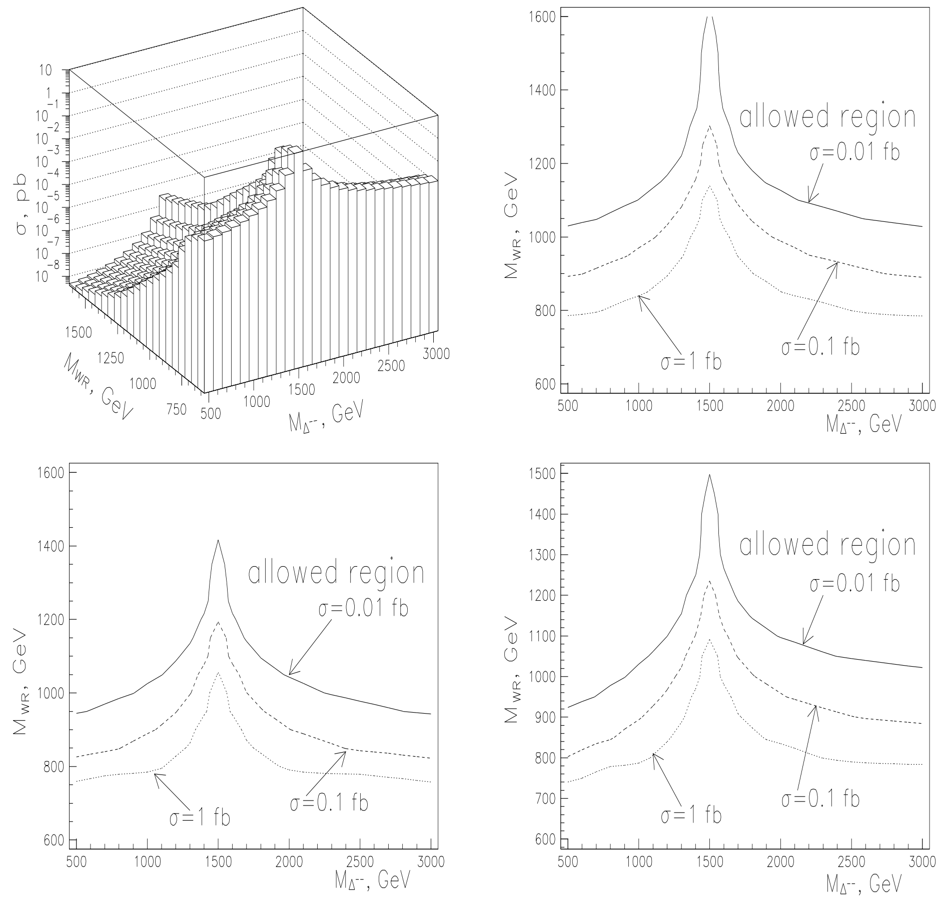
Figure 1: Energy dependence of the full cross section for the process $e^- e^- \to b b \bar{t} \bar{t}$ for different values of Δ_R^{--} mass ($M \equiv M_{\Delta_R^{--}}$) and right-handed neutrino masses: $m_{\nu_2} = 1$ TeV (left upper picture), $m_{\nu_2} = 1.5$ TeV (right upper picture), $m_{\nu_2} = 2$ TeV (lower picture).

achievable limit for M_{W_R} is now about 1.5 TeV at the triplet Higgs resonance and outside the resonance about 1 TeV, a considerable improvement to the present bound. As the cross section is proportional to the mass of neutrino, the larger m_{ν_2} the more stringent are the ensuing constraints. Following the arguments of [20] we apply the following cuts: each b-jet should have energy more than 10 GeV; each t-jet should have energy more than 190 GeV; the opening angle between two detected jets should be greater than 20°; the angle between each detected jet and the colliding axis should be greater than 36°; the total energy of the event should be greater than 400 GeV. We have tested that when these cuts are imposed the following relations hold between the cross sections of the reactions with no, one and two b-jets in the final state:

$$\sigma(0b) \approx \sigma(1b) \approx 4 \cdot \sigma(2b); \tag{10}$$

These relations may be very useful as a test of the LRM.

The SM background can be suppressed to the level 4 orders of magnitude below the process rate if the proper cuts in the phase space are applied, and it can be made



(a) **(b)**

Figure 2: Cross section for the $e^-e^- \to b\,b\,\bar{t}\,\bar{t}$ and it's sensitivity to the masses of W_R and Δ_R^{--} : **(a)** $\sigma = 0.01$ fb (30 events per year), $\sigma = 0.1$ fb (300 events per year), $\sigma = 1$ fb (3000 events per year) for the energy $E = 1.5$ TeV , and the right-handed neutrino mass $m_{\nu_2} = 1.5$ TeV; **(b)** $\sigma = 0.01$ fb (30 events per year), $\sigma = 0.1$ fb (300 events per year), $\sigma = 1$ fb (3000 events per year) for the processes with 1 b-jet or with light-quarks only in the final state (see comments in the text) for the energy $E = 1.5$ TeV, and the right-handed neutrino masses: $m_{\nu_2} = 1.5$ TeV (on the top) and $m_{\nu_2} = 1$ TeV (in the bottom).

even 7 orders of magnitude below the signal level if the full energy of the event can be reconstructed with the accuracy of 50 GeV.

