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Search Strategies for Non-Standard Higgs Bosons at Future e^+e^- Colliders

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Abstract

Already in the simplest two-Higgs-doublet model with CP violation in the Higgs sector, the 3×3 mixing matrix for the neutral Higgs bosons can substantially modify their couplings, thereby endangering the "classical" Higgs search strategies. However, there are sum rules relating Yukawa and Higgs–Z couplings which ensure that the ZZ, $b\bar{b}$ and $t\bar{t}$ couplings of a given neutral 2HDM Higgs boson cannot all be simultaneously suppressed. This result implies that any single Higgs boson will be detectable at an e^+e^- collider if the Z+Higgs, $b\bar{b}$+Higgs *and* $t\bar{t}$+Higgs production channels are all kinematically accessible *and* if the integrated luminosity is sufficient. We explore, as a function of Higgs mass, the luminosity required to guarantee Higgs boson detection, and find that for moderate $\tan\beta$ values the needed luminosity is unlikely to be available for all possible mixing scenarios. Implications of the sum rules for Higgs discovery at the Tevatron and LHC are briefly discussed.

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1 Introduction

Spontaneous gauge symmetry breaking in the Standard Model (SM) is realized by introducing a single CP-even Higgs boson, $h_{\rm SM}$. The "standard" Higgs hunting strategies at an e^+e^- collider rely on the Higgs-strahlung process, $e^+e^- \to Zh_{\rm SM}$, and (for higher energies and heavier Higgs bosons) on the WW fusion process, $e^+e^- \to \nu\bar{\nu}h_{\rm SM}$ (ZZ fusion is smaller by an order of magnitude) [1]. However, even the simplest two-Higgs-doublet model (2HDM) extension of the SM exhibits a rich Higgs sector structure. Moreover, it allows for spontaneous and/or explicit CP violation in the scalar sector [2]. CP violation, which in the SM is achieved only by the Yukawa couplings of the Higgs boson to quarks being explicitly complex [3], could equally well be partially or wholly due to new physics beyond the SM. The possibility that an extended Higgs sector is responsible for CP violation is particularly appealing, especially as a means for obtaining an adequate level of baryogenesis in the early universe [4].

The CP-conserving (CPC) 2HDM predicts [♯1] the existence of two neutral CP-even Higgs bosons (h^0 and H^0, with $m_{h^0} \leq m_{H^0}$ by convention), one neutral CP-odd Higgs (A^0) and a charged Higgs pair ($H^\pm$). The situation is more complex in the 2HDM with CP-violation (CPV) in the scalar sector. There, the physical mass eigenstates, $h_i (i = 1, 2, 3)$, are mixtures (specified by three mixing angles, α_i, $i = 1, 2, 3$) of the real and imaginary components of the original neutral Higgs doublet fields; as a result, the h_i have undefined CP properties.

The absence of any $e^+e^- \to Zh_{\rm SM}$ signal in LEP2 data translates into a lower limit on $m_{h_{\rm SM}}$: the latest analysis of four LEP experiments at $\sqrt{s}$ up to 196 GeV implies $m_{h_{\rm SM}}$ greater than 102.6 GeV [7]. More generally, in e^+e^- collisions, if $m_{h_{\rm SM}} < \sqrt{s} - m_Z$ the $h_{\rm SM}$ will be discovered, assuming sufficient integrated luminosity. In this note, we wish to address the extent to which the neutral Higgs bosons of an extended Higgs sector are guaranteed to be discovered if they are sufficiently light. The possibility of such a guarantee rests on considering not only Z+Higgs production but also Higgs pair production, $b\bar{b}$+Higgs production and $t\bar{t}$+Higgs production and on the existence of sum rules for the Higgs boson couplings controlling the rates for these processes. Our analysis will be performed for a type-II 2HDM, wherein the neutral component of one of the Higgs doublet fields couples only to down-type quarks and leptons and the neutral component of the other couples only to up-type quarks.

We first remind the reader of the 2HDM (CPV or CPC) result [9, 10] that if there are two light Higgs bosons, h_1 and h_2, then at least one will be observable in Zh_1 or Zh_2 production or both in h_1h_2 pair production. This is because of the sum rule [8, 9] for the Higgs boson couplings $C_1^2 + C_2^2 + C_{12}^2 = 1$, where $g_{ZZh_i} \equiv \frac{gm_Z}{c_W}C_i$ and $g_{Zh_ih_j} \equiv \frac{g}{2c_W}C_{ij}$ [$c_W = \cos\theta_W$, g is the SU(2) gauge coupling

[♯1] The same menagerie of pure-CP Higgs bosons is found at the tree level in the minimal supersymmetric model (MSSM) [5]. However, with CP-violating phases of soft-supersymmetry breaking terms, the h^0, H^0 and A^0 will mix beyond the Born approximation [6].

constant]. If none of the three processes are observed, we know that at least one of the two Higgs masses must lie beyond the kinematic limits defined by $\sqrt{s} < m_Z + m_{h_1}, m_Z + m_{h_2}, m_{h_1} + m_{h_2}$. A recent analysis of LEP data shows that the 95% confidence level exclusion region in the (m_{h_1}, m_{h_2}) plane that results from the sum rule is quite significant [11].

Here, we focus on the question of whether a *single* neutral Higgs boson will be observed in e^+e^- collisions if it is sufficiently light, regardless of the masses and couplings of the other Higgs bosons. In general, such a guarantee cannot be established if only the Higgs-strahlung and Higgs-pair production processes are considered. First, there is a "nightmare" scenario in which Higgs-strahlung is inadequate for detection of the lightest Higgs boson h_1 while all other Higgs bosons are too heavy to be kinematically accessible. This is easily arranged by choosing model parameters such that the ZZh_1 coupling is too weak for its detection in Higgs-strahlung production while maintaining consistency with precision electroweak constraints despite the other Higgs bosons being heavy. Of course, if we were to demand that the 2HDM remains perturbative up to energy scales of order $M_X \sim 10^{16}\,\mathrm{GeV}$, then (see [12]) $\sum_i C_i^2 m_{h_i^0}^2 \lesssim m_B^2$, with $m_B \sim 200\,\mathrm{GeV}$ for $M_X \sim 10^{16}\,\mathrm{GeV}$, is required and this scenario could not be realized assuming that $\sqrt{s}$ is substantially larger than m_B. Second, it could happen that there are two light Higgs bosons, h_1 and h_2, but one of them, e.g. the h_2, has full strength ZZh_2 coupling. Then, the sum rule $C_1^2 + C_2^2 + C_{12}^2 = 1$ implies that the ZZh_1 and Zh_1h_2 couplings must both be zero at tree-level. Consequently, the h_2 will be seen in $e^+e^- \to Z^* \to Zh_2$ production but the h_1 will not be discovered in Zh_1 or h_1h_2 production, even when these processes are kinematically accessible. Note that this scenario is completely consistent with the above-noted GUT-scale-perturbativity sum rule. We stress that the above cases can arise regardless of the mixing structure, CPC or CPV, of the neutral Higgs boson sector.

