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Electroweak Precision Tests with GigaZ *

S. Heinemeyer[1 †] and G. Weiglein[2 ‡]

[1] HET, Brookhaven Natl. Lab., Upton, New York 11973, USA

[2] CERN, TH Division, CH-1211 Geneva 23, Switzerland

Abstract. By running the prospective high-energy e^+e^- collider TESLA in the GigaZ mode on the Z resonance, experiments can be performed on the basis of more than 10^9 Z events. This will allow the measurement of the effective electroweak mixing angle to an accuracy of $\delta \sin^2\theta_{\rm eff} \approx \pm 1 \times 10^{-5}$. The W boson mass is likewise expected to be measurable with an error of $\delta M_W \approx \pm 6$ MeV near the W^+W^- threshold. We review the electroweak precision tests that can be performed with these high precision measurements within the Standard Model (SM) and its minimal Supersymmetric extension (MSSM). The complementarity of direct measurements at a prospective linear e^+e^- collider and indirect constraints following from measurements performed at GigaZ is emphasized.

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†) email: Sven.Heinemeyer@bnl.gov
‡) email: Georg.Weiglein@cern.ch

Electroweak Precision Tests at GigaZ

Sven Heinemeyer* and Georg Weiglein†

*HET, Brookhaven Natl. Lab., Upton, New York 11973, USA
†CERN, TH Division, CH-1211 Geneva 23, Switzerland

Abstract. By running the prospective high-energy e^+e^- collider TESLA in the GigaZ mode on the Z resonance, experiments can be performed on the basis of more than 10^9 Z events. This will allow the measurement of the effective electroweak mixing angle to an accuracy of $\delta \sin^2\theta_{\rm eff} \approx \pm 1 \times 10^{-5}$. The W boson mass is likewise expected to be measurable with an error of $\delta M_W \approx \pm 6$ MeV near the W^+W^- threshold. We review the electroweak precision tests that can be performed with these high precision measurements within the Standard Model (SM) and its minimal Supersymmetric extension (MSSM). The complementarity of direct measurements at a prospective linear e^+e^- collider and indirect constraints following from the measurements performed at GigaZ is emphasized.

I THEORETICAL BASIS

The prospective high-energy e^+e^- linear collider TESLA can be operated on the Z boson resonance by adding a bypass to the main beam line [1]. Due to the high luminosity, $\mathcal{L} = 7 \times 10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, about 2×10^9 Z events per year can be generated, which will be referred to as the "GigaZ" mode. By using the Blondel scheme, this results in a measurement of the effective leptonic mixing angle, $\sin^2\theta_{\rm eff}$, of about $\delta \sin^2\theta_{\rm eff} \approx \pm 1 \times 10^{-5}$ [2]. Increasing the collider energy to the W-pair threshold, about $\mathcal{O}(10^6)$ W bosons can be generated resulting in a measurement of the W mass of $\delta M_W \approx \pm 6$ MeV [3]. This increase of precision in $\sin^2\theta_{\rm eff}$ and M_W opens new opportunities for high precision physics in the electroweak sector [4,5].

In this paper we compare the theoretical predictions for M_W and $\sin^2\theta_{\rm eff}$ in the Standard Model (SM) and the Minimal Supersymmetric Standard Model (MSSM) with the expected experimental uncertainties. In order to calculate the W-boson mass in the SM and the MSSM we use

$$M_W^2 = M_Z^2/2 \left[1 + \left(4\pi\alpha/(\sqrt{2}G_F M_Z^2) \times (1 + \Delta r)\right)^{1/2}\right] , \qquad (1)$$

where the loop corrections are summarized in Δr. The quantity $\sin^2\theta_{\rm eff}$ is defined through the effective couplings g_V^f and g_A^f of the Z boson to fermions:

$$\sin^2\theta_{\rm eff} = 1/(4\,|Q_f|) \left[1 - \mathrm{Re}\, g_V^f / \mathrm{Re}\, g_A^f\right] , \qquad (2)$$

where the loop corrections are contained in $g^f_{V,A}$. The theoretical input for M_W and $\sin^2\theta_{\rm eff}$ is described in detail in Ref. [5]. It involves corrections up to $\mathcal{O}(\alpha^2)$ [6] and $\mathcal{O}(\alpha\alpha_s^2)$ [7] in the SM and up to $\mathcal{O}(\alpha\alpha_s)$ in the MSSM [8].

In the SM the Higgs boson mass is a free parameter. Contrary to this, in the MSSM the masses of the neutral $\mathcal{CP}$-even Higgs bosons are calculable in terms of the other MSSM parameters. The largest corrections arise from the t–$\tilde{t}$-sector, where the dominant contribution reads $\Delta m_h^2 \sim m_t^4/M_W^2 \log(m_{\tilde{t}_1}^2\, m_{\tilde{t}_2}^2/m_t^4)$. $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ denote the two stop mass eigenstates. $\theta_{\tilde{t}}$ will later denote the $\tilde{t}$ mixing angle. Since the one-loop corrections are known to be very large, we use the currently most precise two-loop result based on explicit Feynman-diagrammatic calculations [9], where the numerical evaluation is based on Ref. [10]. The relevant observables together with their uncertainties at various colliders and their current experimental value can be found in Tab. 1.

