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The effective fine structure constant at TESLA energies.

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We present a new estimate of the hadronic contribution to the shift in the fine structure constant at LEP and TESLA energies and calculate the effective fine structure constant. Substantial progress in a precise determination of this important parameter is a consequence of substantially improved total cross section measurements by the BES II collaboration and an improved theoretical understanding. In the standard approach which relies to a large extend on experimental data we find $\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) = 0.027896 \pm 0.000395$ which yields $\alpha^{-1}(M_Z^2) = 128.907 \pm 0.054$. A few values at higher energies are given in the following table:

$\sqrt{s}$ GeV	$\Delta\alpha^{(5)}_{\rm had}(s)$	$\alpha^{-1}(s)$
100	0.0283 ± 0.0004	128.790 ± 0.054
300	0.0338 ± 0.0004	127.334 ± 0.054
500	0.0372 ± 0.0004	126.543 ± 0.054
800	0.0417 ± 0.0004	125.634 ± 0.054
1000	0.0436 ± 0.0004	125.229 ± 0.054

Another approach, using the Adler function as a tool to compare theory and experiment, allows us to to extend the applicability of perturbative QCD in a controlled manner. The result in this case reads $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027730 \pm 0.000148$ and hence $\alpha^{-1}(M_Z^2) = 128.930 \pm 0.020$. At TESLA energies a new problem shows up with the definition of an effective charge. A possible solution of the problem is presented. Prospects for further progress in a precise determination of the effective fine structure constant is discussed.

1 Introduction

Precision physics requires appropriate inclusion of higher order effects and the knowledge of very precise input parameters of the electroweak standard model SM. One of the basic input parameters is the fine structure constant which depends logarithmically on the energy scale. Vacuum polarisation effects lead to a partial screening of the charge in the low energy limit (Thomson limit) while at higher energies the strength of the electromagnetic interaction grows. In this note we have in mind future precision physics at TESLA [1] energies as a continuation of LEP [2] experiments and thus consider the effective fine structure constant at energies up to 1 TeV. Very likely, TESLA in addition of being a gauge boson factory like LEP will be a Higgs factory.

Renormalization of the electric charge e by a shift δe at different scales leads to a shift of the fine structure constant by

$$\Delta\alpha = 2\left(\frac{\delta e}{e}(0) - \frac{\delta e}{e}(M_Z)\right) = \Pi'_\gamma(0) - \Pi'_\gamma(M_Z^2) \tag{1}$$

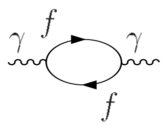
where $\Pi'_\gamma(s)$ is the photon vacuum polarisation function defined via the time-ordered product of two electromagnetic currents $j^\mu_{em}(x)$:

$$i\int d^4x\, e^{iq\cdot x}\langle 0|\mathrm{T}j^\mu_{em}(x)j^\nu_{em}(0)|0\rangle = -(q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi'_\gamma(q^2)\ . \tag{2}$$

The shift $\Delta\alpha$ is large due to the large change in scale going from zero momentum to the Z-mass scale $\mu = M_Z$ and due to the many species of fermions contributing. Zero momentum more precisely means the light fermion mass thresholds.

In perturbation theory the leading light fermion ($m_f \ll M_Z$) contribution is given by

$$\Delta\alpha \;=\; \sum_f$$



$$\begin{aligned} &= \frac{\alpha}{3\pi}\sum_f Q_f^2 N_{cf}\left(\ln\frac{M_Z^2}{m_f^2} - \frac{5}{3}\right) \\ &= \Delta\alpha_{\rm leptons} + \Delta\alpha^{(5)}_{\rm quarks}\ . \end{aligned} \tag{3}$$

A serious problem is the low energy contributions of the five light quarks u,d,s,c and b which cannot be reliably calculated using perturbative quantum chromodynamics (p-QCD). The evaluation of the hadronic contribution $\Delta\alpha^{(5)}_{\rm quarks} \to \Delta\alpha^{(5)}_{\rm hadrons}$ is the main concern of this note. Before I am going into this, let me make a few remarks about its consequences for precision physics.

A major drawback of the partially non-perturbative relationship between $\alpha(0)$ and $\alpha(M_Z)$ is that one has to rely on experimental data exhibiting systematic and statistical errors which implies a non-negligible uncertainty in our knowledge of the effective fine structure constant. In precision predictions of gauge boson properties this has become a limiting factor. Since α , G_μ, M_Z are the most precisely measured parameters, they are used as input parameters for accurate predictions of observables like the effective weak mixing parameter $\sin^2\Theta_f$, the vector v_f and axial-vector a_f neutral current couplings, the W mass M_W the widths Γ_Z and Γ_W of the Z and the W, respectively, etc. However, for physics at higher energies we have to use the effective couplings at the appropriate scale, for physics at the Z–resonance, for example, $\alpha(M_Z)$ is more adequate to use than $\alpha(0)$. Of course this just means that part of the higher order corrections may be absorbed into an effective parameter. If we compare the precision of the basic parameters

$$\begin{array}{llllllll} \frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} & \frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 \times 10^{-4} \\ \frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} & \frac{\delta M_Z}{M_Z} & \sim & 2.4 \times 10^{-5} \end{array} \tag{4}$$

we observe that the uncertainty in $\alpha(M_Z)$ is roughly an order of magnitude worse than the next best, which is the Z–mass. Let me remind the reader that $\Delta\alpha$ enters in electroweak precision physics typically when calculating versions of the weak mixing parameter $\sin^2\Theta_i$ from α, G_μ and M_Z via

$$\sin^2\Theta_i\,\cos^2\Theta_i = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i} \tag{5}$$

where

$$\Delta r_i = \Delta r_i(\alpha,\,G_\mu,\,M_Z, m_H,\,m_{f\neq t},\,m_t) \tag{6}$$

includes the higher order corrections which can be calculated in the SM or in alternative models. It has been calculated for the first time by A. Sirlin in 1980 [3]. In the SM the Higgs mass m_H is the only relevant unknown parameter and by confronting the calculated with the experimentally determined value of $\sin^2\Theta_i$ one obtains the important indirects constraints on the Higgs mass. Δr_i depends on the definition of $\sin^2\Theta_i$. The various definitions coincide at tree level and hence only differ by quantum effects. From the weak gauge boson masses, the electroweak gauge couplings and the neutral current couplings of the charged fermions we obtain

