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A CP sensitive asymmetry in the three–body decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$

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Abstract

We consider the three–body decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$ and propose the asymmetry of the τ polarization perpendicular to the decay plane as a CP sensitive observable. We calculate this asymmetry in the Minimal Supersymmetric Standard Model with the parameters μ and A_t complex. In the parameter domain where the decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$ is important this asymmetry can go up to $\pm 30\%$. We also estimate the event rates necessary to observe this asymmetry at 90% CL.

1 Introduction

The experimental search for supersymmetric (SUSY) particles will have high priority at the upgraded Tevatron and at LHC. The precision determination of the SUSY parameters will be the main goal of a future e^+e^- linear collider [1]. The analysis of scalar top quarks $\tilde{t}_i, i = 1, 2$, will be particularly interesting, because of the large top Yukawa coupling involved in this system. Due to the effects of the top Yukawa coupling the lighter stop may be relatively light if not the lightest charged SUSY particle [2].

A phenomenological study of production and decays of 3rd generation sfermions at an e^+e^- linear collider with cms energy in the range $\sqrt{s} = 0.5 - 1$ TeV has been given in Ref. [3]. There it has been shown that by measuring production cross sections with polarized beams the masses of the top squarks $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and their mixing angle $\theta_{\tilde{t}}$ can be determined. It has also been shown that with an integrated luminosity of $500 fb^{-1}$ a precision of about 1% can be achieved. The precision to be expected for the underlying SUSY parameters has also been estimated. This analysis has been performed in the Minimal Supersymmetric Standard Model (MSSM) with real parameters.

However, the assumption that all MSSM parameters are real may be too restrictive. In principle, the Higgs–higgsino mass parameter μ and the trilinear scalar coupling parameters A_f of the sfermions $\tilde{f}$ may be complex. These complex parameters are new sources of CP violation and may provide potentially large SUSY contributions to the electric dipole moments (EDM) of electron and neutron. The very small experimental upper limits of the electron and neutron EDMs, therefore, may lead to restrictions on the complex phases. Recent analyses have shown that in mSUGRA–type models the phase of μ is restricted to $|\varphi_\mu| \lesssim 0.1 - 0.2$ for a universal scalar mass parameter $M_0 \lesssim$ 400 GeV, whereas the phase of the universal trilinear scalar coupling parameter A_0 is correlated with φ_μ, but otherwise unrestricted [4]. One can conclude that in models with more general parameter specifications the phases of the parameters A_f of the 3rd generation sfermions are not restricted at one–loop level by the electron and neutron EDMs. There may be restrictions at two–loop level [5]. Furthermore, a complex trilinear coupling parameter A_t in the stop system can also lead to interesting CP violating effects in top quark production, as discussed in [6].

For a complete analysis of the stop sector one has to take into account that the parameters μ and A_t may be complex. The parameter $|\mu|$ and its phase φ_μ will presumably be determined by measuring suitable observables of the chargino and neutralino sector [7]. For the determination of $|A_t|$ and its phase φ_{A_t} appropriate observables in the stop sector have to be measured. However, it may be difficult to define a suitable CP sensitive observable in stop decays if the main decay modes are two–body decays.

In the present paper we define a CP sensitive asymmetry in the decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$. As this is a three–body decay the polarization of the τ^+ normal to the decay plane is sensitive to CP violation. The appropriate CP sensitive observable is defined by the asymmetry of the τ polarization perpendicular to the decay plane. As we will show this asymmetry can go up to 30%. Moreover, we show the existence of parameter regions where the decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$ has a sufficient branching ratio allowing for the measurement of this asymmetry. We perform our analysis in the MSSM with μ and A_t complex. We focus on scenarios where only the decays $\tilde{t}_1 \to b\tilde{\nu}_\ell\ell$, $\tilde{t}_1 \to b\tilde{\ell}\nu_\ell$ [8], $\tilde{t}_1 \to c\tilde{\chi}^0_1$ [9] and four body decays [10] are kinematically allowed. We assume that the lightest neutralino $\tilde{\chi}^0_1$ is the lightest SUSY particle (LSP). The CP asymmetry defined above is analogous to that defined in Ref. [11] in case of top quark decays.

In Section 2 we shortly review stop mixing in the presence of complex parameters. In Section 3 we give the formulae of the CP violating observable. In Section 4 we present numerical results for the phase dependences of the CP asymmetry. We give an theoretical estimate of the event rates necessary to observe the CP sensitive asymmetry at an e^+e^- linear collider with $\sqrt{s} = 0.5 - 1$ TeV. Section 5 contains a short summary.

