CERN–TH/2000–109
DESY 00–050
KA–TP–5–2000
UPR–882–T
hep-ph/0005024

Physics Impact of GigaZ

J. Erler[1], S. Heinemeyer[2], W. Hollik[3], G. Weiglein[4] and P.M. Zerwas[2]

[1] *Department of Physics and Astronomy, University of Pennsylvania, Philadelphia PA 19104–6396, USA*

[2] *DESY, Deutsches Elektronen-Synchrotron, D–22603 Hamburg, Germany*

[3] *Institut für Theoretische Physik, Universität Karlsruhe, D–76128 Karlsruhe, Germany*

[4] *CERN, TH Division, CH-1211 Geneva 23, Switzerland*

Abstract

By running the prospective high-energy e^+e^-collider TESLA in the GigaZ mode on the Z resonance, experiments can be performed on the basis of more than 10^9 Z events. They will allow the measurement of the effective electroweak mixing angle to an accuracy of $\delta \sin^2\theta_{\rm eff} \approx \pm 1\times 10^{-5}$. Likewise the W boson mass is expected to be measurable with an error of $\delta M_W \approx \pm 6$ MeV near the W^+W^- threshold. In this note, we study the accuracy with which the Higgs boson mass can be determined from loop corrections to these observables in the Standard Model. The comparison with a directly observed Higgs boson may be used to constrain new physics scales affecting the virtual loops. We also study constraints on the heavy Higgs particles predicted in the Minimal Supersymmetric Standard Model, which are very difficult to observe directly for large masses. Similarly, it is possible to constrain the mass of the heavy scalar top particle.

1 Introduction

The prospective high-energy e^+e^- linear collider TESLA is being designed to operate on top of the Z boson resonance,

$$e^+ e^- \to Z\,, \tag{1}$$

by adding a bypass to the main beam line [1]. Given the high luminosity, $\mathcal{L} = 7 \times 10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, and the cross section, $\sigma_Z \approx 30$ nb [2] (including radiative corrections), about 2×10^9 Z events can be generated in an operational year of 10^7s. We will therefore refer to this option as the GigaZ mode of the machine. Moreover, by increasing the collider energy to the W-pair threshold,

$$e^+ e^- \to W^+W^-\,, \tag{2}$$

about 10^6 W bosons can be generated at the optimal energy point for measuring the W boson mass, M_W, near threshold and about 3×10^6 W bosons at the energy of maximal cross section [3]. The large increase in the number of Z events by two orders of magnitude as compared to LEP 1 and the increasing precision in the measurements of W boson properties, open new opportunities [4] for high precision physics in the electroweak sector [5,6].

By adopting the Blondel scheme [7] for running e^+e^- colliders with polarized beams, the left-right asymmetry, $A_{LR} \equiv 2(1 - 4\sin^2\theta_{\rm eff})/(1 + (1 - 4\sin^2\theta_{\rm eff})^2)$, can be measured with very high precision, $\delta A_{LR} \approx \pm 10^{-4}$ [8], when both electrons and positrons are polarized longitudinally. This accuracy can be achieved since the total cross section, the left-right asymmetry and the polarization factor, $\mathcal{P} = (P_+ + P_-)/(1 + P_+P_-)$, can be measured by individually flipping the electron and positron helicities, generating all $2{\times}2$ spin combinations in $\sigma_{ij}(i,j = L,R)$; only the difference between the moduli $|P_+|$ and $|P_-|$ before and after flipping the polarizations of both the positron and electron beams need to be monitored by laser Compton scattering. From A_{LR} the mixing angle in the effective leptonic vector coupling of the on-shell Z boson, $\sin^2\theta_{\rm eff}$, can be determined to an accuracy

$$\delta\sin^2\theta_{\rm eff} \approx \pm\, 1 \times 10^{-5}, \tag{3}$$

while the W boson mass is expected to be measurable to a precision of

$$\delta M_W \approx \pm\, 6 \text{ MeV}, \tag{4}$$

by scanning the W^+W^- threshold [9].

Besides the improvements in $\sin^2\theta_{\rm eff}$ and M_W, GigaZ has the potential to determine the total Z width within $\delta\Gamma_Z = \pm 1$ MeV; the ratio of hadronic to leptonic partial Z widths with a relative uncertainty of $\delta R_l/R_l = \pm 0.05\%$; the ratio of the $b\bar{b}$ to the hadronic partial widths with a precision of $\delta R_b = \pm 1.4 \times 10^{-4}$; and to improve the b quark asymmetry parameter A_b to a precision of $\pm 1 \times 10^{-3}$ [4,8]. These additional measurements offer complementary information on the Higgs boson mass, M_H, but also on the strong coupling constant, α_s, which enters the radiative corrections in many places. This is desirable in its own right, and in the present context it is important to control α_s effects from higher order loop contributions to avoid confusion with Higgs effects. Indirectly, a well known α_s would also help to control m_t effects, since m_t from a threshold scan at a linear collider will be strongly correlated with α_s. We find that via a precise measurement of R_l, GigaZ would provide a clean determination of α_s with small error

$$\delta\alpha_s \approx \pm\, 0.001, \tag{5}$$

allowing to reduce the error of the top-quark mass from the threshold scan. The anticipated precisions for the most relevant electroweak observables at the Tevatron (Run IIA and IIB), the LHC, a future linear collider, LC, and GigaZ are summarized in Tab. 1.

	now	Tev. Run IIA	Run IIB	LHC	LC	GigaZ
$\delta \sin^2\theta_{\rm eff}(\times 10^5)$	17	50 [10]	13 [10]	21 [10,11]	(6) [10]	1.3 [8]
δM_W [MeV]	42	30 [12]	15 [13]	15 [11,13]	15 [14]	6 [9]
δm_t [GeV]	5.1	4.0 [10]	2.0 [10]	2.0 [10,15]	0.2 [16]	0.13 [16]
δM_H [MeV]	—	—	2000 [17]	100 [17]	50 [17]	50 [17]

