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FeynHiggs and ***FeynHiggsFast*** : Programs for Higher-Order Calculations in the Neutral $\mathcal{CP}$-even Higgs Boson Sector of the MSSM

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Abstract

FeynHiggs and *FeynHiggsFast* are Fortran codes for the calculation of the masses and the mixing angle of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM up to two-loop order. *FeynHiggs* is based on the complete diagrammatic on-shell results at the one-loop level and the dominant diagrammatic corrections at the two-loop level. The Higgs boson masses and the mixing angle are calculated as functions of the MSSM parameters for general mixing in the scalar top sector and arbitrary choices of the parameters in the Higgs sector. *FeynHiggsFast* is based on a compact analytical approximation formula of the complete diagrammatic one-loop result, at the two-loop level the leading logarithmic and non-logarithmic contributions are taken into account. The approximation formula is valid for arbitrary choices of the parameters in the Higgs sector of the model. Comparing its quality to the full diagrammatic result, we find agreement better than 2 GeV for most parts of the MSSM parameter space.

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1 Introduction

One of the most striking phenomenological implications of Supersymmetry (SUSY) is the prediction of a relatively light Higgs boson, which is common to all supersymmetric models whose couplings remain in the perturbative regime up to a very high energy scale [1]. The search for the lightest Higgs boson thus allows a crucial test of SUSY and especially the Minimal Supersymmetric Standard Model (MSSM) [2]. Therefore this search is one of the main goals at the present and the next generation of colliders. A precise knowledge of the dependence of the masses and mixing angles of the Higgs sector of the MSSM on the relevant SUSY parameters is necessary for a detailed analysis of SUSY phenomenology at LEP2, the upgraded Tevatron, and also for the LHC and a future linear e^+e^- collider, where high-precision measurements in the Higgs sector might become possible.

The mass of the lightest $\mathcal{CP}$-even MSSM Higgs boson, m_h, is bounded from above by about $m_h \lesssim 135$ GeV, including radiative corrections up to the two-loop level [3–12]. The mixing angle $\alpha_{\rm eff}$ that diagonalizes the neutral $\mathcal{CP}$-even Higgs sector receives the same kind of corrections. By incorporating $\alpha_{\rm eff}$ into the calculations for Higgs productions or for the Higgs decay widths, the leading electroweak corrections to the production and decay of the neutral $\mathcal{CP}$-even Higgs bosons are taken into account, see Refs. [13, 14] and references therein.

In this note we present the Fortran codes *FeynHiggs* [15] and *FeynHiggsFast* [16]. Using low energy MSSM parameters as input, they evaluate the masses of the neutral $\mathcal{CP}$-even Higgs bosons, m_h and m_H, as well as the corresponding mixing angle, $\alpha_{\rm eff}$, at the two-loop level. In addition the mass of the charged Higgs boson, $m_{H^\pm}$, is evaluated taking into account the dominant one-loop corrections. The ρ-parameter, leading to constraints in the scalar fermion sector of the MSSM, is evaluated up to $\mathcal{O}(\alpha\alpha_s)$, taking into account the gluon and gluino exchange contribution at the two-loop level [17]. In this way experimentally disfavored combinations of squark masses can automatically be excluded. *FeynHiggs* is based on the results of the Feynman-diagrammatic (FD) calculations as given in Refs. [4, 6, 7]. It includes the complete diagrammatic on-shell results at the one-loop level, the dominant diagrammatic two-loop QCD contributions and further subdominant corrections of $\mathcal{O}(G_F^2 m_t^6)$ obtained with renormalization group (RG) methods [9, 10]. The calculation of m_h and m_H is performed for arbitrary values of parameters in the $t - \tilde{t}$ sector and the Higgs sector of the MSSM. *FeynHiggsFast* is based on a compact analytical approximation formula, containing at the two-loop level the dominant logarithmic and non-logarithmic corrections of the full formulas implemented into *FeynHiggs*. In comparison with the full result, the approximation formula is much shorter. Thus *FeynHiggsFast* is about 3×10^4 times faster than *FeynHiggs*. Agreement between these two codes of better than 2 GeV is found for most parts of the MSSM parameter space.

As a subroutine, *FeynHiggs* and *FeynHiggsFast* can be linked to other programs thus incorporating in an easy way a precise prediction for m_h, m_H, $m_{H^\pm}$, the mixing angle $\alpha_{\rm eff}$ and $\Delta\rho_{\rm SUSY}$.

The paper is organized as follows: in section 2 we specify our notations and give a brief outline of the higher-order calculation in the MSSM Higgs sector. A comparison of the approximation formula to the full result is performed. In section 3 the program *FeynHiggs* and in section 4 the program *FeynHiggsFast* are described in detail. Examples of how to use the programs are shown in section 5. The conclusions are given in section 6.

2 The calculational basis

2.1 Calculation of the Higgs boson masses

In the Feynman-diagrammatic approach, the Higgs-boson masses of the neutral $\mathcal{CP}$-even fields h and H in higher orders are derived by finding the poles of the h-H propagator matrix, which, in terms of its inverse, is given by

$$\left(\Delta_{\rm Higgs}\right)^{-1} = -i \left(\begin{array}{cc} q^2 - m^2_{H,{\rm tree}} + \hat{\Sigma}_H(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m^2_{h,{\rm tree}} + \hat{\Sigma}_h(q^2) \end{array} \right). \tag{1}$$

The pole positions are the solutions of the equation

$$\left[q^2 - m^2_{h,{\rm tree}} + \hat{\Sigma}_h(q^2)\right]\left[q^2 - m^2_{H,{\rm tree}} + \hat{\Sigma}_H(q^2)\right] - \left[\hat{\Sigma}_{hH}(q^2)\right]^2 = 0\ . \tag{2}$$

The quantities $\hat{\Sigma}_{h,H,hH}$ denote the renormalized, diagonal and non-diagonal, Higgs-boson self-energies.

The tree-level masses in eq. (1), $m^2_{h,{\rm tree}}, m^2_{H,{\rm tree}}$, are obtained by diagonalizing the lowest-order mass matrix of the neutral $\mathcal{CP}$-even Higgs bosons in the ϕ_1-ϕ_2 basis (expressed in terms of the Z-boson mass, M_Z, the $\mathcal{CP}$-odd Higgs-boson mass, M_A, and $\tan\beta = v_2/v_1$)

$$\begin{aligned} M^{2,{\rm tree}}_{\rm Higgs} &= \left(\begin{array}{cc} m^2_{\phi_1} & m^2_{\phi_1\phi_2} \\ m^2_{\phi_1\phi_2} & m^2_{\phi_2} \end{array} \right) &(3)\\ &= \left(\begin{array}{cc} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{array} \right) &(4)\\ &\xrightarrow{\alpha} \left(\begin{array}{cc} m^2_{H,{\rm tree}} & 0 \\ 0 & m^2_{h,{\rm tree}} \end{array} \right) \end{aligned}$$

by a rotation

$$\left(\begin{array}{c} H^0 \\ h^0 \end{array} \right) = \left(\begin{array}{cc} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{array} \right) \left(\begin{array}{c} \phi_1^0 \\ \phi_2^0 \end{array} \right), \tag{5}$$

with the angle α given in terms of $\tan\beta$ and M_A as follows:

$$\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad -\frac{\pi}{2} < \alpha < 0. \tag{6}$$

2.2 Evaluation of the Higgs boson self-energies

2.2.1 *FeynHiggs*

In *FeynHiggs* [15] the one-loop results for the Higgs boson self-energies $\hat{\Sigma}_s^{(1)}(q^2)$ are calculated according to Ref. [4]. They contain the full one-loop contribution obtained via an explicit Feynman-diagrammatic calculation in the on-shell scheme. The explicit expression are quite lengthy and can be found in Ref. [4]. Here the gaugino parameters M_1 and $M(\equiv M_2)$ enter in the neutralino mass matrix. M_1 is fixed via the GUT relation

$$M_1 = \frac{5}{3}\frac{s_W^2}{c_W^2}M, \tag{7}$$

whereas M is kept as a free input parameter.