In [10] we derived new sum rules relating the Yukawa $g_{f\bar{f}h_i}$ and Higgs–Z couplings of the 2HDM [see Eq. (8)] which guarantee that any h_i that has suppressed ZZh_i coupling must have substantial $t\bar{t}h_i$ and/or $b\bar{b}h_i$ coupling. This result implies that if the h_i is sufficiently light for $t\bar{t}h_i$ to be kinematically allowed and if the luminosity is sufficiently large, then the h_i will be observed in at least one of the Yukawa processes $e^+e^- \to f\bar{f}h_i$ ($f = t$, b and possibly τ), dominated by Higgs radiation from the final state fermions . Therefore, the complete Higgs hunting strategy at e^+e^- colliders, and at hadron colliders as well, must include not only the Higgs-strahlung process and Higgs-pair production but also the Yukawa processes. [♯2] However, our earlier work left open a detailed analysis of just how much integrated luminosity was required.

In this paper, we consider in more detail the 2HDM in the context of future e^+e^- linear colliders ($\sqrt{s} \sim 500 - 800\,\mathrm{GeV}$) with integrated luminosity $L \sim 500 - 1000$

[♯2] In the context of a CP conserving 2HDM, the relevance of the Yukawa processes when $\tan\beta$ is large has been stressed already several times [13, 14, 15].

fb^{-1}, as planned in one-to-two years of running at TESLA. Focusing on the case of a light Higgs boson that cannot be observed in Higgs-strahlung or Higgs-pair production, we determine the L required so that either $b\bar{b}h_1$ or $t\bar{t}h_1$ production will allow h_1 detection. For the worst choices of Higgs mixing angles α_i, the required L is quite large.

The outline of the paper is as follows. In the next section, we discuss the sum rules for Higgs boson couplings in the CPV 2HDM. Then, we present numerical results for Zh_1h_2, $b\bar{b}h_1$ and $t\bar{t}h_1$ cross sections at e^+e^- linear colliders running with $\sqrt{s} = 500$ and 800 GeV and address the question of measurability of Yukawa couplings. In the next section, we determine the portions of parameter space such that unrealistically large integrated luminosity could be required for discovery of a light h_1. In the conclusions, we summarize the main points of the paper and briefly discuss implications of the sum rules for Higgs searches at hadronic accelerators.

2 Higgs boson couplings and sum rules

In the type-II two-Higgs-doublet model, the neutral component of the Φ_1 doublet field couples only to down-type quarks and leptons and the neutral component of Φ_2 couples only to up-type quarks. As usual, we define $\tan\beta \equiv v_2/v_1$, where $|\langle\Phi^0_{1,2}\rangle| = v_{1,2}/\sqrt{2}$. As a result of the mixing (for details see [10]) between real and imaginary parts of neutral Higgs fields, the Yukawa interactions of the h_i mass-eigenstates are not invariant under CP. They are given by:

$$\mathcal{L} = h_i \bar{f}(S_i^f + iP_i^f\gamma_5)f \tag{1}$$

where the scalar (S_i^f) and pseudoscalar (P_i^f) couplings are functions of the mixing angles. For up-type and down-type quarks we have

$$S_i^u = -\frac{m_u}{v s_\beta}R_{i2}, \qquad P_i^u = -\frac{m_u}{v s_\beta}c_\beta R_{i3}, \tag{2}$$

$$S_i^d = -\frac{m_d}{v c_\beta}R_{i1}, \qquad P_i^d = -\frac{m_d}{v c_\beta}s_\beta R_{i3}\,, \tag{3}$$

and similarly for charged leptons. [♯3] For the 2HDM, the R_{ij} are elements of the orthogonal rotation matrix

$$h = R\varphi = \begin{pmatrix} c_1 & -s_1c_2 & s_1s_2 \\ s_1c_3 & c_1c_2c_3 - s_2s_3 & -c_1s_2c_3 - c_2s_3 \\ s_1s_3 & c_1c_2s_3 + s_2c_3 & -c_1s_2s_3 + c_2c_3 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}, \tag{4}$$

[$s_i \equiv \sin\alpha_i$ and $c_i \equiv \cos\alpha_i$] which relates the original neutral degrees of freedom [♯4]

$$(\varphi_1, \varphi_2, \varphi_3) \equiv \sqrt{2}(\mathrm{Re}\phi_1^0,\ \mathrm{Re}\phi_2^0,\ s_\beta\mathrm{Im}\phi_1^0 - c_\beta\mathrm{Im}\phi_2^0) \tag{5}$$

[♯3] $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, and in our normalization $v \equiv \sqrt{v_1^2 + v_2^2} = 2m_W/g = 246\,\text{GeV}$.

[♯4] The remaining degree of freedom, $\sqrt{2}(c_\beta\mathrm{Im}\phi_1^0 + s_\beta\mathrm{Im}\phi_2^0)$, becomes a would-be Goldstone boson which is absorbed in giving mass to the Z gauge boson.

of the two Higgs doublets $\Phi_1 = (\phi_1^+, \phi_1^0)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$ to the physical mass eigenstates h_i ($i = 1, 2, 3$). Without loss of generality, we assume $m_{h_1} \leq m_{h_2} \leq m_{h_3}$.