2 $\tilde{t}_L - \tilde{t}_R$ Mixing

We first give a short account of $\tilde{t}_L - \tilde{t}_R$ mixing in the case the parameters μ and A_t are complex. The masses and couplings of the $\tilde{t}$–squarks follow from the hermitian 2×2 mass matrix which in the basis $(\tilde{t}_L, \tilde{t}_R)$ reads

$$\mathcal{L}_M^{\tilde{t}} = -(\tilde{t}_L^*, \tilde{t}_R^*) \begin{pmatrix} M^2_{\tilde{t}_{LL}} & e^{-i\varphi_{\tilde{t}}}|M^2_{\tilde{t}_{LR}}| \\ e^{i\varphi_{\tilde{t}}}|M^2_{\tilde{t}_{LR}}| & M^2_{\tilde{t}_{RR}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \tag{1}$$

where

$$M^2_{\tilde{t}_{LL}} = M^2_{\tilde{Q}} + (\frac{1}{2} - \frac{2}{3}\sin^2\Theta_W)\cos 2\beta\ m_Z^2 + m_t^2, \tag{2}$$

$$M^2_{\tilde{t}_{RR}} = M^2_{\tilde{U}} + \frac{2}{3}\sin^2\Theta_W \cos 2\beta\ m_Z^2 + m_t^2, \tag{3}$$

$$M^2_{\tilde{t}_{RL}} = (M^2_{\tilde{t}_{LR}})^* = m_t(A_t - \mu^* \cot\beta), \tag{4}$$

$$\varphi_{\tilde{t}} = \arg[A_{\tilde{t}} - \mu^* \cot\beta], \tag{5}$$

where $\tan\beta = v_2/v_1$ with $v_1(v_2)$ being the vacuum expectation value of the Higgs field $H_1^0(H_2^0)$, m_t is the mass of the top quark and Θ_W is the weak mixing angle, μ is the Higgs–higgsino mass parameter and $M_{\tilde{Q}}$, $M_{\tilde{U}}, A_t$ are the soft SUSY–breaking parameters of the stop system. The mass eigenstates $\tilde{t}_i$ are $(\tilde{t}_1, \tilde{t}_2) = (\tilde{t}_L, \tilde{t}_R)\mathcal{R}^{\tilde{t}^T}$ with

$$\mathcal{R}^{\tilde{t}} = \begin{pmatrix} e^{i\varphi_{\tilde{t}}}\cos\theta_{\tilde{t}} & \sin\theta_{\tilde{t}} \\ -\sin\theta_{\tilde{t}} & e^{-i\varphi_{\tilde{t}}}\cos\theta_{\tilde{t}} \end{pmatrix}, \tag{6}$$

with

$$\cos\theta_{\tilde{t}} = \frac{-|M^2_{\tilde{t}_{LR}}|}{\sqrt{|M^2_{\tilde{t}_{LR}}|^2 + (m^2_{\tilde{t}_1} - M^2_{\tilde{t}_{LL}})^2}}, \quad \sin\theta_{\tilde{t}} = \frac{M^2_{\tilde{t}_{LL}} - m^2_{\tilde{t}_1}}{\sqrt{|M^2_{\tilde{t}_{LR}}|^2 + (m^2_{\tilde{t}_1} - M^2_{\tilde{t}_{LL}})^2}}. \tag{7}$$

The mass eigenvalues are

$$m^2_{\tilde{t}_{1,2}} = \frac{1}{2}\left((M^2_{\tilde{t}_{LL}} + M^2_{\tilde{t}_{RR}}) \mp \sqrt{(M^2_{\tilde{t}_{LL}} - M^2_{\tilde{t}_{RR}})^2 + 4|M^2_{\tilde{t}_{LR}}|^2}\right). \tag{8}$$

$\tilde{t}_L - \tilde{t}_R$ mixing is naturally large because of the large top quark mass entering in the off–diagonal elements of the mass matrix (see Eqs. (1) and (4)). This is important for the CP sensitive observable discussed below, because it is proportional to $\sin\theta_{\tilde{t}}\cos\theta_{\tilde{t}}$. Note further that for $|A_t| \gg |\mu|\cot\beta$ we have $\varphi_{\tilde{t}} \approx \varphi_{A_t}$.