From figs. 1 and 2 one can see that the reaction $e^-e^- \to q\,q\,\bar{q}\,\bar{q}$ may be observed at LC for a wide range of reasonable parameter values of the LRM and already below the W_R threshold. For the collision energy $\sqrt{s} = 1.5$ TeV and luminosity $10^{35}\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}$ the lower limit for the mass of the right-handed gauge boson one could reach is $M_{W_R} \sim 1$ TeV. More detailed study of this process one can find in [22].

3.2 Process $e^-e^- \to \mu\,\nu\,q\,\bar{q}$ as a test of the Δ_L Higgs.

The interactions of the Δ_L field described above are the same for both the SM with additional Higgs triplet and for the LR-model. The analysis, presented below is therefore valid for both of these models.

In [23] the production of the singly charged Higgses in e^-e^- collisions was assumed to take place in pairs through a W^-W^- fusion. This process conserves the lepton number. We will consider here production processes that probe the lepton number violating Yukawa couplings. The pair production, which proceeds through t-channel exchange of Majorana neutrinos and s-channel exchange of Δ_L^{--}, is not a suitable process to study in this case. This is because the neutrino exchange is proportional to Majorana mass of the neutrino and hence is suppressed and the $\Delta_L^{--}\Delta_L^-\Delta_L^-$ vertex depends on the self-couplings of scalar potential whose values are unknown. We consider instead a production of a single Δ_L^- in the process $e^-e^- \to \Delta_L^- W_\mu^-$ where the t-channel neutrino exchange is not suppressed as the t-channel neutrino has the same chirality in the both vertices and in the s-channel process the strength of the $\Delta_L^{--}\Delta_L^- W_\mu^-$ vertex does not depend on any unknown parameter of the scalar potential but is determined by the gauge coupling. The experimentally clearest final state to study is the one where Δ_L^- decays to a muon and a muonic neutrino and W_μ^- decays into two quark jets (e.g. d and $\bar{u}$) without missing energy.

In our calculations, made using the CompHEP package [21], we have imposed the following cuts for the final state phase space: each final state particle has energy greater than 10 GeV (including neutrino); the transverse energy of each particle (including missing transverse energy) should be greater than 5 GeV; the opening angle between two quark jets should be more than 20^o; each final state particle should have the outgoing direction more then 10^o away from the beam axis.

In Fig. 3 we present the dependence of the cross section of the process $e^-e^- \to \mu\nu d\bar{u}$ on the collision energy for the different values of masses of singly ($M_{\Delta_L^-}$=100, 400, 700, 1000 GeV) and doubly charged ($M_{\Delta_L^{--}}$ = 100, 400, 700, 1000 GeV) triplet Higgses. The cross sections are dominated by the resonance at $\sqrt{s} = M_{\Delta_L^{--}}$. To estimate the width of the peak we have chosen $\Gamma_{\Delta_L^{--}} = 10^{-3}M$ for the two lepton decays and the $\Delta_L^{--} \to \Delta_L^- W_L^-$ mode was also taken into account [24]. One may conclude that at 0.01 fb level the process $e^-e^- \to \mu\,\nu\,d\,\bar{u}$ may be observed away from the Δ_L^{--} resonance and even below the Δ_L^- threshold.

Fig. 4 presents the sensitivity of the reaction $e^-e^- \to \mu\,\nu\,d\,\bar{u}$ on the masses of the triplet Higgs particles Δ_L^- and Δ_L^{--} for the collision energy $\sqrt{s} = 500$ GeV. We have estimated the values of the running coupling constants at 500 GeV by applying the approximate RG equations of the SM [25]. The influence of the triplet Higgses on the running, which can be expected to be quite small, is not taken into account. Fig. 4a displays the cross section of the $e^-e^- \to \mu\,\nu\,d\,\bar{u}$ process for the different values of the Δ_L^- and Δ_L^{--} masses, with assuming for the Yukawa couplings their maximal allowed values that are in accordance with the present phenomenological constraints [24]:

$$h_{ee}^2 < 10^{-5} \cdot M_{\Delta_L^{--}} \text{ GeV},$$

$$h_{\mu\mu}^2 < 10^{-5} \cdot M_{\Delta_L^{--}} \text{ GeV}. \tag{11}$$

If the mass of doubly charged Higgs is considered to be greater than 100 GeV , then $h_{ee} \cdot h_{\mu\mu} < 0.18$ or $\sqrt{h_{ee} \cdot h_{\mu\mu}} < 0.4$.