Using the above notation, the couplings of neutral Higgs and Z bosons are given by

$$\begin{aligned} C_i &= s_\beta R_{i2} + c_\beta R_{i1} && (6) \\ C_{ij} &= w_i R_{j3} - w_j R_{i3} && (7) \end{aligned}$$

where $w_i = s_\beta R_{i1} - c_\beta R_{i2}$.

The conventional CP-conserving limit can be obtained as a special case: $\alpha_2 = \alpha_3 = 0$. Then, if we take $\alpha_1 = \pi/2 - \alpha$, α is the conventional mixing angle that diagonalizes the mass-squared matrix for $\sqrt{2}\mathrm{Re}\phi_1^0$ and $\sqrt{2}\mathrm{Re}\phi_2^0$. The resulting mass eigenstates are $h_1 = -h^0$ $h_2 = H^0$ and $\sqrt{2}(s_\beta \mathrm{Im}\phi_1^0 - c_\beta \mathrm{Im}\phi_2^0) = -A^0$, where h^0, H^0 (A^0) are the CP-even (CP-odd) Higgs bosons defined earlier for the CPC 2HDM. Of course, there are other CP-conserving limits. For instance, by choosing $\alpha_1 = \alpha_2 = \pi/2$, h_1 becomes pure $\varphi_3 = -A^0$, while it is h_2 and h_3 that are CP-even.

The crucial sum rules that potentially guarantee discovery (assuming sufficient luminosity) of any neutral Higgs boson that is light enough to be kinematically accessible in Higgs-strahlung *and* $b\bar{b}$+Higgs *and* $t\bar{t}$+Higgs are an automatic result of the orthogonality of the R matrix. These sum rules [10] involve a combination of the Yukawa and ZZ couplings of any one Higgs boson and require that at least one of these couplings has to be sizable. In particular, if $C_i \to 0$ (the focus of our paper) then orthogonality of R yields

$$(\hat{S}_i^t)^2 + (\hat{P}_i^t)^2 = \left(\frac{\cos\beta}{\sin\beta}\right)^2, \quad (\hat{S}_i^b)^2 + (\hat{P}_i^b)^2 = \left(\frac{\sin\beta}{\cos\beta}\right)^2 \tag{8}$$

where for convenience we introduce rescaled couplings

$$\hat{S}_i^f \equiv \frac{S_i^f v}{m_f}, \qquad \hat{P}_i^f \equiv \frac{P_i^f v}{m_f}, \tag{9}$$

$f = t, b$. [♯5] Eq. (8) implies that either the $t\bar{t}$ or the $b\bar{b}$ coupling of h_i must be large in the $C_i \to 0$ limit; both cannot be small. Even in the other extreme of $C_i \to \pm 1$, *i.e.* ffull strength ZZh_i coupling, one finds that $(\hat{S}_i)^2 + (\hat{P}_i)^2 \to 1$, for both the top and the bottom quark couplings, in the limit of either very large or very small $\tan\beta$. A completely general result following from orthogonality of R, that is independent of C_i, is

$$\sin^2\beta[(\hat{S}_i^t)^2 + (\hat{P}_i^t)^2] + \cos^2\beta[(\hat{S}_i^b)^2 + (\hat{P}_i^b)^2] = 1\,, \tag{10}$$

[♯5] For obvious reasons we consider the third generation of quarks. Similar expressions hold for for lighter generations.

again implying that the Yukawa couplings to top and bottom quarks cannot be simultaneously suppressed. As a result, if an h_i is sufficiently light, its detection in association with $b\bar{b}$ or $t\bar{t}$ should, in principle, be possible, irrespective of the neutral Higgs sector mixing and regardless of whether or not it is seen in $e^+e^- \to Zh_i$ or Higgs pair production. However, this leaves open the question of just how much luminosity is required to guarantee detection.

3 Higgs boson production in e^+e^- colliders

To treat the three processes: (i) bremsstrahlung off the Z boson ($e^+e^- \to Zh_i$), (ii) Higgs pair production ($e^+e^- \to h_ih_j$), and (iii) the Yukawa processes with Higgs radiation off a heavy fermion line in the final state ($e^+e^- \to f\bar{f}h_i$) on the same footing, we discuss the production of h_1 in association with heavy fermions:

$$e^+e^- \to f\bar{f}h_1\,. \tag{11}$$

Processes (i) and (ii) contribute to this final state when $Z \to f\overline{f}$ and $h_2 \to f\overline{f}$, respectively. If $|C_1|$ is not too near 1, Eqs. (2,3) imply that radiation processes (iii) are enhanced when the Higgs boson is radiated off top quarks for small $\tan\beta$ and off bottom quarks or τ leptons for large values of $\tan\beta$. Since all fermion and Higgs boson masses in the final state must be kept nonzero, the formulae for the cross section are quite involved. The tree level expressions can be found in Ref. [10].

Before turning to the case of a single Higgs boson that is unobservable in Higgs-strahlung or Higgs pair production, we briefly review and extend to higher energy our earlier results regarding the detection of a least one of two light Higgs bosons when $m_{h_1}+m_{h_2}, m_{h_1}+m_Z, m_{h_2}+m_Z < \sqrt{s}$. In particular, suppose that neither is observable in Higgs-strahlung. More precisely, as we scan over the α_i's we require that the number of $e^+e^- \to Zh_1$ and (separately) the number of Zh_2 events both be less than 50 for an integrated luminosity of 500 fb^{-1}. This will mean that $|C_1|$, $|C_2| \ll 1$, which in turn implies that Higgs-pair production is at full strength, $|C_{12}| \sim 1$. In Fig. 1, we show contour plots for the minimum value of the pair production cross section, $\min[\sigma(e^+e^- \to h_1h_2)]$, as a function of Higgs boson masses at $\sqrt{s} = 500$ and 800 GeV obtained by scanning over mixing angles α_i. With integrated luminosity of $500-1000$ fb^{-1}, a large number of events (large enough to allow for selection cuts and experimental efficiencies) is predicted for the above energies over a broad range of Higgs boson masses. If 50 h_1h_2 events before cuts and efficiencies prove adequate, one can probe reasonably close to the kinematic boundary defined above. Thus, we will only need the Yukawa processes for Higgs discovery if (a) there is only one light Higgs boson or (b) there are two light Higgs bosons but one cannot be seen in Higgs-strahlung or Higgs pair production because the other has full SM strength ZZ coupling.

