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Polarized Beams and the Top Threshold

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Abstract

The strong impact of longitudinally polarized electron and positron beams on precision studies of top quark production and decay close to the production threshold is demonstrated. The right-left asymmetry of the production cross section can be measured with a precision of one to two percent. The top quarks will be highly polarized which allows to measure the couplings responsible for the top quark decay. As characteristic examples the decay chains $t \to bW(\to e^+\nu)$ and $t \to H^+b$ will be considered.

The Right Left Asymmetry

The right-left asymmetry

$$A_{RL} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \tag{1}$$

can be measured with polarized beams. (Here R and L correspond to fully polarized right- and lefthanded beams, respectively.) Close to threshold the total cross section for $t\bar{t}$ production is dominated by the S-wave contribution. It is therefore given by the ratio of S-wave amplitudes induced by electromagnetic and neutral current, respectively, and thus independent of uncertainties associated with the hadronic interaction of the $t\bar{t}$ system. In fact, the result for A_{RL} close to threshold [1, 2, 3] is identical to the one on a "would be" toponium resonance [4, 5]. The small P wave admixture to the rate which amounts to about 1% is numerically well under control [6]. A precise measurement of A_{LR} will thus lead to a correspondingly precise determination of the vector coupling of the top quark, assuming, of course that the weak and electromagnetic couplings of the electron are known.

In Born approximation the asymmetry A_{RL} at threshold is given by

$$A_{RL} = -\frac{a_2}{a_1} \tag{2}$$

with the coefficients

$$a_1 = (q_e q_t + v_e v_t d)^2 + (a_e v_t d)^2 \ , \qquad a_2 = 2 a_e v_t d \, (q_e q_t + v_e v_t d) \ , \tag{3}$$

$$d = \frac{1}{16 \sin^2 \theta_W \cos^2 \theta_W} \frac{s}{s - M_Z^2} \tag{4}$$

and the electromagnetic and weak charges

$$\begin{aligned} q_e &= -1\,, \qquad v_e = -1 + 4 \sin^2 \theta_W\,, \qquad a_e = -1\,, \\ q_t &= 2/3\,, \qquad v_t = 1 - 8/3 \sin^2 \theta_W\,, \qquad a_t = 1\,. \end{aligned} \tag{5}$$

At threshold $s = 4m_t^2$.

Numerically one finds (for $\sin^2 \theta = 0.23$ and $m_t = 175$ GeV) $A_{RL} = -0.426$. Electroweak corrections have been evaluated in [5, 1] and modify this result to $A_{LR} = -0.405$. The corresponding results for the Two-Higgs-Doublett Model can be found in [7].

For experiments with polarized electron and positron beams the effective polarization is given by

$$P_{eff} = \frac{P_+ - P_-}{1 - P_+ P_-}\,. \tag{6}$$

The values $|P_-| = 0.8$ and $|P_+| = 0.6$ have been quoted for TESLA, corresponding to $P_{eff} = 0.946$.

The relative contamination of the top signal by the background is denoted by $\eta = \sigma_{bg}/\sigma_{t\bar{t}}$ and the effective asymmetry thus given by

$$\langle A_{RL} \rangle = \frac{A_{RL} + A_{RL}^{bg} \eta}{1 + \eta}\,. \tag{7}$$

For the analysis below we shall assume that the background is dominated by W-pairs and for simplicity take $A_{RL}^{bg} = -1$, the worst case. (The results are hardly affected by this assumption, as long as η remains small.) The statistical error expected for a measurement of A_{RL} is given by [8]

$$\Delta A_{RL} = \frac{(1 + \eta)}{\sqrt{N}} \sqrt{\left(\frac{1}{P_{eff}^2} - \langle A_{RL}^2 \rangle \right) + \eta^2 (\langle A_{RL} \rangle - A_{RL}^{bg})^2}\,. \tag{8}$$

with

$$N = N_{RL} + N_{LR} = (\sigma_{bg} + \sigma_{t\bar{t}}) \frac{1 + |P_+ P_-|}{2} \int \mathcal{L} \tag{9}$$

We assume that positive and negative polarizations of electrons are chosen with equal probability, similarly those of positrons. Electron and positron polarisations are chosen uncorrelated. The asymmetry measurement, however, is only based on the two samples with dominantly righthanded electrons and lefthanded positrons (RL) and vice versa (LR). Operating only with these properly correlated combinations the rate would be enhanced by a factor two.

