LC-TH-2001-021
University of Bergen, Department of Physics
Scientific/Technical Report No. 2001-01
ISSN 0803-2696
hep-ph/0101150
January 2001

Contact interactions and polarized beams at a Linear Collider

A.A. Babich[a], P. Osland[b], A.A. Pankov[a,c] and N. Paver[c]

[a] Pavel Sukhoi Technical University, Gomel, 246746 Belarus

[b] Department of Physics, University of Bergen,

Allégaten 55, N-5007 Bergen, Norway

[c] Dipartimento di Fisica Teorica, Università di Trieste and

Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Trieste, Italy

Abstract

We discuss contact-interaction searches in the processes $e^+e^- \to \mu^+\mu^-$, $b\bar{b}$ and $c\bar{c}$ at an e^+e^- Linear Collider with c.m. energy $\sqrt{s} = 0.5$ TeV and with longitudinally polarized beams. The measurement of polarized cross sections allows to study the individual helicity cross sections, and consequently to derive separate, model-independent, constraints on the four-fermion contact interaction couplings. We evaluate the reach on those parameters foreseeable in the case of both electron and positron polarization fixed at some reference values, and compare it with the situation where only electron polarization is available. The analysis is based on polarized integrated cross sections with optimal kinematical cuts that can improve the sensitivity to the relevant couplings. While electron polarization would by itself allow such an analysis, the additional positron polarization (with no loss of beam intensity) and optimization can have a crucial role in improving the sensitivity to the new interactions.

1 Introduction

The possibility of longitudinally polarizing electron and positron beams at the Linear Collider (LC) is considered with great interest in connection with the physics programme to be performed at such a facility [1]. Indeed, this situation would enable to probe with enhanced sensitivity the chiral structure of electroweak interactions and, in particular, to set stringent, and model-independent, constraints on new interactions by looking for deviations of the data from the Standard Model (SM) predictions.

Here, we will consider the process of fermion pair production ($f \neq e$, t)

$$e^+ + e^- \to f + \bar{f} \tag{1}$$

at the LC with: (i) one beam (electron) polarized, and (ii) both beams polarized. For both cases, and for different values of the luminosity, we discuss the sensitivity of the measurable helicity cross sections to the $SU(3) \times SU(2) \times U(1)$ symmetric $eeff$ contact-interaction effective Lagrangian with helicity-conserving and flavor-diagonal fermion currents [2]:

$$\mathcal{L}_{\text{C.I.}} = \sum_{\alpha\beta} \frac{g^2_{\text{eff}}}{\Lambda^2_{\alpha\beta}} \eta_{\alpha\beta} \left(\bar{e}_\alpha \gamma_\mu e_\alpha\right) \left(\bar{f}_\beta \gamma^\mu f_\beta\right). \tag{2}$$

In Eq. (2), generation and color indices have been suppressed, $\alpha, \beta = \text{L}, \text{R}$ indicate left- or right-handed helicities, and $\eta_{\alpha\beta} = \pm 1, 0$ depending on the chiral structure of the individual interactions. Also, one takes $g^2_{\text{eff}} = 4\pi$ to remind that such new interaction, originally proposed for compositeness, would become strong at $\sqrt{s} \sim \Lambda_{\alpha\beta}$. However, more generally $\mathcal{L}$ should be considered as the 'low-energy' parameterization of some non-standard interaction acting at the much larger energy scales Λ not attainable by the machine. Examples are the exchanges in the different channels of extremely heavy objects such as Z' with a few TeV mass [3] and leptoquarks [4].

Clearly, the new coupling constants in Eq. (2) (equivalently, the mass scales $\Lambda_{\alpha\beta}$) are *a priori* free parameters that induce deviations of observables from the SM predictions, and the attainable constraints are assessed by the numerical comparison of such deviations to the expected experimental accuracies.

For a given final fermion species f, Eq. (2) defines eight individual, independent models corresponding to the combinations of the four chiralities α, β with the $\pm$ signs of the η's. Therefore, in the most general case where the observed contact interaction is a combination of these models, one faces the complication that the aforementioned deviations simultaneously depend on all four-fermion effective couplings in Eq. (2). The simplest procedure consists in assuming a non-zero value for only one coupling at a time (or a specific combination of them) and a 1-parameter χ^2 fit to the data, which leads to tests of the models mentioned above [3, 5].

On the other hand, a general, model-independent, analysis must simultaneously account for all non-zero couplings as free parameters and, at the same time, allow the derivation of separate constraints. This possibility is offered by initial beam polarization, that enables the extraction from the data of the individual helicity cross sections of process (1), each one being directly related to a single $eeff$ contact term that, accordingly, can be disentangled.

Actually, we shall adopt here as basic observables two particular, polarized, integrated cross sections that allow to reconstruct the four helicity amplitudes from linear combinations of measurements at different values of the beam polarizations. Integrated observables should be of some advantage in the case of limited experimental statistics. Also, in principle, a significant improvement can be obtained by defining optimally chosen kinematical regions of integration that lead to maximal sensitivity of the analysis to the four-fermion couplings [6, 7].

In the sequel, after giving the main definitions and briefly reviewing the procedure, we shall assess the reach on $\Lambda_{\alpha\beta}$ for the LC with $\sqrt{s} = 0.5$ TeV as a function of the luminosity, for reference values of the electron and positron longitudinal polarizations with given uncertainties, and making standard assumptions on the expected systematic uncertainties on the cross sections for process (1).