In the following, the quantities $\hat{\Sigma}^{(i)}$ denote the one-loop ($i = 1$) and the two-loop ($i = 2$) contributions to the renormalized Higgs-boson self-energies. The two-loop results $\hat{\Sigma}_s^{(2)}$ are taken from Refs. [6, 7]. The leading two-loop corrections have been obtained by calculating the $\mathcal{O}(\alpha\alpha_s)$ contribution of the t–$\tilde{t}$ sector to the renormalized Higgs boson self-energies at zero external momentum from the Yukawa part of the theory. These two-loop QCD corrections are expected to constitute the most sizable part of the full set of two-loop corrections. In Refs. [6, 7] the self-energies have been computed first in the ϕ_1–ϕ_2 basis and afterwards rotated into the h–H basis according to eq. (5):

$$\begin{aligned}
\hat{\Sigma}_H^{(2)}(q^2) &= \cos^2\alpha\,\hat{\Sigma}_{\phi_1}^{(2)}(q^2) + \sin^2\alpha\,\hat{\Sigma}_{\phi_2}^{(2)}(q^2) + 2\sin\alpha\,\cos\alpha\,\hat{\Sigma}_{\phi_1\phi_2}^{(2)}(q^2)\\
\hat{\Sigma}_h^{(2)}(q^2) &= \sin^2\alpha\,\hat{\Sigma}_{\phi_1}^{(2)}(q^2) + \cos^2\alpha\,\hat{\Sigma}_{\phi_2}^{(2)}(q^2) - 2\sin\alpha\,\cos\alpha\,\hat{\Sigma}_{\phi_1\phi_2}^{(2)}(q^2)\\
\hat{\Sigma}_{hH}^{(2)}(q^2) &= -\sin\alpha\,\cos\alpha\,\left(\hat{\Sigma}_{\phi_1}^{(2)}(q^2) - \hat{\Sigma}_{\phi_2}^{(2)}(q^2)\right) + (\cos^2\alpha\, - \sin^2\alpha\,)\hat{\Sigma}_{\phi_1\phi_2}^{(2)}(q^2).
\end{aligned} \tag{8}$$

Thus, for our results up to the two-loop level, the matrix (1) contains the renormalized Higgs boson self-energies

$$\hat{\Sigma}_s(q^2) = \hat{\Sigma}_s^{(1)}(q^2) + \hat{\Sigma}_s^{(2)}(0), \quad s = h, H, hH \tag{9}$$

(in the one-loop contribution the full momentum dependence is kept). The calculation is performed for arbitrary parameters of the Higgs and the scalar-top sector and for arbitrary gluino mass $m_{\tilde{g}}$. Thus the accuracy of the calculation does not depend on how the parameters of the $\tilde{t}$ sector $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and $\theta_{\tilde{t}}$ are chosen, e.g. on the ammount of mixing.

Starting from the unrotated tree-level mass matrix eq. (3) and neglecting the external momentum, $\hat{\Sigma}_s(q^2) \to \hat{\Sigma}_s(0)$, $s = \phi_1, \phi_2, \phi_1\phi_2$, the Higgs boson mass matrix can be diagonalized

$$\begin{pmatrix} m_{\phi_1}^2 - \hat{\Sigma}_{\phi_1}(0) & m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}(0) \\ m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}(0) & m_{\phi_2}^2 - \hat{\Sigma}_{\phi_2}(0) \end{pmatrix} \xrightarrow{\alpha_{\rm eff}} \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix}, \tag{10}$$

involving the mixing angle $\alpha_{\rm eff}$,

$$\alpha_{\rm eff} = \arctan\left[\frac{-(M_A^2 + M_Z^2)\sin\beta\,\cos\beta\, - \hat{\Sigma}_{\phi_1\phi_2}(0)}{M_Z^2\cos^2\beta\, + M_A^2\sin^2\beta\, - \hat{\Sigma}_{\phi_1}(0) - m_h^2}\right], \quad -\frac{\pi}{2} < \alpha_{\rm eff} < \frac{\pi}{2}. \tag{11}$$

This higher-order corrected mixing angle is also calculated by *FeynHiggs*. By substituting $\alpha \to \alpha_{\rm eff}$ into tree-level expressions, the leading corrections from the Higgs-propagator corrections can be implemented into other calculations, see e.g. Refs. [13, 14]

In order to fix the notation, we shortly list our conventions for the MSSM scalar-top sector. The mass matrix in the basis of the current eigenstates $\tilde{t}_L$ and $\tilde{t}_R$ is given by

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + \cos 2\beta\,(\frac{1}{2} - \frac{2}{3}s_W^2)M_Z^2 & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + \frac{2}{3}\cos 2\beta\, s_W^2 M_Z^2 \end{pmatrix}, \qquad (12)$$

with

$$m_t X_t = m_t (A_t - \mu \cot\beta\,)\ . \qquad (13)$$

Diagonalizing the $\tilde{t}$ mass matrix yields the mass eigenvalues $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and the $\tilde{t}$ mixing angle $\theta_{\tilde{t}}$, which relates the current eigenstates to the mass eigenstates:

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{t}} & \sin\theta_{\tilde{t}} \\ -\sin\theta_{\tilde{t}} & \cos\theta_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}\ . \qquad (14)$$

In order to take into account the leading electroweak two-loop contribution to the mass of the lightest Higgs boson, we have implemented the leading Yukawa correction of $\mathcal{O}(G_F^2 m_t^6)$, which gives a sizable contribution only for m_h. The formula is taken over from the result obtained by renormalization group methods [9, 10]:

$$\hat{\Sigma}^{(2)}_{\phi_2}(0) = -\frac{9}{16\pi^4}\frac{G_F^2 m_t^6}{\sin^2\beta}\left[\tilde{X} t + t^2\right] \qquad (15)$$

$$\text{with} \quad \tilde{X} = \Bigg[\left(\frac{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}{4 m_t^2}\sin^2 2\theta_{\tilde{t}}\right)^2 \left(2 - \frac{m^2_{\tilde{t}_2} + m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}\log\left(\frac{m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1}}\right)\right) + \frac{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}{2 m_t^2}\sin^2 2\theta_{\tilde{t}}\log\left(\frac{m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1}}\right)\Bigg], \qquad (16)$$

$$t = \frac{1}{2}\log\left(\frac{m^2_{\tilde{t}_1} m^2_{\tilde{t}_2}}{m_t^4}\right). \qquad (17)$$

The most recent calculation of the Yukawa corrections, containing also non-logarithmic contributions [12], may enhance m_h by another GeV. Since this calculation has not been confirmed by an independent calculation[1] it has not been taken into account in *FeynHiggs* and *FeynHiggsFast*.

Furthermore, incorporated into *FeynHiggs* are leading QCD corrections beyond two-loop order by using the $\overline{\rm MS}$ top mass

$$\overline{m}_t = \overline{m}_t(m_t) \approx \frac{m_t}{1 + \frac{4}{3\pi}\alpha_s(m_t)} \qquad (18)$$

[1] Ref. [12] disagrees also in the logarithmic contributions with the existing calculations [9, 10].

for the two-loop contributions of $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(G_F^2 m_t^6)$ instead of the pole mass, $m_t = 175$ GeV.