3 Tau Polarization Asymmetry

The parts of the Lagrangian relevant for the three–body decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$ are

$$\mathcal{L}_{b\tilde{\chi}_j\tilde{t}_1} = g\bar{b}(k^{\tilde{t}}_{1j}P_L + l^{\tilde{t}}_{1j}P_R)\tilde{\chi}^{c+}_j\tilde{t}_1 + h.c., \tag{9}$$

$$\mathcal{L}_{\tau\tilde{\chi}_j\tilde{\nu}_\tau} = g\overline{\tilde{\chi}^{c+}_j}(k^{\tilde{\nu}_\tau\,*}_j P_R + l^{\tilde{\nu}_\tau\,*}_j P_L)\tau\;\tilde{\nu}^*_\tau + h.c., \tag{10}$$

where

$$l^{\tilde{t}}_{1j} = -e^{-i\varphi_{\tilde{t}}}\cos\theta_{\tilde{t}}V_{j1} + Y_t\sin\theta_{\tilde{t}}V_{j2}, \qquad k^{\tilde{t}}_{1j} = Y_b e^{-i\varphi_{\tilde{t}}}\cos\theta_{\tilde{t}}U^*_{j2}, \tag{11}$$

$$l^{\tilde{\nu}_\tau}_j = -V_{j1}, \qquad k^{\tilde{\nu}_\tau}_j = Y_\tau U^*_{j2}. \tag{12}$$

g is the weak coupling constant, $P_{R,L} = 1/2(1 \pm \gamma_5)$ and the Yukawa couplings are $Y_t = m_t/\sqrt{2}m_W\sin\beta$, $Y_b = m_b/\sqrt{2}m_W\cos\beta$, $Y_\tau = m_\tau/\sqrt{2}m_W\cos\beta$. The unitary 2×2 matrices U and V diagonalize the chargino mass matrix [12]. A convenient parametrization of U and V for complex parameters can be found in [13].

The three body decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$ proceeds via exchange of charginos $\tilde{\chi}^\pm_i, i = 1, 2$. We consider the polarization of the outgoing τ^+ perpendicular to the decay plane, which is sensitive to CP violation. We define the unit vector

$$\vec{e}_N = \frac{\vec{p}_\tau \times \vec{p}_b}{|\vec{p}_\tau \times \vec{p}_b|}. \tag{13}$$

The average polarization of the outgoing τ^+ in the direction $\vec{e}_N$ in the decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$ is given by

$$\mathcal{P}^{\tau^+}_N = \frac{B(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+(\vec{e}_N)) - B(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+(-\vec{e}_N))}{B(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+(\vec{e}_N)) + B(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+(-\vec{e}_N))}. \tag{14}$$

For the CP conjugated process we get

$$\mathcal{P}^{\tau^-}_N = \frac{B(\bar{\tilde{t}}_1 \to \bar{b}\bar{\tilde{\nu}}_\tau\tau^-(\vec{e}_N)) - B(\bar{\tilde{t}}_1 \to \bar{b}\bar{\tilde{\nu}}_\tau\tau^-(-\vec{e}_N))}{B(\bar{\tilde{t}}_1 \to \bar{b}\bar{\tilde{\nu}}_\tau\tau^-(\vec{e}_N)) + B(\bar{\tilde{t}}_1 \to \bar{b}\bar{\tilde{\nu}}_\tau\tau^-(-\vec{e}_N))}. \tag{15}$$

Note that $\mathcal{P}^{\tau^-}_N = -\mathcal{P}^{\tau^+}_N$ since the couplings are complex conjugate to each other. The observables $\mathcal{P}^{\tau^+}_N$ and $\mathcal{P}^{\tau^-}_N$ are odd under naive time reversal $T_{\mathcal{N}}$, where only the polarization and momentum vectors are reversed but initial and final states are not interchanged. We can define a CP sensitive asymmetry of the form

$$\mathcal{A}_{CP} = \frac{1}{2}(\mathcal{P}^{\tau^+}_N - \mathcal{P}^{\tau^-}_N). \tag{16}$$

In order to obtain a CP asymmetry which for practical reasons is more useful, we have to take into account the subsequent decay $\tilde{\nu}_\tau \to \tilde{\chi}^0_1 \nu_\tau$. The complete decay chain is then $\tilde{t}_1 \to b\tilde{\nu}_\tau \tau^+ \to b\tau^+\tilde{\chi}^0_1\nu_\tau$. Another decay chain leading to the same final state is $\tilde{t}_1 \to b\tilde{\tau}_i\nu_\tau \to b\tau\tilde{\chi}^0_1\nu_\tau$, where in the second step the $\tilde{\tau}_i$ decays into $\tau\tilde{\chi}^0_1$. In the $\tilde{\tau}_i$ rest system $\tilde{\tau}_i \to \tau\tilde{\chi}^0_1$ has an isotropic decay distribution. The decay mode $\tilde{t}_1 \to b\tilde{\tau}_i\nu_\tau \to b\tau\tilde{\chi}^0_1\nu_\tau$ can be easily incorporated in our consideration.

