LC-TH-2003-046

Discriminating graviton exchange effects from other new physics scenarios in e^+e^- collisions

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Abstract

We study the possibility of uniquely identifying the effects of graviton exchange from other new physics in high energy e^+e^- annihilation into fermion-pairs. For this purpose, we use as basic observable a specific asymmetry among integrated differential distributions, that seems particularly suitable to directly test for such gravitational effects in the data analysis.

1 Introduction

All types of new physics (NP) scenarios are determined by non-standard dynamics involving new building blocks and forces mediated by exchange corresponding to heavy states with mass scales Λ much greater than M_W. Unambiguous confirmation of such dynamics would require the experimental discovery of the envisaged new heavy objects and the measurement of their coupling constants to ordinary quarks and leptons. While there is substantial belief that the supersymmetric partners of the Standard Model (SM) particles should be directly produced, and identified, at future proton–proton and electron–positron high energy colliders such as the LHC and the Linear Collider (LC), in the other cases the current experimental limits on the new, heavy particles are so high, of the order of several (or tens of) TeV, that one cannot expect them to be directly produced at the energies foreseen for these machines. In this situation, the new interactions can manifest themselves only by indirect, virtual, effects represented by deviations of the measured observables from the SM numerical predictions. The problem, then, is to identify from the data analysis the possible new interactions, because different NP scenarios can in principle cause similar measurable deviations, and for this purpose suitable observables must be defined.

At "low" energies (compared to the above-mentioned large mass scales) the physical effects of the new interactions are conveniently accounted for, in reactions involving the familiar quarks and leptons, by effective *contact-interaction* (CI) Lagrangians that provide the expansion of the relevant transition amplitudes to leading order in the small ratio $\sqrt{s}/\Lambda$ ($\sqrt{s}$ being the c.m. energy).

Familiar classes of contact interactions are represented by composite models of quarks and leptons [1, 2]; exchanges of very heavy Z' with a few TeV mass [3, 4] and of scalar and vector heavy leptoquarks [5]; in the SUSY context, R-parity breaking interactions mediated by sneutrino exchange [6, 7]; bi-lepton boson exchanges [8]; anomalous gauge boson couplings (AGC) [9]; virtual Kaluza–Klein (KK) graviton exchange in the context of gravity propagating in large extra dimensions, exchange of gauge boson KK towers or string excitations, *etc.* [10–15]. Of course, this list is not exhaustive, because other kinds of *contact interactions* may well exist.

In this note, we briefly discuss the deviations induced by *contact interactions* in the electron–positron annihilation into fermion pairs at the planned Linear Collider energies [16, 17]. In particular, we propose a simple observable that can be used to unambiguously identify graviton KK tower exchange effects in the data, relying on its spin-two character and by "filtering" out contributions of other NP interactions.

If deviations from the SM predictions were effectively measured, the identification of the NP source could be attempted by Monte Carlo best fits of the observed effects, and this would apply also to graviton exchange [14]. Alternatively, moments of the differential cross section folded with Legendre polynomial weights appear to be a promising technique to pin down NP effects in the case of electron–positron reactions induced at the SM level by s-channel exchanges [18]. Here, we shall consider a suitably defined combination of integrated cross sections, the so-called "center–edge" asymmetry A_{CE}, that allows to disentangle the graviton exchange in a very simple, and efficient, way. Specifically, in Sect. 2 we present the required kinematical details and discuss the properties of $A_{\rm CE}$, in Sect. 3 we discuss beam polarization, in Sect. 4 we evaluate the sensitivity of this observable to the characteristic mass parameter of the graviton KK tower exchange, in Sect. 5 we find the corresponding identification reaches and discuss an application to sneutrino exchange, differentiating it from KK graviton exchange and, finally, Sect. 6 is devoted to some comments and conclusive remarks.