The results obtained from *FeynHiggs* have been compared to those of previous calculations based on RG-improved effective field theory methods. Good agreement has been found in the case of no mixing in the $\tilde{t}$ sector, i.e. $X_t = 0$ GeV, whereas sizable deviations can occur when mixing in the $\tilde{t}$ sector is taken into account. This has been discussed in detail in Refs. [6, 7]. The main difference at the two-loop level is due to new non-logarithmic contributions present in the FD, but not in the RG calculation [19].

2.2.2 *FeynHiggsFast*

In *FeynHiggsFast* [16] the calculation is performed in a similar way as in *FeynHiggs*, but approximate formulae are used for the renormalized self-energies. They allow for a much faster evaluation of the Higgs boson masses.

The analytical expressions arising from the $t - \tilde{t}$ sector are given at the one-loop level as follows:

$$
\begin{aligned}
\hat{\Sigma}^{(1)}_{\phi_1}(0) &= \frac{G_F\sqrt{2}}{\pi^2} M_Z^4 \Lambda \cos^2\beta \, \log\left(\frac{\overline{m}_t^2}{M_S^2}\right) , && (19)\\
\hat{\Sigma}^{(1)}_{\phi_1\phi_2}(0) &= -\frac{G_F\sqrt{2}}{\pi^2} M_Z^2 \cot\beta \left[-\frac{3}{8}\overline{m}_t^2 + M_Z^2 \Lambda \sin^2\beta\right] \log\left(\frac{\overline{m}_t^2}{M_S^2}\right) , && (20)\\
\hat{\Sigma}^{(1)}_{\phi_2}(0) &= \frac{G_F\sqrt{2}}{\pi^2} \frac{\overline{m}_t^4}{8\sin^2\beta} \Bigg\{ -2\frac{M_Z^2}{\overline{m}_t^2} + \frac{11}{10}\frac{M_Z^4}{\overline{m}_t^4} \\
&\quad + \left[12 - 6\frac{M_Z^2}{\overline{m}_t^2}\sin^2\beta + 8\frac{M_Z^4}{\overline{m}_t^4}\Lambda\sin^4\beta\right] \log\left(\frac{\overline{m}_t^2}{M_S^2}\right) \\
&\quad + \frac{X_t^2}{M_S^2}\left[-12 + 4\frac{M_Z^2}{\overline{m}_t^2} + 6\frac{\overline{m}_t^2}{M_S^2}\right] + \frac{X_t^4}{M_S^4}\left[1 - 4\frac{\overline{m}_t^2}{M_S^2} + 3\frac{\overline{m}_t^4}{M_S^4}\right] \\
&\quad + \frac{X_t^6}{M_S^6}\left[\frac{3}{5}\frac{\overline{m}_t^2}{M_S^2} - \frac{12}{5}\frac{\overline{m}_t^4}{M_S^4} + 2\frac{\overline{m}_t^6}{M_S^6}\right] \\
&\quad + \frac{X_t^8}{M_S^8}\left[\frac{3}{7}\frac{\overline{m}_t^4}{M_S^4} - \frac{12}{7}\frac{\overline{m}_t^6}{M_S^6} + \frac{3}{2}\frac{\overline{m}_t^8}{M_S^8}\right]\Bigg\} , && (21)
\end{aligned}
$$

with

$$
\Lambda = \left(\frac{1}{8} - \frac{1}{3}s_W^2 + \frac{4}{9}s_W^4\right), \qquad s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \qquad (22)
$$

The two-loop contributions in $\mathcal{O}(\alpha\alpha_s)$ read:

$$
\begin{aligned}
\hat{\Sigma}^{(2)}_{\phi_1}(0) &= 0 , \\
\hat{\Sigma}^{(2)}_{\phi_1\phi_2}(0) &= 0 , \\
\hat{\Sigma}^{(2)}_{\phi_2}(0) &= \frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\frac{\overline{m}_t^4}{\sin^2\beta}\Bigg[4 + 3\log^2\left(\frac{\overline{m}_t^2}{M_S^2}\right) + 2\log\left(\frac{\overline{m}_t^2}{M_S^2}\right) - 6\frac{X_t}{M_S}
\end{aligned}
$$

$$
- \frac{X_t^2}{M_S^2} \left\{ 3 \log\left(\frac{\overline{m}_t^2}{M_S^2}\right) + 8 \right\} + \frac{17}{12} \frac{X_t^4}{M_S^4} \Bigg] . \tag{23}
$$

The two-loop Yukawa correction in this approximation reads:

$$
\hat{\Sigma}^{(2)}_{\phi_2}(0) = -\frac{9}{16\pi^4} \frac{G_F^2 \overline{m}_t^6}{\sin^2\beta} \left[\log^2\left(\frac{\overline{m}_t^2}{M_S^2}\right) - 2\frac{X_t^2}{M_S^2} \log\left(\frac{\overline{m}_t^2}{M_S^2}\right) + \frac{1}{6} \frac{X_t^4}{M_S^2} \log\left(\frac{\overline{m}_t^2}{M_S^2}\right) \right] . \tag{24}
$$

M_S has to be chosen according to

$$
M_S = \begin{cases} \sqrt{m_{\tilde{q}}^2 + \overline{m}_t^2} & : \quad M_{\tilde{t}_L} = M_{\tilde{t}_R} = m_{\tilde{q}} \\ \left[M_{\tilde{t}_L}^2 M_{\tilde{t}_R}^2 + \overline{m}_t^2 (M_{\tilde{t}_L}^2 + M_{\tilde{t}_R}^2) + \overline{m}_t^4 \right]^{\frac{1}{4}} & : \quad M_{\tilde{t}_L} \neq M_{\tilde{t}_R} \end{cases} \tag{25}
$$

For the one-loop corrections from the other sectors of the MSSM the logarithmic approximation given in Ref. [10] has been used for *FeynHiggsFast*, see also Ref. [20]. Thus, for our results up to the two-loop level the approximated self-eneries are calculated as

$$
\hat{\Sigma}_s^{\text{approx}} = \hat{\Sigma}_s^{(1)}(0) + \hat{\Sigma}_s^{(2)}(0), \;\; s = \phi_1, \phi_2, \phi_1\phi_2, \tag{26}
$$

using eqs. (19)-(24). The masses m_h and m_H are then obtained by diagonalizing the higher-order mass matrix

$$
\begin{pmatrix} m_{\phi_1}^2 - \hat{\Sigma}_{\phi_1}^{\text{approx}} & m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}^{\text{approx}} \\ m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}^{\text{approx}} & m_{\phi_2}^2 - \hat{\Sigma}_{\phi_2}^{\text{approx}} \end{pmatrix} \xrightarrow{\alpha_{\text{eff}}(\text{approx})} \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix} , \tag{27}
$$

involving the mixing angle $\alpha_{\text{eff}}(\text{approx})$,

$$
\begin{aligned} \alpha_{\text{eff}}(\text{approx}) \;\; &= \;\; \arctan\left[\frac{-(M_A^2 + M_Z^2)\sin\beta\,\cos\beta \, - \hat{\Sigma}_{\phi_1\phi_2}^{\text{approx}}}{M_Z^2\cos^2\beta \, + M_A^2 \sin^2\beta \, - \hat{\Sigma}_{\phi_1}^{\text{approx}} - m_h^2} \right] , \\ & -\frac{\pi}{2} < \alpha_{\text{eff}}(\text{approx}) < \frac{\pi}{2} . \end{aligned} \tag{28}
$$

It has been shown that these self-energies (26) approximate the full result for m_h and m_H better than 2 GeV for most parts of the MSSM parameter space [20]. For illustration of the quality of the compact approximation, we compare m_h and α_{eff} with the results from the complete Feynman-diagrammatic calculation included in *FeynHiggs*. For the Standard Model parameters we use $M_Z = 91.187$ GeV, $M_W = 80.39$ GeV, $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}, $\alpha_s(m_t) = 0.1095$, and $m_t = 175$ GeV. In the numerical evaluation we have furthermore chosen the triple couplings in the scalar top and bottom sector to be $A_b = A_t$. This fixes together with the choice of μ (the Higgs mixing parameter) the mixing in the scalar bottom sector. The parameter M appearing in the plots is the $SU(2)$ gaugino mass parameter, it enters as an independent parameter in the full result only, see also Ref. [7].