However, the decay $\tilde{t}_1 \to bW\tilde{\chi}^0_1 \to b\tau\tilde{\chi}^0_1\nu_\tau$ also leads to the same final state [14]. The decay $\tilde{t}_1 \to bW\tilde{\chi}^0_1 \to b\tau\tilde{\chi}^0_1\nu_\tau$ is more involved, because the W polarization leads to a non-vanishing correlation between the $\vec{p}_\tau - \vec{p}_b$ plane and the τ polarization. While this may also lead to CP sensitive effects, in the present paper we confine ourselves to the discussion of the CP asymmetry in the decay chains $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau \to b\tau\tilde{\chi}^0_1\nu_\tau$ and $\tilde{t}_1 \to b\tilde{\tau}_i\nu_\tau \to b\tau\tilde{\chi}^0_1\nu_\tau$, assuming that $m_{\tilde{t}_1} < m_W + m_{\tilde{\chi}^0_1} + m_b$.

We define a CP asymmetry similar to Eq. (16), but for the final state $b\tau\tilde{\chi}^0_1\nu_\tau$:

$$\mathcal{A}'_{CP} = \frac{1}{2}(\mathcal{P}'^{\tau^+}_N - \mathcal{P}'^{\tau^-}_N), \tag{17}$$

with

$$\mathcal{P}'^{\tau^+}_N = \frac{B(\tilde{t}_1 \to f(\vec{e}_N)) - B(\tilde{t}_1 \to f(-\vec{e}_N))}{B(\tilde{t}_1 \to f(\vec{e}_N)) + B(\tilde{t}_1 \to f(-\vec{e}_N))}, \quad \mathcal{P}'^{\tau^-}_N = \frac{B(\bar{\tilde{t}}_1 \to \bar{f}(\vec{e}_N)) - B(\bar{\tilde{t}}_1 \to \bar{f}(-\vec{e}_N))}{B(\bar{\tilde{t}}_1 \to \bar{f}(\vec{e}_N)) + B(\bar{\tilde{t}}_1 \to \bar{f}(-\vec{e}_N))}, \tag{18}$$

where we have introduced a shorthand notation $f(\vec{e}_N) \equiv \tilde{\chi}^0_1\bar{\nu}_\tau b\, \tau^+(\vec{e}_N)$ and $\bar{f}(\vec{e}_N) \equiv \tilde{\chi}^0_1\nu_\tau\bar{b}\, \tau^-(\vec{e}_N)$. The branching ratio for the decay $\tilde{t}_1 \to \tilde{\chi}^0_1\nu_\tau b\, \tau$ is to a good approximation given by

$$B(\tilde{t}_1 \to \tilde{\chi}^0_1\nu_\tau b\, \tau) \simeq B(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau)B(\tilde{\nu}_\tau \to \tilde{\chi}^0_1\nu_\tau) + \sum_{i=1}^{2} B(\tilde{t}_1 \to b\tilde{\tau}_i\nu_\tau)B(\tilde{\tau}_i \to \tilde{\chi}^0_1\tau). \tag{19}$$

This can also be seen by using the formulas for the 4–body stop decays given in [10]. Relation (19) holds if $|m_{\tilde{\tau}_i} - m_{\tilde{\nu}_\tau}| \gg \Gamma_{\tilde{\tau}_i} + \Gamma_{\tilde{\nu}_\tau}$, which is naturally fulfilled. If $B(\tilde{\nu}_\tau \to \tilde{\chi}^0_1\nu_\tau) \simeq 1$ and $B(\tilde{\tau}_i \to \tilde{\chi}^0_1\tau^+) \simeq 1$, then Eq.(17) can be rewritten as

$$\mathcal{A}'_{CP} \simeq \frac{\Gamma(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+(\vec{e}_N)) - \Gamma(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+(-\vec{e}_N)) - \Gamma(\bar{\tilde{t}}_1 \to \bar{b}\bar{\tilde{\nu}}_\tau\tau^-(\vec{e}_N)) + \Gamma(\bar{\tilde{t}}_1 \to \bar{b}\bar{\tilde{\nu}}_\tau\tau^-(-\vec{e}_N))}{2\, \Gamma_{\mathrm{unpol}}}, \tag{20}$$