In Fig. 4b we show the dependence of the cross section on the Δ_L^{--} mass in the case that Δ_L^- is effectively decoupled. Supposing that the mass of Δ_L^{--} is known, one can conservatively estimate, by setting for the Yukawa coupling $h_{\mu\mu}$ the largest phenomenologically allowed value, the contribution of the Δ_L^{--} mediated processes on the total cross section. When this is subtracted from the total cross section, what is left is the contribution of the t-channel neutrino exchange process alone. This has a threshold behaviour and its strength gives direct information on the product $h_{ee} \cdot h_{\mu\mu}$ of Yukawa couplings.

In Fig. 4c we display the 0.03 fb (30 events per year) discovery contours on the $\left(M_{\Delta_L^-}, M_{\Delta_L^{--}}\right)$ −plane, corresponding to the cross section of the isolated t-channel process, for the different values (0.1, 0.4 and 1.0) of "average" Yukawa couplings ($h_{\text{Yuk}} = \sqrt{h_{ee} \cdot h_{\mu\mu}}$). In the plot the collision energy is taken as $\sqrt{s} = 500$ GeV. It is seen from the figure that the process $e^-e^- \to \mu\,\nu\,d\,\bar{u}$ might probe the the mass $M_{\Delta_L^-}$ to much larger values than what is the production threshold, providing that the average Yukawa coupling is larger than 0.1 and the collision does not happen in the vicinity of the Δ_L^{--} pole. If these conditions are not met Δ_L^- would have detectable effects only when it is produced as a real particle.

The main SM background to the reaction $e^-e^- \to \mu\,\nu\,d\,\bar{u}$ is due to the process $e^-e^- \to W^-W^-\nu\nu$ studied in [26]. Reconstructing the invariant squared mass of the muon and neutrino pair it would be possible to separate background in the cases when

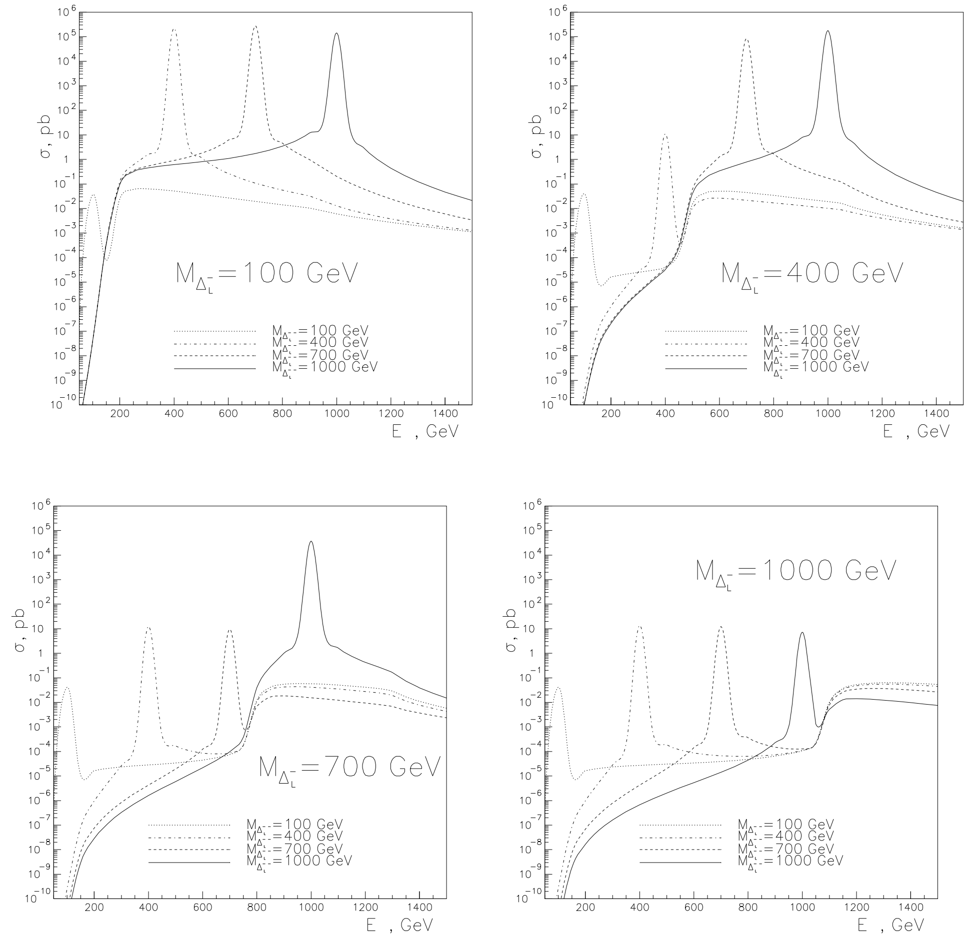
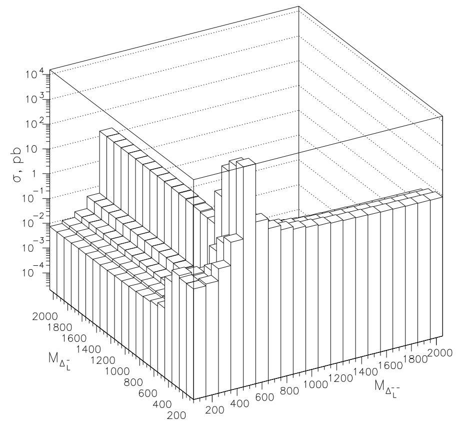