$$\sin^2\Theta_W = 1-\frac{M_W^2}{M_Z^2} \tag{7}$$

$$\sin^2\Theta_g = e^2/g^2 = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_W^2} \tag{8}$$

$$\sin^2\Theta_f = \frac{1}{4|Q_f|}\left(1-\frac{v_f}{a_f}\right)\;,\quad f\neq\nu\;, \tag{9}$$

for the most important cases and the general form of Δr_i reads

$$\Delta r_i \quad = \quad \Delta\alpha - f_i(\sin^2\Theta_i)\,\Delta\rho + \Delta r_{i\,\mathrm{remainder}} \tag{10}$$

with a universal term $\Delta\alpha$ which affects the predictions for M_W, A_{LR}, A^f_{FB}, Γ_f, etc. The oder terms can be calculated safely in perturbation theory. $\Delta\rho$ is the famous correction to the ρ–parameter, first calculated by M. Veltman in 1977 [4][1], exhibiting the leading top mass correction

$$\Delta\rho \simeq \frac{\sqrt{2}G_\mu}{16\pi^2}\,3m_t^2 \quad ; \quad m_t \gg m_b \tag{11}$$

which allowed LEP experiments to obtain a rather good indirect estimate of the top quark mass prior to the discovery at the TEVATRON [5]. Note that in (10) $f_W = c_W^2/s_W^2 \simeq 3.35$ is substantially enhanced relative to $f_f = 1$. The "remainder" term although sub-leading is very important for the interpretation of the precision experiments at LEP and includes part of the leading Higgs mass dependence. For a heavy Higgs particle we obtain the simple expression

$$\Delta r_i^{\mathrm{Higgs}} \simeq \frac{\sqrt{2}G_\mu M_W^2}{16\pi^2}\left\{c_i^H\,(\ln\frac{m_H^2}{M_W^2} - \frac{5}{6})\right\} \quad ; \quad m_M \gg M_W \tag{12}$$

where $c_f^H = (1+9\sin^2\Theta_f)/(3\sin^2\Theta_f)$ and $c_W^H = 11/3$, for example.

The uncertainty $\delta\Delta\alpha$ implies uncertainties δM_W, $\delta\sin^2\Theta_f$ given by

$$\frac{\delta M_W}{M_W} \quad \sim \quad \frac{1}{2}\frac{\sin^2\Theta_W}{\cos^2\Theta_W - \sin^2\Theta_W}\,\delta\Delta\alpha \sim 0.23\,\delta\Delta\alpha \tag{13}$$

$$\frac{\delta\sin^2\Theta_f}{\sin^2\Theta_f} \quad \sim \quad \frac{\cos^2\Theta_f}{\cos^2\Theta_f - \sin^2\Theta_f}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha \tag{14}$$

which obscure in particular the indirect bounds on the Higgs mass obtained from electroweak precision measurements. A summary of the present status and future expectations will be presented below. Once the Higgs boson will have been discovered and its mass is known, precision measurements of the Δr_i, which would be possible

[1]The article for the first time established non–decoupling heavy particle effects in spontaneously broken gauge theories and presented the calculation of heavy fermion contributions. While $\Delta\rho$ measures the the weak iso-spin breaking proportional to the sum of $|m^2_{t'} - m^2_{b'}|$, where t' and b' denote the top and bottom components of weak iso–doublets and thus vanishes for mass degenerate doublets,the latter contribute to another parameter Δ_1 which is then given by $\Delta_1 = \frac{\alpha}{24\pi s_W^2}\,N_d$ and directly measures the number N_d of degenerate heavy doublets. $\Delta\rho$ and Δ_1 nowadays go under the labels ε_1 or αT and ε_3 or $\frac{\alpha S}{4 s_W^2}$, respectively ($s_W^2 \equiv \sin^2\Theta_W$, $c_W^2 = 1 - s_W^2$).

with the Giga-Z option of TESLA [1], would provide excellent possibilities to establish new physics contributions beyond the SM. Similar tests would be possible by confronting the effective parameters

$$\hat{G}_\mu = \frac{12\pi\Gamma_{W\ell\nu}}{\sqrt{2}M_W^3} \quad \text{and} \quad \hat{\rho} = \frac{M_W^3}{M_Z^3}\frac{2\Gamma_{Z\nu\nu}}{\Gamma_{W\ell\nu}} \tag{15}$$

which are the high energy versions of G_μ and $\rho \equiv G_{\rm NC}(0)/G_\mu$ which are not plagued by uncertainties from $\Delta\alpha$. Here, $G_{\rm NC}(0)$ denotes the low energy effective neutral current coupling.

2 The hadronic contributions to $\alpha(s)$

The effective QED coupling constant at scale $\sqrt{s}$ is given by the renormalization group resummed running fine structure constant

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)} \tag{16}$$

with

$$\Delta\alpha(s) = -4\pi\alpha\mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right] \ . \tag{17}$$

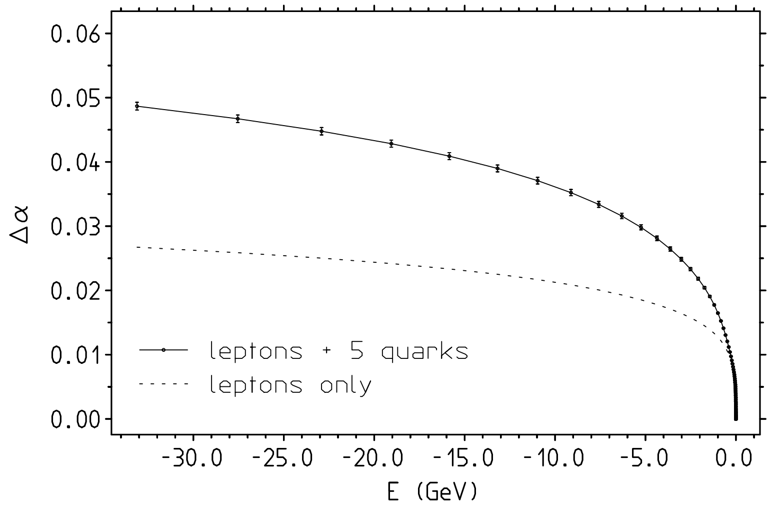


Figure 1: The running of α. The "negative" E axis is chosen to indicate space-like momentum transfer. The vertical bars at selected points indicate the uncertainty.