2 The center–edge asymmetry $A_{\rm CE}$

We consider the process (with $f \neq e, t$)

$$e^+ + e^- \to f + \bar{f}, \tag{1}$$

and, neglecting all fermion masses with respect to $\sqrt{s}$, we can write the differential angular distribution for unpolarized e^+e^- beams in terms of s-channel γ and Z exchanges plus any *contact-interaction* terms in the following form [19]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{1}{4}\left(\frac{\mathrm{d}\sigma_{\rm LL}}{\mathrm{d}z} + \frac{\mathrm{d}\sigma_{\rm RR}}{\mathrm{d}z} + \frac{\mathrm{d}\sigma_{\rm LR}}{\mathrm{d}z} + \frac{\mathrm{d}\sigma_{\rm RL}}{\mathrm{d}z}\right). \tag{2}$$

Here, $z \equiv \cos\theta$, with θ the angle between the incoming electron and the outgoing fermion in the c.m. frame, and $\mathrm{d}\sigma_{\alpha\beta}/\mathrm{d}\cos\theta$ ($\alpha, \beta = \mathrm{L}, \mathrm{R}$) are the helicity cross sections given by:

$$\frac{\mathrm{d}\sigma_{\alpha\beta}}{\mathrm{d}z} = N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,|\mathcal{M}_{\alpha\beta}|^2\,(1 \pm z)^2, \tag{3}$$

where the two signs $\pm$ correspond to the LL, RR, and LR, RL, helicity configurations, respectively, and $N_C \simeq 3(1 + \alpha_s/\pi)$ represents the number of colours of the final state, including the first-order QCD correction. The helicity amplitudes $\mathcal{M}_{\alpha\beta}$ can be written as

$$\mathcal{M}_{\alpha\beta} = \mathcal{M}_{\alpha\beta}^{\rm SM} + \Delta_{\alpha\beta} = Q_e Q_f + g_\alpha^e\, g_\beta^f\, \chi_Z + \Delta_{\alpha\beta}, \tag{4}$$

where: $\chi_Z = s/(s - M_Z^2 + iM_Z\Gamma_Z) \approx s/(s - M_Z^2)$ represents the Z propagator; $g_{\rm L}^f = (I_{3L}^f - Q_f s_W^2)/s_W c_W$ and $g_{\rm R}^f = -Q_f s_W/c_W$ are the SM left- and right-handed fermion couplings of the Z with $s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W$; Q_e and Q_f are the fermion electric charges. The $\Delta_{\alpha\beta}$ functions represent the contact interaction contributions coming from TeV-scale physics.

The structure of the differential cross section (2)–(4) is particularly interesting in that it is equally valid for a wide variety of New Physics (NP) models listed in Table 1. Note that only graviton exchange induces a modified angular dependence to the differential cross section via its z-dependence of $\Delta_{\alpha\beta}$.

We define the generalized center–edge asymmetry $A_{\rm CE}$ as [20]:

$$A_{\rm CE} = \frac{\sigma_{\rm CE}}{\sigma}, \tag{5}$$

in terms of the difference between the central and edge parts of the cross section

$$\sigma_{\rm CE} = \left[\int_{-z^*}^{z^*} - \left(\int_{-1}^{-z^*} + \int_{z^*}^{1}\right)\right] \frac{{\rm d}\sigma}{{\rm d}z}\, {\rm d}z, \tag{6}$$

and the total cross section

$$\sigma = \int_{-1}^{1} \frac{{\rm d}\sigma}{{\rm d}z}\, {\rm d}z, \tag{7}$$

and $0 < z^* < 1$.[1]

Table 1: Parametrization of the $\Delta_{\alpha\beta}$ functions in different models ($\alpha, \beta = {\rm L}, {\rm R}$).