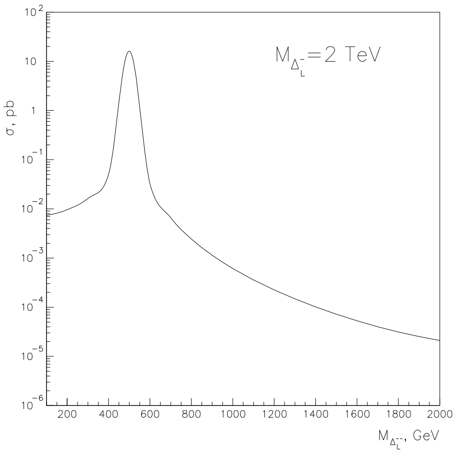
Figure 3: Energy dependence of the cross section of the $e^-e^- \to \mu\,\nu_\mu\,d\,\bar{u}$ for different values of the masses of singly charged (Δ_L^-) and doubly charged (Δ_L^{--}) triplet Higgses.

the mass difference between Δ_L^- and W^- is greater than invariant mass resolution (for $M_{\Delta_L^-} > 100$ GeV this should be possible). But even in the cases when $M_{\Delta_L^-} \simeq M_W$ it is possible to compare the cross sections of $e^-e^- \to \mu\,\nu\,d\,\bar{u}$ and $e^-e^- \to d\,\bar{u}\,s\,\bar{c}$ which should be equal in the SM. Any substantial difference between these cross sections would be a signal of the new physics. In other words, in order to get rid of the SM background one should consider the ratio of the cross sections of $e^-e^- \to d\ \bar{u}\ s\ \bar{c}$ and $e^-e^- \to \mu\ \nu\ d\ \bar{u}$.

From figs 3 and 4 one can conclude that the process $e^-e^- \to \mu\ \nu\ d\ \bar{u}$ provides a good test for lepton flavor non-conservation of the singly charged scalars. At the collision energy 500 GeV the process may be seen well below Δ_L^- and/or Δ_L^{--} thresholds for a wide range of the lepton number violating Yukawa couplings. The influence of Δ_L^- contribution (below its threshold) may be extracted from the process, if colliding energy is away from the Δ_L^{--} resonance. The present bounds on the Yukawa couplings may be significantly



(a)



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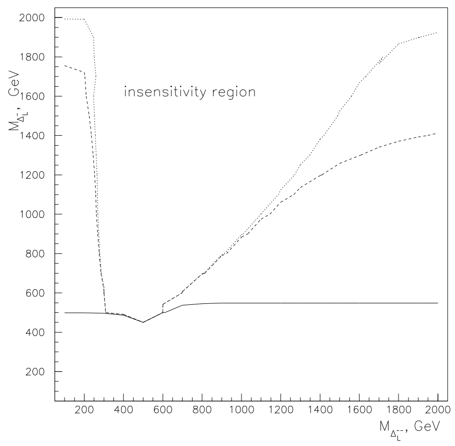

(c)

Figure 4: The cross section for the $e^-e^- \to \mu\,\nu_\mu\,d\,\bar{u}$ process (a) for the different values of $M_{\Delta_L^-}$ and $M_{\Delta_L^{--}}$, (b) as a function of $M_{\Delta_L^{--}}$ in the limit $M_{\Delta_L^-} >> M_{\Delta_L^{--}}$. (c) The contour plots for the difference between cross sections of the process with finite and infinite $M_{\Delta_L^-}$ for 0.03 fb (30 events per year), for different values of Yukawa couplings (solid line for $h = 0.1$, dashed line for $h = 0.4$, and dotted line for $h = 1$). Collision energy is taken to be $\sqrt{s} = 500$ GeV. In Fig. (c) the region above curves is outside the reach of experiment.

improved. More detailed study of this process one can find in [27].