TABLE 1. Expected precision at various colliders for $\sin^2\theta_{\rm eff}$, M_W, m_t and the (lightest) Higgs boson mass, M_h. "now" refers to the present accuracy obtained at LEP, SLD and the Tevatron RunI. "LHC" here and in the following also includes Tev. RunII. See Ref. [5] for a detailed list or references.

	now	LHC	LC	GigaZ	current central value
$\delta\sin^2\theta_{\rm eff}(\times 10^5)$	17	17	6	1.3	0.23146
δM_W [MeV]	37	15	15	6	80.436 GeV
δm_t [GeV]	5.1	2	0.2	0.13	174.3 GeV
δM_H [MeV]	–	200	50	50	–

II COMPARISON OF SM AND MSSM

In Fig. 1 the theoretical predictions for M_W and $\sin^2\theta_{\rm eff}$ obtained in the SM and the MSSM are compared with their experimental values. In the left plot of Fig. 1 the bands in the m_t–M_W plane allowed in the SM and the MSSM are compared to the (prospective) experimental precisions at LEP/Tevatron, LHC/LC and GigaZ. The SM band arises from the unknown value of the Higgs boson mass, where the upper boundary is obtained from the lower bound set by LEP, $M_H \gtrsim 113$ GeV [11]. The band in the MSSM is due to the unknown masses of the SUSY particles. The upper boundary corresponds to light SUSY, the lower boundary corresponds to heavy SUSY, i.e. the MSSM is SM like. In the overlap area the SM has a Higgs boson in the SUSY range, i.e. $M_H \lesssim 130$ GeV. The plot shows a slight preference of the present data for the MSSM at the 68% CL.

The right plot of Fig. 1 shows the M_W–$\sin^2\theta_{\rm eff}$ plane. The allowed area in the SM and the MSSM is compared with the experimental precision at LEP/SLD/Tevatron, LHC/LC and GigaZ. For the SM area, the Higgs boson mass has been varied between 113 GeV $\le M_H \le$ 400 GeV. The top quark mass has been varied between 170 GeV $\le m_t \le$ 180 GeV. Both models possess an allowed parameter space at the 68% CL.

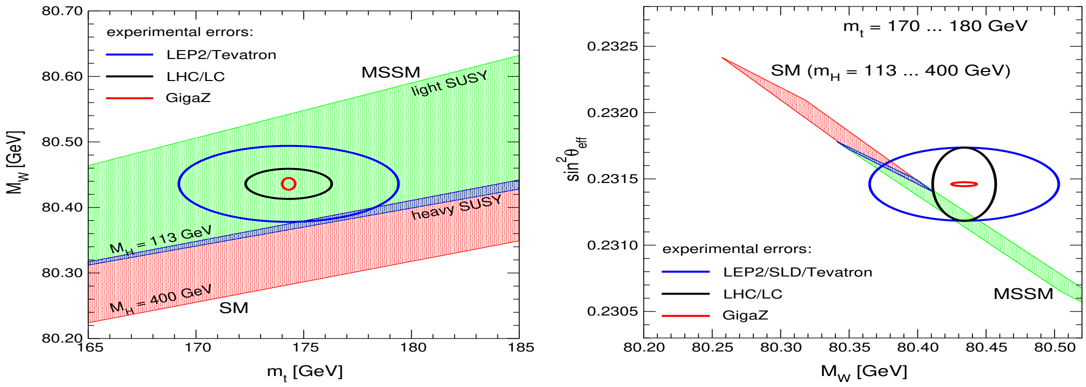

FIGURE 1. The theoretical prediction of M_W and $\sin^2\theta_{\rm eff}$ is compared to the experimental measured values with the current LEP/SLD/Tevatron precision and with the prospective accuracies at the LHC/LC and at GigaZ.