Figure 1 shows the running of α at low space-like momentum transfer. The leptonic contributions are calculable in perturbation theory where at leading order the free lepton loops yield

$$\begin{aligned}
&\Delta\alpha_{\rm leptons}(s) = \\
&= \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[-\frac{8}{3}+\beta_\ell^2-\frac{1}{2}\beta_\ell(3-\beta_\ell^2)\ln\left(\frac{1-\beta_\ell}{1+\beta_\ell}\right)\right] \\
&= \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[\ln\left(s/m_\ell^2\right)-\frac{5}{3}+O\left(m_\ell^2/s\right)\right] \;\text{ for } |s| \gg m_\ell^2 \\
&\simeq 0.03142 \text{ for } s = M_Z^2
\end{aligned} \tag{18}$$

where $\beta_\ell = \sqrt{1-4m_\ell^2/s}$. This leading contribution is affected by small electromagnetic corrections only in the next to leading order. The leptonic contribution is actually known to three loops [6,7] at which it takes the value

$$\Delta\alpha_{\rm leptons}(M_Z^2) \;\simeq\; 314.98 \;\times\; 10^{-4}. \tag{19}$$

In contrast the corresponding free quark loop contribution gets substantially modified be low energy strong interaction effects, which cannot be obtained by p-QCD. Fortunately one can evaluate this hadronic term $\Delta\alpha^{(5)}_{\rm hadrons}$ from hadronic e^+e^-- annihilation data by using a dispersion relation. The relevant vacuum polarisation amplitude satisfies the convergent dispersion relation

$$Re\Pi'_\gamma(s)-\Pi'_\gamma(0)=\frac{s}{\pi}Re\int_{s_0}^\infty ds'\frac{Im\Pi'_\gamma(s')}{s'(s'-s-i\varepsilon)}$$

and using the optical theorem (unitarity) one has

$$Im\Pi'_\gamma(s)=\frac{s}{e^2}\sigma_{tot}(e^+e^-\to\gamma^*\to\text{hadrons})(s)\;.$$

In terms of the cross-section ratio

$$R(s)=\frac{\sigma_{tot}(e^+e^-\to\gamma^*\to\text{hadrons})}{\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)}\;,$$

where $\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)=\frac{4\pi\alpha^2}{3s}$ at tree level, we finally obtain

$$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)\;\;=\;\;-\frac{\alpha M_Z^2}{3\pi}Re\int_{4m_\pi^2}^\infty ds\frac{R(s)}{s(s-M_Z^2-i\varepsilon)}\;.$$

Using the experimental data for $R(s)$ up to $\sqrt{s}=E_{cut}=5.5$ GeV and for the Υ resonances region between 9.6 and 11 GeV and perturbative QCD from 5.5 to 9.6 GeV and for the high energy tail [8–10] above 11 GeV we get as an update of [11] including the recent new data from CMD [12] and BES [13]

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) &= 0.027896\pm0.000395 \\
\alpha^{-1}(M_Z^2) &= 128.907\pm0.054\;\;.
\end{aligned}$$

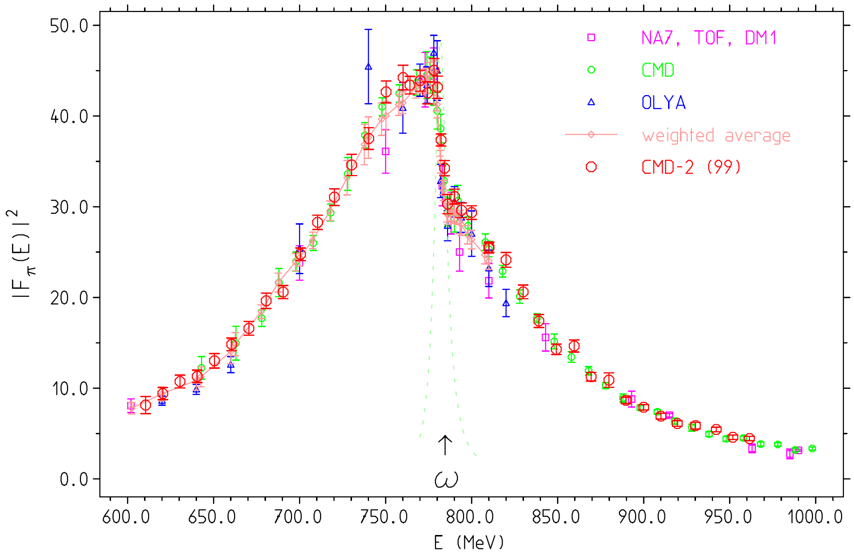


Figure 2: Recent CMD-2 results [12].

at $M_Z = 91.19$ GeV. The CMD-2 experiment at Novosibirsk has continued and substantially improved the $\sigma(e^+e^- \to \mathrm{hadrons})$ measurements below 1.4 GeV [12] and the BES-II experiment at Beijing has published a new measurement, which in the region from 2 to 5 GeV improves the evaluation from 15% to 20% systematic error to about 6.6% [13]. As a consequence we observe a dramatic reduction of the error with respect to our 1995 evaluation 0.0280 ± 0.0007 [11] mainly due to the new BES data. The latter result has been independently confirmed earlier in [14,15]. For an evaluation which yields a quite different answer see [16] and Tab. 2 below.

From the BES-II data we have subtracted the narrow resonance $\Upsilon(3)$ (6 points) because this resonance contribution is calculated as a Breit–Wigner resonance using the parameters of the particle data table [17]. The BES-II data in the J/psi resonance range from 3.6 GeV to 5 GeV is integrated and combined with results for integral obtained from other experiments. Below the resonance we calculate the weighted average with the older data points and integrate the weighted average. This procedure has been motivated and tested in [11].

For details about our evaluation procedure of we refer to [11]. In our *standard approach* we take data serious as published and combine them according to rules suggested by the particle data group [17]. In Figs. 2 and 3 we show the new data in comparison to the older ones.

Below we will present another result obtained with the *Euclidean approach*, which is based on comparing experimental data and theory (i.e. p–QCD) by means of the

Adler function.