In addition one has to take into account the systematic error from the uncertainty in the beam polarization

$$\Delta\langle A_{RL}\rangle = \langle A_{RL}\rangle \frac{\Delta P_{eff}}{P_{eff}} \tag{10}$$

and from the uncertainty in the signal to background ratio

$$\Delta A_{RL} = |\langle A_{RL}\rangle - A_{LR}^{bg}|\Delta\eta \tag{11}$$

The subsequent analysis will be based on the extraction of the top signal described in [9]. Using appropriate cuts the efficiency for extracting the top signal is quoted to be 41.4 %. The remaining background cross section of 8.4 fb is constant throughout the threshold region, the signal to background ratio varies between 6 at the lowest scan point and 60 at the highest scan point At $E_{cm} = 2m_t$ it amounts to 21. For simplicity I will use the value 20 for the signal to background ratio throughout (whence $\langle A_{RL}\rangle = 0.433$,) and assume, furthermore, an integrated luminosity of 100 fb^{-1}. With these assumptions, and using eq.9

$$N_{t\bar{t}} = 8.4fb \times 20 \times (1 + 0.8 * 0.6)/2 \times 100\ fb^{-1} = 12400\,. \tag{12}$$

from the RL plus LR samples. We do not employ the potential improvement in background rejection from the difference in the polarization asymmetries of signal and background (dominantly W-pairs with A_{RL} close to -1).

For the evaluation of the statistical error (eq. 8) the second term under the squareroot can safely be neglected. In total one finds

$$\Delta A_{RL} \approx \frac{1}{\sqrt{N}} \approx 0.0093\,. \tag{13}$$

With a difference of 0.02 between the prediction of A_{RL} at tree level and in one loop approximation the measurement is evidently sensitive to the quantum corrections.

In order to match this statistical error, a relative precision in P_{eff} better than 2% is required (cf. eq. 10) and an accuracy in η of $\Delta\eta \leq \Delta A_{RL}/(\langle A_{RL}\rangle - A_{LR}^{bg}) \approx 1.64\%$ (cf. eq. 11), corresponding to a relative error of $\Delta\eta/\eta = 30\%$ in the signal to background ratio.

The relative errors of A_{RL} and of the top vector coupling v_t are related through

$$\frac{\Delta A_{RL}}{A_{RL}} = \left[1 + v_t d\frac{(v_e + a_e A_{RL})}{(q_e q_t + v_e v_t d)} + \frac{v_e}{a_e}A_{RL}\right]\frac{\Delta v_t}{v_t} \approx 0.87\frac{\Delta v_t}{v_t}\,. \tag{14}$$

In total one thus expects a measurement of v_t with a relative precision of about 2%. Adopting more optimistic assumptions, e.g. an integrated luminosity of 300 fb^{-1} invested for properly correlated electron and positron polarisations only, precisions in A_{RL} and v_t of about 0.4% and 1% respectively can be achieved without drastic requirements on the systematic error of the polarization.

Decays of Polarized Top Quarks at Rest: Measuring the Lorentz Structure of Charged Currents and the Handedness of a Charged Higgs Boson

Close to threshold the orbital angular momentum of top quarks is dominantly $J = 0$ and the helicity of electrons and positrons is transferred to the spins of top quark and antiquark [11]. [The angular dependent transverse and normal components of the top quark, resulting from S-P-wave interference, have been studied in [2, 3]. They are sensitive to the axial coupling of the top quark and to t-$\bar{t}$ rescattering. Beam polarization plays an important role also in this case. A detailed analysis of the statistical and systematical error in such an analysis is beyond the scope of this note.] Fully polarized beams would thus allow to study the decay of fully polarized top quarks at rest. The formation time of top mesons is far longer than the life time of the quark. Angular distributions of its decay products thus reflect those predicted for free quarks [11].

In practice one will deal with only partially polarized beams. For dominantly left handed electrons ($P_- = -0.8$) and right handed positrons ($P_+ = +0.6$) the effective beam polarization $P_{eff} = 0.946$ is close to one. The quark polarizations are thus given by

$$P_t = P_{\bar{t}} = \frac{P_{eff} + A_{RL}}{1 + P_{eff} A_{RL}} = -0.978\,, \tag{15}$$

and are thus very close to the maximal value. For both beam polarizations reversed, P_{eff} changes its sign and with

$$P_t = P_{\bar{t}} = 0.877\,, \tag{16}$$

the quark polarizations are only slightly degraded.

For an integrated luminosity $\int \mathcal{L} = 100 fb^{-1}$ distributed evenly among the four helicity combinations one obtains the event rate

$$\begin{aligned} N_{RL} &= \frac{1 + |P_{eff}| A_{RL}}{2} \sigma_{t\bar{t}} \frac{1 + |P_+ P_-|}{2} \int \mathcal{L} \\ N_{LR} &= \frac{1 - |P_{eff}| A_{RL}}{2} \sigma_{t\bar{t}} \frac{1 + |P_+ P_-|}{2} \int \mathcal{L} \end{aligned} \tag{17}$$

with a relative rate $N_{RL}/N_{LR} = 0.446$.

Two decay channels will be discussed in the following, which illustrate the power of measurements with polarized beams: the angular distribution of leptons from the decay chain $t \to bW^+(\to e^+\nu)$ and the angular distribution of a charged Higgs boson from the decay $t \to H^+b$. The former will allow to determine the Lorentz structure of the top quark coupling to the charged current, the latter gives the relative strength of the charged Yukawa coupling to right handed versus left handed b quarks, which is a direct measure of $\tan\beta$.