Table 1: *Expected precision at various colliders for $\sin^2\theta_{\rm eff}$, M_W, m_t and the (lightest) Higgs boson mass, M_H, at the reference value $M_H = 110$ GeV. Run IIA refers to 2 fb^{-1} integrated luminosity per experiment collected at the Tevatron with the Main Injector, while Run IIB assumes the accumulation of 30 fb^{-1} by each experiment. LC corresponds to a linear collider without the GigaZ mode. (The entry in parentheses assumes a fixed target polarized Møller scattering experiment using the e^- beam.) The present uncertainty on M_W will be improved by the end of the LEP2 program. δm_t from the Tevatron and the LHC is the error for the top pole mass, while at the top threshold in e^+e^- collisions the $\overline{\rm MS}$ top-quark mass can be determined. The smaller value of δm_t at GigaZ compared to the LC is due to the prospective reduced uncertainty of α_s, which affects the relation between the mass parameter directly extracted at the top threshold and the $\overline{\rm MS}$ top-quark mass.*

In this note, we study the potential impact of such measurements on the parameters of the Standard Model (SM) and its minimal supersymmetric extension (MSSM) [18]. Higgs boson masses and SUSY particle masses affect the high precision observables through loop corrections. These loop corrections are evaluated in this note at the presently available level of theoretical accuracy, still leaving many refinements to be worked out in the coming years [19]. Even though a complete set of calculations is lacking at the present time, the essential features of the GigaZ physics potential can nevertheless be studied in first exploratory steps. In Ref. [4] it has been demonstrated that very stringent consistency tests of the SM and the MSSM will become feasible with the GigaZ precision, and the prospects for b physics at GigaZ have been discussed. The latter topic has been studied in more detail in Ref. [8]. In the present note, we will focus in a systematic way on the Higgs sectors of the SM and the MSSM, and also on the scalar top sector of the MSSM.

2 Higgs Sector of the SM

In the canonical form of the SM, the precision observables measured at the Z peak are affected by two high mass scales in the model: the top quark mass, m_t, and the Higgs boson mass, M_H. They enter as virtual states in loop corrections to various relations between electroweak observables. For example, the radiative corrections entering the relation between M_W and M_Z, and between M_Z and $\sin^2\theta_{\rm eff}$, have a strong quadratic dependence on m_t and a logarithmic dependence on M_H. We mainly focus on the two electroweak observables that are expected to be measurable with the highest accuracy at GigaZ, M_W and $\sin^2\theta_{\rm eff}$. Our analysis is based on the results for the electroweak precision observables including higher order electroweak [20, 21] and QCD [22, 23] corrections. The current theoretical uncertainties [24] are dominated by the parametric uncertainties from the errors in the input parameters m_t (see Tab. 1) and $\Delta\alpha$. The latter denotes the QED-induced shift in the fine structure constant, $\alpha \to \alpha(M_Z)$, originating from charged-lepton and light-quark photon vacuum polarization diagrams. The hadronic contribution to $\Delta\alpha$ currently introduces an uncertainty of $\delta\Delta\alpha = \pm 2 \times 10^{-4}$ [25]. Forthcoming low-energy e^+e^-annihilation experiments may reduce this uncertainty to about $\pm 5 \times 10^{-5}$ [26]. Combining this value with future (indistinguishable) errors from unknown higher order corrections, we assign the total uncertainty of $\delta\Delta\alpha = \pm 7 \times 10^{-5}$ to $\Delta\alpha$, which is used throughout the paper unless otherwise stated. For the future theoretical uncertainties from unknown higher-order corrections (including the uncertainties from $\delta\Delta\alpha$) we assume,

$$\delta M_W(\text{theory}) = \pm 3 \text{ MeV}, \quad \delta \sin^2\theta_{\text{eff}}(\text{theory}) = \pm 3 \times 10^{-5} \quad (\text{future}). \tag{6}$$

Given the high precision of GigaZ, also the experimental error in M_Z, $\delta M_Z = \pm 2.1$ MeV [27], results in non-negligible uncertainties of $\delta M_W = \pm 2.5$ MeV and $\delta \sin^2\theta_{\text{eff}} = \pm 1.4 \times 10^{-5}$. The experimental error in the top-quark mass, $\delta m_t = \pm 130$ MeV, induces further uncertainties of $\delta M_W = \pm 0.8$ MeV and $\delta \sin^2\theta_{\text{eff}} = \pm 0.4 \times 10^{-5}$. Thus, while currently the experimental error in M_Z can safely be neglected, for the GigaZ precision it will actually induce an uncertainty in the prediction of $\sin^2\theta_{\text{eff}}$ that is larger than its experimental error.

(a) The relation between $\sin^2\theta_{\text{eff}}$ and M_Z can be written as

$$\sin^2\theta_{\text{eff}} \cos^2\theta_{\text{eff}} = \frac{A^2}{M_Z^2(1 - \Delta r_Z)}, \tag{7}$$

where $A = [(\pi\alpha)/(\sqrt{2}G_F)]^{1/2} = 37.2805(2)$ GeV is a combination of two precisely known low-energy coupling constants, the Fermi constant, G_F, and the electromagnetic fine structure constant, α. The quantity Δr_Z summarizes the loop corrections, which at the one-loop level can be decomposed as

$$\Delta r_Z = \Delta\alpha - \Delta\rho^{\text{t}} + \Delta r_Z^{\text{H}} + \cdots. \tag{8}$$

The leading top contribution to the ρ parameter [28], quadratic in m_t, reads

$$\Delta\rho^{\text{t}} = \frac{3G_F m_t^2}{8\pi^2\sqrt{2}}. \tag{9}$$

The Higgs boson contribution is screened, being logarithmic for large Higgs boson masses

$$\Delta r_Z^{\text{H}} = \frac{G_F M_W^2}{8\pi^2\sqrt{2}} \frac{1 + 9s_W^2}{3c_W^2} \log\frac{M_H^2}{M_W^2} + \cdots. \tag{10}$$

(b) An independent analysis can be based on the precise measurement of M_W near threshold. The M_W–M_Z interdependence is given by,

$$\frac{M_W^2}{M_Z^2}\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{A^2}{M_Z^2(1 - \Delta r)}, \tag{11}$$

where the quantum correction Δr has the one-loop decomposition,

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho^{\text{t}} + \Delta r^{\text{H}} + \cdots, \tag{12}$$

$$\Delta r^{\text{H}} = \frac{G_F M_W^2}{8\pi^2\sqrt{2}} \frac{11}{3} \log\frac{M_H^2}{M_W^2} + \cdots, \tag{13}$$

with $\Delta\alpha$ and $\Delta\rho^{t}$ as introduced above.