where $\Gamma_{\mathrm{unpol}} = \Gamma(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau) + \sum_{i=1}^2 \Gamma(\tilde{t}_1 \to b\tilde{\tau}_i\nu_\tau)$. A very useful approximation for $\mathcal{A}'_{CP}$ can be obtained in the limit $\varphi_\mu \to 0$

$$\mathcal{A}'_{CP} \simeq \frac{g^4}{\Gamma_{\mathrm{unpol}}}\; 2\, m_{\tilde{t}_1} Y_t Y_\tau |\sin 2\theta_{\tilde{t}}|\; |\mu|\; \sin\varphi_{\tilde{t}}\; \mathcal{I}, \tag{21}$$

where

$$\mathcal{I} = \frac{1}{2m_{\tilde{t}_1}} \int \frac{d^3p_b}{(2\pi)^3 2E_b}\frac{d^3p_\tau}{(2\pi)^3 2E_\tau}\frac{d^3p_{\tilde{\nu}_\tau}}{(2\pi)^3 2E_{\tilde{\nu}_\tau}}(2\pi)^4\delta(p_{\tilde{t}_1} - p_b - p_\tau - p_{\tilde{\nu}_\tau})\frac{|\vec{p}_\tau||\vec{p}_b|\sin\theta_{b\tau}}{(p^2_{\tilde{\chi}^\pm} - m^2_{\tilde{\chi}^\pm_1})(p^2_{\tilde{\chi}^\pm} - m^2_{\tilde{\chi}^\pm_2})}, \tag{22}$$

where $p_{\tilde{t}_1} = (m_{\tilde{t}_1}, \vec{0}), p_{\tilde{\chi}^\pm} = p_{\tilde{t}_1} - p_b$, $m_{\tilde{\chi}^\pm_{1,2}}$ are the masses of the charginos and $\sin\theta_{b\tau}$ is the angle between the b quark and the τ lepton. Eq. (21) is very instructive, because it exhibits the main features of the behaviour of $\mathcal{A}'_{CP}$. For example, the sign of $\mathcal{A}'_{CP}$ is given by $sgn(\mathcal{A}'_{CP}) = sgn(\varphi_{\tilde{t}})$. The unpolarized decay width Γ_{unpol} appearing in Eq. (20) and (21) is given in [8] for real parameters and in [15] for complex parameters.

Finally, we want to remark that the absorptive part of the amplitude caused by the Breit–Wigner form of the chargino propagators is strongly suppressed, because we have assumed that the decay $\tilde{t}_1 \to \tilde{\chi}^+_1 b$ is not accessible. Therefore the rate asymmetry is expected to be at most of the order of $\approx 10^{-3}$.

4 Numerical Results

In the following we present numerical results for the CP sensitive asymmetry $\mathcal{A}'_{CP}$ defined in Eqs. (17) and (18). Our input parameters are $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{\nu}_\tau}, |A_t|, \varphi_{A_t}, A_\tau, \tan\beta, M_2, |\mu|$, assuming $M_1 = 5/3\tan^2\Theta_W\, M_2$, with M_1 and M_2 real. For simplicity we set $\varphi_\mu = 0$. We impose the approximate necessary condition for the tree–level vacuum stability $|A_t|^2 < 3(M_{\tilde{Q}}^2 + M_{\tilde{U}}^2 + (m_A^2 + m_Z^2)\cos^2\beta - m_Z^2/2)$ [16]. For the pseudoscalar Higgs mass which appears in this condition we choose for definiteness $m_A = 150$ GeV. We have checked that in the numerical examples studied below the restrictions from the electron and neutron EDMs at two–loop level [5] are fulfilled.