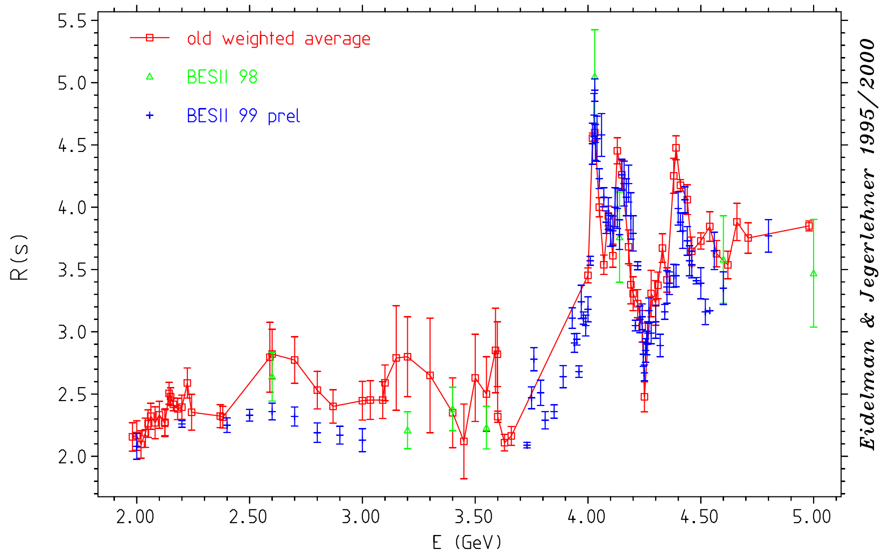


Figure 3: Recent BES-II results [13].

The contributions from different energy ranges are shown in Tab. 1

Note that the ρ contribution slightly increased due to the new CMD data. The new BES data imply a small increase of the contribution from the range (3.6, 5.0) GeV, while, as is obvious from Fig. 3, the contribution in the range (2.0,3.6) is lower as compared to previous results. In total the shift in the central value is very moderate, while the uncertainly has become smaller.

3 Theoretical progress

In view of the increasing precision LEP experiments have achieved during the last few years, more accurate theoretical prediction became desirable. As elaborated in the introduction, one of the limiting factors is the hadronic uncertainty of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$. Because of the large uncertainties in the data, many authors advocated to extend the use of perturbative QCD in place of data [19–23]. The assumption that p-QCD may be reliable to calculate

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s) \tag{20}$$

down to energies as low as 1.8 GeV seems to be supported by

final state	energy range (GeV)	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ (stat) (syst)	$\Delta\alpha_{\rm had}^{(5)}(-s_0)$ (stat) (syst)
χPT	(0.28, 0.32)	0.03 (0.00) (0.00)	0.03 (0.00) (0.00)
ρ	(0.28, 0.81)	25.67 (0.21) (0.42)	23.81 (0.20) (0.39)
ω	(0.42, 0.81)	2.96 (0.04) (0.08)	2.70 (0.03) (0.07)
ϕ	(1.00, 1.04)	5.14 (0.07) (0.12)	4.41 (0.06) (0.10)
J/ψ		11.90 (0.58) (0.64)	4.21 (0.20) (0.20)
Υ		1.24 (0.05) (0.07)	0.07 (0.00) (0.00)
hadrons	(0.81, 1.40)	13.92 (0.11) (0.71)	11.91 (0.09) (0.59)
hadrons	(1.40, 3.10)	26.75 (0.10) (1.83)	15.56 (0.06) (1.13)
hadrons	(3.10, 3.60)	5.26 (0.05) (0.17)	1.89 (0.02) (0.06)
hadrons	(3.60, 9.46)	50.73 (0.24) (2.97)	8.34 (0.04) (0.44)
hadrons	(9.46,12.00)	13.47 (0.16) (1.19)	0.72 (0.01) (0.06)
perturb	(12.0,∞)	121.67 (0.00) (0.12)	1.27 (0.00) (0.00)
data	(0.28,12.00)	157.05 (0.70) (3.84)	73.61 (0.31) (1.43)
total		278.72 (0.70) (3.84)	74.89 (0.31) (1.43)

Table 1: Distribution of uncertainties for $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)^{\rm data}$ and for $\Delta\alpha_{\rm had}^{(5)}(-s_0)^{\rm data}$ in comparison to $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)^{\rm data}$ ($\sqrt{s_0}=2.5$ GeV).

- the apparent applicability of p-QCD to τ physics. In fact the running of $\alpha_s(M_\tau)\to\alpha_s(M_Z)$ from the τ mass up to LEP energies agrees well with the LEP value. The estimated uncertainty may be debated, however.
- the smallness [19] (see also: [24]) of non–perturbative (NP) effects if parametrized as prescribed by the operator product expansion (OPE) of the electromagnetic current correlator [26]

$$\begin{aligned}
\Pi_\gamma^{'\rm NP}(Q^2) &= \frac{4\pi\alpha}{3}\sum_{q=u,d,s} Q_q^2 N_{cq}\cdot\left[\frac{1}{12}\left(1-\frac{11}{18}a\right)\frac{<\frac{\alpha_s}{\pi}GG>}{Q^4}\right. \\
&+ 2\left(1+\frac{a}{3}+\left(\frac{11}{2}-\frac{3}{4}l_{q\mu}\right)a^2\right)\frac{<m_q\bar{q}q>}{Q^4} \\
&+ \left.\left(\frac{4}{27}a+\left(\frac{4}{3}\zeta_3-\frac{257}{486}-\frac{1}{3}l_{q\mu}\right)a^2\right)\sum_{q'=u,d,s}\frac{<m_{q'}\bar{q}'q'>}{Q^4}\right] \\
&+ \cdots
\end{aligned} \tag{21}$$

where $a \equiv \alpha_s(\mu^2)/\pi$ and $l_{q\mu} \equiv \ln(Q^2/\mu^2)$. $< \frac{\alpha_s}{\pi} GG >$ and $< m_q \bar{q} q >$ are the scale-invariantly defined condensates.

Progress in p-QCD comes mainly from [27]. In addition an exact two–loop calculation of the renormalization group (RG) in the background field MOM scheme (BF-MOM) is available [28]. This allows us to treat "threshold effects" closer to physics than in the $\overline{\mathrm{MS}}$ scheme. The BF-MOM scheme respects the QCD Slavonv-Taylor identities (non-Abelian gauge symmetry) but in spite of that is gauge parameter (ξ) dependent[2]. Except from Ref. [20] which is based on [27] most other "improved" calculations utilize older results, mainly, the well known massless result [8] plus some leading mass corrections. For a recent critical review of the newer estimates of vacuum polarization effects see [29] and Tab.2 below.