Owing to the different dependences of $\sin^2\theta_{\rm eff}$ and M_W on m_t and M_H, the high precision measurements of these quantities at GigaZ (combined with the other supplementary electroweak observables) can determine the mass scales m_t and M_H. The expected accuracy in the indirect determination of M_H from the radiative corrections within the SM is displayed in Fig. 1. To obtain these contours, the error projections in Tab. 1 are supplemented by central values equal to the current SM best fit values for the entire set of current high precision observables [29]. For the theoretical uncertainties, Eq. (6) is used, while the parametric uncertainties, such as from α_s and M_Z, are automatically accounted for in the fits. The allowed bands in the m_t–M_H plane for the GigaZ accuracy are shown separately for $\sin^2\theta_{\rm eff}$ and M_W. By adding the information on the top-quark mass, with $\delta m_t \lesssim 130$ MeV obtained from measurements of the $t\bar{t}$ production cross section near threshold, an accurate determination of the Higgs boson mass becomes feasible from both, M_W and $\sin^2\theta_{\rm eff}$. If the two values are found to be consistent, they can be combined and compared to the Higgs boson mass measured in direct production through Higgs-strahlung [30] (see the last row in Tab. 1). In Fig. 1 this is shown by the shaded area labeled as "GigaZ (1σ errors)", where the measurements of other Z boson properties as anticipated for GigaZ are also included (the best fit value for m_t is assumed to coincide with the central m_t value in Fig. 1). For comparison, the area in the m_t–M_H plane corresponding to the current experimental accuracies, labeled as "now (1σ errors)", is also shown.

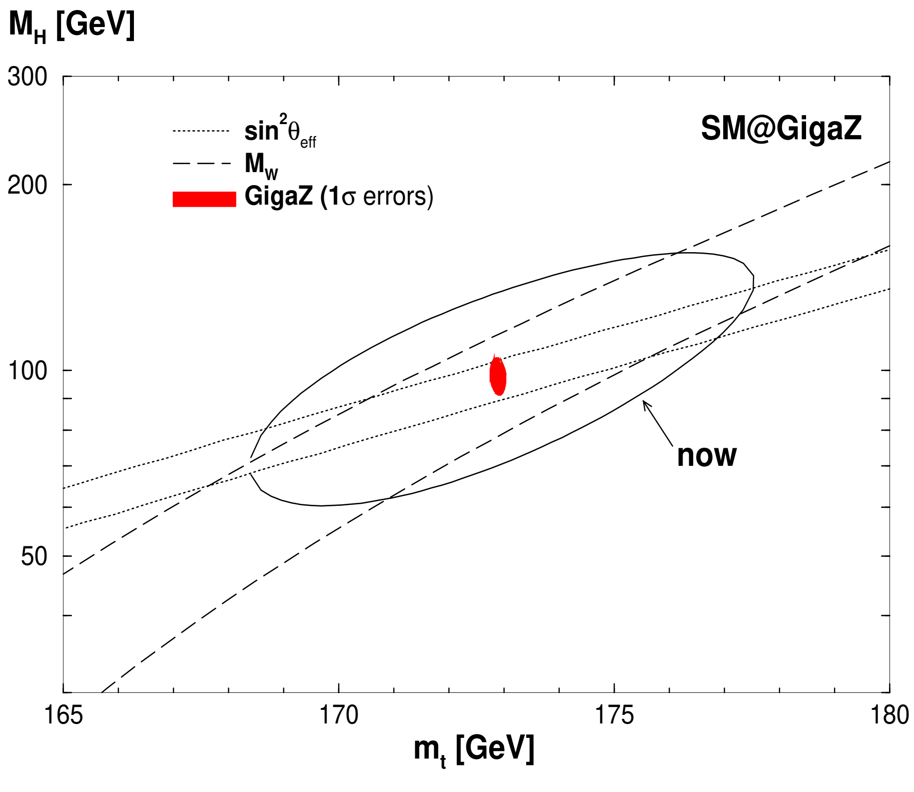

Figure 1: *1σ allowed regions in the m_t-M_H plane taking into account the anticipated GigaZ precisions for $\sin^2\theta_{\rm eff}, M_W, \Gamma_Z, R_l, R_q$ and m_t (see text). The presently allowed region (full curve labeled 'now') is shown for comparison.*

The results can be summarized by calculating the accuracy with which M_H can be determined indirectly. The expectations for $\delta M_H/M_H$ in each step until GigaZ are collected in Tab. 2. Extending an earlier analysis [4], where $\delta M_H/M_H$ was separately determined from M_W and $\sin^2\theta_{\rm eff}$, we additionally employ here the full set of precision observables for our analysis. Concerning the experimental errors, the values from Tab. 1 are taken. For the uncertainty in $\Delta\alpha$, we use $\delta\Delta\alpha = \pm 7\times 10^{-5}$ (yielding Eq. (6) upon combination with our estimate of unknown higher order corrections), except for the first row displaying the present situation, where $\delta\Delta\alpha = \pm 2\times 10^{-4}$ [25] is employed.

It is apparent that GigaZ, reaching $\delta M_H/M_H = \pm 7\%$, triples the precision in M_H relative to the anticipated LHC status. On the other hand, a linear collider without the high luminosity option would provide only a modest improvement.

$\delta M_H/M_H$ *from:*	M_W	$\sin^2\theta_{\rm eff}$	all
now	229 %	62 %	61 %
Tevatron Run IIA	77 %	46 %	41 %
Tevatron Run IIB	39 %	28 %	26 %
LHC	28 %	24 %	21 %
LC	18 %	20 %	15 %
GigaZ	12 %	7 %	7 %

Table 2: *Cumulative expected precisions of indirect Higgs mass determinations, $\delta M_H/M_H$, given the error projections in Tab. 1. Theoretical uncertainties and their correlated effects on M_W and $\sin^2\theta_{\rm eff}$ are taken into account (see text). The last column shows the indirect Higgs mass determination from the full set of precision observables.*