So now let us turn to the case of a light Higgs boson that cannot be seen in Zh_1 production ($|C_1| \ll 1$) or Higgs pair production ($|C_{1i}| \ll 1$, $i = 2,3$ and/or

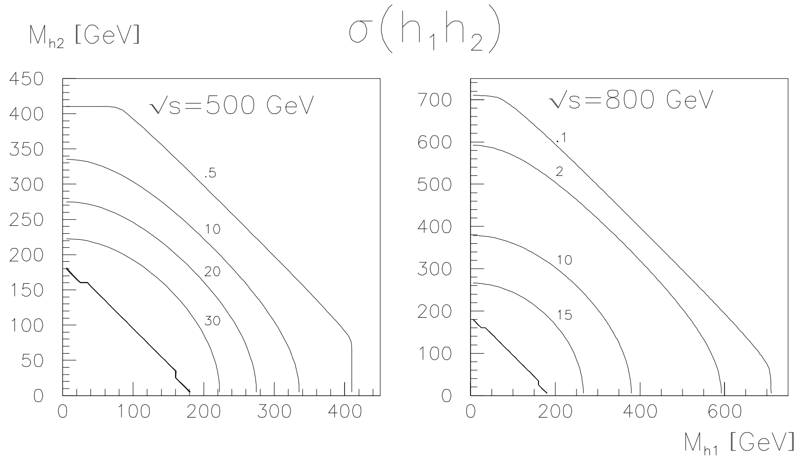

Figure 1: Contour lines for $\min[\sigma(e^+e^- \to h_1h_2)]$ in fb units, obtained by scanning over the α_i while requiring ≤ 50 Zh_1 or Zh_2 events for $L = 500$ fb^{-1}, are plotted in (m_{h_1}, m_{h_2}) parameter space for the indicated $\sqrt{s}$ values and for $\tan\beta = 0.5$. The plots are virtually unchanged for larger values of $\tan\beta$. The contour lines overlap in the inner corner of each plot as a result of excluding mass choices inconsistent with experimental constraints from LEP2 data.

$m_{h_2}, m_{h_3} > \sqrt{s} - m_{h_1}$). The question is whether the sum rules (8) imply that Yukawa couplings are sufficiently large to allow detection of the h_1 in $t\bar{t}h_1$ and/or $b\bar{b}h_1$ production (assuming both are kinematically allowed). In Fig. 2, we plot the minimum and maximum values of $\sigma(e^+e^- \to f\bar{f}h_1)$ for $f = t, b$ as a function of the Higgs boson mass, where we scan over the mixing angles α_1 and α_2 [♯6] at a given $\tan\beta$ while requiring fewer than 50 Zh_1 events for $L = 500$ fb^{-1}.[♯7] We see that, if m_{h_1} is not large and $\tan\beta$ is either very small or very large, we are guaranteed that there will be sufficient events in either the $b\bar{b}h_1$ or the $t\bar{t}h_1$ channel to allow h_1 discovery. However, if $\tan\beta$ is of moderate size, the reach in m_{h_1} is quite limited if the α_i's are such that $\sigma(t\bar{t}h_1)$ is minimal. For example, at $\sqrt{s} = 500$ GeV let us take 50 events (before cuts and efficiencies) as the observability criteria.[♯8] For $L = 500$ fb^{-1}, ≥ 50 events then requires $\sigma \geq 0.1$ fb. From Fig. 2, we see the

♯6 If only the h_1 is light, we only need to scan over α_1 and α_2 since all the couplings of the h_1 depend only upon these two mixing angles.

♯7 We note that, if $C_1 \sim 0$, then the minimal and maximal $b\bar{b}h_1$ cross sections are almost equal.

♯8 For $\tan\beta \ll 1$ and a light h_1, requiring 50 $t\bar{t}h_1$ events might not be sufficient since the h_1 will decay predominantly into $c\bar{c}$, and the resulting $t\bar{t}c\bar{c}$ final states will have a large background from ordinary $t\bar{t}$+multijet events. On the other hand, the $t\bar{t}h_1$ cross section is substantially enhanced when $\tan\beta \ll 1$ and, unless there is severe phase space suppression, we will have substantially more than 50 events.