In the rest frame of the top quark the distribution of leptons from the decay $t \to bW^+(\to e^+\nu)$ factorizes into an angular and an energy dependent part [12], with

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta} = \frac{1 + P_t\alpha\cos\theta}{2}\,. \tag{18}$$

and $\alpha = 1$ in the Standard Model. This angular dependence is hardly affected by QCD corrections[13]. The smearing of this angular distribution from the momentum distribution [14] of the top quark due to the interaction between t and $\bar{t}$ is relatively unimportant ($\beta_t \approx 0.1$) and will be ignored. It can be either unfolded by measuring the top momentum or included in a more detailed Monte Carlo simulation.

In the case where the background contribution can be neglected, the statistical error for a measurement of the coefficient α is given by

$$\Delta\alpha = \frac{1}{\sqrt{N}}\sqrt{C/P_t^2 - \alpha^2} \tag{19}$$

where $C = 3$ if the measurement is based on an evaluation of the moment

$$\alpha = 3\langle\cos\theta\rangle/P_t \tag{20}$$

and $C = 4$ if it is based on the lepton forward backward asymmetry

$$\alpha = \frac{2}{P_t}\frac{N_F - N_B}{N_F + N_B}\,. \tag{21}$$

For the channel where one quark decays semileptonically and the other one into four jets, a signal to background ratio of $S/B = 55$ has been quoted [9], with a signal cross section of 77 fb. For an integrated luminosity of 100 fb^{-1}, distributed evenly among the four combinations of beam polarizations, one finds $N_{RL} = 1760$ and $N_{LR} = 3940 fb^{-1}$. The statistical precision

$$\Delta\alpha = 0.02 \tag{22}$$

is largely dominated by the LR-sample. A precision of $\delta P_{eff}/P_{eff} = 2\%$ is required to match this accuracy. Investing even an integrated luminosity of 300 fb^{-1} specifically into the LR and RL combinations, $\Delta\alpha = 0.008$ can be achieved, again without drastic requirements on the control of P_{eff}. In addition one may also study the (energy dependent) angular distribution of the neutrino, whose missing momentum can well be reconstructed in these measurements. A precision of one to two percent in the coefficient α in combination with the neutrino analysis will indeed severly restrict anomalous tbW couplings [16].

The analysis of "Non-Standard-Decays" of polarized top quarks allows to explore in a fairly direct way the Lorentz structure of the couplings responsible for these unconventional modes. Let us discuss, as a characteristic example, the transition $t \to H^+ b$ in the framework of the two-Higgs doublet model. From the amplitude

$$\mathcal{A} = \sqrt{\frac{G_f}{\sqrt{2}}}\,\bar{b}[m_t \cot\beta(1-\gamma_5) + m_b \tan\beta(1+\gamma_5)]t \tag{23}$$

one obtains [4]

$$\frac{\mathrm{d}N(H^+)}{\mathrm{d}\cos\theta} = \frac{1}{2}(1 + P_t\,\alpha_H \cos\theta) \tag{24}$$

with

$$\alpha_H = \frac{(m_t^2 \cot^2\beta - m_b^2 \tan^2\beta)}{(m_t^2 \cot^2\beta + m_b^2 \tan^2\beta)} \tag{25}$$

The determination of α_H is thus complementary to the measurement of the branching ratio proportional to $(m_t^2 \cot^2\beta + m_b^2 \tan^2\beta)$. In principle the anlysis proceeds just as in the case of semileptonic decay. Let us concentrate on the case of large $\tan\beta$, which allows for a sizeable branching ratio $\mathrm{Br}(t \to H^+ b) = \mathcal{O}(10\%)$, if the charged Higgs boson is sufficiently light [15]. For $\tan\beta \geq 3$ the mode $H^+ \to \tau^+\nu$ dominates the charged Higgs decays. With good vertex detectors it should be possible to identify the τ lepton and thus a large fraction of these final states. The signal characteristics are similar to those of the semileptonic decay $t \to bW(\to \tau\nu)$, albeit with a different energy of the b jet and a different missing momentum. Hence we shall assume that the background can similarly be rejected to a large extent, and the signal retained with an efficiency of 30%. From eq.19 and with $N_{RL} + N_{LR} = 2000$ (corresponding again to $\mathcal{L} = 100 fb^{-1}$), using furthermore $C = 3$ and $P_t^2 \approx 1$ one finds

$$\Delta\alpha \approx 0.04\,. \tag{26}$$

For the optimistic asumptions on luminosity and correlated beam polarizations even $\Delta\alpha = 0.016$ seems within reach. From the relation between the errors in α_H and $\tan\beta$

$$\Delta\alpha_H = \frac{8\frac{m_b^2}{m_t^2}\tan^4\beta}{(1+\frac{m_b^2}{m_t^2}\tan^4\beta)^2}\,\frac{\delta\tan\beta}{\tan\beta} \tag{27}$$

one may expect a fairly precise determination of $\tan\beta$ if $m_b^2/m_t^2 \tan\beta$ is of order one. Otherwise one would confirm the dominance of right- or of lefthanded couplings and thus provide an important test of the Lorentz structure of the Yukawa couplings.

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