III INDIRECT CONSTRAINTS FROM GIGAZ

Often the indirect constraints on observables obtained at GigaZ could be complementary to their direct measurements at the Tevatron RunII, the LHC or at an LC. As an example we present an analysis for the scalar top sector. The direct information on the stop sector parameters, $m_{\tilde{t}_1}$ and $\theta_{\tilde{t}}$, can be obtained from the process $e^+e^- \to \tilde{t}_1\tilde{t}_1$ to a precision of $\mathcal{O}(1\%)$ [12]. These direct measurements can be combined with the indirect information from requiring consistency of the MSSM with a precise measurement of the Higgs boson mass, m_h, and the electroweak precision observables. This is shown in Fig. 2, where the allowed parameter space according to measurements of m_h, M_W and $\sin^2\theta_{\rm eff}$ are displayed in the plane of the heavier stop mass, $m_{\tilde{t}_2}$, and $|\cos\theta_{\tilde{t}}|$ for the accuracies at a LC with and without the GigaZ option and at the LHC (see Tab. 1). For $m_{\tilde{t}_1}$ the central value and experimental error of $m_{\tilde{t}_1} = 180 \pm 1.25$ GeV are taken for LC/GigaZ, while for the LHC an uncertainty of 10% in $m_{\tilde{t}_1}$ is assumed. The other parameters have been chosen according to the mSUGRA reference scenario 2 [13], with the following accuracies: $M_A = 257 \pm 10$ GeV, $\mu = 263 \pm 1$ GeV, $M_2 = 150 \pm 1$ GeV, $m_{\tilde{g}} = 496 \pm 10$ GeV. For the top-quark mass an error of 0.2 GeV has been used for GigaZ/LC and of 2 GeV for the LHC. For $\tan\beta$ a lower bound of $\tan\beta > 10$ has been taken. For the future theory uncertainty of m_h from unknown higher-order corrections an error of 0.5 GeV has been assumed. The central values for M_W and $\sin^2\theta_{\rm eff}$ have been chosen in accordance with a non-zero contribution to the precision observables from SUSY loops.

As one can see in Fig. 2, the allowed parameter space in the $m_{\tilde{t}_2}$–$|\cos\theta_{\tilde{t}}|$ plane is significantly reduced from the LHC to the LC, in particular in the GigaZ scenario. Using the direct information on $|\cos\theta_{\tilde{t}}|$ from Ref. [12] allows an indirect determination of $m_{\tilde{t}_2}$ with a precision of better than 5% in the GigaZ case. By comparing this indirect prediction for $m_{\tilde{t}_2}$ with direct experimental information on the mass of this particle, the MSSM could be tested at its quantum level in a sensitive and highly non-trivial way.

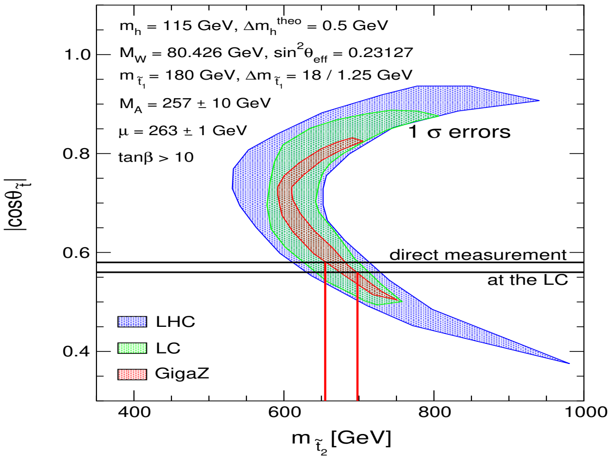

FIGURE 2. Indirect constraints on the MSSM parameter space in the $m_{\tilde{t}_2}$–$|\cos\theta_{\tilde{t}}|$ plane from measurements of m_h, M_W, $\sin^2\theta_{\text{eff}}$, m_t and $m_{\tilde{t}_1}$ in view of the prospective accuracies for these observables at a LC with and without GigaZ option and at the LHC. The direct information on the mixing angle from a measurement at the LC is indicated together with the corresponding indirect determination of $m_{\tilde{t}_2}$.

REFERENCES

1. R. Brinkmann, Talk at the ECFA/DESY Linear Collider Workshop, Orsay 1999.
2. R. Hawkings and K. Mönig, *EPJdirect* **C8** (1999) 1.
3. G. Wilson, Proceedings, Linear Collider Workshop, Sitges 1999.
4. S. Heinemeyer, Th. Mannel and G. Weiglein, hep-ph/9909538.
5. J. Erler, S. Heinemeyer, W. Hollik, G. Weiglein and P.M. Zerwas, *Phys. Lett.* **B486** (2000) 125.
6. A. Freitas et al., *Phys. Lett.* **B495** (2000) 338; A. Freitas et al., hep-ph/0007129.
7. K. Chetyrkin, J. H. Kühn and M. Steinhauser, *Phys. Rev. Lett.* **75** (1995) 3394.
8. A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger and G. Weiglein, *Phys. Rev. Lett.* **78** (1997) 3626; *Phys. Rev.* **D57** (1998) 4179.
9. S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev.* **D58** (1998) 091701; *Phys. Lett.* **B440** (1998) 296; *Eur. Phys. Jour.* **C9** (1999) 343.
10. S. Heinemeyer, W. Hollik and G. Weiglein, *Comp. Phys. Comm.* **124** (2000) 76; hep-ph/0002213 (`http://www.feynhiggs.de`)
11. S. Andringa et al., ALEPH 2000-074 CONF 2000-051, DELPHI 2000-148 CONF 447, L3 Note 2600, OPAL Technical Note TN661.
12. A. Bartl, H. Eberl, S. Kraml, W. Majerotto and W. Porod, hep-ph/9909378; M. Berggren, R. Keränen, H. Nowak and A. Sopczak, hep-ph/9911345.
13. `http://wwwhephy.oeaw.ac.at/susy/lcws.html`