2 Determination of helicity cross sections

Limiting ourselves to the cases $f \neq e, t$ and neglecting the fermion mass with respect to the c.m. energy $\sqrt{s}$, the differential cross section of process (1) with polarized electron and positron beams reads, in the Born approximation [8]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{3}{8}\left[(1+\cos\theta)^2\sigma_+ + (1-\cos\theta)^2\sigma_-\right]. \tag{3}$$

Here, θ is the angle between the incoming electron and the outgoing fermion in the c.m. frame and, with P_e and $P_{\bar{e}}$ the longitudinal polarizations of the beams, σ_+ and σ_- can be expressed in terms of the helicity cross sections as

$$\begin{aligned}\sigma_+ &= \frac{1}{4}\left[(1-P_e)(1+P_{\bar{e}})\,\sigma_{\mathrm{LL}} + (1+P_e)(1-P_{\bar{e}})\,\sigma_{\mathrm{RR}}\right] \\ &= \frac{D}{4}\left[(1-P_{\mathrm{eff}})\,\sigma_{\mathrm{LL}} + (1+P_{\mathrm{eff}})\,\sigma_{\mathrm{RR}}\right], \qquad (4) \\ \sigma_- &= \frac{1}{4}\left[(1-P_e)(1+P_{\bar{e}})\,\sigma_{\mathrm{LR}} + (1+P_e)(1-P_{\bar{e}})\,\sigma_{\mathrm{RL}}\right] \\ &= \frac{D}{4}\left[(1-P_{\mathrm{eff}})\,\sigma_{\mathrm{LR}} + (1+P_{\mathrm{eff}})\,\sigma_{\mathrm{RL}}\right], \qquad (5)\end{aligned}$$

where

$$P_{\mathrm{eff}} = \frac{P_e - P_{\bar{e}}}{1 - P_e P_{\bar{e}}} \tag{6}$$

is the effective polarization [9], satisfying $|P_{\mathrm{eff}}| \leq 1$, and $D = 1 - P_e P_{\bar{e}}$. It should be noted that with $P_{\bar{e}} \neq 0$, $|P_{\mathrm{eff}}|$ can be larger than $|P_e|$. Moreover, with $\alpha, \beta = \mathrm{L}, \mathrm{R}$, in Eqs. (4) and (5):

$$\sigma_{\alpha\beta} = N_C \sigma_{\mathrm{pt}} |A_{\alpha\beta}|^2, \tag{7}$$

where $N_C \approx 3(1+\alpha_s/\pi)$ for quarks and $N_C = 1$ for leptons, respectively, and $\sigma_{\rm pt} \equiv \sigma(e^+e^- \to \gamma^* \to l^+l^-) = (4\pi\alpha^2)/(3s)$. Including the γ, Z exchanges and the contribution of $\mathcal{L}_{\rm C.I.}$ according to Eq. (2), the helicity amplitudes $A_{\alpha\beta}$ can be written as

$$A_{\alpha\beta} = Q_e Q_f + g_\alpha^e\, g_\beta^f\, \chi_Z + \frac{s\eta_{\alpha\beta}}{\alpha\Lambda^2_{\alpha\beta}}, \tag{8}$$

where $\chi_Z = s/(s - M_Z^2 + iM_Z\Gamma_Z)$ is the Z boson propagator, $g^f_{\rm L,R}$ are the SM left- and right-handed fermion couplings of the Z, and Q_f are the fermion electric charges. The above relations clearly show the direct relation of helicity cross sections to the individual contact interactions in Eq. (2) with definite chiralities, that allows the desired model-independent analysis with all contact-interactions taken into account simultaneously as free parameters. The various contributions in Eqs. (4) and (5) can be disentangled by making measurements at two different values of the polarizations (a minimum of four measurements is needed). For this purpose we use the set of values $P_e = \pm P_1$ and $P_{\bar e} = \mp P_2$ $(P_{1,2} > 0)$ or, alternatively, $P_{\rm eff} = \pm P$ with D fixed. Correspondingly, from Eqs. (4) and (5):

$$\sigma_{\rm LL} = \frac{1}{D}\left[-\frac{1-P}{P}\sigma_+(P) + \frac{1+P}{P}\sigma_+(-P)\right], \tag{9}$$

$$\sigma_{\rm RR} = \frac{1}{D}\left[\frac{1+P}{P}\sigma_+(P) - \frac{1-P}{P}\sigma_+(-P)\right], \tag{10}$$

with $\sigma_{\rm LR}$ and $\sigma_{\rm RL}$ obtained from $\sigma_{\rm LL}$ and $\sigma_{\rm RR}$, respectively, replacing σ_+ by σ_-.

Actually, for the purpose of optimizing the resulting bounds on $\Lambda_{\alpha\beta}$, one can more generally define the polarized cross sections integrated over the *a priori* arbitrary kinematical ranges $(-1,\ z^*)$ and $(z^*,\ 1)$ [6]:

$$\sigma_1(z^*, P, D) \equiv \int_{z^*}^{1} \frac{{\rm d}\sigma}{{\rm d}\cos\theta}{\rm d}\cos\theta = \frac{1}{8}\left\{\left[8 - (1+z^*)^3\right]\sigma_+ + (1-z^*)^3\sigma_-\right\}, \tag{11}$$

$$\sigma_2(z^*, P, D) \equiv \int_{-1}^{z^*} \frac{{\rm d}\sigma}{{\rm d}\cos\theta}{\rm d}\cos\theta = \frac{1}{8}\left\{(1+z^*)^3\sigma_+ + \left[8 - (1-z^*)^3\right]\sigma_-\right\}, \tag{12}$$

and take $\sigma_{1,2}(z^*, P, D)$ as the basic set of integrated polarized observables to be measured.[1] The basic reason this procedure, with $z^* \neq 0$, can be advantageous, is that the SM amplitude, against which the contact interaction term interferes, is *not* forward-backward symmetric. By solving Eqs. (11) and (12) one obtains σ_+ and σ_- from the measurement of σ_1 and σ_2:

$$\sigma_+ = \left[a(z^*)\sigma_1(z^*, P, D) + b(z^*)\sigma_2(z^*, P, D)\right], \tag{13}$$

$$\sigma_- = \left[b(-z^*)\sigma_1(z^*, P, D) + a(-z^*)\sigma_2(z^*, P, D)\right], \tag{14}$$

[1] For simplicity of notations, the polarization dependence of $\sigma_\pm$ on the right-hand sides of Eqs. (11) and (12) has been suppressed.

where

$$a(z^*) = \frac{8-(1-z^*)^3}{6(1-z^{*2})}, \qquad b(z^*) = -\frac{(1-z^*)^3}{6(1-z^{*2})}. \tag{15}$$

The experimental values of the helicity cross sections $\sigma_{\alpha\beta}$ are finally determined from the linear system of equations (9), (10).