(c) A direct formal relation between M_W and $\sin^2\theta_{\rm eff}$ can be established by combining the two relations Eqs. (7) and (11) as

$$M_W^2 = \frac{A^2}{\sin^2\theta_{\rm eff}(1-\Delta r_W)}. \tag{14}$$

The quantum correction Δr_W is independent of $\Delta\rho^{\rm t}$ in leading order and has the one-loop decomposition

$$\Delta r_W = \Delta\alpha - \Delta r_W^{\rm H} + \cdots, \tag{15}$$

$$\Delta r_W^{\rm H} = \frac{G_F M_Z^2}{24\pi^2\sqrt{2}}\log\frac{M_H^2}{M_W^2} + \cdots. \tag{16}$$

Relation (14) can be evaluated by inserting the measured value of the Higgs boson mass as predetermined at the LHC and the LC. This is visualized in Fig. 2, where the present and future theoretical predictions for $\sin^2\theta_{\rm eff}$ and M_W (for different values of M_H) are compared with the experimental accuracies at various colliders. Besides the independent predictions of $\sin^2\theta_{\rm eff}$ and M_W within the SM, the $M_W - \sin^2\theta_{\rm eff}$ contour plot in Fig. 2 can be interpreted as an additional indirect determination of M_W from the measurement of $\sin^2\theta_{\rm eff}$. Given the expected negligible error in M_H, this results in an uncertainty of

$$\delta M_W(\text{indirect}) \approx \pm 2\ \text{MeV} \pm 3\ \text{MeV}. \tag{17}$$

The first uncertainty reflects the experimental error in $\sin^2\theta_{\rm eff}$, while the second is the theoretical uncertainty discussed above (see Eq. (6)). The combined uncertainty of this indirect prediction is about the same as the one of the SM prediction according to Eq. (11) and is close to the experimental error expected from the W^+W^- threshold given in Eq. (4).

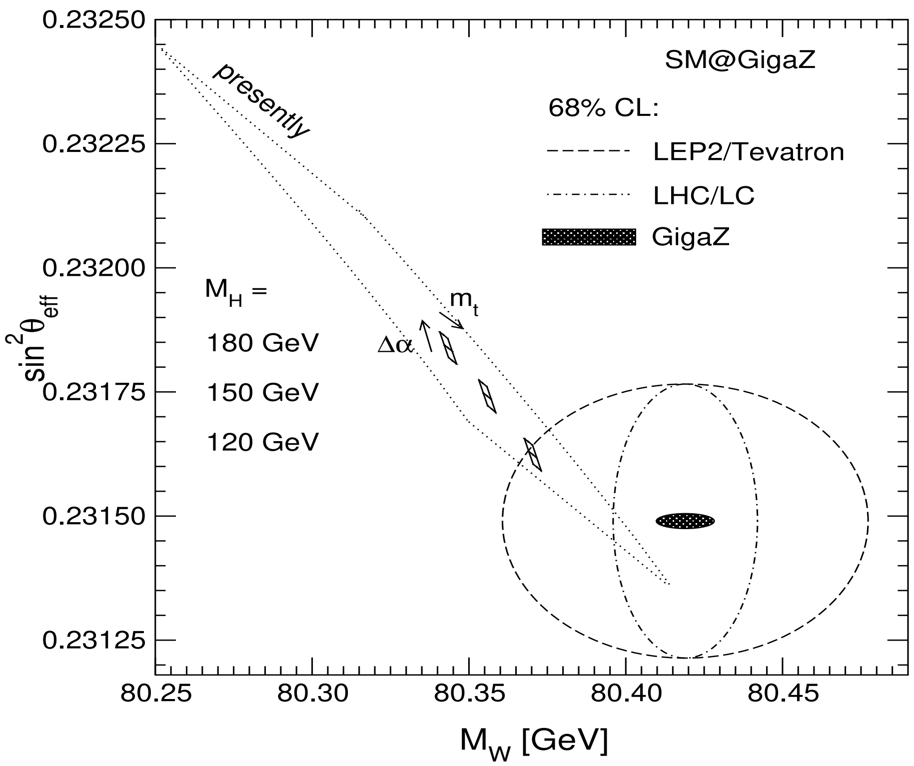

Figure 2: *The theoretical prediction for the relation between* $\sin^2\theta_{\rm eff}$ *and* M_W *in the SM for Higgs boson masses in the intermediate range is compared to the experimental accuracies at LEP 2/Tevatron (Run IIA), LHC/LC and GigaZ (see Tab. 1). For the theoretical prediction an uncertainty of* $\delta\Delta\alpha = \pm 7\times 10^{-5}$ *and* $\delta m_t = \pm 200$ MeV *is taken into account.*

Consistency of all the theoretical relations with the experimental data would be the ultimate precision test of the SM based on quantum fluctuations. The comparison between theory and experiment can also be exploited to constrain possible physics scales beyond the SM. These additional contributions can conveniently be described in terms of the S,T,U [31] or ϵ parameters [32]. Adopting the notation of Ref. [29], the errors with which they can be measured at GigaZ are given as follows:

$$\begin{aligned} \Delta S &= \pm 0.05, & \Delta\hat\epsilon_3 &= \pm 0.0004, \\ \Delta T &= \pm 0.06, & \Delta\hat\epsilon_1 &= \pm 0.0005, \\ \Delta U &= \pm 0.04, & \Delta\hat\epsilon_2 &= \pm 0.0004. \end{aligned} \tag{18}$$

The oblique parameters in Eq. (18) are strongly correlated. On the other hand, many types of new physics predict $U = \hat\epsilon_2 = 0$ or very small (see Ref. [29] and references therein). With the U ($\hat{\epsilon}_2$) parameter known, the anticipated errors in S and T would decrease to about ± 0.02, while the errors in $\hat{\epsilon}_1$ and $\hat{\epsilon}_3$ would be smaller than ± 0.0002.

In the context of a spontaneously broken gauge theory, the above mentioned comparisons shed light on the basic theoretical components for generating the masses of the fundamental particles. On the other hand, an observed inconsistency would be a clear indication for the existence of a new physics scale.

3 Supersymmetry

The second step in this GigaZ analysis is based on the assumption that supersymmetry would be discovered at LEP 2, the Tevatron, or the LHC, and further explored at an e^+e^- linear collider. The high luminosity expected at TESLA can be exploited to determine supersymmetric particle masses and mixing angles with errors from $\mathcal{O}(1\%)$ down to one per mille [33], provided they reside in the kinematical reach of the collider, which we assume to be about 1 TeV. In this context we will address two problems arising in the Higgs sector and the scalar top sector within the MSSM.