In Ref. [30] a different approach of p-QCD improvement was proposed, which relies on the fact that the vacuum polarization amplitude $\Pi(q^2)$ is an analytic function in q^2 with a cut in the s–channel $q^2 = s \geq 0$ at $s \geq 4m_\pi^2$ and a smooth behavior in the t–channel (space-like or Euclidean region). Thus, instead of trying to calculate the complicated function $R(s)$, which obviously exhibits non-perturbative features like resonances, one considers the simpler Adler function in the Euclidean region. In [30] the Adler function was investigated and p-QCD was found to work very well above 2.5 GeV, provided the exact three–loop mass dependence was used (in conjunction with the background field MOM scheme). The Adler function may be defined as a derivative

$$D(-s) = -(12\pi^2)\, s\, \frac{d\Pi'_\gamma(s)}{ds} = \frac{3\pi}{\alpha} s \frac{d}{ds} \Delta\alpha_{\mathrm{had}}(s) \tag{22}$$

of (20) which is the hadronic contribution to the shift of the fine structure constant. It is represented by

$$D(Q^2) = Q^2 \left(\int_{4m_\pi^2}^{E^2_{\mathrm{cut}}} \frac{R^{\mathrm{data}}(s)}{(s+Q^2)^2} ds \; + \; \int_{E^2_{\mathrm{cut}}}^{\infty} \frac{R^{\mathrm{pQCD}}(s)}{(s+Q^2)^2} ds \right) \tag{23}$$

in terms of the experimental e^+e^-–data. The standard evaluation ([11]) of (23) then yields the non–perturbative "experimental" Adler function, as displayed in Figs. 4 and 5.

For the p-QCD evaluation it is mandatory to utilize the calculations with massive quarks which are available up to three–loops [27]. The four-loop corrections are known in the approximation of massless quarks [8]. The outcome of this analysis is pretty surprising and is shown in Figs. 4 and 5. For a discussion we refer to the original

[2]In applications considered below all numerical calculations have been performed in the "Landau gauge" $\xi = 0$.

paper [30].

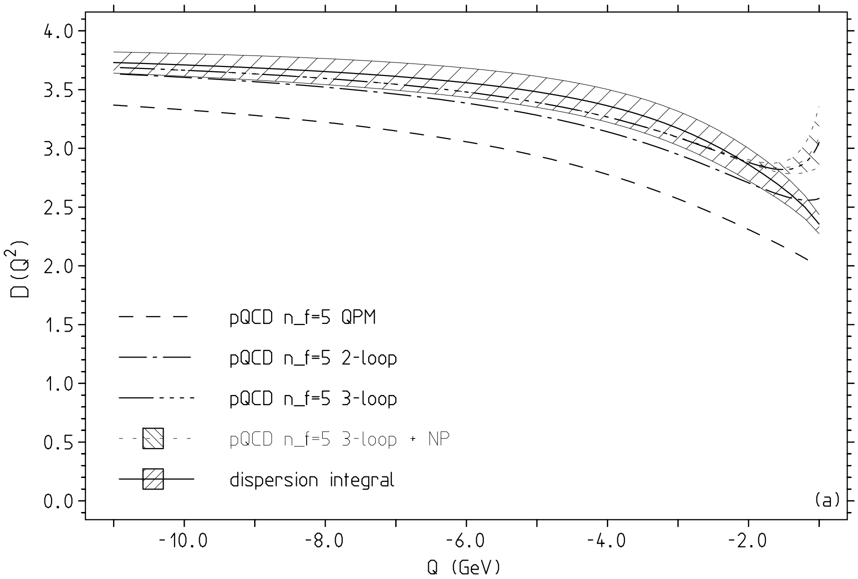


Figure 4: Adler function: theory vs. experiment (a) [30].

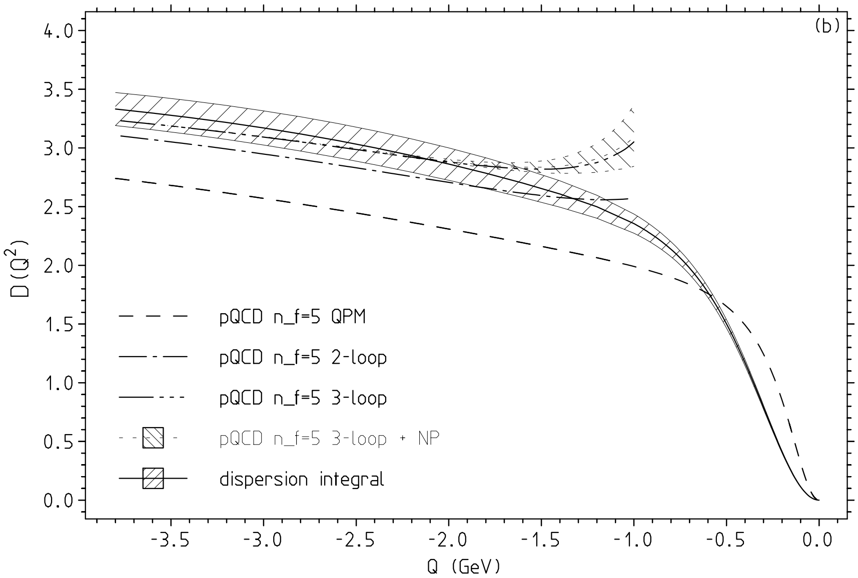


Figure 5: Adler function: theory vs. experiment (b) [30].

According to (22), we may compute the hadronic vacuum polarization contribution to the shift in the fine structure constant by integrating the Adler function. In the region where p-QCD works fine we integrate the p-QCD prediction, in place of the data. We thus calculate in the Euclidean region

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm p-QCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} \;. \qquad (24)$$

A save choice is $s_0 = (2.5\,\mathrm{GeV})^2$ where we obtain

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007489 \pm 0.000146 \qquad (25)$$

from the evaluation of the dispersion integral (20). With the results presented above we find

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027685 \pm 0.000146^{*} \qquad (26)$$

for the Euclidean (t–channel) effective fine structure constant.

Remaining problems are the following:
a) contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included. The scheme dependence $\overline{\mathrm{MS}}$ versus background field MOM has been discussed in Ref. [28].
b) The effective fine structure constant in the time–like region (s–channel), as required for e^+e^-–collider physics may be obtained from the Euclidean one by adding the difference

$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000045 \pm 0.000002\,, \qquad (27)$$

which may be calculated perturbatively or directly from the "non–perturbative"[3] dispersion integral. It accounts for the $i\pi$–terms

$$\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$$

from the logs.
c) One may ask the question whether these terms should be resummed at all, i.e., included in the running coupling. Usually such terms tend to cancel against constant rational terms which are not included in the renormalization group (RG) evolution. It should be stressed that the Dyson summation (propagator bubble summation) in general is not a systematic resummation of leading, sub-leading etc. terms as the RG resummation is.