The advantage of this, rather elaborate, procedure is that the actual value of z^*, representing an input parameter related to given experimental conditions, can be tuned to achieve an optimization of the constraints on the mass scales $\Lambda_{\alpha\beta}$.

Of course, electron *or* positron polarization is a *necessity* in order to disentangle the helicity cross sections and evaluate separate, and model-independent, constraints on the corresponding contact-interaction couplings. However, one can expect on statistical grounds a significant increase of the sensitivity due to the polarization of positrons, provided the luminosity in this case remains the same or is only moderately reduced, and the polarization is known very precisely.

In the following analysis, cross sections will be evaluated including initial- and final-state radiation by means of the program ZFITTER [10], which has to be used along with ZEFIT, adapted to the present discussion, with $m_{\rm top} = 175$ GeV and $m_H = 120$ GeV. One-loop SM electroweak corrections are accounted for by improved Born amplitudes [11, 12], such that the form of the previous formulae remains the same. Concerning initial-state radiation, a cut on the energy of the emitted photon $\Delta = E_\gamma/E_{\rm beam} = 0.9$ is applied for $\sqrt{s} = 0.5$ TeV in order to avoid the radiative return to the Z peak, and increase the signal originating from the contact interaction contribution [13].

3 Sensitivity of polarized observables

Current bounds on $\Lambda_{\alpha\beta}$, of the order of several TeV [14], are such that for the LC c.m. energy $\sqrt{s} = 0.5$ TeV the characteristic suppression factor s/Λ^2 in Eq. (8) is quite small. Therefore, we can only look at indirect manifestations of the contact interaction (2) as deviations of measured helicity cross sections from the SM predictions, and assess the corresponding reach on the $\Lambda_{\alpha\beta}$ on the basis of the foreseen initial beam polarizations and the experimental accuracies.

We can define the 'sensitivity' to contact interactions of each helicity cross section as the ratio

$$\mathcal{S}(\sigma_{\alpha\beta}) = \frac{|\Delta\sigma_{\alpha\beta}|}{\delta\sigma_{\alpha\beta}}, \tag{16}$$

where $\Delta\sigma_{\alpha\beta}$ is the deviation from the SM prediction due to (2), dominated for $\sqrt{s} \ll \Lambda_{\alpha\beta}$ by the linear interference term

$$\Delta\sigma_{\alpha\beta} \equiv \sigma_{\alpha\beta} - \sigma_{\alpha\beta}^{\rm SM} \simeq 2N_C\,\sigma_{\rm pt}\left(Q_e\,Q_f + g_\alpha^e\,g_\beta^f\,\chi_Z\right)\frac{s\eta_{\alpha\beta}}{\alpha\Lambda_{\alpha\beta}^2}, \tag{17}$$

and $\delta\sigma_{\alpha\beta}$ denotes the expected experimental uncertainty on $\sigma_{\alpha\beta}$, combining statistical and systematic uncertainties. The reach on $\Lambda_{\alpha\beta}$ can be obtained from a χ^2 analysis,

$$\chi^2 \equiv \mathcal{S}^2 = \left(\frac{\Delta\sigma_{\alpha\beta}}{\delta\sigma_{\alpha\beta}}\right)^2, \tag{18}$$

by imposing, as a criterion to constrain the allowed values of the contact-interaction parameters from the non-observation of the corresponding deviations within the expected uncertainty $\delta\sigma_{\alpha\beta}$, that:

$$\chi^2 < \chi^2_{\rm CL}, \tag{19}$$

where the actual value of $\chi^2_{\rm CL}$ specifies the desired 'confidence' level. Since, as (17) shows, the deviation $\Delta\sigma_{\alpha\beta}$ depends on a single 'effective' non-standard parameter represented by the product of the known relevant SM coupling times the contact-interaction coupling one wants to constrain, in such a χ^2 analysis of data one effective parameter is involved, and we take $\chi^2_{\rm CL} = 3.84$ corresponding to 95% C.L. with a one-parameter fit.[2]

To proceed to numerical evaluations of the bounds, an assessment of the expected experimental uncertainty $\delta\sigma_{\alpha\beta}$ is needed. To obtain an indication, we combine all uncertainties in quadrature, and separate for convenience the systematic uncertainty of the initial positron and electron polarizations, essentially by considering $\sigma_{1,2}$, P_e and $P_{\bar{e}}$ in Eqs. (9) and (10) as if they were independent measurables. This is clearly an approximation, but it should exhibit the main dependence on the uncertainties of the polarizations. Thus,

$$(\delta\sigma_{\alpha\beta})^2 = \left(\overline{\delta}\sigma_{\alpha\beta}\right)^2 + \left(\delta\sigma^{\rm pol}_{\alpha\beta}\right)^2. \tag{20}$$