For the SUSY contributions to M_W and $\sin^2\theta_{\rm eff}$ we use the complete one-loop results in the MSSM [34] as well as the leading higher order QCD corrections [35]. The recent electroweak two-loop results of the SM part in the MSSM [21] have not been taken into account, since no genuine MSSM counterpart is available so far. As above, concerning the future theoretical uncertainties of M_W and $\sin^2\theta_{\rm eff}$ we use Eq. (6).

In contrast to the Higgs boson mass in the SM, the lightest $\mathcal{CP}$-even MSSM Higgs boson mass, M_h, is not a free parameter but can be calculated from the other SUSY parameters. In the present analysis, the currently most precise result based on Feynman-diagrammatic methods [36] is used, relating M_h to the pseudoscalar Higgs boson mass, M_A. The numerical evaluation has been performed with the Fortran code *FeynHiggs* [37]. In our analysis we assume a future uncertainty in the theoretical prediction of M_h of ± 0.5 GeV.

(a) The relation between M_W and $\sin^2\theta_{\rm eff}$ is affected by the parameters of the supersymmetric sector, especially the $\tilde{t}$-sector. At the LHC [38] and especially at a prospective LC, the mass of the light $\tilde{t}$, $m_{\tilde{t}_1}$, and the $\tilde{t}$-mixing angle, $\theta_{\tilde{t}}$, may be measurable very well, particularly in the process $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$ [39]. On the other hand, background problems at the LHC and insufficient energy at the LC may preclude the analysis of the heavy $\tilde{t}$-particle, $\tilde{t}_2$.

In Fig. 3 it is demonstrated how in this situation limits on $m_{\tilde{t}_2}$ can be derived from measurements of M_h, M_W and $\sin^2\theta_{\rm eff}$. As experimental values we assumed $M_h = 115$ GeV, $M_W = 80.40$ GeV and $\sin^2\theta_{\rm eff} = 0.23140$, with the experimental errors given in the last column of Tab. 1. We consider two cases for $\tan\beta$, the ratio of the vacuum expectation values of the two Higgs doublets in the MSSM: the low $\tan\beta$ region, where we assume a band, $2.5 < \tan\beta < 3.5$, and the high $\tan\beta$ region where we assume a lower bound, $\tan\beta \geq 10$, as can be expected from measurements in the gaugino sector (see e.g. Ref. [40]). As for the other parameters, the following values are assumed, with uncertainties as expected from LHC [41] and TESLA [42]: $m_{\tilde{t}_1} = 500 \pm 2$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 0.014$, $A_b = A_t \pm 10\%$, $\mu = -200 \pm 1$ GeV, $M_2 = 400 \pm 2$ GeV and $m_{\tilde{g}} = 500 \pm 10$ GeV. ($A_{b,t}$ are trilinear soft SUSY-breaking parameters, μ is the Higgs mixing parameter, M_2 is one of the soft SUSY-breaking parameter in the gaugino sector, and $m_{\tilde{g}}$ denotes the gluino mass.)

For low $\tan\beta$ the heavier $\tilde{t}$-mass, $m_{\tilde{t}_2}$, can be restricted to 760 GeV $\lesssim m_{\tilde{t}_2} \lesssim$ 930 GeV from the M_h, M_W and $\sin^2\theta_{\rm eff}$ precision measurements. The mass M_A varies between

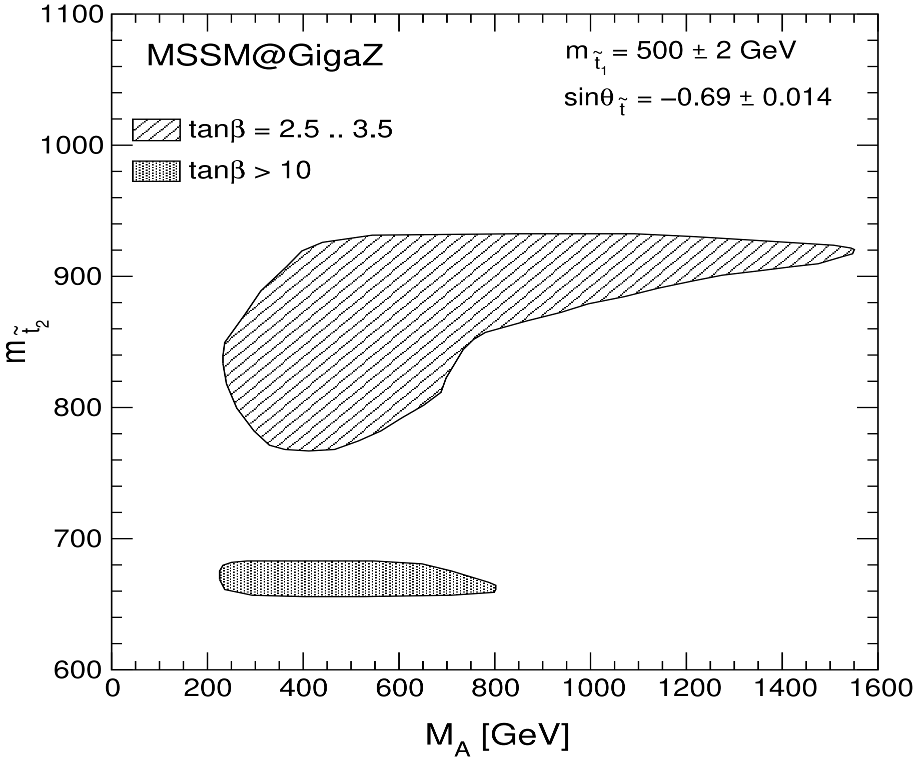

Figure 3: *The region in the $M_A - m_{\tilde{t}_2}$ plane, allowed by $1\,\sigma$ errors obtained from the GigaZ measurements of M_W and $\sin^2\theta_{\rm eff}$: $M_W = 80.40$ GeV, $\sin^2\theta_{\rm eff} = 0.23140$, and from the LC measurement of M_h: $M_h = 115$ GeV. The experimental errors for the SM parameters are given in Tab. 1. $\tan\beta$ is assumed to be experimentally constrained by $2.5 < \tan\beta < 3.5$ or $\tan\beta > 10$. The other parameters including their uncertainties are given by $m_{\tilde{t}_1} = 500 \pm 2$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 0.014$, $A_b = A_t \pm 10\%$, $\mu = -200 \pm 1$ GeV, $M_2 = 400 \pm 2$ GeV and $m_{\tilde{g}} = 500 \pm 10$ GeV. For the uncertainties of the theoretical predictions we use Eq. (6).*