First we consider the influence of the parameter φ_{A_t}. In Figs. 1a and 1b we show $\mathcal{A}'_{CP}$ as a function of φ_{A_t} for $M_{\tilde{Q}} > M_{\tilde{U}}$ ($|\cos\theta_{\tilde{t}}| < |\sin\theta_{\tilde{t}}|$) and $M_{\tilde{Q}} < M_{\tilde{U}}$ ($|\cos\theta_{\tilde{t}}| > |\sin\theta_{\tilde{t}}|$), respectively. We display the asymmetry for the four scenarios ($|\mu| = 400$ GeV, $\tan\beta = 3$), ($|\mu| = 400$ GeV, $\tan\beta = 10$), ($|\mu| = 700$ GeV, $\tan\beta = 3$) and ($|\mu| = 700$ GeV, $\tan\beta = 10$), taking $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $M_2 = 350$ GeV. We fix the mass of $\tilde{\tau}_1$ by taking $M_{\tilde{E}} = 0.9 M_{\tilde{L}}$. As can be seen, the CP asymmetry $\mathcal{A}'_{CP}$ is much larger for $M_{\tilde{Q}} > M_{\tilde{U}}$ than for $M_{\tilde{Q}} < M_{\tilde{U}}$. The reason is, that for $M_{\tilde{Q}} < M_{\tilde{U}}$ the $\tilde{t}_1$ has a larger $\tilde{t}_L$ component than for $M_{\tilde{Q}} > M_{\tilde{U}}$, implying a stronger coupling to the gaugino component of the charginos. This in turn implies a larger $\Gamma_{\rm unpol}$ for $M_{\tilde{Q}} < M_{\tilde{U}}$. Cearly $\mathcal{A}'_{CP}$ has to vanish for the CP conserving case $\varphi_{A_t} = 0, \pm\pi$ (see the corresponding factor in Eq.(21)). For better understanding of the value of $\mathcal{A}'_{CP}$ in the region $|\varphi_{A_t}| \lesssim \pi/4$, we show in Fig. 2 the branching ratio $B(\tilde{t}_1 \to \tilde{\chi}_1^0 \nu_\tau b\tau)$ as a function of φ_{A_t}, for the two scenarios ($|\mu| = 400$ GeV, $\tan\beta = 3$) and ($|\mu| = 700$ GeV, $\tan\beta = 3$), keeping the other parameters as in Fig. 1a. The decay width $\Gamma(\tilde{t}_1 \to \tilde{\chi}_1^0 \nu_\tau b\tau)$ has a similar behaviour. The minimum at $\varphi_{A_t} = 0$ can be traced back to the fact that $|l_{11}^{\tilde{t}}|$ (see Eq. (11)) has a minimum there, due to a negative interference between the gaugino and higgsino contribution.

In Fig. 3 we show $\mathcal{A}'_{CP}$ for three values of $|A_t| = 600$ GeV, 1000 GeV and 1300 GeV, assuming $M_{\tilde{Q}} > M_{\tilde{U}}$. The other parameters are $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $A_\tau = 0$, $M_2 = 350$ GeV, $|\mu| = 600$ GeV and $\tan\beta = 3$. For $|A_t| \lesssim 1000$ GeV $\mathcal{A}'_{CP}$ increases with increasing $|A_t|$, because $|\sin 2\theta_{\tilde{t}}|$ increases (see Eq. (21)). The decrease of $\mathcal{A}'_{CP}$ for $|A_t| \gtrsim 1000$ GeV is explained by the fact that $\Gamma_{\rm unpol}$ increases stronger than $|\sin 2\theta_{\tilde{t}}|$.

The polarization of the τ is analysed through its decay distributions. Usually the decay modes $\tau \to \pi\nu, \rho\nu, a_1\nu, \mu\nu\bar{\nu}, e\nu\bar{\nu}$ are used as analyzers. As we are interested in the transverse polarisation of the τ lepton, we take only the $\rho\nu$ and $a_1\nu$ final states for our analysis. The sum of the branching ratios of these two decay modes is $\approx 34\%$. We take for the sensitivities for measuring the polarization of the τ lepton the values quoted in Ref. [17] for an ideal detector. Moreover, the numbers quoted are for longitudinal tau polarization and it is expected that the sensitivities for transversely polarized tau leptons are somewhat smaller. To account for both effects we assume a reduction of the sensitivity S of 10% [18]. Furthermore, we assume that the direction of flight of the τ lepton can be reconstructed. Following [17], the error in measuring the polarization asymmetry is given by

$$\delta\mathcal{P}'^{\tau^-}_N = \frac{1}{S_{\rm red}\sqrt{N_\tau}}, \tag{23}$$

where $S_{\rm red} = 0.3, N_\tau = B(\tilde{t}_1 \to \tilde{\chi}_1^0 \nu_\tau b\tau) N_{\tilde{t}_1}$, with $N_{\tilde{t}_1}$ being the number of produced $\tilde{t}_1\bar{\tilde{t}}_1$ pairs in the process $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$.

The minimum number $N_{\tilde{t}_1}$ of produced $\tilde{t}_1$ pairs necessary to observe $\mathcal{A}'_{CP}$ at 90% confidence level (CL) is then given by

$$N_{\tilde{t}_1} = \frac{1}{B(\tilde{t}_1 \to \tilde{\chi}_1^0 \nu_\tau b\tau)} \left(\frac{1.64}{S_{\rm red}|\mathcal{A}'_{CP}|}\right)^2. \tag{24}$$

In Table 1 we display the values of the asymmetry $\mathcal{A}'_{CP}$ and the numbers $N_{\tilde{t}_1}$ needed to observe this asymmetry at 90% CL.