With $\sigma_{1,2}$ our basic observables (see Eqs. (9)–(15)), one can write:

$$\begin{aligned}\left(\overline{\delta}\sigma_{\rm LL}\right)^2 &= a^2(z^*)\left[\left(\frac{1-P}{PD}\right)^2(\delta\sigma_1(z^*,P))^2 + \left(\frac{1+P}{PD}\right)^2(\delta\sigma_1(z^*,-P))^2\right]\\ &+ b^2(z^*)\left[\left(\frac{1-P}{PD}\right)^2(\delta\sigma_2(z^*,P))^2 + \left(\frac{1+P}{PD}\right)^2(\delta\sigma_2(z^*,-P))^2\right],\end{aligned} \tag{21}$$

$$\begin{aligned}\left(\overline{\delta}\sigma_{\rm LR}\right)^2 &= b^2(-z^*)\left[\left(\frac{1-P}{PD}\right)^2(\delta\sigma_1(z^*,P))^2 + \left(\frac{1+P}{PD}\right)^2(\delta\sigma_1(z^*,-P))^2\right]\\ &+ a^2(-z^*)\left[\left(\frac{1-P}{PD}\right)^2(\delta\sigma_2(z^*,P))^2 + \left(\frac{1+P}{PD}\right)^2(\delta\sigma_2(z^*,-P))^2\right],\end{aligned} \tag{22}$$

[2]The signs of the η's in (2) turn out to be numerically unimportant for the determination of constraints on the $\Lambda_{\alpha\beta}$. Indeed, for given helicities $\alpha\beta$, different signs of η's yield practically identical results for the mass scales $\Lambda_{\alpha\beta}$ as long as, in the chosen kinematical configurations, the non-standard effects are largely dominated by the interference (17) between contact-interaction and SM terms.

where $P = |P_{\rm eff}|$. Explicit expressions for $\overline{\delta}\sigma_{\rm RR}$ and $\overline{\delta}\sigma_{\rm RL}$ can be derived from $\overline{\delta}\sigma_{\rm LL}$ and $\overline{\delta}\sigma_{\rm LR}$, respectively, replacing in the above equations $\pm P \to \mp P$ in $\delta\sigma_i(z^*, \pm P)$, but not in the corresponding prefactors. For simplicity of notations, the dependence of $\delta\sigma_{1,2}$ on D has not been explicitly indicated.

The expected smallness of deviations from the SM allows the use, to a very good approximation, of the SM predictions for the cross sections $\sigma_{1,2}$ to assess the expected $\delta\sigma_1$ and $\delta\sigma_2$ in (21), (22) and, accordingly, to write:

$$(\delta\sigma_i)^2 \simeq (\delta\sigma_i^{\rm SM})^2 = \frac{\sigma_i^{\rm SM}}{\epsilon\,\mathcal{L}_{\rm int}} + \left(\delta^{\rm sys}\sigma_i^{\rm SM}\right)^2, \qquad i = 1, 2. \tag{23}$$

In Eq. (23), $\mathcal{L}_{\rm int}$ is the integrated luminosity, and ϵ is the efficiency for detecting the final state under consideration. For our numerical analysis we shall assume the commonly used reference values of the identification efficiencies ϵ and the systematic uncertainties $\delta^{\rm sys}$ [15]: $\epsilon = 95\%$ and $\delta^{\rm sys} = 0.5\%$ for l^+l^-; $\epsilon = 60\%$ and $\delta^{\rm sys} = 1\%$ for $b\bar{b}$; $\epsilon = 35\%$ and $\delta^{\rm sys} = 1.5\%$ for $c\bar{c}$. Notice that, as a simplification, we take the same $\delta^{\rm sys}$ for both $i = 1$ and 2, and independent of z^* in the relevant angular range. Concerning the statistical uncertainty, we shall vary $\mathcal{L}_{\rm int}$ from 50 to 500 ${\rm fb}^{-1}$ (half for each polarization orientation) to study the relative roles of statistical and systematic uncertainties, and a fiducial experimental angular range $|\cos\theta| \leq 0.99$.

Let us now turn to a discussion of the systematic uncertainty of the initial beam polarization. Finite values of δP_e and of $\delta P_{\bar{e}}$ will influence the extraction of the helicity cross sections $\sigma_{\alpha\beta}$ through the prefactors of Eqs. (9), (10), (13) and (14), as well as through the dependence of σ_1 and σ_2 on P and D. Lacking at present sufficiently detailed knowledge of the individual sources of uncertainty needed for a complete assessment, for simplicity we model the systematic uncertainty by assuming the latter effect to be included in the $\delta^{\rm sys}\sigma_i$ introduced in Eq. (23). Under the above assumptions, we obtain

$$\begin{aligned}
\left(\delta\sigma_{\rm LL}^{\rm pol}\right)^2 &= [f(z^*,P)(1+P_{\bar{e}}P^2) - f(z^*,-P)(1-P_{\bar{e}}P^2)]^2\left(\frac{\delta P_e}{D^2P^2}\right)^2 \\
&+ [f(z^*,P)(1-P_eP^2) - f(z^*,-P)(1+P_eP^2)]^2\left(\frac{\delta P_{\bar{e}}}{D^2P^2}\right)^2, \\
\left(\delta\sigma_{\rm RR}^{\rm pol}\right)^2 &= [f(z^*,P)(1-P_{\bar{e}}P^2) - f(z^*,-P)(1+P_{\bar{e}}P^2)]^2\left(\frac{\delta P_e}{D^2P^2}\right)^2 \\
&+ [f(z^*,P)(1+P_eP^2) - f(z^*,-P)(1-P_eP^2)]^2\left(\frac{\delta P_{\bar{e}}}{D^2P^2}\right)^2,
\end{aligned} \tag{24}$$