200 GeV and 1600 GeV. A reduction of this interval to $M_A \geq 500$ GeV by its non-observation at the LHC and the LC does not improve the bounds on $m_{\tilde{t}_2}$. If $\tan\beta \geq 10$, the second theoretically preferred range [43], the allowed region turns out to be much smaller (660 GeV $\lesssim m_{\tilde{t}_2} \lesssim$ 680 GeV), and the mass M_A is restricted to $M_A \lesssim 800$ GeV. In deriving the bounds on $m_{\tilde{t}_2}$, both the constraints from M_h (see Ref. [44]) and $\sin^2\theta_{\rm eff}$ play an important role. For the bounds on M_A, the main effect comes from $\sin^2\theta_{\rm eff}$. We have assumed a value for $\sin^2\theta_{\rm eff}$ slightly different from the corresponding value obtained in the SM limit. For this value the (logarithmic) dependence on M_A is still large enough so that in combination with the high precision in $\sin^2\theta_{\rm eff}$ at GigaZ an *upper limit* on M_A can be set. For an error as obtained at an LC without the GigaZ mode (see Tab. 1) no bound on M_A could be inferred.

(b) A similar problem of high interest occurs in the sector of the MSSM Higgs particles. It is well known, that the heavy Higgs bosons A, H and $H^\pm$, are increasingly difficult to observe at the LHC with rising mass [41]. At e^+e^- linear colliders heavy Higgs particles are produced primarily in pairs (HA) and (H^+H^-) so that they cannot be analyzed for mass values beyond the beam energy of 500 GeV in the first phase of such a machine. It has been demonstrated though that the ratio of the decay branching ratios of the light Higgs boson h

is sensitive to M_A up to values of 700 GeV to 1 TeV [45]. Since any such analysis is difficult, it is suggestive to search for complementary channels in which new limits may be derived from other high precision measurements.

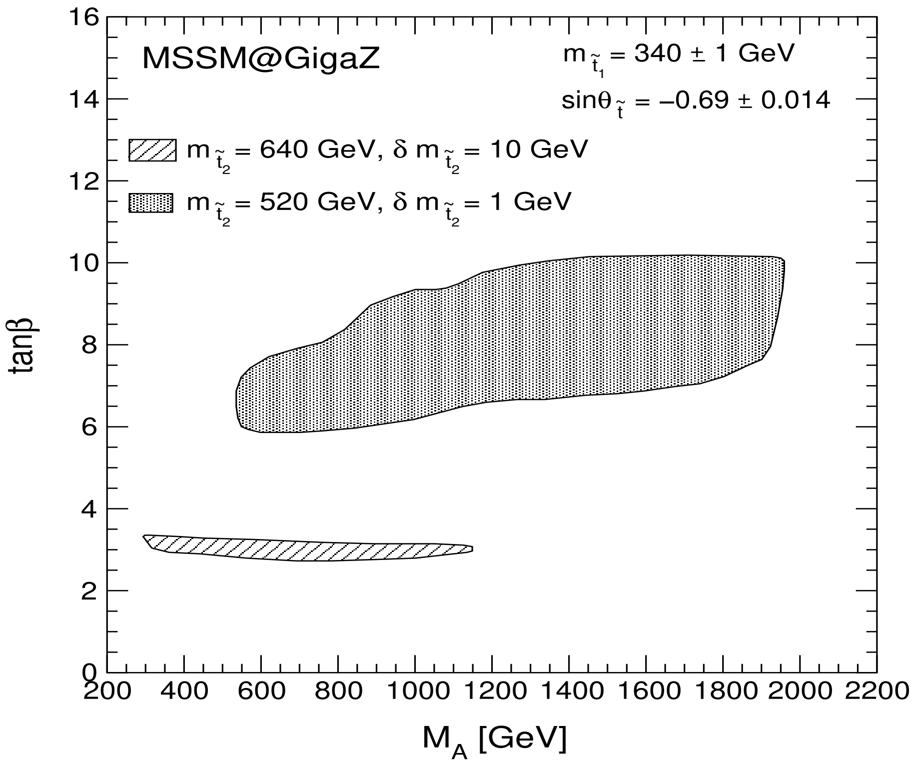

Figure 4: *The region in the $M_A - \tan\beta$ plane, allowed by $1\,\sigma$ errors by the GigaZ measurements of M_W and $\sin^2\theta_{\mathrm{eff}}$: $M_W = 80.40$ GeV, $\sin^2\theta_{\mathrm{eff}} = 0.23138$, and by the LC measurement of M_h: $M_h = 110$ GeV. The experimental errors for the SM parameters are given in Tab. 1. The other parameters including their uncertainties are given by $m_{\tilde{t}_1} = 340 \pm 1$ GeV, $m_{\tilde{t}_2} = 640 \pm 10$ GeV or $m_{\tilde{t}_2} = 520 \pm 1$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 0.014$, $A_b = -640 \pm 60$ GeV, $\mu = 316 \pm 1$ GeV, $M_2 = 152 \pm 2$ GeV and $m_{\tilde{g}} = 496 \pm 10$ GeV. For the uncertainties of the theoretical predictions we use Eq. (6).*

The result of such a study is presented in Fig. 4, based on the expected errors for M_h, $m_{\tilde{t}_1}$, and $\theta_{\tilde{t}}$ from LC measurements, and assuming either a rough measurement of the heavy $\tilde{t}$-mass, $m_{\tilde{t}_2}$, at the LHC, or a precise determination of $m_{\tilde{t}_2}$ at an LC. Fig. 4 shows the exclusion contours in the $M_A - \tan\beta$ plane based on the following scenario (inspired by the mSUGRA(1) reference scenario studied e.g. in Ref. [33]): $M_h = 110 \pm 0.05$ GeV from LC measurements, $M_W = 80.400 \pm 0.006$ GeV and $\sin^2\theta_{\mathrm{eff}} = 0.23138 \pm 1 \times 10^{-5}$ from GigaZ measurements, $m_{\tilde{t}_1} = 340 \pm 1$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 0.014$ from the LC, $m_{\tilde{t}_2} = 640 \pm 10$ GeV from the LHC or alternatively $m_{\tilde{t}_2} = 520 \pm 1$ GeV from LC measurements; furthermore $A_b = -640 \pm 60$ GeV, $\mu = 316 \pm 1$ GeV, $M_2 = 152 \pm 2$ GeV, $m_{\tilde{g}} = 496 \pm 10$ GeV based on LHC or LC runs.