Table 1: The CP asymmetry $\mathcal{A}'_{CP}$ defined in Eq.(17) and the number $N_{\tilde{t}_1}$ of $\tilde{t}_1\bar{\tilde{t}}_1$ pairs required to measure this CP asymmetry at 90% CL, choosing $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $M_2 = 350$ GeV and $|A_t| = 1000$ GeV. For $M_{\tilde{Q}} > M_{\tilde{U}}$ we take ($|\mu| = 700$ GeV, $\tan\beta = 3$) and for $M_{\tilde{Q}} < M_{\tilde{U}}$ we take ($|\mu| = 400$ GeV, $\tan\beta = 10$).

$\mathcal{A}'_{CP}$	$N_{\tilde{t}_1} \times 10^{-3}$	φ_{A_t}	
0.05	31	$\pi/2$	$M_{\tilde{Q}} > M_{\tilde{U}}$
0.11	7	$\pi/4$	$M_{\tilde{Q}} > M_{\tilde{U}}$
0.21	4	$\pi/8$	$M_{\tilde{Q}} > M_{\tilde{U}}$
0.033	48	$\pi/2$	$M_{\tilde{Q}} < M_{\tilde{U}}$
0.030	58	$\pi/4$	$M_{\tilde{Q}} < M_{\tilde{U}}$
0.018	165	$\pi/8$	$M_{\tilde{Q}} < M_{\tilde{U}}$

The cross section for $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$ for $m_{\tilde{t}_1} = 240$ GeV is about $5fb$ at $\sqrt{s} = 500$ GeV and in the range 15 – $120fb$ at $\sqrt{s} = 800$ GeV, depending on stop mixing and beam polarization. If the branching ratio $B(\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+)$ is of the order of 5% or larger, then there are good prospects to measure the CP asymmetry $\mathcal{A}'_{CP}$ at a linear collider with an integrated luminosity of $500fb^{-1}$.

5 Summary

In this paper we have studied the CP violating observable defined by the asymmetry of the τ polarization perpendicular to the decay plane in the three–body decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$. In the parameter domain $m_{\tilde{t}_1} < m_{\tilde{\chi}_1^\pm} + m_b$, $m_{\tilde{t}_1} < m_W + m_{\tilde{\chi}_1^0} + m_b$ the branching ratio for this decay mode can be quite large. We have calculated this CP asymmetry and the branching ratios in the MSSM with the parameters μ and A_t complex. We give numerical predictions for the case of an e^+e^- linear collider with cms energy $\sqrt{s} = 0.5$ – 1 TeV. The asymmetry can reach values up to $\pm 30\%$. We give a theoretical estimate of the number of produced $\tilde{t}_1\bar{\tilde{t}}_1$ pairs necessary for measuring this asymmetry at 90% CL. There are good prospects to measure this CP asymmetry at a linear collider with an integrated luminosity of 500 fb^{-1}, if the branching ratio of the decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$ is about 5% or larger.

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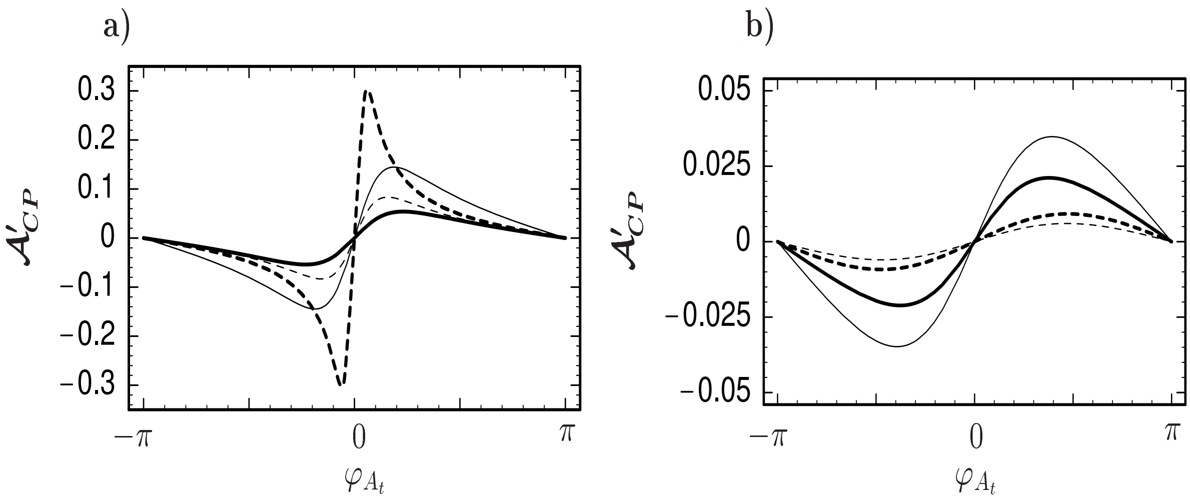