with

$$f(z^*,P) = a(z^*)\sigma_1(z^*,P) + b(z^*)\sigma_2(z^*,P). \tag{25}$$

Furthermore, $\delta\sigma_{\rm LR}^{\rm pol}$ and $\delta\sigma_{\rm RL}^{\rm pol}$ are obtained from $\delta\sigma_{\rm LL}^{p}$ and $\delta\sigma_{\rm RR}^{p}$, respectively, by substituting $a(z^*) \leftrightarrow b(-z^*)$. Numerically, for explicit evaluations of the reach in $\Lambda_{\alpha\beta}$, we shall work out the example of $|P_e| = 0.8$ with $\delta P_e/P_e = 0.5\%$, as currently achieved [16], and $|P_{\bar{e}}| = 0.0$, 0.4 and 0.6 with $\delta P_{\bar{e}}/P_{\bar{e}} = 0.5\%$, assuming no loss of luminosity compared to the case of no positron polarization. With these values of the longitudinal polarizations, $|P_{\text{eff}}| = P = 0.8$, $D = 1$; $P = 0.909$, $D = 1.32$; $P = 0.946$, $D = 1.48$, respectively.

As far as the proposed optimization procedure is concerned, from the previous formulae we observe that the z^* dependence of σ_1 and σ_2, as defined in Eqs. (11) and (12), translates into a z^* dependence of the uncertainties $\delta\sigma_{\alpha\beta}$ that appear in (16) and (18). Since the deviation $\Delta\sigma_{\alpha\beta}$ is independent of z^*, see Eq. (17), the full sensitivity of each helicity cross section to the relevant contact-interaction coupling constant is determined not only by the size but also by the z^* behavior of the corresponding uncertainty $\delta\sigma_{\alpha\beta}$. Therefore, an optimization would be obtained by choosing for z^* the value z^*_{opt} where the uncertainty $\delta\sigma_{\alpha\beta}$ has a minimum, *i.e.*, where the corresponding sensitivity Eq. (16) has a maximum.

For the minimization of the statistical uncertainty, the first term on the right-hand side of Eq. (23), one may use the explicit expressions of the SM cross sections. The equation determining the relevant z^* is:

$$z^* = -3\frac{1 - r_{\alpha\beta}}{1 + r_{\alpha\beta}}\frac{z^{*4} - 6z^{*2} - 3}{z^{*4} - 2z^{*2} - 23}, \tag{26}$$

where

$$r_{\text{LL}} = r_{\text{LR}} = \frac{(1 + 3P^2_{\text{eff}})\sigma^{SM}_{\text{LR}} + (1 - P^2_{\text{eff}})\sigma^{SM}_{\text{RL}}}{(1 + 3P^2_{\text{eff}})\sigma^{SM}_{\text{LL}} + (1 - P^2_{\text{eff}})\sigma^{SM}_{\text{RR}}}, \tag{27}$$

and $r_{\text{RR}} = r_{\text{RL}}$ is obtained by replacing $\text{L} \leftrightarrow \text{R}$ in (27). As one can see, the location of z^* that minimizes the statistical uncertainty only depends on the SM parameters and P_{eff} and for each final-state fermion in (1) is independent of the luminosity and the efficiency of reconstruction ϵ. In a left-right symmetric theory, the above ratios $r_{\alpha\beta}$ would all be 1, and in this case $z^* = 0$. However, in the SM, depending on flavour and energy, $r_{\alpha\beta}$ may be less than, or larger than unity. Since the z^*-dependent fraction in (26) is positive for $z^{*2} \leq 1$, it follows that the solutions satisfy $z^* < 0$ if $r_{\alpha\beta} < 1$ and *vice versa*. We also note that the location is the same for the LL and LR configurations, and likewise for RR and RL, while numerically the sensitivities are different. Clearly, the values of z^* determined from the above SM formulae can be regarded as a simple, first determination of z^*_{opt} in the case where the expected uncertainty is dominated by the statistical one (*e.g.*, for low luminosities). In the cases where statistical and systematic uncertainties are comparable, the z^*_{opt} must be determined by a more elaborate numerical analysis that includes all different sources of experimental uncertainties. A more extended discussion and a set of numerical results can be found in Refs. [6, 7].

4 Bounds on $\Lambda_{\alpha\beta}$ and concluding remarks

We assume half the total integrated luminosity for each value of the effective polarization, $P_{\text{eff}} = \pm P$, and the same time of operation in the different polarization configurations.

From the procedure and the inputs outlined in the previous section, we find for the discovery limits on the mass scales $\Lambda_{\alpha\beta}$ *vs.* $\mathcal{L}_{\rm int}$ the results represented by the curves in Fig. 1. We recall that the sensitivity (16), *via* (18) and its square root, determines the reach in $\Lambda_{\alpha\beta}$.

In the (simpler) example of polarized electrons and unpolarized positrons, the relative uncertainties $\delta\sigma_{\alpha\beta}/\sigma_{\alpha\beta} \simeq \delta\sigma^{\rm SM}_{\alpha\beta}/\sigma^{\rm SM}_{\alpha\beta}$ compared to the case of same P_e, but $\delta P_e = 0$, has been discussed for variable δP_e in [7]. For the values considered here, the contribution of δP_e to the overall $\delta\sigma_{\alpha\beta}$ is really negligible for the $\mu^+\mu^-$ final state (much less than a fraction of a %), and the origin for such a strong suppression can be easily understood on using the SM-values for σ_1 and σ_2 in the parameterization (24) and (25). For quarks, the contribution of δP_e to $\delta\sigma_{\alpha\beta}$ turns out to be potentially larger, especially for the LL and RR cases of $b\bar{b}$ final states, but is still insignificant for $\delta P_e/P_e = 0.5\%$ (see Fig. 2 of [7]).