It is worthwhile to stress here that the running coupling is **not** a true function of q^2 (or even an analytic function of q^2) but a function of the RG scale μ^2. The coupling as it appears in the Lagrangian in any case must be a constant, albeit a μ^2–dependent one,

[3]Since we utilize p-QCD for the high energy tail in the dispersion integral, $\Delta(s)$ for large s is dominated by the tail and thus in fact is perturbative.

if we do not want to end up in conflict with basic principles of quantum field theory. The effective identification of μ^2 with a particular value of q^2 must be understood as a subtraction (reference) point.
The above result was obtained using the background–field MOM renormalization scheme, mentioned before. In the transition from the $\overline{\mathrm{MS}}$ to the MOM scheme we adapt the rescaling procedure described in [28], such that for large μ

$$\overline{\alpha}_s((x_0\mu)^2) = \alpha_s(\mu^2) + 0 + O(\alpha_s^3)\ .$$

This means that x_0 is chosen such that the couplings coincide to leading and next–to–leading order at asymptotically large scales. Numerically we find $x_0 \simeq 2.0144$. Due to this normalization by rescaling the coefficients of the Adler–function remain the same in both schemes up to three–loops. In the MOM scheme we automatically have the correct mass dependence of full QCD, i.e., we have automatic decoupling and do not need decoupling by hand and matching conditions like in the $\overline{\mathrm{MS}}$ scheme. For the numerical evaluation we use the pole quark masses [17] $m_c = 1.55\mathrm{GeV},\ m_b = 4.70\mathrm{GeV},\ m_t = 173.80\mathrm{GeV}$ and the strong interaction coupling $\alpha^{(5)}_{s\,\overline{\mathrm{MS}}}(M_Z) = 0.120 \pm 0.003$. For further details we refer to [30].

Since Δ Eq. (27) is small we may include it in the resummation without further worrying and thus obtain

$$\begin{aligned} \Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) &= 0.027730 \pm 0.000148^* \\ \alpha^{-1}(M_Z^2) &= 128.930 \pm 0.020^*\ . \end{aligned} \tag{28}$$

The asterisk on the error indicates that this error does not yet include the uncertainty of the p-QCD contribution which is still under investigation. The alternative evaluation by the Euclidean approach is compared with the standard evaluation in Tab. 1.

Our alternative procedure to evaluate $\Delta\alpha^{(5)}_{\mathrm{had}}(-M_Z^2)$ in the Euclidean region has several advantages as compared to other approaches used so far: The virtues of our analysis are the following:

- no problems with the physical threshold and resonances
- p-QCD is used only in the Euclidean region and not below 2.5 GeV. For lower scales p-QCD ceases to describe properly the functional dependence of the Adler function [30] (although the p-QCD answer remains within error bands down to about 1.6 GeV).
- no manipulation of data must be applied and we need not refer to global or even local duality. That power corrections of the type Eq. (21) are negligible has been known for a long time. This, however, does not proof the absence of

other kind of non-perturbative effects. Therefore our conservative choice of the minimum Euclidean energy seems to be necessary.

- as we shall see our non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data, which changes the chances of possible future experimental improvement dramatically, as illustrated in Fig. 6.

The two methods (standard vs. Euclidean) of evaluating $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ are also compared in Fig. 6

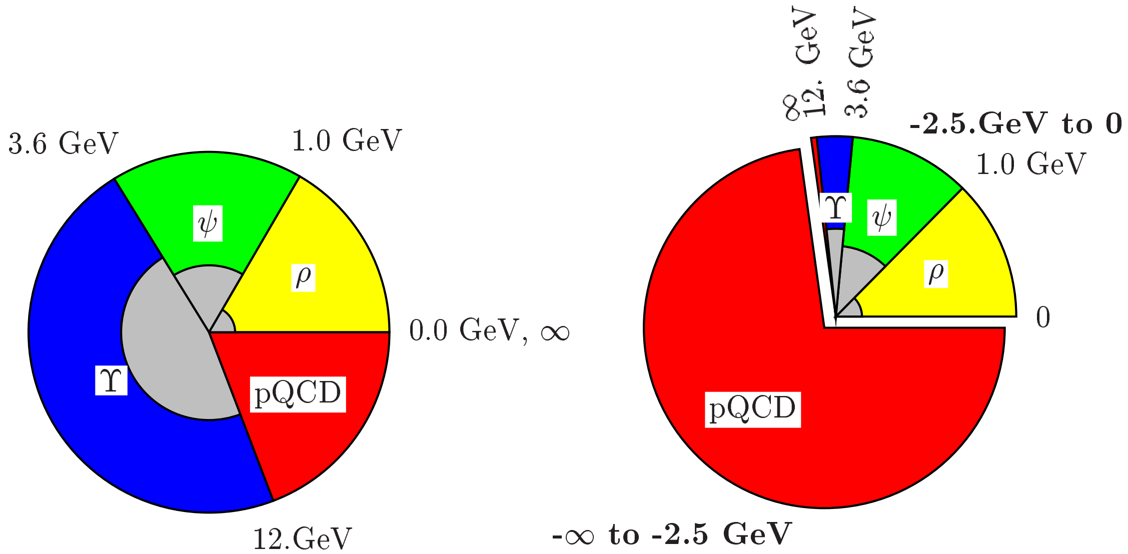


Figure 6: Comparison of the distribution of contributions and errors (shaded areas scaled up by 10) in the standard (left) and the Adler function based approach (right), respectively.

While the uncertainties to $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ in the standard approach are coming essentially from everywhere below M_Υ, which would make a new scan over all energies for a precision measurement of $\sigma_{\rm had} \equiv \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$ unavoidable, the new approach leads to a very different situation. The uncertainty of $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is completely dominated by the uncertainties of data below $M_{J/\psi}$ and thus new data on $\sigma_{\rm had}$ are only needed below about 3.6 GeV which could be covered by a tunable "τ–charm facility".

Table 2 compares our results with results obtained by other authors which obtain smaller errors because they are using p-QCD in a less controlled manner.