Figure 1: The CP sensitive asymmetry $\mathcal{A}'_{CP}$ as a function of φ_{A_t}. The input parameters are $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $M_2 = 350$ GeV, $|A_t| = 1000$ GeV, ($\tan\beta = 3, |\mu| = 400$ GeV; thick solid line), ($\tan\beta = 10, |\mu| = 400$ GeV; thin solid line), ($\tan\beta = 3, |\mu| = 700$ GeV; thick dashed line), ($\tan\beta = 10, |\mu| = 700$ GeV; thin dashed line), for the cases: a)$M_{\tilde{Q}} > M_{\tilde{U}}$ and b)$M_{\tilde{Q}} < M_{\tilde{U}}$;

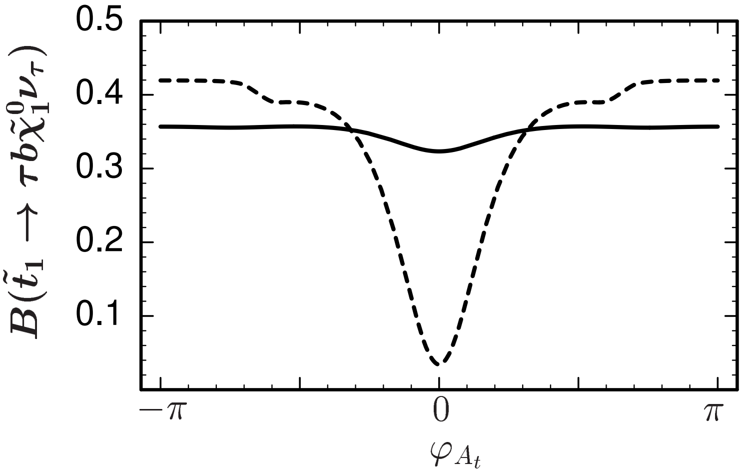

Figure 2: The branching ratio of $\tilde{t}_1 \to \tau b\tilde{\chi}_1^0\nu_\tau$ as a function of φ_{A_t}, with $M_{\tilde{Q}} > M_{\tilde{U}}, m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $\tan\beta = 3, M_2 = 350$ GeV, $|A_t| = 1000$ GeV for the two cases $|\mu| = 400$ GeV (solid line), $|\mu| = 700$ GeV (dashed line).

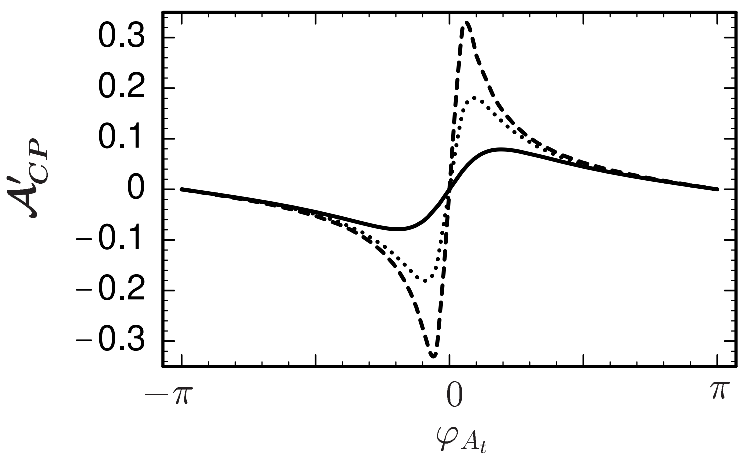

Figure 3: The CP sensitive asymmetry $\mathcal{A}'_{CP}$ as a function of φ_{A_t}. The input parameters are $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV,$m_{\tilde{\nu}_\tau} = 200$ GeV, $M_2 = 350$ GeV, $\tan\beta = 3, |\mu| = 600$ GeV for $|A_t| = 600$ GeV (solid line), $|A_t| = 1000$ GeV (dashed line), $|A_t| = 1300$ GeV (dotted line).