Turning to the case of both electron and positron longitudinal polarization, and referring to Eqs. (4) and (5), in the chosen helicity configuration where $P_eP_{\bar{e}} < 0$, one has $D > 1$ and $|P_{\rm eff}| > \max(|P_e|, |P_{\bar{e}}|)$, and consequently an increase of the sensitivity, provided the luminosity remains the same. However, this improvement from positron polarization is obtained up to a maximum value of $\delta P_{\bar{e}}/P_{\bar{e}}$, above which there would be no benefit, but, actually, a worsening of the sensitivity (see Figs. 3 and 4 of [7]).[3] This is not the case for the present input value of $\delta P_{\bar{e}}/P_{\bar{e}}$ and, indeed, Fig. 1 shows a clear benefit from positron polarization in improving the reach on $\Lambda_{\alpha\beta}$, by about 20–40% depending on the helicity configuration and the final fermion state.

As an indication of the influence of $\delta P_{\bar{e}}$, we report in Fig. 2 the results on $\Lambda_{\alpha\beta}$ obtained by increasing $\delta P_{\bar{e}}/P_{\bar{e}}$ up to 2%, with the same value of $\delta P_e/P_e$ as in Fig. 1, and for integrated luminosity $\mathcal{L}_{\rm int} = 50$ and 500 fb^{-1}. The anticipated worsening of the constraints for increasing $\delta P_{\bar{e}}$ is well represented in Fig. 2. Clearly, the point at which the benefit from positron polarization would be lost, is determined by the relative sizes of the uncertainties due to $\delta P_{\bar{e}}/P_{\bar{e}}$ and the other sources of experimental uncertainties, in particular by the specific reference values adopted in the parameterization of the $\delta\sigma_i$.[4] On the other hand, there is some confidence that P_e and $P_{\bar{e}}$ could be measured with the same kind of precision [17], so that full benefit from positron polarization should be obtained.

Fig. 1 shows that, at higher luminosity, all curves become less steep. This is a reflection of the fact that the statistical uncertainty decreases with respect to the other ones, including those due to polarization uncertainties. Therefore, one can expect a saturation of these curves when the integrated luminosity is such that the statistical uncertainty becomes negligible (of course, this depends on the individual channels and helicity combination), unless the systematic uncertainties are diminished accordingly. For example, for $\mathcal{L}_{\rm int} = 10^3$ fb^{-1}, we would obtain for $P_{\bar{e}} = 0.6$, and for $\mu^+\mu^-$ final states, the lower bounds $\Lambda_{\rm LL} = 60$ TeV, $\Lambda_{\rm LR} = 73$ TeV, $\Lambda_{\rm RL} = 73$ TeV and $\Lambda_{\rm RR} = 62$ TeV. The corresponding numbers for the hadronic channels are, respectively: 53, 72, 86 and 72 TeV for $b\bar{b}$; 39, 56, 62 and 46 TeV for $c\bar{c}$.

In this regard, for completeness one should discuss at the same time also the dependence

[3] Also this dependence can be qualitatively understood from (24) and (25).

[4] Of course, a similar discussion applies to the role of δP_e.

of the bounds on $\Lambda_{\alpha\beta}$ from the systematic uncertainty $\delta^{\rm sys}$ of Eq. (23). Basically, the effect of $\delta^{\rm sys}$ variations around the chosen input values in the derivation of Figs. 1 and 2 is rather small in the LR and RL cases where the statistical uncertainty is the dominant one for the luminosities considered here, but can be appreciable in the LL and RR cases where statistical and systematic uncertainties are comparable.

One can note that the bounds on $\Lambda_{\alpha\beta}$, although derived for the most general case, where all the contact interaction couplings of Eq. (2) simultaneously appear as free parameters, are numerically comparable to those obtained by allowing the presence of just one specific helicity channel at a time [5]. In this connection, a significant role is played by the optimization procedure introduced previously, i.e., the introduction of the optimal kinematical value of z^* in the definition of σ_1 and σ_2 of Eqs. (11) and (12). Indeed, the results on $\Lambda_{\rm LL}$ and $\Lambda_{\rm RR}$ found at such $z^*_{\rm opt}$ show a rather modest improvement over those derived, for the same helicity combinations, from the more conventional choice $z^*=0$ (that assumes the polarized total cross section and forward-backward asymmetry as fundamental observables). Conversely, the choice $z^*=z^*_{\rm opt}$ allows a dramatic improvement of the sensitivity in the LR and RL cases, and substantially increases the bounds on $\Lambda_{\rm LR}$ and $\Lambda_{\rm RL}$, by about 20–30%. For the sake of making a model-independent analysis, this improvement certainly justifies the elaborate procedure of determining $z^*_{\rm opt}$ from the analysis of the z^* dependence of the experimental uncertainty on $\sigma_{\alpha\beta}$ as measured *via* σ_1 and σ_2, prior to the application of the χ^2 procedure for the derivation of constraints on the $\Lambda_{\alpha\beta}$.

In conclusion, although the numerical support is based on the specific example worked out here on hypothetical values and assumptions on the initial beam polarizations and the values and properties of the experimental uncertainties, the above considerations should hold in general. Clearly, in practice, definite quantitative statements should await a clarification of the realistic experimental situation, in particular concerning the different sources, and relative roles, of expected experimental errors.

Acknowledgements
This research has been supported by the Research Council of Norway, and by MURST (Italian Ministry of University, Scientific Research and Technology).

References

[1] E. Accomando et al. (ECFA/DESY LC Physics Working Group), Phys. Rept. **299** (1998) 1.

[2] E. J. Eichten, K. D. Lane, M. E. Peskin, Phys. Rev. Lett. **50** (1983) 811;
R. Rückl, Phys. Lett. **B 129** (1983) 363.