Model	$\Delta_{\alpha\beta}$
composite fermions [2]	$\pm \frac{s}{\alpha_{\rm e.m.}} \frac{1}{\Lambda^2_{\alpha\beta}}$
extra gauge boson Z' [3, 4]	$g'^{\,e}_\alpha\, g'^{\,f}_\beta\, \chi_{Z'}$
AGC ($f = \ell$) [9]	$\Delta_{\rm LL} = s\left(\frac{\tilde{f}_{DW}}{2s^2_W} + \frac{2\tilde{f}_{DB}}{c^2_W}\right),\ \frac{\Delta_{\rm RR}}{2} = \Delta_{\rm LR} = \Delta_{\rm RL} = s\frac{4\tilde{f}_{DB}}{c^2_W}$
TeV-scale extra dim. [14, 15]	$(Q_e Q_f + g^e_\alpha\, g^f_\beta)\, \frac{\pi^2}{3\, M^2_C}$
ADD model [10, 12]	$\Delta_{\rm LL} = \Delta_{\rm RR} = f_G\,(1 - 2\,z),\ \Delta_{\rm LR} = \Delta_{\rm RL} = -f_G\,(1 + 2\,z)$

In Table 1 $\Lambda_{\alpha\beta}$ are compositeness scales; $\chi_{Z'}$ is the Z' propagator defined according to χ_Z; $\tilde{f}_{DW}$ and $\tilde{f}_{DB}$ are related to f_{DW} and f_{DB} of ref. [9] by $\tilde{f} = f/m_t^2$ (f_{DW} and f_{DB} parametrize new-physics effects associated with the SU(2) and hypercharge currents, respectively); M_C is the compactification scale; finally, $f_G = \lambda\, s^2/(4\pi\alpha_{\rm e.m.} M_H^4)$ parametrizes the strength associated with massive graviton exchange with M_H the cut-off scale in the KK graviton tower sum. Note that, compared with, e.g., the composite fermion case, the KK graviton effect is suppressed by the (larger) power $(\sqrt{s}/M_H)^4$, so that a lower reach on M_H can be expected in comparison to the constraints obtainable, at the same c.m. energy, on Λ's. The effect of the extra dimensional model [14] is s-independent, and the sign of $\Delta_{\alpha\beta}$ is fixed.

First, let us consider graviton exchange effects. For definiteness we consider the ADD model [10]. From Eqs. (2)–(7) and Table 1 one can derive the asymmetry $A_{\rm CE}$ for the process (1) including graviton tower exchange:

$$A_{\rm CE} = \frac{\sigma^{\rm SM}_{\rm CE} + \sigma^{\rm INT}_{\rm CE} + \sigma^{\rm NP}_{\rm CE}}{\sigma^{\rm SM} + \sigma^{\rm INT} + \sigma^{\rm NP}}, \tag{8}$$

[1] The center–edge asymmetry $A_{\rm CE}$ for W-pair production and fixed $z^* = 0.5$ has been introduced in [21].

where "SM", "INT" and "NP" refer to "Standard Model", "Interference" and (pure) "New Physics" contributions. Explicitly, we have

$$\begin{aligned}
\sigma_{\rm CE}^{\rm SM} =& N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,\frac{1}{4}\left[(\mathcal{M}_{\rm LL}^{\rm SM})^2+(\mathcal{M}_{\rm RR}^{\rm SM})^2+(\mathcal{M}_{\rm LR}^{\rm SM})^2+(\mathcal{M}_{\rm RL}^{\rm SM})^2\right]\,\frac{4}{3}\left[z^*(z^{*2}+3)-2\right],\\
\sigma_{\rm CE}^{\rm INT} =& N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,2\,f_G\,\frac{1}{4}\left[\mathcal{M}_{\rm LL}^{\rm SM}+\mathcal{M}_{\rm RR}^{\rm SM}-\mathcal{M}_{\rm LR}^{\rm SM}-\mathcal{M}_{\rm RL}^{\rm SM}\right]\,4z^*(1-z^{*2}),\\
\sigma_{\rm CE}^{\rm NP} =& N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,f_G^2\,\frac{4}{5}\left[4z^{*5}+5z^*(1-z^{*2})-2\right],
\end{aligned}\tag{9}$$