$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	$\delta\Delta\alpha$	$\delta\sin^2\Theta_f$	δM_W	Method	Ref.
0.0280	0.00065	0.000232	12.0	data $<$ 12. GeV	[11]
0.02777	0.00017	0.000061	3.2	data $<$ 1.8 GeV	[20]
0.02763	0.00016	0.000057	3.0	data $<$ 1.8 GeV	[22]
0.027730	0.000148	0.000089	4.6	Euclidean $>$ 2.5 GeV	[29]
0.027426	0.000190	0.000070	3.6	scaled data, pQCD 2.8-3.7, 5-∞	[16]
0.027649	0.000214	0.000078	4.0	same but "exclusive"	[16]
0.027896	0.000391	0.000139	7.2	[11] + new data CMD & BES	[25]
-	0.00007	0.000025	1.3	$\delta\sigma \lesssim 1\%$ up to J/ψ	
-	0.00005	0.000018	0.9	$\delta\sigma \lesssim 1\%$ up to Υ	
world average		0.000160	22.0	Osaka 2000	

Table 2: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ and its uncertainties in different evaluations. Two entries show what can be reached by increasing the precision of cross section measurements to 1%. δM_W in MeV.

4 The running electric charge at high energies

Beyond the Z peak not only the fermions contribute to the vacuum polarisation but also the bosonic degrees of freedom in particular the charged W–boson. However, if we try to "define" the running charge in terms of the photon propagator simply, we get into troubles. The analogy of a fermion loop, which is gauge invariant at the one–loop level at least, the W–boson loop is not gauge invariant. In fact one cannot measure self-energy contributions in isolation. What experimentalists can measure are cross sections, in the simplest case for a "$2 \to 2$ fermions" process with contributions from self–energy-, vertex- and box-diagrams. A physically more acceptable definition of a running charge seems to be via the electromagnetic form–factor of the electron, for example, but also this is true only in an energy range where the one–photon exchange approximation is accurate, such that we face a factorisation of the cross section like in Thomson scattering at low energies. At high energies (far off–shell) a form–factor is not any longer accessible directly by experiment. We then may adapt a formal definition, like the $\overline{\rm MS}$ scheme, which is unphysical because heavy degree of freedom do not decouple automatically in spite of the fact that heavy states cannot affect the physics at much lower energies. In the $\overline{\rm MS}$ scheme one has to perform decoupling "by hand" therefore, i.e., one only counts degrees of freedom which are lighter than a given scale. That this may cause problems is not very surprising since one tends to switch off individual members of gauge group multiplets.

Our analysis above, which includes non–perturbative effects from low energy hadrons, is more in the spirit of on–shell renormalization, which is more physical with respect to its decoupling behaviour. The latter comes out for free in an on–shell scheme, because on–shell renormalization exhibits the correct physical threshold structure. But, as mentioned above, probing an on–shell electron by an off–shell photon of virtuality q^2 is not physical, and in fact not gauge invariant in the non–Abelian SM [31]. Still, a reasonable *convention* is possible by requiring the photon to satisfy Maxwell's equations, which is not automatic. The reason is that in the SM, adopting the standard lowest order definition, the photon field

$$A_\mu = (gB_\mu + g' W_{\mu 3})/\sqrt{g'^2+g^2} \tag{29}$$

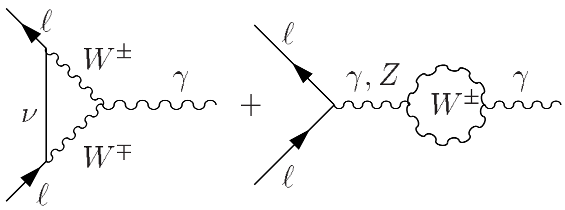
has a non-Abelian component. This fact at higher orders causes problems which do not appear in pure QED. A manifestly Abelian photon may be defined by [31]

$$\begin{aligned} A^a_\mu &= (gB_\mu + g' W^s_{\mu 3})/\sqrt{g'^2+g^2} \\ &= (\sqrt{g'^2+g^2}/g)\, B_\mu + (g'/g)\, Z^s_\mu \end{aligned} \tag{30}$$

where Z^s_μ defined by

$$\begin{aligned} Z^s_\mu &= (gW^s_{\mu 3} - g' B_\mu)/\sqrt{g'^2+g^2} \\ &= -(i/\sqrt{g'^2+g^2})\, (\Phi^+ D_\mu \Phi - \text{h.c.})/(\Phi^+\Phi) \end{aligned} \tag{31}$$

obviously is a singlet, with respect to the SM gauge group, and $W^s_{\mu 3}$ is Abelian. Φ is the Higgs doublet field and $D_\mu \Phi$ its covariant derivative. In the *unitary gauge* A_μ and A^a_μ coincide, which means that in the unitary gauge we automatically are dealing with the Abelian photon field, which satisfies the correct Maxwell equations. The gauge dependent part is originating at the one–loop level solely from the W–pair excitation, described by the diagrams



Because we are mainly interested in the high energy behaviour and in order to avoid lengthy expressions, we present the results for $|q^2| \gg M_W^2$ only. The one–loop contributions to the singlet form–factor may be written as

$$\Delta\alpha = \Delta\alpha_W + \Delta\alpha_Z + \Delta\alpha_\gamma + \Delta\alpha_f \tag{32}$$

with contributions from W, Z, photon and the fermions. For the "renormalized" virtual $W^\pm$ contribution one finds

$$\Delta\alpha_W(q^2) = \frac{\alpha}{16\pi s_\theta^2 c_\theta^2}\left\{a_0 + b_0 \ln\frac{|q^2|}{M_W^2} + a_1 \frac{q^2}{M_W^2}\left(\ln\frac{|q^2|}{\mu^2} - \frac{8}{3}\right) - i\pi\theta(q^2 - 4M_W^2)\left(b_0 + a_1\frac{q^2}{M_W^2}\right)\right\} \quad (33)$$

in the $\overline{\mathrm{MS}}$ subtraction scheme. Introducing the notation $c_\theta^2 \doteq M_W^2/M_Z^2$, $s_\theta^2 \doteq 1 - c_\theta^2$ and $g(c_\theta^2) \doteq \sqrt{4c_\theta^2 - 1}\;$ arc ctg$\sqrt{4c_\theta^2 - 1}$, the coefficients are given by

$$\begin{aligned}
a_0 &= -32c_\theta^6 - \frac{56}{3}c_\theta^4 + \frac{253}{6}c_\theta^2 + \frac{1}{2} \\
&\quad -(-32c_\theta^6 - \frac{64}{3}c_\theta^4 + \frac{134}{3}c_\theta^2 - \frac{22}{3} - \frac{1}{2}c_\theta^{-2})g(c_\theta^2) \\
b_0 &= \frac{5}{3}c_\theta^2 - \frac{19}{6} - \frac{1}{4}c_\theta^{-2} \\
a_1 &= \frac{1}{6}c_\theta^2 - \frac{1}{4}\ .
\end{aligned}$$