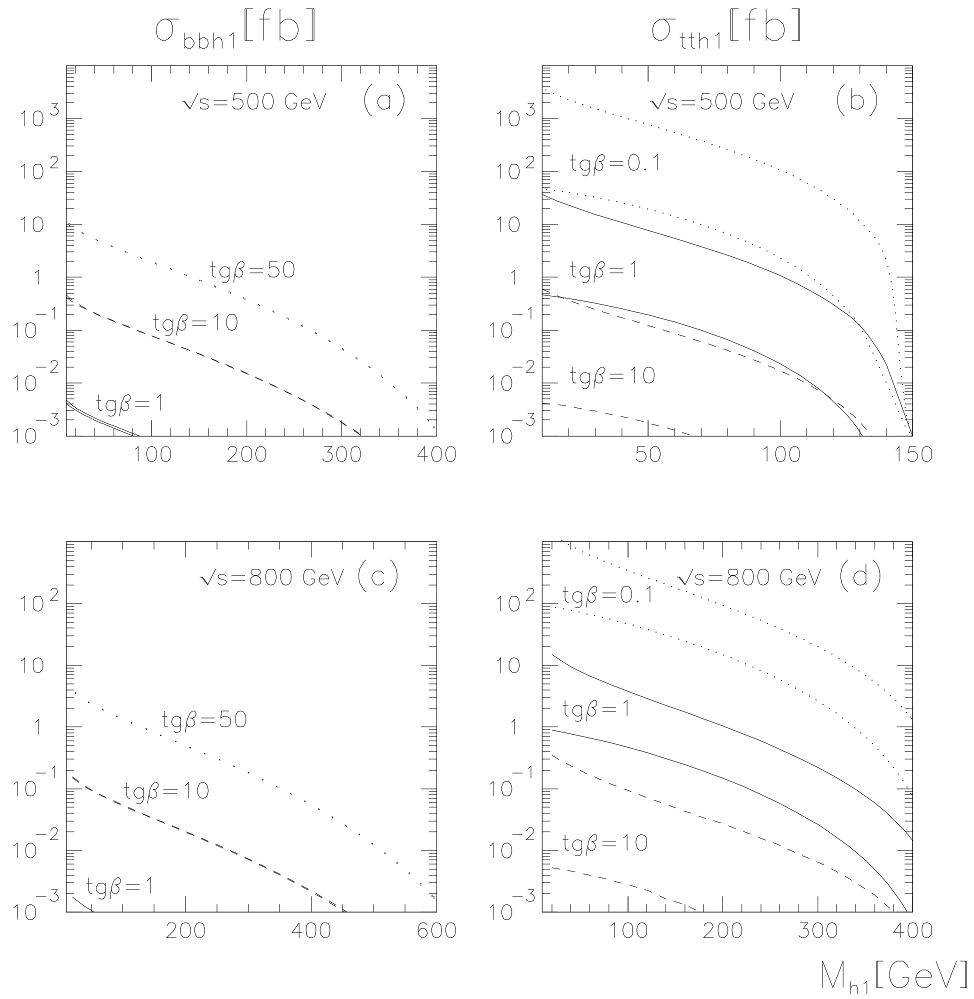

Figure 2: The minimal and maximal values of $\sigma(b\bar{b}h_1)$ and $\sigma(t\bar{t}h_1)$, obtained by scanning over α_1 and α_2 (see footnote 6) while requiring $\leq$ 50 Zh_1 events for $L = 500$ fb^{-1}, are plotted for $\sqrt{s} = 500$ and 800 GeV. For a given value of $\tan\beta$, the same type of line (dots for $\tan\beta = 0.1$ and $t\bar{t}h_1$, solid for $\tan\beta = 1$, dashes for $\tan\beta = 10$, dots for $\tan\beta = 50$ and $b\bar{b}h_1$) is used for the minimal and maximal values of the cross sections. In the case of $b\bar{b}h_1$, the minimal and maximal values of the cross sections are almost the same. Masses of the remaining Higgs bosons are assumed to be 1000 GeV.

following.

- At $\tan\beta = 1$, $\sigma(b\bar{b}h_1) \ll 0.1$ fb for all m_{h_1}, while $\sigma_{\rm min}(t\bar{t}h_1)$ falls below 0.1 fb for $m_{h_1} > 70\,{\rm GeV}$. Thus, all but quite light h_1's would elude discovery.

- At $\tan\beta = 10$, $\sigma_{\rm min}(t\bar{t}h_1) \ll 0.1$ fb and $\sigma_{\rm min}(b\bar{b}h_1) \simeq \sigma_{\rm max}(b\bar{b}h_1)$ falls below 0.1 fb for $m_{h_1} > 80\,{\rm GeV}$.

A $\sqrt{s} = 800$ GeV machine considerably extends the mass reach for $\tan\beta = 1$: the h_1 will be observable for $m_{h_1} \lesssim 230\,{\rm GeV}$ (requiring $\sigma_{\rm min}(t\bar{t}h_1) \geq 0.1$ fb). However, for $\tan\beta = 10$, $\sigma_{\rm min}(t\bar{t}h_1)$ is again very small while $\sigma(b\bar{b}h_1)$ actually declines faster, falling below 0.1 fb already at $m_{h_1} \sim 50\,{\rm GeV}$. Obviously, for $\tan\beta$ somewhat less than 10, only a very light h_1 is guaranteed to be observable with only $L = 500$ fb^{-1} of integrated luminosity.

For the most part, the minimum cross sections obtained above when the number of Zh_1 events is small and $\tan\beta$ is moderate in size correspond to α_i choices such that $C_1 = 0$ exactly. Further, when $C_1 = 0$, the minimum cross sections are achieved for a purely CP-odd h_1 ($\alpha_1 = \alpha_2 = \pi/2$ and variants thereof), even though $C_1 = -s_\beta s_1 c_2 + c_\beta c_1$ is exactly zero for any choices of α_1 and α_2 such that $c_2 = \cot\beta c_1/s_1$ (see discussion in [10]).

For more extreme $\tan\beta$ values than those illustrated in Fig. 2, there is, however, an alternative — we can actually zero one of the Yukawa process, namely the one that is already suppressed, while keeping the Zh_1 cross section small. For example, if $\tan\beta$ is large, implying small c_β, C_1 can be small enough to satisfy a finite experimental limit on the number of Zh_1 events if $\alpha_1 = 0$ ($s_1 = 0$, $c_1 = 1$), so that the first term in $C_1 = -s_\beta s_1 c_2 + c_\beta c_1$ is 0. In this extreme, the $t\bar{t}h_1$ cross section will be zero (irrespective of α_2) since $S_1^t \propto -s_1 c_2/s_\beta$ and $P_1^t \propto s_1 s_2 \cot\beta$ are both 0. In this limit, h_1 is purely φ_1, the neutral Higgs component that couples only to bottom quarks. If $\tan\beta$ is small (s_β small), the converse situation arises. C_1 can be kept small by taking $c_1 = 0$ ($\alpha_1 = \pi/2$) irrespective of the value of α_2. One can then zero the $b\bar{b}h_1$ cross section by choosing $\alpha_2 = 0$. This is the limit in which h_1 is purely φ_2, the neutral Higgs component that couples to top quarks.

To illustrate, consider $\sqrt{s} = 800\,{\rm GeV}$ and large $\tan\beta$. If we require that $\sigma(Zh_1) < 0.1$ fb (corresponding to fewer than 50 events for $L = 500$ fb^{-1}), then for $\tan\beta = 10, 15, 20$ we can choose $\alpha_1 = 0$, i.e. $\sigma(t\bar{t}h_1) = 0$, for $m_{h_1} \geq 410, 90, 0\,{\rm GeV}$, respectively. Note that $\tan\beta = 10$ is just on the border for which this extreme of zeroing $t\bar{t}h_1$ becomes relevant. In fact, the $\tan\beta = 10$ minimum cross section curve at $\sqrt{s} = 800\,{\rm GeV}$ in Fig. 2 lies below that which would be obtained for a purely CP-odd h_1 and corresponds to a slight compromise between exactly zeroing the $t\bar{t}h_1$ cross section and the requirement of keeping $\sigma(Zh_1) < 0.1$ fb.