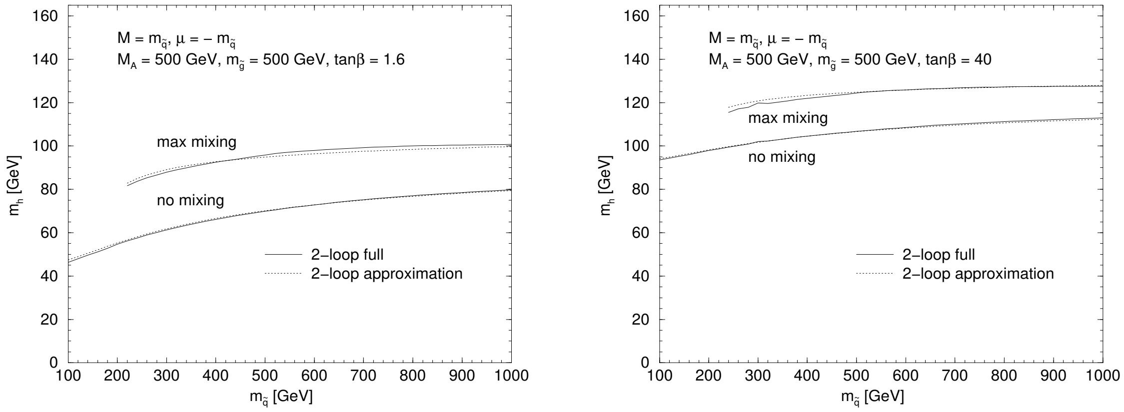

Figure 1: m_h as a function of $m_{\tilde{q}}$, calculated from the full formula and from the approximation formula for $M_A = 500$ GeV, $m_{\tilde{g}} = 500$ GeV and $\tan\beta = 1.6$ or 40.

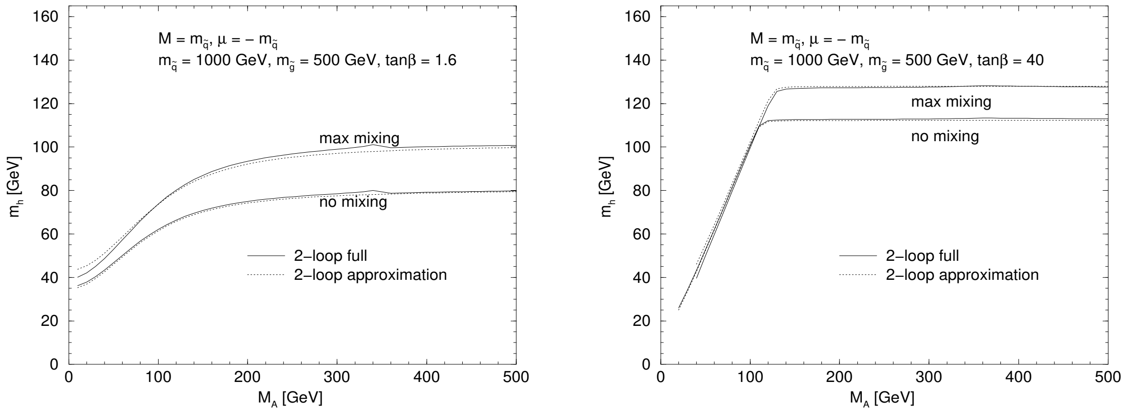

Figure 2: m_h as a function of M_A, calculated from the full formula and from the approximation formula for $m_{\tilde{q}} = 1000$ GeV, $m_{\tilde{g}} = 500$ GeV and $\tan\beta = 1.6$ or 40.

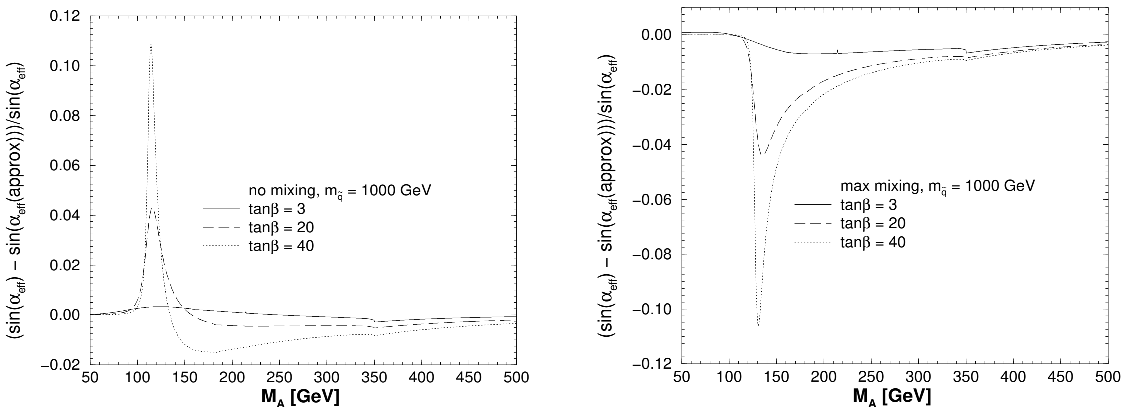

Figure 3: The relative difference $(\sin\alpha_{\text{eff}} - \sin\alpha_{\text{eff}}(\text{approx}))/\sin\alpha_{\text{eff}}$ is shown as a function of M_A for three values of $\tan\beta$ in the no mixing and the maximal mixing scenario. The other parameters are $\mu = -100$ GeV, $M = m_{\tilde{q}}$, $m_{\tilde{g}} = 500$ GeV and $A_b = A_t$.

In Refs. [7, 20] it has been shown that the lightest Higgs boson mass m_h as a function of X_t reaches a maximum at around $|X_t/m_{\tilde{q}}| \approx 2$. This case we refer to as 'maximal mixing'. A minimum of m_h is reached for $X_t \approx 0$, which we refer to as 'no mixing'.

Fig. 1 displays the dependence of m_h on $m_{\tilde{q}}$ for the cases of no mixing and maximal mixing, and we have set $M_A = 500$ GeV. For $\tan\beta$ we have chosen two typical values: $\tan\beta = 1.6$ as a low and $\tan\beta = 40$ as a typical high value. Very good agreement is found in the no-mixing scenario as well as in the maximal-mixing scenario, the deviations lie below 2 GeV.

The dependence on M_A is shown in Fig. 2. The quality of the approximation is typically better than 1 GeV for the no-mixing case and better than 2 GeV for the maximal-mixing case. Only for very small (and experimentally already excluded) values of M_A a deviation of 5 GeV occurs. The peaks in the plot for $\tan\beta = 1.6$ in the full result are due to the threshold $M_A = 2\,m_t$ in the one-loop contribution, originating from the top-loop diagram in the A self-energy. This peak does not occur in the approximation formula (where the momentum dependence of the A self-energy has been neglected) and can thus lead to a larger deviation around the threshold.