with

$$\begin{aligned}
\sigma^{\rm SM} =& N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,\frac{1}{4}\left[(\mathcal{M}_{\rm LL}^{\rm SM})^2+(\mathcal{M}_{\rm RR}^{\rm SM})^2+(\mathcal{M}_{\rm LR}^{\rm SM})^2+(\mathcal{M}_{\rm RL}^{\rm SM})^2\right]\,\frac{8}{3},\\
\sigma^{\rm INT} =& 0, \qquad \sigma^{\rm NP}=N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,f_G^2\,\frac{8}{5}.
\end{aligned}\tag{10}$$

Note that, at $z^*=0$ and 1, $\sigma_{\rm CE}=\mp\sigma$, respectively.

In the case of the SM the center–edge asymmetry $A_{\rm CE}^{\rm SM}$ can be obtained from Eqs. (8)–(10) taking $f_G=0$:

$$A_{\rm CE}^{\rm SM}=\frac{\sigma_{\rm CE}^{\rm SM}}{\sigma^{\rm SM}}=\frac{1}{2}\,z^*\,(z^{*2}+3)-1. \tag{11}$$

It is interesting to note that in Eq. (11) the helicity amplitudes in the numerator and denominator cancel and only a ratio of kinematical factors remains in the limit of neglecting external fermion masses. In addition, $A_{\rm CE}^{\rm SM}$ is independent of energy and of the flavour of the final-state fermions. It contains *only* the kinematical variable z^*. Fig. 1 shows $A_{\rm CE}^{\rm SM}$ as a function of z^*. From Eq. (11) one can determine the value of z^* where $A_{\rm CE}^{\rm SM}$ vanishes [22],

$$z_0^*=(\sqrt{2}+1)^{1/3}-(\sqrt{2}-1)^{1/3}=0.596, \tag{12}$$

corresponding to $\theta=53.4^\circ$ (see the solid curve in Fig. 1).

Graviton exchange in the ADD model affects A_{CE} inducing a deviation from the SM prediction:

$$\Delta A_{\rm CE}=A_{\rm CE}-A_{\rm CE}^{\rm SM}. \tag{13}$$

For $(s/M_H^2)^2\ll 1$, it will be $\sigma_{\rm CE}^{\rm INT}$ which will produce the largest deviation from the expectations of the SM, since this term is of order $(\sqrt{s}/M_H)^4$, whereas the pure NP contribution proportional to f_G^2 in Eqs. (9) and (10) is of the much higher order $(\sqrt{s}/M_H)^8$. Taking into account only SM-NP interference terms, one derives:

$$\Delta A_{\rm CE}\simeq f_G\,\frac{\mathcal{M}_{\rm LL}^{\rm SM}+\mathcal{M}_{\rm RR}^{\rm SM}-\mathcal{M}_{\rm LR}^{\rm SM}-\mathcal{M}_{\rm RL}^{\rm SM}}{\left[(\mathcal{M}_{\rm LL}^{\rm SM})^2+(\mathcal{M}_{\rm RR}^{\rm SM})^2+(\mathcal{M}_{\rm LR}^{\rm SM})^2+(\mathcal{M}_{\rm RL}^{\rm SM})^2\right]}\,3\,z^*\,(1-z^{*2}). \tag{14}$$

For the lepton pair production process in the ADD model, the corresponding $A_{\rm CE}$ is shown in Fig. 1 for $M_H=1$ TeV and $\lambda=\pm1$. As one can see from Fig. 1, $\Delta A_{\rm CE}=0$ for