If the scenario with lower $\tilde{t}$ masses is realized, this would, due to the dependence of the Higgs boson mass on the MSSM parameters, correspond to higher values for both M_A and $\tan\beta$. Despite the precise measurement of $m_{\tilde{t}_2}$ up to 1 GeV at the LC, in the example considered here the restrictions placed on the Higgs sector would be relatively weak. The constraints would be 450 GeV $\lesssim M_A \lesssim$ 1950 GeV and $6 \lesssim \tan\beta \lesssim 10$. However, if the higher $\tilde{t}_2$ mass is realized in nature, corresponding to larger mixing in the $\tilde{t}$ sector, in spite of the relatively rough measurement of $m_{\tilde{t}_2}$, in our example the allowed parameter range is reduced to 250 GeV $\lesssim M_A \lesssim$ 1200 GeV and $2.5 \lesssim \tan\beta \lesssim 3.5$. Again, $\sin^2\theta_{\rm eff}$ plays an important role (cf. discussion of Fig. 3); without the high precision measurement of $\sin^2\theta_{\rm eff}$ no upper limit on M_A could be set.

Thus the high precision measurements of M_W, $\sin^2\theta_{\rm eff}$ and M_h do not improve on the direct lower bound on the mass of the pseudoscalar Higgs boson A. Instead they enable us to set an *upper bound* on this basic parameter of the supersymmetric Higgs sector, derived from the requirement of consistency of the electroweak precision data with the MSSM.

4 Conclusions

The opportunity to measure electroweak observables very precisely in the GigaZ mode of the prospective e^+e^- linear collider TESLA, in particular the electroweak mixing angle $\sin^2\theta_{\rm eff}$ and the W boson mass, opens new areas for high precision tests of electroweak theories. We have analyzed in detail two generic examples: (i) The Higgs mass of the Standard Model can be extracted to a precision of a few percent from loop corrections. By comparison with the direct measurements of the Higgs mass, bounds on new physics scales can be inferred that may not be accessible directly. (ii) The masses of particles in supersymmetric theories, which for various reasons may not be accessible directly neither at the LHC nor at the LC, can be constrained. Typical examples are the heavy Higgs bosons and the heavy scalar top quark. In the scenarios studied here, a sensitivity of up to order 2 TeV for the mass of the pseudoscalar Higgs boson and an upper bound of 1 TeV for the heavy scalar top quark could be expected from the virtual loop analyses of the high precision data.

Opening windows to unexplored energy scales renders these analyses of virtual effects an important tool for experiments in the GigaZ mode of a future e^+e^- linear collider.

Acknowledgements

We thank P. Langacker for a critical reading of the manuscript and A. Hoang and T. Teubner for helpful discussions. Parts of the calculation have been performed on the QCM cluster at the University of Karlsruhe, supported by the Deutsche Forschungsgemeinschaft (Forschergruppe “Quantenfeldtheorie und Computeralgebra”).

References

[1] R. Brinkmann, Talk at the ECFA/DESY Linear Collider Workshop, Orsay 1999.

[2] G. Altarelli, R. Kleiss and C. Verzegnassi (conv.) *et al.*, Proceedings, *Z Physics at LEP 1*, CERN 89-08.

[3] Z. Kunszt and W.J. Stirling (conv.) *et al.*, Proceedings, *Physics at LEP 2*, CERN 96-01.

[4] S. Heinemeyer, Th. Mannel and G. Weiglein, Proceedings, Linear Collider Workshop Sitges 1999, e-print `hep-ph/9909538`.

[5] S.L. Glashow, *Nucl. Phys.* **22** (1961) 579; A. Salam, in *Elementary Particle Theory* (ed. N. Svartholm), Stockholm 1968; S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264.

[6] G. 't Hooft, *Nucl. Phys.* **B35** (1971) 167; G. 't Hooft and M. Veltman, *Nucl. Phys.* **B44** (1972) 189.

[7] A. Blondel, *Phys. Lett.* **B202** (1988) 145.

[8] R. Hawkings and K. Mönig, *EPJdirect* **C8** (1999) 1.

[9] G. Wilson, Proceedings, Linear Collider Workshop, Sitges 1999.

[10] U. Baur and M. Demarteau, e-print `hep-ph/9611334`, and references therein.

[11] S. Haywood, P.R. Hobson, W. Hollik, Z. Kunzst et al., *Electroweak Physics*, e-print `hep-ph/0003275`, in CERN-YR-2000/01, eds. G. Altarelli and M.L. Mangano and references therein.

[12] K. del Signore for the CDF and DØ Collaborations, *The Future Collider Physics Program at Fermilab: Run II and TeV 33*, preprint FERMILAB-Conf-98/221-E, published in the Proceedings of the 33rd Rencontres de Moriond: QCD and High Energy Hadronic Interactions, Les Arcs, France, 1998.

[13] S. Keller and J. Womersley, e-print `hep-ph/9711304`.

[14] H. Haber *et al.*, e-print `hep-ph/9703391`.

[15] M. Beneke, I. Efthymiopoulos, M.L. Mangano, J. Womersley, *Top Quark Physics*, e-print `hep-ph/0003033`, in CERN-YR-2000/01, eds. G. Altarelli and M.L. Mangano and references therein.

[16] R. Frey *et al.*, e-print `hep-ph/9704243`; P. Igo-Kemenes *et al.*, Proceedings, LC Workshop 1991/93, DESY 92/93-123; A. Hoang *et al.*, to appear in *EPJdirect* **C**, e-print `hep-ph/0001286`.

[17] J.F. Gunion *et al.*, e-print `hep-ph/9703330`.

[18] H. Haber and G. Kane, *Phys. Rep.* **117** (1985) 75; H.P. Nilles, *Phys. Rep.* **110** (1984) 1.