In Fig. 3 we display the quality of the short analytical approximation formula for the effective mixing angle. The relative difference between $\alpha_{\rm eff}$, see eq. (11), obtained from the full calculation (i.e. calculating the renormalized Higgs self-energies $\hat{\Sigma}_s(0)$ without any approximation) and the angle obtained with the help of the approximated Higgs self-energies used in *FeynHiggsFast*, $\alpha_{\rm eff}$(approx) of eq. (28), is shown as a function of M_A. We use three values of $\tan\beta$, $\tan\beta = 3, 20, 40$. Sizable deviations occur only in the region 100 GeV $\leq M_A \leq$ 150 GeV, especially for large $\tan\beta$. In this region of parameter space the values of m_h and m_H are very close to each other. This results in a high sensitivity to small deviations in the Higgs boson self-energies entering the Higgs-boson mass matrix (10), (11). Otherwise the relative difference stays below 3%.

2.3 Calculation of the ρ parameter

We have also implemented the calculation of the MSSM contributions to $\Delta\rho$ [17, 21]. Here the corrections arising from $\tilde{t}/\tilde{b}$ loops up to $\mathcal{O}(\alpha\alpha_s)$ have been taken into account. The result is valid for arbitrary parameters in the $\tilde{t}$ and $\tilde{b}$ sector, also taking into account the mixing in the $\tilde{b}$ sector which can have a non-negligible effect in the large $\tan\beta$ scenario [21].

The two-loop result is separated into the pure gluon-exchange contribution, which can be expressed by a very compact formula that allows a very fast evaluation, and the pure gluino-exchange contribution, which is given by a rather lengthy expression. The latter correction goes to zero with increasing gluino mass and can thus be discarded for a heavy gluino[2]. Concerning the implementation of the ρ-parameter into *FeynHiggsFast*, we have neglected the gluino exchange contribution. The ρ parameter can be used as an additional constraint (besides the experimental bounds) on the squark masses. A value of $\Delta\rho$ outside

[2]An additional contribution to $\Delta\rho$, arising from a shift in the squark masses when the soft SUSY breaking parameters are used as input (due to the $SU(2)$ invariance of these parameters in the squark sector), is also implemented in *FeynHiggs*. This correction is described in detail in Ref. [17].

the experimentally preferred region of $\Delta\rho^{\rm SUSY} \approx 2 \times 10^{-3}$ [22] indicates experimentally disfavored $\tilde{t}$ and $\tilde{b}$ masses.

3 The Fortran program *FeynHiggs*

3.1 The main structure

The complete program *FeynHiggs* consists of about 50.000 lines Fortran code, where the main part belongs to the formulas for the renormalized Higgs self-energies at the two-loop level and the gluino contribution to $\Delta\rho$. The executable file fills about 3 MB disk space. The calculation for one set of parameters, including the $\Delta\rho$ constraint, with the highest accuracy takes less than 2 seconds on a Sigma station (Alpha processor, 600 MHz processing speed, 512 MB RAM).

There exists a Home page for *FeynHiggs*: `www.feynhiggs.de` .
Here a uu-encoded version of the code can be obtained, together with a short instruction, information about bug fixes etc.

FeynHiggs consists of several subprograms which are listed in Table 1. We now describe the different subprograms in detail.

- **FeynHiggs.f** is the front-end for the whole program. Here all variables are set, all options are chosen, the subprogram **FeynHiggsSub.f** is called, and the results for the Higgs masses are printed out. For an easy use of *FeynHiggs* this front-end can be manipulated at will, the rest of the program need not to be changed.

- **FeynHiggsSub.f**: Here the actual calculation is carried out. The various results for the renormalized Higgs boson self-energies (9) are put together. Eq. (2) is solved numerically, the refinement terms (the leading two-loop Yukawa contribution and the leading QCD corrections beyond two-loop order) are incorporated.

- **Hhmasssr2.f**: The results for the complete one-loop self-energies are evaluated.

- **varcom.h**: The variables needed for the files ***code.f** are defined and grouped in common blocks.

- **bc.f, lamspen.f** contain mathematical functions needed for the one- and two-loop self-energies.

- **def2.f**: The masses of the stops and sbottoms are calculated from the parameters in the squark mass matrices. The opposite way is also possible.

- **P1setlcode.f** contains the complete code for the two-loop contribution to the renormalized ϕ_1 self-energy in $\mathcal{O}(\alpha\alpha_s)$. This code was first calculated with the help of *Mathematica* packages (see [6, 7]) and was afterwards transformed automatically into this Fortran code. The complete code has been split up into 13 smaller functions in order not to exceed the maximal number of continuation lines of the Fortran compiler.

subprogram	**function of the program**
FeynHiggs.f	front-end
FeynHiggsSub.f	main part of *FeynHiggs* : calculation of m_h, m_H
Hhmasssr2.f	one-loop self-energies
varcom.h	definition of variables
bc.f	one-loop functions
lamspen.f	further mathematical functions
def2.f	definitions for the MSSM parameters
P1secode.f	Fortran code for $\hat{\Sigma}^{(1)}_{\phi_1}(0)$
P1sesum.f	putting together the different parts of $\hat{\Sigma}^{(1)}_{\phi_1}(0)$
P1sevar.f	definition of variables for $\hat{\Sigma}^{(1)}_{\phi_1}(0)$
P2se[code,sum,var].f	same for $\hat{\Sigma}^{(1)}_{\phi_2}(0)$
P1P2se[code,sum,var].f	same for $\hat{\Sigma}^{(1)}_{\phi_1\phi_2}(0)$
P1setl[code,sum,var].f	same for $\hat{\Sigma}^{(2)}_{\phi_1}(0)$
P2setl[code,sum,var].f	same for $\hat{\Sigma}^{(2)}_{\phi_2}(0)$
P1P2setl[code,sum,var].f	same for $\hat{\Sigma}^{(2)}_{\phi_1\phi_2}(0)$
delrhosub.f	main part for the calculation of $\Delta\rho$
delrhoGluino[code,sum,var].f	subroutines for the gluino-exchange contribution to $\Delta\rho$

Table 1: The subprograms of *FeynHiggs*.

- **P1setlsum.f**: The sum of 13 functions for the renormalized ϕ_1 self-energy at the two-loop level, contained in **P1setlcode.f**, is put together.

- **P1setlvar.f** contains the variable definition for the 13 subfunctions described above.

- The above three files exist in an analogous way for the other renormalized Higgs boson self-energies at the one- and at the two-loop level. The one-loop self-energies in these files are given in the approximation explained in detail in [6, 7] and are only needed internally for consistency checks. For the real calculation of m_h and m_H the complete one-loop self-energies are calculated in **Hhmasssr2.f**.

- **delrhosub.f** is needed for the calculation of the MSSM contribution to $\Delta\rho$. In this program the evaluation of the leading one-loop contribution is performed. In addition also the gluon-exchange correction in $\mathcal{O}(\alpha\alpha_s)$ is calculated. The $\mathcal{O}(\alpha\alpha_s)$ contribution to $\Delta\rho$ is completed by including the gluino-exchange contribution by calling the subprogram **delrhoGluinosum.f**.

- **delrhoGluino[code,sum,var].f**: The code for the gluino exchange contribution to $\Delta\rho$ is implemented in the same way as it is described for the renormalized Higgs boson self-energies.

3.2 Options and setting of variables

FeynHiggs can be run in several ways, determined by the choice of several options.

- **Depth of calculation** allows to choose to what extent the refinements described in section 2.1 should be applied.

- **Selection of input parameters:** One can either use the physical parameters of the $\tilde{t}$ mass matrix ($m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and $\theta_{\tilde{t}}$) or the soft SUSY breaking parameters ($M_{\tilde{t}_L}, M_{\tilde{t}_R}$ and X_t) as input parameters. In order to allow a correct combination of *FeynHiggs* with RG based codes, the $\overline{\mathrm{MS}}$ parameters of the $\tilde{t}$ mass matrix can also be used as input. They have to be supplemented with the appropriate scale choice so as to make the correct transition from $\overline{\mathrm{MS}}$ to on-shell parameters possible [19].