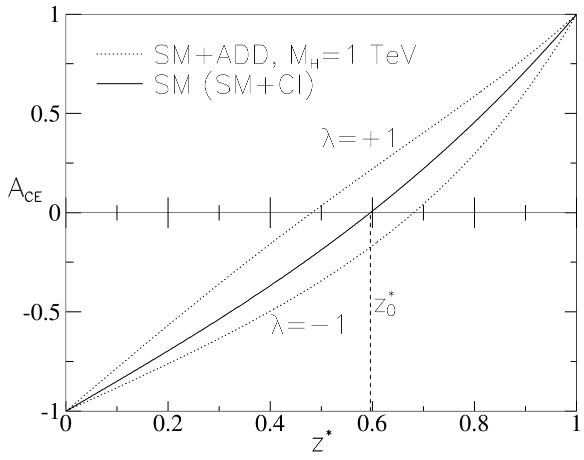

Figure 1: Tree-diagram result for A_{CE} for the process $e^+e^- \to l^+l^-$ $(l = \mu, \tau)$ as a function of z^* in the SM and in the ADD model with $M_H = 1$ TeV and $\lambda = \pm 1$.

$z^* = 0$ and 1. Clearly, in contrast to $A_{\rm CE}^{\rm SM}$, the $\Delta A_{\rm CE}$ of Eq. (14) depends on the flavour of the final-state fermion f.

To illustrate the effect of graviton exchange on the center–edge asymmetry, we show in Fig. 2 the z^*-distributions of the deviation $\Delta A_{\rm CE}$, taking as examples the values of M_H indicated in the caption. The deviation $\Delta A_{\rm CE}$ [including also the pure NP term in addition to the simple result of Eq. (14)] is compared to the expected statistical uncertainties, $\delta A_{\rm CE}$, represented by the vertical bars and given by

$$\delta A_{\rm CE} = \sqrt{\frac{1-(A_{\rm CE}^{\rm SM})^2}{\epsilon_f\,\mathcal{L}_{\rm int}\,\sigma^{\rm SM}}}. \tag{15}$$

Here, $\mathcal{L}_{\rm int}$ is the integrated luminosity, and ϵ_f the efficiency for reconstruction of $f\bar{f}$ pairs. We will assume that the efficiencies of identifying the final state fermions are rather high: 100% for $l = \mu$, τ, 80% for $f = b$, and 60% for $f = c$. Fig. 2 qualitatively indicates that, for the chosen values of the c.m. energy $\sqrt{s}$ and $\mathcal{L}_{\rm int}$, the reach on M_H will be of the order of 2.5 TeV.

Now, let us consider the conventional *contact-interaction*-like effects parametrized by z-independent $\Delta_{\alpha\beta}$ summarized in Table 1. Application of Eq. (5) to composite-like contact interactions is straightforward, the result can be written as:

$$A_{\rm CE}^{\rm SM+CI} = \frac{\sigma_{\rm CE}^{\rm SM+CI}}{\sigma^{\rm SM+CI}}, \tag{16}$$

where

$$\sigma_{\rm CE}^{\rm SM+CI} = N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,\frac{1}{4}\left[(\mathcal{M}_{\rm LL})^2 + (\mathcal{M}_{\rm RR})^2 + (\mathcal{M}_{\rm LR})^2 + (\mathcal{M}_{\rm RL})^2\right]\,\frac{4}{3}\left[z^*(z^{*2}+3)-2\right], \tag{17}$$