4 The supersymmetric left-right models

The supersymmetrized versions of the left-right model [28]-[37] have been actively studied in recent years. From the supersymmetric point of view, an important motivation for the left-right models is due to their gauge group. It has been noted [38, 39] that if the gauge symmetry of MSSM is suitably extended to contain $U(1)_{B-L}$, the R-parity is automatically conserved in the Lagrangian of the theory. Thus one of the major problematic features of the MSSM is explained by the gauge symmetry. Here we will concentrate on the supersymmetric left-right model (SLRM) based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

The particle content of the supersymmetric left-right model is enlarged compared to the MSSM. Because of the extended symmetry, there are superfields containing gauge bosons connected to $SU(2)_R$. Due to the more symmetric treatment of the fermions in the model, also the right-handed neutrino superfield is included. The breaking of the extended symmetry requires a new set of Higgs bosons. The Higgs sector of the SLRM can be chosen in many ways, but with triplets in the spectrum, one can have the conventional see-saw mechanism for neutrino mass generation. This will be the choice for the $SU(2)_R$ breaking here. The $SU(2)_L$ will be broken mainly by bidoublets which contain the doublets of the MSSM:

$$\Phi = \begin{pmatrix} \Phi_1^0 & \Phi_1^+ \\ \Phi_2^- & \Phi_2^0 \end{pmatrix},\ \chi = \begin{pmatrix} \chi_1^0 & \chi_1^+ \\ \chi_2^- & \chi_2^0 \end{pmatrix} \sim (1,2,2,0),$$
$$\Delta_R \sim (1,1,3,-2),\ \delta_R \sim (1,1,3,2),\ \Delta_L \sim (1,3,1,-2),\ \delta_L \sim (1,3,1,2). \tag{12}$$

In (12) it is assumed that the gauge symmetry is supplemented by a discrete left-right symmetry. The $SU(2)_L$ triplets Δ_L and δ_L make the Lagrangian fully symmetric under $L \leftrightarrow R$ transformation, although these are not needed for symmetry breaking, or for the see-saw mechanism.

The most general gauge invariant superpotential involving these superfields can be written as (generation indices suppressed)

$$\begin{aligned} W_{min} &= h_{\Phi Q} Q^T i\tau_2 \Phi Q^c + h_{\chi Q} Q^T i\tau_2 \chi Q^c + h_{\Phi L} L^T i\tau_2 \Phi L^c + h_{\chi L} L^T i\tau_2 \chi L^c \\ &\quad + h_{\delta_L} L^T i\tau_2 \delta_L L + h_{\Delta_R} L^{cT} i\tau_2 \Delta_R L^c + \mu_1 \mathrm{Tr}(\mathrm{i}\tau_2 \Phi^{\mathrm{T}} \mathrm{i}\tau_2 \chi) + \mu_1' \mathrm{Tr}(\mathrm{i}\tau_2 \Phi^{\mathrm{T}} \mathrm{i}\tau_2 \Phi) \\ &\quad + \mu_1'' \mathrm{Tr}(\mathrm{i}\tau_2 \chi^{\mathrm{T}} \mathrm{i}\tau_2 \chi) + \mathrm{Tr}(\mu_{2\mathrm{L}} \Delta_{\mathrm{L}} \delta_{\mathrm{L}} + \mu_{2\mathrm{R}} \Delta_{\mathrm{R}} \delta_{\mathrm{R}}). \end{aligned} \tag{13}$$

The general form of the vacuum expectation values of the various scalar fields which preserve the $U(1)_{\rm em}$ gauge invariance can be written as

$$\langle \Phi \rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & e^{i\phi_1}\kappa_1' \end{pmatrix},\ \langle \chi \rangle = \begin{pmatrix} e^{i\phi_2}\kappa_2' & 0 \\ 0 & \kappa_2 \end{pmatrix},$$
$$\langle \Delta_R^0 \rangle = v_{\Delta_R},\ \langle \delta_R^0 \rangle = v_{\delta_R},\ \langle \Delta_L^0 \rangle = v_{\Delta_L},\ \langle \delta_L^0 \rangle = v_{\delta_L},\ \langle \tilde{\nu}_L \rangle = \sigma_L,\ \langle \tilde{\nu}_L^c \rangle = \sigma_R. \tag{14}$$

From the heavy gauge boson mass limits the triplet vacuum expectation values v_{Δ_R} and v_{δ_R} are in the range $v_{\Delta_R}, v_{\delta_R} \gtrsim 1$ TeV. κ'_1 and κ'_2 contribute to the mixing of the charged gauge bosons and to the flavour changing neutral currents, and are usually assumed to vanish. As was pointed out before, in order to have the tree level value of the electroweak ρ parameter close to unity the left-triplet vacuum expectation values v_{Δ_L} and v_{δ_L} must be small.

With the minimal field content, the only way to preserve the $U(1)_{em}$ gauge symmetry unbroken is to have a nonzero sneutrino VEV [29, 31]. Thus the R-parity is spontaneously broken in the SLRM with minimal particle content and renormalizable interactions.