- **m_t in the $\tilde{t}$ mass matrix:** If the soft SUSY breaking parameters are used as input parameters, one can choose whether the top pole mass, m_t, or the running top mass, $\overline{m}_t$, should be used in the $\tilde{t}$ mass matrix in order to determine the masses of the eigenstates $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and $\theta_{\tilde{t}}$.

- **Limit for $\Delta\rho^{\mathrm{SUSY}}$:** One can specify the maximally allowed value for the MSSM contribution to $\Delta\rho$. If $\Delta\rho^{\mathrm{SUSY}}$ exceeds this limit a warning is printed out.

- **Selection for the one-loop accuracy:** Before the calculation of the Higgs masses starts, one has to specify to what accuracy the one-loop renormalized self-energies should be evaluated. One can either take into account the top sector only, one can choose to use the top *and* the bottom sector, or one can select the option that the complete MSSM should be taken into account.

In **FeynHiggs.f** the Standard Model (SM) variables are set to their present experimental values. New values for these variables can be implemented by the user into the code of the file **FeynHiggs.f** easily. The MSSM variables can be chosen by the user at will. In addition, since the dependence of the Higgs masses on the top mass is very strong, also m_t can be chosen at will. In Table 2 the meaning of the different parameters *FeynHiggs* asks for is explained[3]. All these variables are transferred to the different subprograms by common blocks.

[3]Some MSSM variables exist internally in *FeynHiggs*, but are not input parameters. These variables have no entry in the left column of Table 2.

input for ***FeynHiggs***	**expression in the MSSM**	**internal expr. in *FeynHiggs***
`tan(beta)`	$\tan\beta$	`ttb`
`Msusy_top_L`	$M_{\tilde{t}_L}$	`msusytl`
`Msusy_top_R`	$M_{\tilde{t}_R}$	`msusytr`
	$M_{\tilde{b}_L}$	`msusybl`
`Msusy_bot_R`	$M_{\tilde{b}_R}$	`msusybr`
`Xt`	$X_t \equiv M_t^{LR}$	`mtlr`
`Xb`	$X_b \equiv M_b^{LR}$	`mblr`
`MSt1`	$m_{\tilde{t}_1}$	`mst1`
`MSt2`	$m_{\tilde{t}_2}$	`mst2`
`sin(theta_stop)`	$\sin\theta_{\tilde{t}}$	`stt`
`MSb1`	$m_{\tilde{b}_1}$	`msb1`
`MSb2`	$m_{\tilde{b}_2}$	`msb2`
`sin(theta_sbot)`	$\sin\theta_{\tilde{b}}$	`stb`
`Mtop` or `mt`	m_t	`mmt`
`Mbottom`	m_b	`mbb`
`Mgluino`	$m_{\tilde{g}}$	`mgl`
`Mue`	μ	`mmue`
`M2`	$M \equiv M_2$	`mmm`
`MA`	M_A	`mma`

Table 2: The meaning of the different MSSM variables which have to be entered into *FeynHiggs*.

4 The Fortran program *FeynHiggsFast*

The complete program *FeynHiggsFast* consists of about 1300 lines Fortran code. The executable file fills about 65 KB disk space. The calculation for one set of parameters, including the $\Delta\rho$ constraint, takes about 2×10^{-5} seconds on a Sigma station (Alpha processor, 600 MHz processing speed, 512 MB RAM). The program can be obtained from the *FeynHiggs* home page: `www.feynhiggs.de` .
Here the code is available, together with a short instruction, information about bug fixes etc.

FeynHiggsFast consists of a front-end (`program FeynHiggsFast`) and the main part where the evaluation is performed (starting with `subroutine feynhiggsfastsub`). The front-end can be manipulated by the user at will, whereas the main part should not be changed. In this way *FeynHiggsFast* can be accommodated as a subroutine to existing programs, thus providing an extremely fast and for many purposes sufficiently accurate evaluation for the masses and mixing angles in the MSSM Higgs sector.

The input of *FeynHiggsFast* are the low energy SUSY parameters, listed in Tab. 3. Concerning the $\tilde{t}$ sector, the user has the option to enter either the physical parameters, i.e. the masses and the mixing angle ($m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ and $\sin\theta_{\tilde{t}}$) or the unphysical parameters $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$ and X_t. From these input parameters *FeynHiggsFast* calculates the masses and the mixing angle of the neutral $\mathcal{CP}$-even Higgs sector, as well as the mass of the charged Higgs boson and the ρ-parameter.

input for *FeynHiggsFast*	**MSSM expr.**	**internal expr. in *FeynHiggsFast***
`tan(beta)`	$\tan\beta$	`ttb`
`Msusy_top_L`	$M_{\tilde{t}_L}$	`msusytl`
`Msusy_top_R`	$M_{\tilde{t}_R}$	`msusytr`
`MtLR`	$X_t \equiv M_t^{LR}$	`mtlr`
`MSt2`	$m_{\tilde{t}_2}$	`mst2`
`delmst`	$\Delta m_{\tilde{t}} = m_{\tilde{t}_2} - m_{\tilde{t}_1}$	`delmst`
`sin(theta_stop)`	$\sin\theta_{\tilde{t}}$	`stt`
`MT`	m_t	`mmt`
`Mue`	μ	`mmue`
`MA`	M_A	`mma`

Table 3: The meaning of the different MSSM variables which have to be entered into *FeynHiggsFast*.

5 How to use *FeynHiggs* and *FeynHiggsFast*

5.1 *FeynHiggs*

In this section we will give two examples of how *FeynHiggs* is used. As stated before, the front-end **FeynHiggs.f** can be manipulated by the user at will, whereas the subprogram **FeynHiggsSub.f** and all the other subprograms should not be changed. Concerning a modification of the front-end one has to note that all variables have to be defined in the front-end. The Higgs masses are then obtained by calling the subroutine via

```
call feynhiggssub(mh1,mh2,mh12,mh22) ,
```

where `mh1` and `mh2` are the one-loop corrected values for m_h and m_H, respectively. `mh12` and `mh22` contain the values for the two-loop corrected Higgs masses, including the refinement terms as it has been specified in the options.

In the following two examples are given, how the application of *FeynHiggs* looks like, using the given front-end of the currently distributed version. The user's input is given in **bold face** letters.

5.1.1 Example 1

>FeynHiggs.exe

```
... Introduction ...
---------------------------------------------------

SETUP:      ( * indicates one input)
======

* depth of calculation ?
1: full 1-loop + 2-loop QCD
2: same as 1, but in addition with
             running mt at 2-loop
3: same as 2, but in addition with
             two-loop Yukawa term included
(3 gives most accurate result)
* Select input:
1: On-Shell: Msusy_top_L,R, Xt, Msusy_bot_L,R, Xb
2: On-Shell: MSt1, MSt2, stt, MSb1, MSb2, stb
3: MSbar   : MSt1, MSt2, stt, Msusy_bot_R, Xb
4: MSbar   : MSt1, MSt2, stt, MSb1, MSb2, stb

* mt in the OS stop mass matrix at 2-loop ?
--> irrelevant if input = 2, 3, 4 has been chosen
1: mt = pole mass
2: mt = running mass
* Are you interested in the 'fast and approximate
result from the short approximation formula?
1: no, I prefer the full result only
2: yes, I'd like to get this additional information
* Limit for Delta rho = 1.3 * 10^ -3 ? (0 = ok)

What additional information do you want?
*    Test output of your input parameters? (0: yes // 1: no)
*    Sfermion masses etc? (0: yes // 1: no)
*    Neutralino/Chargino masses etc?(0: yes // 1: no)
```