[19] F. Jegerlehner *et al.*, *"The LoopVerein"*, ECFA/DESY Linear Collider Workshop, Obernai, 1999.

[20] J. van der Bij and F. Hoogeveen, *Nucl. Phys.* **B283** (1987) 477; M. Consoli, W. Hollik and F. Jegerlehner, *Phys. Lett.* **B227** (1989) 167; R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci and A. Vicere, *Phys. Lett.* **B288** (1992) 95; *E:* **B312** (1993) 511; *Nucl. Phys.* **B409** (1993) 105; J. Fleischer, O.V. Tarasov and F. Jegerlehner, *Phys. Lett.* **B319** (1993) 249.

[21] G. Degrassi, P. Gambino and A. Vicini, *Phys. Lett.* **B383** (1996) 219; G. Degrassi, P. Gambino and A. Sirlin, *Phys. Lett.* **B394** (1997) 188; S. Bauberger and G. Weiglein, *Phys. Lett.* **B419** (1998) 333.

[22] A. Djouadi and C. Verzegnassi, *Phys. Lett.* **B195** (1987) 265; A. Djouadi, *Nuovo Cim.* **A100** (1988) 357; B. Kniehl, *Nucl. Phys.* **B347** (1990) 89.

[23] K. Chetyrkin, J. H. Kühn and M. Steinhauser, *Phys. Lett.* **B351** (1995) 331; *Phys. Rev. Lett.* **75** (1995) 3394; L. Avdeev, J. Fleischer, S. Mikhailov and O. Tarasov, *Phys. Lett.* **B336** (1994) 560, E: *Phys. Lett.* **B349** (1995) 597.

[24] D. Bardin, M. Grünewald and G. Passarino, e-print `hep-ph/9802452`.

[25] M. Davier and A. Höcker, *Phys. Lett.* **B435** (1998) 427; J. Kühn and M. Steinhauser, *Phys. Lett.* **B437** (1998) 425; J. Erler, *Phys. Rev.* **D59** (1999) 054008; F. Jegerlehner, in Proceedings of the IVth International Symposium on Radiative Corrections, ed. J. Solà (World Scientific, Singapore, 1999), e-print `hep-ph/9901386`.

[26] F. Jegerlehner, *Sigma Hadronic and Precision Tests of the SM*, Talk presented at the LNF Spring School, Frascati, Italy, 1999, `http://www.ifh.de/~fjeger/Frascati99.ps.gz`.

[27] The LEP collaborations ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group and the SLD Flavor and Electroweak Group, CERN-EP/2000-016.

[28] M. Veltman, *Nucl. Phys.* **B123** (1977) 89.

[29] J. Erler and P. Langacker, *Electroweak Model and Constraints on New Physics*, in: C. Caso *et al.*, *Eur. Phys. J.* **C3**, 1 (1998) and 1999 off-year partial update for the 2000 edition available on the PDG WWW pages (URL: `http://pdg.lbl.gov/`).

[30] E. Accomando *et al.*, *Phys. Rep.* **299** (1998) 1; P.M. Zerwas, Lectures Summer Institute Cargèse 1999, e-print `hep-ph/0003221`.

[31] M. Peskin and T. Takeuchi, *Phys. Rev.* **D46**, 381 (1992).

[32] G. Altarelli and R. Barbieri, *Phys. Lett.* **B253**, 161 (1990).

[33] G. Blair and U. Martyn, Proceedings, Linear Collider Workshop Sitges 1999, e-print `hep-ph/9910416`.

[34] P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski and J. Rosiek, *Nucl. Phys.* **B417** (1994) 101; D. Garcia and J. Solà, *Mod. Phys. Lett.* **A9** (1994) 211; D. Garcia, R. Jiménez and J. Solà, *Phys. Lett.* **B347** (1995) 309; A. Dabelstein, W. Hollik and W. Mösle, e-print `hep-ph/9506251`; P. Chankowski and S. Pokorski, *Nucl. Phys.* **B475** (1996) 3; W. de Boer, A. Dabelstein, W. Hollik, W. Mösle and U. Schwickerath, *Z. Phys.* **C75** (1997) 625.

[35] A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger and G. Weiglein, *Phys. Rev. Lett.* **78** (1997) 3626; *Phys. Rev.* **D57** (1998) 4179.

[36] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev.* **D58** (1998) 091701; *Phys. Lett.* **B440** (1998) 296; *Eur. Phys. Jour.* **C9** (1999) 343.

[37] S. Heinemeyer, W. Hollik and G. Weiglein, *Comp. Phys. Comm.* **124** (2000) 76.

[38] W. Beenakker, M. Krämer, T. Plehn, M. Spira and P.M. Zerwas, *Nucl. Phys.* **B519** (1998) 85.

[39] A. Bartl, H. Eberl, S. Kraml, W. Majerotto, W. Porod, and A. Sopczak, *Z. Phys.* **C76** (1997) 549; A. Bartl, H. Eberl, S. Kraml, W. Majerotto and W. Porod, Proceedings, Linear Collider Workshop Sitges 1999, e-print `hep-ph/9909378`; M. Berggren, R. Keränen, H. Nowak and A. Sopczak, Proceedings, Linear Collider Workshop Sitges 1999, e-print `hep-ph/9911345`.

[40] S.Y. Choi, A. Djouadi, M. Guchait, J. Kalinowski and P.M. Zerwas, *Eur. Phys. Jour.* **C**, in press, e-print `hep-ph/0002033`.

[41] ATLAS Collaboration, "Detector and Physics Performance Technical Design Report", CERN/LHCC/99-15 (1999); CMS Collaboration, Technical Design Reports, CMS TDR 1-5 (1997/98).

[42] R. Brinkmann et al. (editors), *Conceptual Design of a 500 GeV e^+e^-Linear Collider with Integrated X-ray Laser Facility*, DESY 97-048.

[43] M. Carena, S. Pokorski and C. Wagner, *Nucl. Phys.* **B406** (1993) 59; W. de Boer *et al.*, *Z. Phys.* **C71** (1996) 415.

[44] G. Weiglein, e-print `hep-ph/0001044`.

[45] M. Battaglia, Proceedings, Linear Collider Workshop Sitges 1999, e-print `hep-ph/9910271`; G. Borisov and F. Richard, *ibid.*, e-print `hep-ph/9905413`.