An alternative to the minimal left-right supersymmetric model described above involves additional triplet fields, $\Omega_L(1,3,1,0)$ and $\Omega_R(1,1,3,0)$ to the minimal model [33]. In these extended models the breaking of $SU(2)_R$ is achieved in two stages via an intermediate symmetry $SU(2)_L \times U(1)_R \times U(1)_{B-L}$. In this theory the parity-breaking minimum respects the electromagnetic gauge invariance without a sneutrino VEV. The superpotential for these models contains additional terms involving the triplet fields Ω_L and Ω_R:

$$\begin{aligned} W_\Omega &= W_{min} + \frac{1}{2}\mu_{\Omega_L}\mathrm{Tr}\,\Omega_L^2 + \frac{1}{2}\mu_{\Omega_R}\mathrm{Tr}\,\Omega_R^2 + a_L\mathrm{Tr}\,\Delta_L\Omega_L\delta_L + a_R\mathrm{Tr}\,\Delta_R\Omega_R\delta_R \\ &\quad +\mathrm{Tr}\,\Omega_L\left(\alpha_L\Phi i\tau_2\chi^T i\tau_2 + \alpha_L{}'\Phi i\tau_2\Phi^T i\tau_2 + \alpha_L{}''\chi i\tau_2\chi^T i\tau_2\right) \\ &\quad +\mathrm{Tr}\,\Omega_R\left(\alpha_R i\tau_2\Phi^T i\tau_2\chi + \alpha_R{}' i\tau_2\Phi^T i\tau_2\Phi + \alpha_R{}'' i\tau_2\chi^T i\tau_2\chi\right), \end{aligned} \tag{15}$$

where W_{min} is the superpotential (13) of the minimal left-right model. In these models the see-saw mechanism takes its canonical form with $m_\nu \simeq m_D^2/M_{BL}$, where m_D is the neutrino Dirac mass. In this case the low-energy effective theory is the MSSM with unbroken R-parity, and contains besides the usual MSSM states, a triplet of Higgs scalars much lighter than the $B-L$ breaking scale.

Another possibility is to add non-renormalizable terms to the Lagrangian of the minimal left-right supersymmetric model, while retaining the minimal Higgs content [39, 33]. The superpotential for these models can be written as

$$\begin{aligned} W_{NR} &= W_{min} + \frac{a_L}{2M}(\mathrm{Tr}\,\Delta_L\delta_L)^2 + \frac{a_R}{2M}(\mathrm{Tr}\,\Delta_R\delta_R)^2 + \frac{c}{M}\mathrm{Tr}\,\Delta_L\delta_L\mathrm{Tr}\,\Delta_R\delta_R \\ &\quad +\frac{b_L}{2M}\mathrm{Tr}\,\Delta_L^2\mathrm{Tr}\,\delta_L^2 + \frac{b_R}{2M}\mathrm{Tr}\,\Delta_R^2\mathrm{Tr}\,\delta_R^2 + \frac{1}{M}\left[d_1\mathrm{Tr}\,\Delta_L^2\mathrm{Tr}\,\delta_R^2 + d_2\mathrm{Tr}\,\delta_L^2\mathrm{Tr}\,\Delta_R^2\right] \\ &\quad +\frac{\lambda_{ijkl}}{M}\mathrm{Tr}\,i\tau_2\Phi_i^T i\tau_2\Phi_j\mathrm{Tr}\,i\tau_2\Phi_k^T i\tau_2\Phi_l + \frac{\alpha_{ijL}}{M}\mathrm{Tr}\,\Delta_L\delta_L\Phi_i i\tau_2\Phi_j^T i\tau_2 \end{aligned}$$

$$
\begin{aligned}
&+\frac{\alpha_{ijR}}{M}\mathrm{Tr}\,\Delta_R\delta_R i\tau_2\Phi_i^T i\tau_2\Phi_j + \frac{1}{M}\mathrm{Tr}\,\tau_2\Phi_i^T\tau_2\Phi_j[\beta_{ijL}\mathrm{Tr}\,\Delta_L\delta_L + \beta_{ijR}\mathrm{Tr}\,\Delta_R\delta_R] \\
&+\frac{\eta_{ij}}{M}\mathrm{Tr}\,\Phi_i\Delta_R i\tau_2\Phi_j^T i\tau_2\delta_L + \frac{\overline{\eta}_{ij}}{M}\mathrm{Tr}\,\Phi_i\delta_R i\tau_2\Phi_j^T i\tau_2\Delta_L \\
&+\frac{k_{ql}}{M}Q^T i\tau_2 L\, Q^{cT} i\tau_2 L^c + \frac{k_{qq}}{M}Q^T i\tau_2 Q\, Q^{cT} i\tau_2 Q^c + \frac{k_{ll}}{M}L^T i\tau_2 L\, L^{cT} i\tau_2 L^c \\
&+\frac{1}{M}[j_L Q^T i\tau_2 Q\, Q^T i\tau_2 L + j_R Q^{cT} i\tau_2 Q^c\, Q^{cT} i\tau_2 L^c]. \qquad (16)
\end{aligned}
$$

It has been shown that the addition of non-renormalizable terms suppressed by a high scale such as Planck mass, $M_{Pl} \sim 10^{19}$ GeV, with the minimal field content ensures the correct pattern of symmetry breaking in the supersymmetric left-right model with the intermediate scale $M_R \gtrsim 10^{10} - 10^{11}$ GeV, and R-parity remains exact.