3,1,1,2,0,0,1,1

```
INPUT:
======

tan(beta)
Msusy_top_L
Msusy_top_R (0: Msusy_top_R = Msusy_top_L)
Msusy_bot_R (0: Msusy_bot_R = Msusy_top_L)
Xt == MtLR
Xb == MbLR (1: Ab = At)
Mtop(pole) (0: Mtop = 175 // 1: Mtop = 174.3)
Mbottom (0: Mbot = 4.5 // 1: Mbot = 2.97)
Mgluino (0: Mgl = 500 // 1: Mgl = Msusy_top_L)
Mue (1: Mue = -200 // 2: Mue = -Msusy_top_L)
M2 (1: M2 = 400 // 2: M2 = Msusy_top_L)
MA
Accuracy for the one-loop calculation:
1 = top only, 2 = top/bottom only, 3 = all
```

3,1000,0,0,500,1,1,1,800,200,1,500,3

```
Your OS parameters:
MT, Msusy(top-left), Msusy(top-right), Xt, At
MB, Msusy(bot-left), Msusy(bot-right), Xb, Ab
tb, Mgl, Mu, M2, MA
174.3000       1000.000       1000.000        500.0000       566.6667
2.970000       1000.000       1000.000       -33.33333       566.6667
3.000000       800.0000       200.0000        400.0000       500.0000

-------------------------------------------------
-------------------------------------------------
Performing the calculation...
using running mt for two-loop contribution:   166.665430375750
including two-loop Yukawa term
-------------------------------------------------
-------------------------------------------------
----------------------------------------------------
----------------------------------------------------
The results:  light Higgs     heavy Higgs     alpha
----------------------------------------------------
mh-tree :      72.50753       503.0484     -0.3378600
----------------------------------------------------
mh-1loop:
 --> BEST  :   113.4450       504.3510     -0.3490313
```

```
short form.:   112.2561       503.9977     -0.3488834
Yuk-approx :   117.5207       504.2240
----------------------------------------------------
mh-2loop:
 --> BEST  :   102.4391       504.0138     -0.3454219
short form.:   101.0657       503.6877     -0.3453216
Yuk-approx :   107.2035       503.8850
----------------------------------------------------
----------------------------------------------------
charged Higgs, tree:   506.4213
             1-loop:   506.1459
----------------------------------------------------
----------------------------------------------------
Delta rho 1-loop         :   5.633068085824507E-005
Delta rho 2-loop (gluon) :   5.811323997046967E-006
Delta rho 2-loop (gluino):   8.324327227139254E-006
Delta rho total          :   7.046633208243128E-005
----------------------------------------------------

collected WARNINGS:

----------------------------------------------------


What next? (0: new calculation // 1: setup)
```

In this example the unphysical parameters in the $\tilde{t}$ sector in the on-shell scheme have been chosen as input parameters. The calculation has been performed at the highest level of accuracy. No warnings have come up.

5.1.2 Example 2

>FeynHiggs.exe

```
... Introduction ...
----------------------------------------------------

SETUP:      ( * indicates one input)
======

* depth of calculation ?
1: full 1-loop + 2-loop QCD
2: same as 1, but in addition with
             running mt at 2-loop
3: same as 2, but in addition with
             two-loop Yukawa term included
(3 gives most accurate result)
* Select input:
1: On-Shell: Msusy_top_L,R, Xt, Msusy_bot_L,R, Xb
2: On-Shell: MSt1, MSt2, stt, MSb1, MSb2, stb
3: MSbar   : MSt1, MSt2, stt, MSb1, MSb2, stb

* mt in the OS stop mass matrix at 2-loop ?
--> irrelevant if input = 2, 3 has been chosen
1: mt = pole mass
2: mt = running mass
* Are you interested in the 'fast and approximate
result from the short approximation formula?
1: no, I prefer the full result only
2: yes, I'd like to get this additional information
* Limit for Delta rho = 1.3 * 10^ -3 ? (0 = ok)

What additional information do you want?
*    Test output of your input parameters? (0: yes // 1: no)
*    Sfermion masses etc? (0: yes // 1: no)
*    Neutralino/Chargino masses etc? (0: yes // 1: no)
```

3,3,2,2,0.000005,0,1,1

```
INPUT:
tan(beta)
MSt1, scale of MSt1 (1: q = MSt1)
MSt2, scale of MSt2 (1: q = MSt2)
```

```
sin(theta_stop) (1: stt = sin(-Pi/4))
scale of sine (1: MSt1 // 2: MSt2 // 3: Sqrt[MSt1 MSt2] )
MSb1, scale of MSb1 (1: q = MSb1)
MSb2, scale of MSb2 (1: q = MSb2)
sin(theta_sbot) (1: stb = sin(+Pi/4))
scale of sine (1: MStb // 2: MSb2 // 3: Sqrt[MSb1 MSb2] )
Mtop(pole) (0: Mtop = 175 // 1: Mtop = 174.3)
Mbottom (0: Mbot = 4.5 // 1: Mbot = 2.97)
Mgluino (0: Mgl = 500 // 1: Mgl = 1000)
Mue (1: Mue = -200 // 2: Mue = -1000)
M2 (1: M2 = 400 // 2: M2 = 1000)
MA
Accuracy for the one-loop calculation:
1 = top only, 2 = top/bottom only, 3 = all
```

40,900,1,1000,1,-0.7,3,950,1,960,1,1,3,176,3,800,1,1,1000,3

```
MSbar to OnShell
================

MSt1, MSt2, stt
900.0000       1000.000     -0.7000000
945.1918       1025.727     -0.6989895
-83387.63      -52115.14     -1.0105076E-03

MSb1, MSb2, stb
950.0000       960.0000      0.7071068
983.9677       994.1149      0.7071068
-65692.46      -66664.53     -3.5655791E-11

Transition to unphysical OS parameters completed

Your OS parameters:
MT, Msusy(top-left), Msusy(top-right), Xt, At
MB, Msusy(bot-left), Msusy(bot-right), Xb, Ab
tb, Mgl, Mu, M2, MA
176.0000       971.0214       972.0188       450.8129       445.8129
3.000000       987.2608       988.7380      -3345.344      -11345.34
40.00000       800.0000      -200.0000       400.0000       1000.000

-------------------------------------------------
-------------------------------------------------
Performing the calculation...
```

```
using running mt for two-loop contribution:   168.300379152987
including two-loop Yukawa term
------------------------------------------------
------------------------------------------------
---------------------------------------------------
---------------------------------------------------
The results:  light Higgs     heavy Higgs     alpha
---------------------------------------------------
mh-tree :      91.07214       1000.010     -2.5413230E-02
---------------------------------------------------
mh-1loop:
 --> BEST  :   135.8726       999.9851     -2.5282357E-02
short form.:   125.0990       1000.013     -2.5605066E-02
Yuk-approx :   130.6613       1000.010
---------------------------------------------------
mh-2loop:
 --> BEST  :   123.8153       999.9789     -2.5262697E-02
short form.:   114.9872       1000.012     -2.5542060E-02
Yuk-approx :   121.2963       1000.010
---------------------------------------------------
---------------------------------------------------
charged Higgs, tree:   1003.226
             1-loop:   1003.958
---------------------------------------------------
---------------------------------------------------
WARNING: Delta rho > experimental limit
Delta rho 1-loop         :   8.626183665401147E-006
Delta rho 2-loop (gluon) :   8.928295652882673E-007
Delta rho 2-loop (gluino):   0.000000000000000E+000
Delta rho total          :   9.519013230689415E-006
---------------------------------------------------

collected WARNINGS:
Delta rho > experimental limit

---------------------------------------------------

What next? (0: new calculation // 1: setup)
```

Here the $\overline{\rm MS}$ parameters of the $\tilde{t}/\tilde{b}$ sector have been chosen as input. They are transformed into the corresponding on-shell parameters automatically. In this example only a very small SUSY contribution to $\Delta\rho$ is allowed and the value of $\Delta\rho^{\rm SUSY}$ for the chosen parameters exceeds the above specified maximal value (a warning is printed out.)