The virtual Z contribution reads

$$\Delta\alpha_Z(q^2) = \frac{\alpha}{16\pi s_\theta^2 c_\theta^2}\left\{a_0' + b_0' \ln\frac{|q^2|}{M_Z^2} + \frac{2}{3}b_0'\left(\mathrm{Sp}\left(1 + \frac{q^2}{M_Z^2}\right) - \frac{\pi^2}{6}\right) - i\pi\theta(q^2 - 4m_e^2)\, b_0'\right\} \quad (34)$$

which in contrast to the $W^\pm$ contribution is finite (i.e., μ–independent). The coefficients are given by

$$\begin{aligned}
a_0' &= -14c_\theta^4 + 21c_\theta^2 - \frac{35}{4} \\
b_0' &= 12c_\theta^4 - 18c_\theta^2 + \frac{15}{2}\ ,
\end{aligned}$$

and the Spence function Sp is asymptotically given by

$$\mathrm{Sp}\left(1 + \frac{q^2}{M_Z^2}\right) - \frac{\pi^2}{6} \simeq \begin{cases} \frac{\pi^2}{6} - \frac{1}{2}\ln^2\frac{q^2}{M_Z^2} + i\pi\ln\frac{q^2}{M_Z^2}\ ; & q^2 \gg M_Z^2 \\ -\frac{\pi^2}{3} - \frac{1}{2}\ln^2\left(-\frac{q^2}{M_Z^2}\right)\ ; & -q^2 \gg M_Z^2\ . \end{cases} \quad (35)$$

The QED electron vertex + self–energy contributions exhibit the well known infrared problem with soft and collinear logs which only become physical after combining them with the soft real photon radiation. Virtual + soft QED corrections together are related to definition of the initial and/or final state and are therefore taken into account in a different way. They have nothing to do with the running of the charge or vacuum polarization effects. We therefore apply the convention to set $\Delta_\gamma = 0$ in the calculation of $\Delta\alpha$.

Another problem is due to $\gamma - Z$ mixing. At higher energies the mixing effects have to be taken into account. We have seen that this is crucial for the W–pair creation and reabsorption but in fact also applies to the fermion contributions, once $\gamma \to Z \to \gamma$ transitions become relevant at sufficiently high energies, we must include

$$\Delta\alpha_f(q^2) = -2a \frac{M_Z^2}{q^2 - M_Z^2} \, (A_{1r}^{\gamma Z})_f - (ev)^2 \frac{1}{q^2} \, (A_{1r}^{\gamma\gamma})_f$$

where

$$-(ev)^2 \frac{1}{q^2} \, (A_{1r}^{\gamma\gamma})_f = \frac{\alpha}{3\pi} \sum_f Q_f^2 H_f \left(4m_f^2/q^2\right)$$

is the renormalized QED vacuum polarization and

$$(A_{1r}^{\gamma Z})_f = -\frac{\alpha}{24\pi s_\theta^2} \frac{q^2}{M_W^2} \sum_f (4a_f Q_f) \left[H_f\left(4m_f^2/q^2\right) - H_f\left(4m_f^2/M_Z^2\right)\right]$$

is the $\gamma - Z$ mixing contribution. The function H_f is given by

$$H_f(y_f) = \frac{5}{3} + y_f + \left(1 + \frac{y_f}{2}\right) \sqrt{1 - y_f} \ln \frac{\sqrt{1 - y_f} - 1}{\sqrt{1 - y_f} + 1}$$

and $a_f = -Q_f s_\theta^2 \pm \frac{1}{4}$ for the upper and lower components of the weak iso-doublets, respectively. The one–loop perturbative fermion formula also is appropriate to take into account the **top quark** contribution. At M_Z we have included in $\alpha^{-1}(M_Z)$

$$\Delta\alpha_{\mathrm{top}}(M_Z^2) = -0.76 \times 10^{-4} \quad .$$

Numerical results for the SM contributions in the singlet form–factor definition of the effective charge will be presented elsewhere.

5 Concluding Remarks

Experimental efforts to measure very precisely the total cross section $\sigma(e^+e^- \to hadrons)$ at low energies are mandatory for the future of electroweak precision physics.

Taking into account recent theoretical progress, these "low energy" measurements are not only important for testing the muon anomalous magnetic moment a_μ but as well for the effective fine structure constant $\alpha(M_Z)$. A real breakthrough would be possible by measuring $\sigma(e^+e^- \to hadrons)$ at 1% accuracy below the τ–threshold. We once more refer to Tab. 2 for the status and future perspectives.

Fortunately there is work in progress which can help to further reduce the uncertainties of theoretical predictions: (i) VEPP-2M Novosibirsk (CMD-2, SND): can further improve to 1.5% up to 1.4 GeV. An upgrade of the machine and the detectors is under consideration. (ii) DAΦNE Frascati (KLOE): within one year of running we expect a measurement below the ϕ resonance which is expected to be competitive to the Novosibirsk data. Since the KLOE experiment is very different in the technology from the Novosibirsk experiments this will provide a very important cross check of older results. (iii) BEPC Beijing (BES): can still improve in the important J/Ψ region and down to 2 GeV. (iv) In future a possible "τ–charm facility" tunable between 2 GeV and 3.6 GeV would settle the remaining problems essentially.

If we adopt the Euclidean approach of calculating $dahz$ via $\Delta\alpha^{(5)}_{\rm had}(-s_0)$, in future the Adler function is an ideal object for direct simulation in lattice QCD. Of course, there is a long way to go, in order to achieve an accuracy which is competitive with present evaluations from the e^+e^-–data (see Figs. 4 and 5). However, a purely theoretical prediction of $\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)$ seems to be feasible in future. Continuous progress in theory and experiment let us expect that the necessary improvements required for the future of precision physics will be realized. This is particularly important for the electroweak precision physics which would be possible with the Giga-Z–option at TESLA.

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