In addition to the lightest neutral CP-even Higgs, it has been known for quite some time [31] that the lightest doubly charged Higgs boson occurring in triplets of the SLRM may be light. The detection of a doubly charged Higgs was discussed extensively in [17]. Also the fermionic sector of the Higgs sector and its use in identifying the model was considered in [17]. While the lightest doubly charged Higgs or its fermionic partner may offer best possibilities to identify the triplet, the singly charged chargino production may be most suitable for finding out the R-parity violation in the model [36]. Here we will first concentrate on the experimentally interesting results on the masses of the lightest neutral and doubly charged Higgs bosons, and then describe new results in analysing different processes.

4.1 The upper limit on the lightest CP-even Higgs

In the case of the SLRM we have many new couplings and also new scales in the model and it is not obvious, what is the upper limit on the lightest CP-even Higgs boson mass. This mass bound is a very important issue, since the experiments are approaching the upper limit of the lightest Higgs boson mass in the MSSM.

A general method to find an upper limit for the lightest Higgs mass was presented in [40]. This method has been applied to the mass of the lightest Higgs, m_h, of SLRM [37] in three cases: (A) R-parity is spontaneously broken (sneutrinos get VEVs), (B) R-parity is conserved because of additional triplets, and (C) R-parity is conserved because of nonrenormalizable terms.

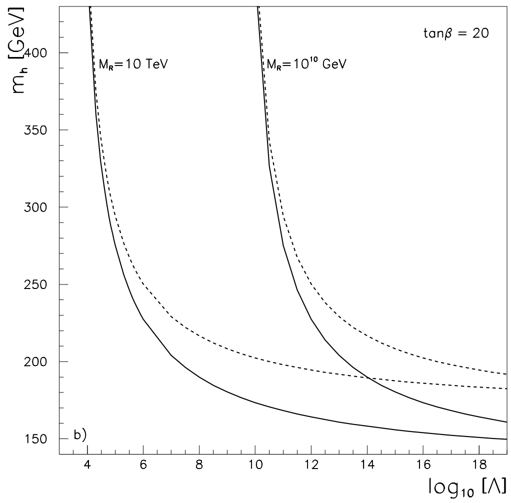

Figure 5: The upper bound on the mass of the lightest neutral Higgs boson. The soft supersymmetry breaking parameters are 1 TeV (solid) and 10 TeV (dashed).

For the minimal model, case (A), the upper bound on m_h is [37]

$$m_h^2 \leq \frac{1}{2v^2}\left[g_L^2(\omega_\kappa^2+\sigma_L^2)^2+g_R^2\omega_\kappa^4+g_{B-L}^2\sigma_L^4+8(h_{\Phi L}\kappa_1'+h_{\chi L}\kappa_2)^2\sigma_L^2+8h_{\Delta_L}^2\sigma_L^4\right], \quad (17)$$

where $v^2=\kappa_1^2+\kappa_1'^2+\kappa_2^2+\kappa_2'^2+\sigma_L^2$ and $\omega_\kappa^2=\kappa_1^2-\kappa_2^2-\kappa_1'^2+\kappa_2'^2$. The addition of extra triplets does not change this bound. Thus, the bound for the case (B), can be obtained from (17) by taking the limit $\sigma_L\to 0$. The total number of nonrenormalizable terms in case (C) is large. However, the contribution to the Higgs mass bound from these terms is found to be [37] typically numerically negligible. Therefore the upper bound for this class of models is essentially the same as in the case (B).

The radiative corrections to the lightest Higgs mass are significant. For the SLRM lightest Higgs they have been calculated in detail [37]. For nearly degenerate stop masses, the radiative corrections on m_h in the SLRM differ in form from the MSSM upper bound only because of new supersymmetric Higgs mixing parameters.

The upper bound on the mass of the lightest Higgs is plotted in Fig.5 as a function of the scale Λ up to which the SLRM remains perturbative. The upper bound is shown for two different values of the $SU(2)_R$ breaking scale, $M_R = 10$ TeV and $M_R = 10^{10}$ GeV, and for two values of soft supersymmetry breaking mass parameter, $M_s = 1$ TeV and $M_s = 10$ TeV. For large values of Λ the upper bound is below 200 GeV.

Another relevant issue concerning the lightest Higgs is its branching ratios to fermions.

Connected to that in the left-right symmetric models are problems with FCNC, which are expected if several light Higgs bosons exist [41], unless $m_{H_{FCNC}} \gtrsim \mathcal{O}(1 \text{ TeV})$. Thus the relevant limit to discuss is the one in which all the neutral Higgs bosons, except the lightest one, are heavy. It has been shown that in the decoupling limit the Yukawa couplings of the τ's are the same in the SM and the SLRM even if the τ's contain a large fraction of gauginos or higgsinos [37]. If a neutral Higgs boson, which couples to fermions very differently than the Standard Model Higgs, is found, the model most probably is not left-right symmetric.