Measuring the Yukawa couplings:

From the Yukawa sum rules and Fig. 2, it is clear that the value of C_i that makes it easiest to measure at least one of the h_i Yukawa couplings is very $\tan\beta$ dependent.

If $\tan\beta$ is either $\ll 1$ or $\gg 10$, then $C_i = 0$ seems to be the most optimistic. This is because the largest of the minimum cross section values (whether $t\bar{t}h_i$ for $\tan\beta \ll 1$ or $b\bar{b}h_i$ for $\tan\beta \gg 1$) is typically substantially enhanced if $|C_i| \sim 0$, whereas if $|C_i|$ is not small the sum rules imply that less enhancement is possible. In particular, if $|C_i| \sim 1$ (as would be known if the Zh_i rate is full strength), then, as outlined earlier, both $(\hat{S}_i^t)^2 + (\hat{P}_i^t)^2$ and $(\hat{S}_i^b)^2 + (\hat{P}_i^b)^2$ will be of order unity, approaching 1 exactly if $\tan\beta$ is either very large or very small. This implies minimum cross section values close to those found for $\tan\beta = 1$. From the $\tan\beta = 1$ minimum cross section curves of Fig. 2 for $\sqrt{s} = 500\,\mathrm{GeV}$ ($\sqrt{s} = 800\,\mathrm{GeV}$), one finds that $L = 500$ fb^{-1} would not be sufficient for a measurement of the $b\bar{b}h_i$ coupling via $b\bar{b}h_i$ production, and, if m_{h_i} is significantly above $70\,\mathrm{GeV}$ ($230\,\mathrm{GeV}$), it would also be difficult to measure the $t\bar{t}h_i$ Yukawa coupling.

The situation is quite different if $\tan\beta$ is moderate in size. In this regime of $\tan\beta$, Fig. 2 shows that a relatively light h_i may not even be observable for $L = 500$ fb^{-1} at $\sqrt{s} = 800\,\mathrm{GeV}$ when $|C_i| \ll 1$. However, it might very well be observable in one of the Yukawa processes if $|C_i| = 1$. For example, consider $m_{h_1} = 200\,\mathrm{GeV}$ in Fig. 2. If $\tan\beta = 10$, $\sigma_{\min}(b\bar{b}h_1)$ and $\sigma_{\min}(t\bar{t}h_1)$ of Fig. 2 are both below 0.1 fb if $|C_1| \ll 1$, whereas if $|C_1| = 1$ then the $\tan\beta = 1$ curves of Fig. 2 become relevant, from which we see that $\sigma_{\min}(t\bar{t}h_1) \sim 0.2$ fb, implying that one could obtain a reasonably good measurement of the $t\bar{t}h_1$ coupling.

4 Worst case scenarios

The discussion of the previous section raises the interesting question of just how much luminosity is required as a function of $\sqrt{s}$ and m_{h_1} in order to absolutely guarantee discovery of a light h_1 in at least one of the three modes, Zh_1, $t\bar{t}h_1$ or $b\bar{b}h_1$. We consider only h_1 masses such that both Yukawa modes are kinematically allowed. In our first plot, Fig. 3, we impose the requirements (I) that there be ≤ 50 Zh_1 events for $L = 500$ fb^{-1} and (II) that LEP/LEPII upper bounds [16] on the ZZh_1 coupling be satisfied. For each m_{h_1}, we scan over $\tan\beta$, to determine the $\tan\beta$ at which $\min_{(\alpha_1,\alpha_2)}\left\{\max\left[\sigma(b\bar{b}h_1), \sigma(t\bar{t}h_1)\right]\right\}$ is smallest. Here, $\max\left[\sigma(b\bar{b}h_1), \sigma(t\bar{t}h_1)\right]$ is the larger of $\sigma(b\bar{b}h_1)$ and $\sigma(t\bar{t}h_1)$ for any given (α_1, α_2) choice, and $\min_{(\alpha_1,\alpha_2)}$ refers to the minimum value of this maximum after scanning over all (α_1, α_2) values (see footnote 6) satisfying the constraints (I) and (II). We then look for the $\tan\beta$ value at which this minimum is smallest. For any other $\tan\beta$ choice, one or the other cross section will be larger than this minimum for *all* choices of (α_1, α_2) and the corresponding mode easier to observe. This defines the 'worst case' $\tan\beta$ choice for which a light Higgs boson that is unobservable in the Zh_1 mode will be most difficult to see by virtue of neither the $b\bar{b}h_1$ nor the $t\bar{t}h_1$ cross section being enhanced relative to the other. In Fig. 3, we plot the worst case choice of $\tan\beta$ and the corresponding value of $\sigma_{\min} \equiv \min_{\tan\beta}\left(\min_{(\alpha_1,\alpha_2)}\left\{\max\left[\sigma(b\bar{b}h_1), \sigma(t\bar{t}h_1)\right]\right\}\right)$. Results are presented for both $\sqrt{s} = 500\,\mathrm{GeV}$ and $\sqrt{s} = 800\,\mathrm{GeV}$.