[3] V. Barger, K. Cheung, K. Hagiwara, D. Zeppenfeld, Phys. Rev. D **57** (1998) 391;
D. Zeppenfeld, K. Cheung, preprint MADPH-98-1081, hep-ph/9810277.

[4] G. Altarelli, J. Ellis, G. F. Giudice, S. Lola, M. L. Mangano, Nucl. Phys. **B 506** (1997) 3;

R. Casalbuoni, S. De Curtis, D. Dominici, R. Gatto, Phys. Lett. **B 460** (1999) 135; V. Barger, K. Cheung, UCD-2000-8, hep-ph/0002259.

[5] S. Riemann, presentation at Linear Collider Workshop 2000 - LCWS 2000, FNAL, October 2000.

[6] A.A. Babich, P. Osland, A.A. Pankov, N. Paver, Phys. Lett. **B 476** (2000) 95.

[7] A.A. Babich, P. Osland, A.A. Pankov, N. Paver, Phys. Lett. **B 481** (2000) 263.

[8] B. Schrempp, F. Schrempp, N. Wermes, D. Zeppenfeld, Nucl. Phys. **B 296** (1988) 1.

[9] K. Flöttmann, preprint DESY 95-064; K.Fujii, T.Omori, KEK preprint 95-127.

[10] S. Riemann, FORTRAN program ZEFIT Version 4.2; D. Bardin et al., preprint DESY 99-070, hep-ph/9908433.

[11] M. Consoli, W. Hollik, F. Jegerlehner, *in* Z physics at LEP1, G. Altarelli, R. Kleiss, C. Verzegnassi (Eds.), vol.1, p.7, 1989.

[12] G. Altarelli, R. Casalbuoni, D. Dominici, F. Feruglio, R. Gatto, Nucl. Phys. **B 342** (1990) 15.

[13] A. Djouadi, A. Leike, T. Riemann, D. Schaile, C. Verzegnassi, Z. Phys. **C 56** (1992) 289.

[14] See, e.g.: D. Abbaneo et al., The LEP Collaborations, CERN-EP-2000-016; J. Breitenweg et al., ZEUS Collaboration, DESY-99-058; C. Adloff et al., H1 Collaboration, DESY-00-027.

[15] C. J. S. Damerell, D.J. Jackson, in *Proceedings of the 1996 DPF/DPB Summer Study on New Directions for High Energy Physics* (Snowmass 96), Edited by D.G. Cassel, L. Trindle Gennari, R.H. Siemann (SLAC, 1997) p. 442.

[16] M. Swartz, 19th International Symposium on Lepton and Photon Interactions at High-Energies (LP 99), Stanford, CA, 9-14 Aug 1999.

[17] P. Schüler, presentation at Linear Collider Workshop 2000 - LCWS 2000, FNAL, October 2000.