4.2 The lightest doubly charged Higgs

Whether the lightest doubly charged Higgs is observable in experiments is an interesting issue, since this particle may both reveal the nature of the gauge group and help to determine the particular supersymmetric left-right model in question. The chances for detection depend strongly on the mass of the particle. This will be our main concern in this section, but we'll also shortly review the processes discussed more thoroughly in [17].

There are four doubly charged Higgs bosons in the SLRM, of which two are right-handed and two left-handed. The masses of the left-handed triplets are expected to be of the same order as the soft terms. The mass matrix for the right-handed triplets depends on the right-triplet VEV. Nevertheless, it was noticed in [31] that in the SLRM with broken R-parity one right-handed doubly charged scalar tends to be light. Also, in the nonrenormalizable case it is possible to have light doubly charged scalars [34]. On the other hand, in the nonsupersymmetric left-right model all the doubly charged scalars typically have a mass of the order of the right-handed scale [42]. This is also true in the SLRM with enlarged particle content [33]. Thus a light doubly charged Higgs would be a strong indication of a supersymmetric left-right model with minimal particle content.

In Figure 6 a) an example of H^{++} masses with broken R-parity is shown as a function of A_Δ for fixed σ_R. The soft masses and right-handed breaking scale, are of the order of 10 TeV. The maximum triplet Yukawa coupling allowed by positivity of the mass eigenvalues in this case is $h_\Delta \sim 0.4$. Even in the maximal case the mass of the doubly charged scalar $m_{H^{++}} \sim 1$ TeV. In Fig. 6 b) $m_{H^{++}}$ is plotted in the model containing nonrenormalizable terms as a function of the nonrenormalizable b_R-parameter for $v_R^2/M = 10^2$ GeV.

The collider phenomenology of the doubly charged scalars has been actively studied,

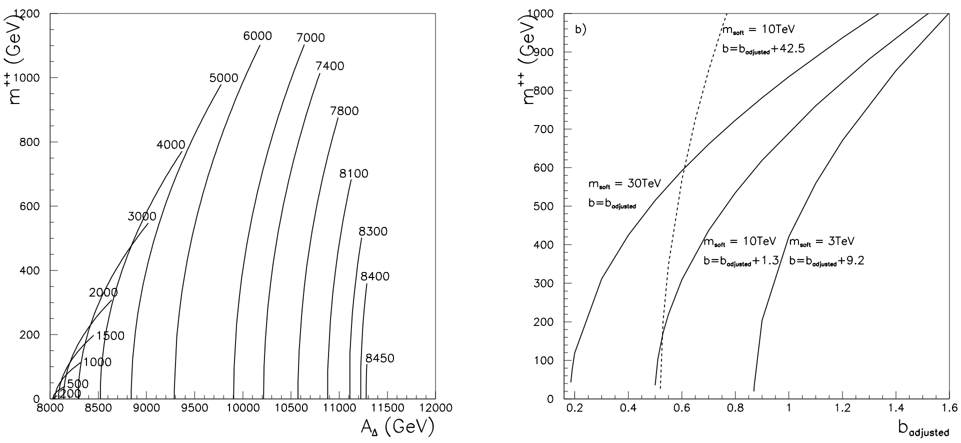

Figure 6: The mass $m_{H^{++}}$ of the lightest doubly charged Higgs. In a) the mass is as a function of the soft trilinear coupling A_Δ. The allowed σ_R varies between 100 GeV and 8.45 TeV. In b) the mass is as a function of the $b_{adjusted}$-parameter related to b_R as denoted. In b) $D = (3 \text{ TeV})^2$ (solid) except for $m_{soft} = 10$ TeV also $D = 10 \text{ TeV}^2$ is shown (dashed). The soft supersymmetry breaking parameters are marked in the figure. $\tan\beta = 50$.

since they appear in several extensions of the Standard Model, can be relatively light and have clear signatures. The main decay modes for relatively light doubly charged Higgs are [24] $H^{--} \to l_1^- l_2^-$, where $l_{1,2}$ denote leptons. Thus the experimental signature of the decay is a same sign lepton pair with no missing energy, including lepton number violating final states.

Since the left-right models contain many extra parameters when compared with the MSSM, a great advantage of the pair production is that it is relatively model independent. The doubly charged Higgses can be produced in $f\bar{f} \to \gamma^*, Z^* \to H^{++}H^{--}$ both at lepton and hadron colliders, if kinematically allowed, even if W_R is very heavy, or the triplet Yukawa couplings are very small. The pair production cross section at a linear collider has been given in [43]. The cross section remains sufficiently large close to the kinematical limit for the detection to be possible.

Kinematically, production of a single doubly charged scalar would be favoured. This option is more model dependent, but for reasonable parameter range the kinematical reach is approximately doubled compared to the pair production.

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