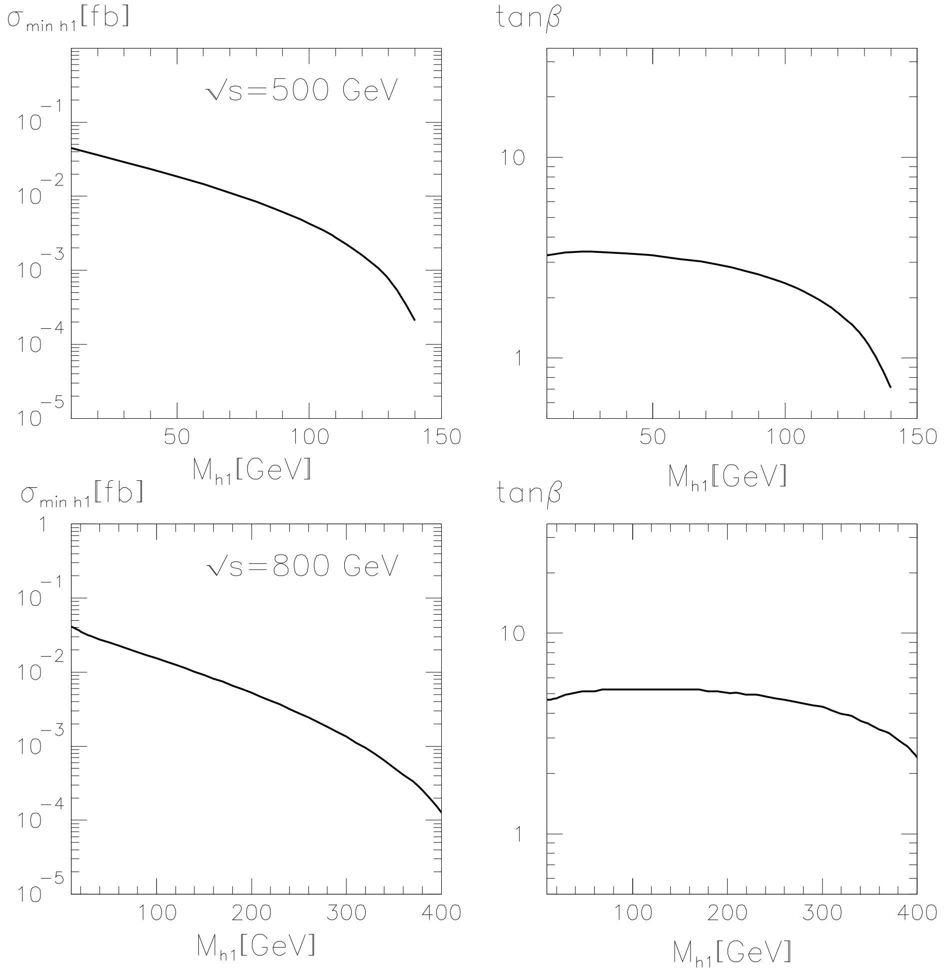

Figure 3: For $\sqrt{s} = 500\,\mathrm{GeV}$ and $\sqrt{s} = 800\,\mathrm{GeV}$, we present as a function of m_{h_1} the value of $\sigma_{\min} \equiv \min_{\tan\beta}\left(\min_{(\alpha_1,\alpha_2)}\left\{\max[\sigma_{\min}(b\bar{b}h_1), \sigma_{\min}(t\bar{t}h_1)]\right\}\right)$, and the corresponding value of $\tan\beta$, as obtained by scanning over $\tan\beta$ and (α_1, α_2) parameter space subject to constraints (I) and (II) — see text for details. Masses of the remaining Higgs bosons are assumed to be 1000 GeV.

We observe that the integrated luminosity required for the worst case cross section to yield 50 events in each of the Yukawa modes is always greater than $L = 500$ fb^{-1}. Even for small $m_{h_1} \sim 10\,\mathrm{GeV}$, $\sigma_{\rm min} \sim 4 - 5 \times 10^{-2}$ fb at these energies, implying that $L \gtrsim 1000$ fb^{-1} would be required for just $40 - 50$ events in each. As we increase m_{h_1}, the worst case cross section for $\sqrt{s} = 500\,\mathrm{GeV}$ falls dramatically and detection of the h_1 would not be possible for any reasonable L. However, at $\sqrt{s} = 800\,\mathrm{GeV}$ the worst case cross section has only fallen to about 1.7×10^{-2} fb at $m_{h_1} = 100\,\mathrm{GeV}$ for which $L = 3000$ fb^{-1} would yield about 50 events in the $b\overline{b}h_1$ and $t\overline{t}h_1$ modes, each (while still not guaranteeing as many as 50 Zh_1 events). Possibly, such a large L could be achieved after several years of running.

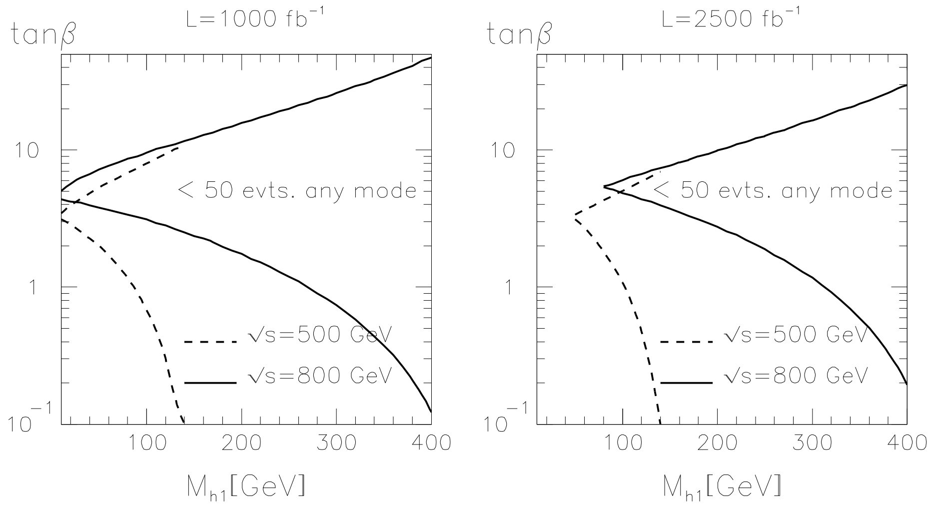

Figure 4: For $\sqrt{s} = 500\,\mathrm{GeV}$ (dashes) and $\sqrt{s} = 800\,\mathrm{GeV}$ (solid) we present as a function of m_{h_1} the maximum and minimum $\tan\beta$ values between which $t\overline{t}h_1$, $b\overline{b}h_1$ and Zh_1 final states can (for some choice of (α_1, α_2) consistent with constraint (II) — see text) all have fewer than 50 events assuming (a) $L = 1000$ fb^{-1} or (b) $L = 2500$ fb^{-1}. Masses of the remaining Higgs bosons are assumed to be 1000 GeV.