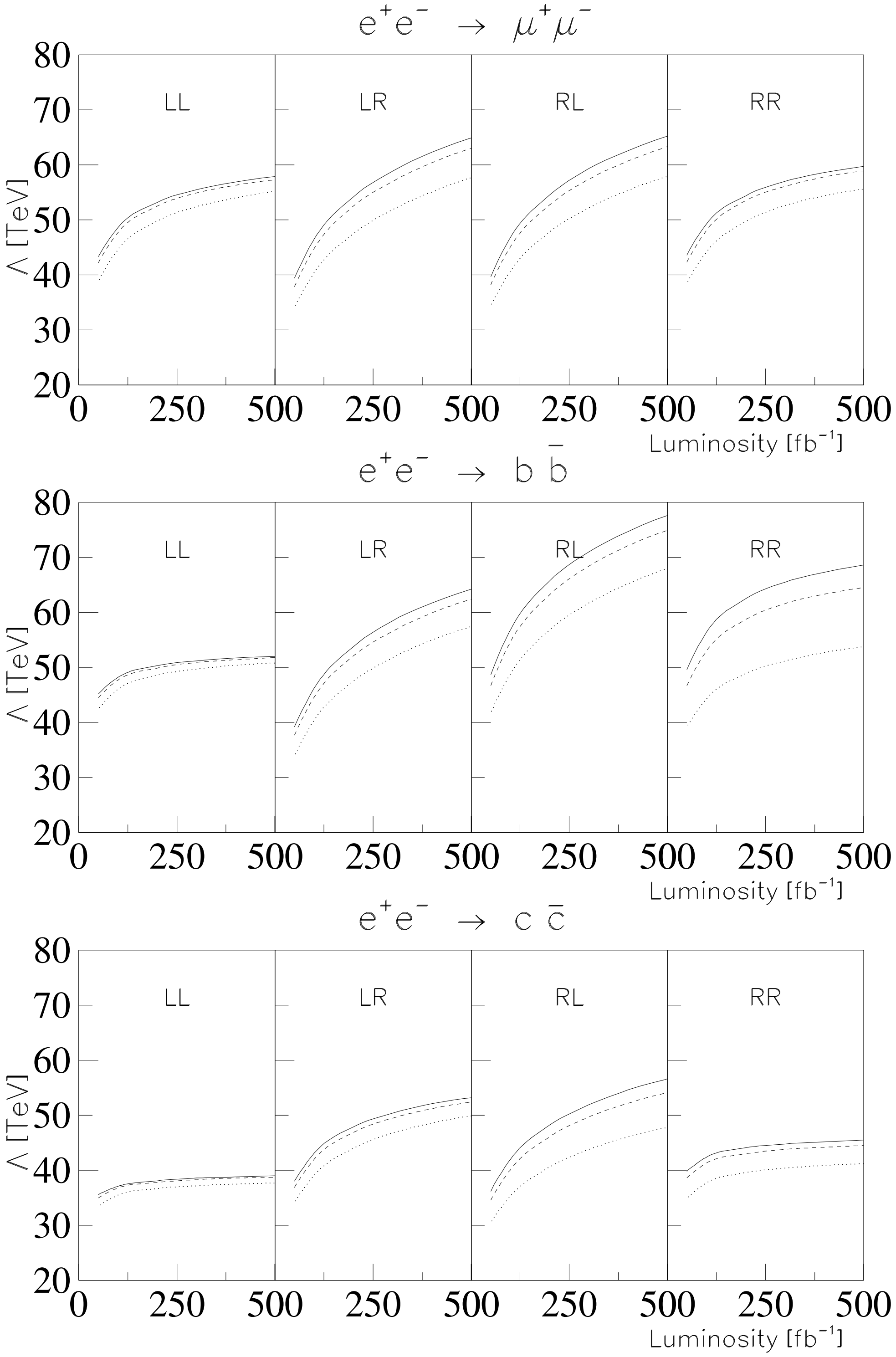

Figure 1: Reach in $\Lambda_{\alpha\beta}$ at 95% C.L., for the proposed model-independent analysis, for $e^+e^- \to \mu^+\mu^-, b\bar{b}$ and $c\bar{c}$ *vs.* $\mathcal{L}_{\rm int}$. Dotted: $P_{\bar{e}} = 0.0$; dashed: $P_{\bar{e}} = 0.4$; solid: $P_{\bar{e}} = 0.6$.

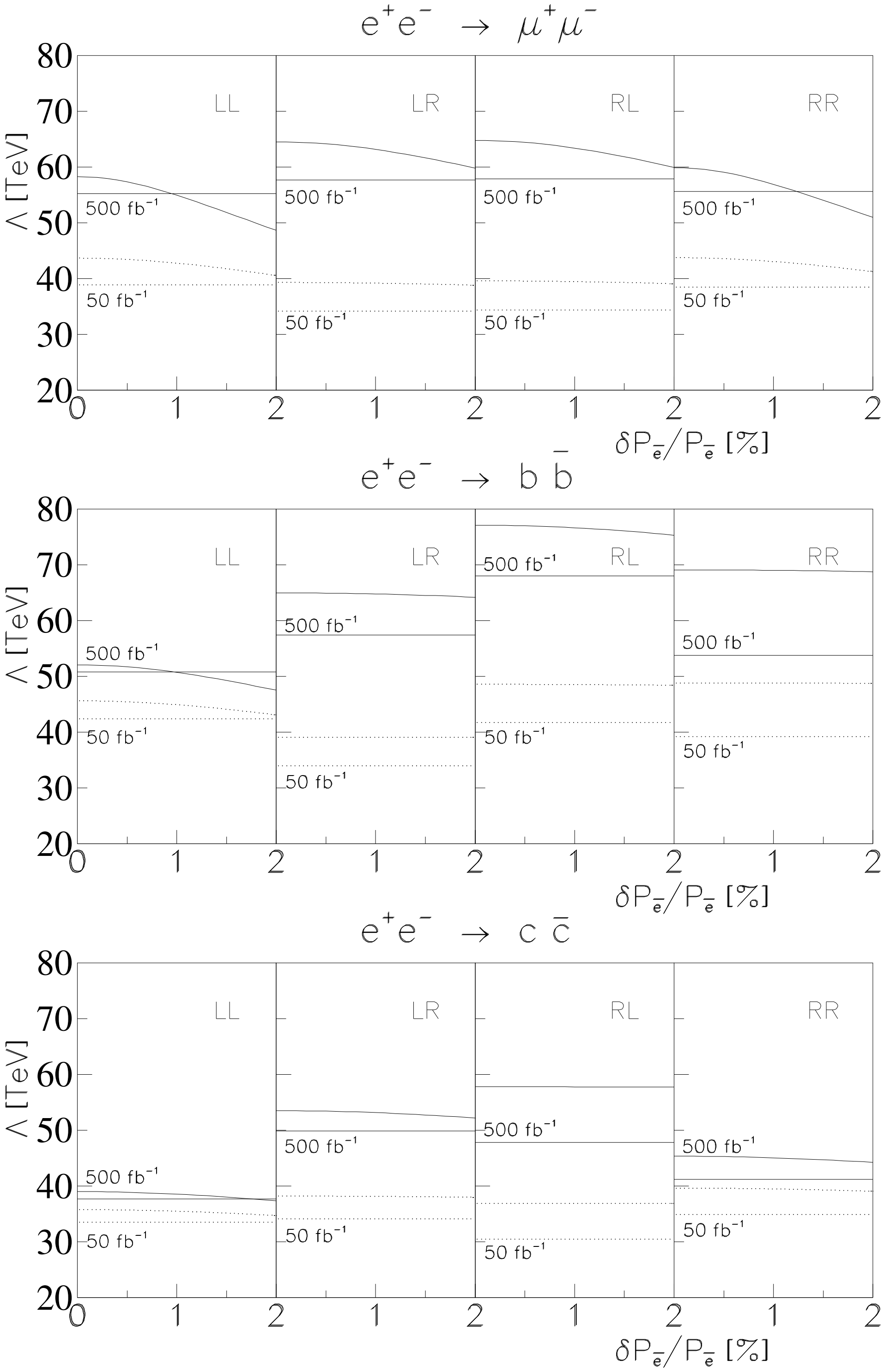

Figure 2: Reach in $\Lambda_{\alpha\beta}$ *vs.* uncertainty in positron polarization, $\delta P_{\bar{e}}/P_{\bar{e}}$ for $\mu^+\mu^-$, $b\bar{b}$ and $c\bar{c}$ final states. Dashed: 50 fb^{-1}, solid: 500 fb^{-1}. Horizontal lines: no positron polarization.