5.2 *FeynHiggsFast*

In this section we will give two examples of how *FeynHiggsFast* is used. As stated before, the front-end can be manipulated by the user at will, whereas the subprogram (starting with `subroutine feynhiggsfastsub()` and all the other subprograms should not be changed. Concerning a modification of the front-end one has to note that all variables have to be defined in the front-end. The Higgs masses are then obtained by calling the subroutine via

$$\texttt{call feynhiggsfastsub}()\ .$$

In the following two examples are given, how the application of *FeynHiggs* looks like, using the given front-end of the currently distributed version. The user's input is given in **bold face** letters.

5.2.1 Example 1

>FeynHiggsFast.exe

```
... Introduction ...
--------------------------------------------------

depth of calculation ?
1: full 1-loop + 2-loop QCD with top pole mass
2: same as 1, but in addition with
              Yukawa term added for light Higgs
3: full 1-loop + 2-loop QCD with running top mass
4: same as 3, but in addition with
              Yukawa term added for light Higgs
The choice for the top mass (pole/running) will
also be applied on the top mass appearing in
the stop mass matrix
```

4

```
Select input:
1: Msusy, MtLR, ...
2: MSt2, delmst, stt, ...
```

1

```
Limit for Delta rho = 1.3 * 10^ -3 ? (0 = ok)
```

0

```
tan(beta) = ?
```

3

```
Msusy_top_L = ?
```

1000

```
Msusy_top_R = ? (0: Msusy_top_R = Msusy_top_L)
```

0

```
top pole mass = 175 ? (0 = ok)
```

174

```
Mue = -Msusy ? (0 = ok)
```

-200

```
MA = ?
```

200

```
MtLR = ?
```

2000

```
Your parameters:
Msusy(top-left), Msusy(top-right), MtLR
tb, MT, MA, Mue
1000.000       1000.000       2000.000
3.000000       174.0000       200.0000      -200.0000

using running top mass:   166.376917095558
adding Yukawa term
-------------------------------------------------
-------------------------------------------------
The results: light Higgs    heavy Higgs     alpha
-------------------------------------------------
mh-tree :    70.02447       208.3546     -0.4261538
-------------------------------------------------
mh-1loop:    119.0707                                 (formula for MA >> MZ)
mh-1loop:    116.5500       214.3274     -0.5604230   (matrix diag.)
-------------------------------------------------
mh-2loop:    113.3472                                 (formula for MA >> MZ)
mh-2loop:    111.2652       213.2732     -0.5379112   (matrix diag.)
-------------------------------------------------
charged Higgs, tree:    215.5517
             1-loop:    215.0103
-------------------------------------------------
-------------------------------------------------
Delta rho 1-loop         :   5.075509248081155E-005
Delta rho 2-loop (gluon) :   6.675786383508177E-006
Delta rho total          :   5.743087886431972E-005
-------------------------------------------------

tan(beta) = ?
```

In this example the unphysical parameters in the $\tilde{t}$ sector in the on-shell scheme have been chosen as input parameters. The calculation has been performed at the highest level of accuracy. Concerning the evaluation of m_h and m_H, two different methods have been used: the compact formula valid only for $M_A \gg M_Z$ and the matrix diagonalization, which is valid for arbitrary parameters. Since $M_A = 200$ GeV has been chosen, there is a finite difference between the two methods concerning m_h. For details see Ref. [20]. (Within *FeynHiggs* the latter method has been implemented.)

5.2.2 Example 2

>FeynHiggsFast.exe

```
... Introduction ...
---------------------------------------------------

depth of calculation ?
1: full 1-loop + 2-loop QCD with top pole mass
2: same as 1, but in addition with
              Yukawa term added for light Higgs
3: full 1-loop + 2-loop QCD with running top mass
4: same as 3, but in addition with
              Yukawa term added for light Higgs
The choice for the top mass (pole/running) will
also be applied on the top mass appearing in
the stop mass matrix
```
4

```
Select input:
1: Msusy, MtLR, ...
2: MSt2, delmst, stt, ...
```
2

```
Limit for Delta rho = 1.3 * 10^ -3 ? (0 = ok)
```
0

```
tan(beta) = ?
```
30
```
MSt2 = ?
```
1000
```
delmst = ?
```
100
```
sin(theta_stop) = ?
(0: stt = 0 // 1: stt = sin(-Pi/4))
```
0.1
```
top pole mass = 175 ? (0 = ok)
```
0
```
Mue = -1000 ? (0 = ok)
```
0
```
MA = ?
```
500

```
Your parameters:
MSt1, MSt2, stt
tb, MT, MA, Mue
900.0000       1000.000      0.1000000
30.00000       175.0000       500.0000      -1000.000

using running top mass:   167.338636349986
adding Yukawa term
-------------------------------------------------
-------------------------------------------------
The results: light Higgs    heavy Higgs     alpha
-------------------------------------------------
mh-tree :    90.97765       500.0381     -3.5606291E-02
-------------------------------------------------
mh-1loop:    117.5601                                   (formula for MA >> MZ)
mh-1loop:    117.5599       500.0406     -3.6348902E-02 (matrix diag.)
-------------------------------------------------
mh-2loop:    111.8071                                   (formula for MA >> MZ)
mh-2loop:    111.8071       500.0389     -3.6146898E-02 (matrix diag.)
-------------------------------------------------
charged Higgs, tree:    506.4213
             1-loop:    508.1403
-------------------------------------------------
-------------------------------------------------
Delta rho 1-loop         :   9.907417158686958E-005
Delta rho 2-loop (gluon) :   1.027513491549583E-005
Delta rho total          :   1.093493065023654E-004
-------------------------------------------------


tan(beta) = ?
```

Here the physical parameters of the $\tilde{t}$ sector have been chosen as input. With $M_A = 500$ GeV nearly no difference between the the methods for the evaluation of m_h and m_H occurs.

6 Conclusions

FeynHiggs and *FeynHiggsFast* are Fortran codes for the calculation of the masses of the neutral $\mathcal{CP}$-even Higgs bosons of the MSSM. In addition they provide also the mixing angle $\alpha_{\rm eff}$ and the masses of the charged Higgs bosons. The programs are based on results which have been obtained using the Feynman-diagrammatic approach. *FeynHiggs* consist of the full one-loop and the dominant two-loop QCD corrections. Subdominant two-loop contributions are also implemented. *FeynHiggsFast* is based on short approximation formulas for the one- and the two-loop contributions, thus allowing for a much faster evaluation. It has been shown that these short formulas give an approximation better than 2 GeV for m_h for most parts of the MSSM parameter space.

The programs are available via the WWW page `www.feynhiggs.de` .

The code consists of a front-end and a subroutine. The front-end can be manipulated at the user's will. The different parts of the code have been described in detail, and the meaning of the variables used in the code has been explained. We have given two examples of how to use *FeynHiggs* and *FeynHiggsFast*.

The subroutines are self-contained and can be linked as an independent part to other existing programs[4] thus providing a precise prediction for m_h and m_H, which can then be used for further computations.

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[4]This has been carried out, for example, for the program HZHA [23, 24], into which *FeynHiggs* has been incorporated as a subroutine or for the programs HDECAY [25, 26] or SUSYGEN [27, 28], into which *FeynHiggsFast* has been implemented.

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