To illustrate all of this more completely, we have determined, as a function of m_{h_1}, the $\tan\beta$ range for which constraint (II) above is satisfied while the Zh_1, the $b\overline{b}h_1$ and the $t\overline{t}h_1$ cross section *each* yield fewer than 50 events for at least one (α_1, α_2) choice assuming (a) $L \leq 1000\,\mathrm{fb}^{-1}$ or (b) $L \leq 2500\,\mathrm{fb}^{-1}$ and $\sqrt{s} = 500\,\mathrm{GeV}$ or, separately, $\sqrt{s} = 800\,\mathrm{GeV}$. These $\tan\beta$ ranges are represented in Fig. 4 by the wedge of $\tan\beta$ between the solid ($\sqrt{s} = 800\,\mathrm{GeV}$) or dashed ($\sqrt{s} = 500\,\mathrm{GeV}$) lines. For $\tan\beta$ values above (below) the upper (lower) line, $b\overline{b}h_1$ ($t\overline{t}h_1$) will be observable for all (α_1, α_2) choices. We see that, even after combining $\sqrt{s} = 500\,\mathrm{GeV}$

and $\sqrt{s} = 800\,\mathrm{GeV}$ running, the $L = 1000$ fb^{-1} wedge begins at $m_{h_1} \sim 25\,\mathrm{GeV}$ and widens rapidly with increasing m_{h_1}. For $L = 2500$ fb^{-1}, the wedge begins at a higher m_{h_1} value ($\sim 80\,\mathrm{GeV}$ for $\sqrt{s} = 800\,\mathrm{GeV}$), but still expands rapidly as m_{h_1} increases further. Thus, it is apparent that, despite the sum rules guaranteeing significant fermionic couplings for a light 2HDM Higgs boson that is unobservable in Z+Higgs production, $\tan\beta$ and the α_i mixing angles can be chosen so that the cross section magnitudes of the two Yukawa processes are simultaneously so small that detection of such an h_1 cannot be guaranteed for integrated luminosities that are expected to be available.

On a final technical note, we have found that the h_1 is, for the most part, either exactly, or almost exactly, CP-odd for the (α_1, α_2) parameters corresponding to the curves plotted in Figs. 3 and 4. The only exception is for m_{h_1} values between $\sim 160\,\mathrm{GeV}$ ($\sim 240\,\mathrm{GeV}$) and $\sim 270\,\mathrm{GeV}$ ($\sim 300\,\mathrm{GeV}$) for $L = 1000$ fb^{-1} ($L = 2500$ fb^{-1}) at $\sqrt{s} = 800\,\mathrm{GeV}$ along the upper lines in Fig. 4. For this range, α_1 can be chosen close to 0 to minimize $\sigma(t\bar{t}h_1)$ while continuing to satisfy the Zh_1 event number constraint (see the discussion at the end of the previous section).

5 Discussion and conclusions

The sum rules, Eq. (8), relating the Yukawa and Higgs-ZZ couplings of a general CP-violating two-Higgs-doublet model have important implications for Higgs boson discovery at an e^+e^- collider. In particular, for any h_i, if the ZZh_i coupling is small, then the $t\bar{t}h_i$ or $b\bar{b}h_i$ Yukawa coupling must be substantial. This means that any one of the three neutral Higgs bosons that is light enough to be produced in $e^+e^- \to t\bar{t}h_i$ (implying that $e^+e^- \to Zh_i$ and $e^+e^- \to b\bar{b}h_i$ are also kinematically allowed) will normally be found at an e^+e^- linear collider if the integrated luminosity is sufficient. However, we have found that the mass reach in m_{h_i} may fall well short of the $\sqrt{s} - 2m_t$ kinematic limit for moderate $\tan\beta$ values and anticipated luminosities. We have made a precise determination of the value of $\tan\beta$ (as a function of m_{h_i}) for which the smallest (common) value of the $t\bar{t}h_i$ and $b\bar{b}h_i$ cross sections is attained when the ZZh_i coupling is suppressed. From this, we have computed as a function of m_{h_i} the minimum luminosity required in order to detect such an h_i. Even at $\sqrt{s} = 800\,\mathrm{GeV}$, to guarantee detection of a Higgs boson with small ZZ coupling for the worst possible choice of $\tan\beta$ and neutral Higgs sector mixing angles would require an integrated luminosity in excess of 1000 fb^{-1} starting at $m_{h_i} \sim 10\,\mathrm{GeV}$. Further, the minimum L required to guarantee detection for the worst choices of $\tan\beta$ and mixing angles increases rapidly as m_{h_i} increases, as does the band of $\tan\beta$ in which $L > 1000$ fb^{-1} is required.

We also discussed the case of an h that *is* observed in the Zh final state but also light enough to be seen in $t\bar{t}h$ and, by implication, $b\bar{b}h$. We have noted that if Zh production proceeds with SM strength, then the same sum rules can be used to show that measurement of its $b\bar{b}$ coupling will be impossible for any conceivably achievable integrated luminosity, while measurement of its $t\bar{t}$ coupling may only be possible for m_h up to values significantly below the $\sqrt{s}-2m_t$ phase space limit (the exact reach depending upon the integrated luminosity and Higgs sector mixing angles).

Finally, we note that detection 'guarantees' for the 2HDM model are likely to apply over an even more restricted range of model parameter space in the case of the Tevatron and LHC hadron colliders. In the case of the Tevatron, the small rate for $t\bar{t}$+Higgs production is a clear problem. In the case of the LHC, a detailed study is needed to determine what cross section level is required in order that Higgs detection in the $t\bar{t}$+Higgs channel will be possible. Existing studies in the context of supersymmetric models can be used to point to parameter regions that are problematical because of large backgrounds and/or signal dilution due to sharing of available coupling strength. Almost certainly, very small (large) $\tan\beta$ values will be needed in order to be certain that the $t\bar{t}$+Higgs ($b\bar{b}$+Higgs) modes will be viable. Still, it is clear that the sum rules do imply that difficult parameter regions are of limited extent.

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