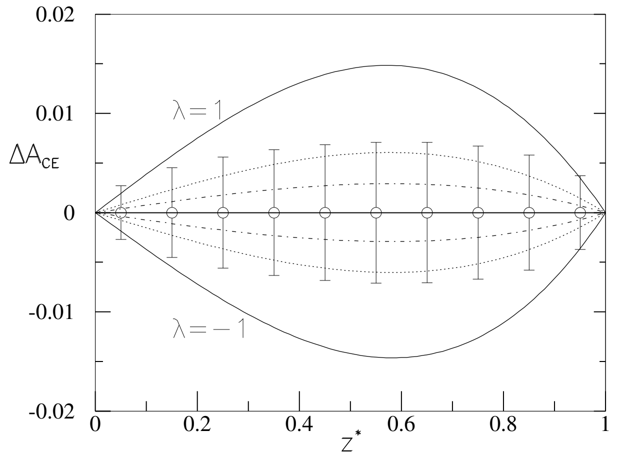

Figure 2: The deviation of $A_{\rm CE}$ [cf. Eq. (13)] from the SM (or SM+CI) expectations (at tree level) as a function of z^* for the process $e^+e^- \to l^+l^-$ for $M_H = 2$ (solid), 2.5 (dotted), and 3 TeV (dash-dotted), $\lambda = \pm 1$ and $\sqrt{s} = 0.5$ TeV. The expected statistical uncertainties at $\mathcal{L}_{\rm int} = 50$ fb^{-1} are shown as error bars.

and

$$\sigma^{\rm SM+CI} = N_C\,\frac{\pi\alpha^2_{\rm e.m.}}{2s}\,\frac{1}{4}\left[(\mathcal{M}_{\rm LL})^2 + (\mathcal{M}_{\rm RR})^2 + (\mathcal{M}_{\rm LR})^2 + (\mathcal{M}_{\rm RL})^2\right]\frac{8}{3}. \tag{18}$$

From Eqs. (16)–(18), one has

$$A_{\rm CE}^{\rm SM+CI} = \frac{1}{2}\,z^*\,(z^{*2}+3) - 1. \tag{19}$$

This result is *identical* to $A_{\rm CE}^{\rm SM}$ defined by Eq. (11)! In other words, $A_{\rm CE}$ has the form (19) in the SM and will remain so even if *contact-interaction*-like effects are present. Thus, conventional contact-interaction effects, being described by current–current interactions, yield *the same* center–edge asymmetry as the Standard Model. The reason is simply that both these interactions are described by vector currents, as opposed to the tensor couplings of gravity. The deviation of $A_{\rm CE}$ from the SM (and SM+CI) prediction is clearly a signal of the spin-2 particle exchange. Thus, it is clear that a non-zero value of $\Delta A_{\rm CE}$ can provide a clean signature for graviton, or more generally, spin-2 exchange in the process $e^+e^- \to \bar{f}f$.

3 Polarized beams

Let us now consider the case of longitudinally polarized beams, with P and $\bar{P}$ the degrees of polarization of the electron and positron beams, respectively. The polarized differential cross section can then be written as

$$\frac{{\rm d}\sigma}{{\rm d}z} = \frac{D}{4}\left[(1-P_{\rm eff})\left(\frac{{\rm d}\sigma_{\rm LL}}{{\rm d}z} + \frac{{\rm d}\sigma_{\rm LR}}{{\rm d}z}\right) + (1+P_{\rm eff})\left(\frac{{\rm d}\sigma_{\rm RR}}{{\rm d}z} + \frac{{\rm d}\sigma_{\rm RL}}{{\rm d}z}\right)\right], \tag{20}$$

where $D = 1 - P\bar{P}$ and $P_{\rm eff} = (P - \bar{P})/(1 - P\bar{P})$ is the effective polarization [23]. For example, $P_{\rm eff} = \pm 0.95$ and $D \approx 1.5$ for $P = \pm 0.8$ and $\bar{P} = \mp 0.6$.

In addition, in the case of a reduced kinematical region, with cuts around the beam pipe, $|z| \leq z_{\rm cut}$ $(0 < z_{\rm cut} < 1)$, one can define the generalized center–edge asymmetry $A_{\rm CE}$ as above, with Eqs. (6) and (7) replaced by

$$\sigma_{\rm CE} = \left[\int_{-z^*}^{z^*} - \left(\int_{-z_{\rm cut}}^{-z^*} + \int_{z^*}^{z_{\rm cut}}\right)\right] \frac{{\rm d}\sigma}{{\rm d}z}\,{\rm d}z, \tag{21}$$

and

$$\sigma = \int_{-z_{\rm cut}}^{z_{\rm cut}} \frac{{\rm d}\sigma}{{\rm d}z}\,{\rm d}z, \tag{22}$$

with $0 < z^* < z_{\rm cut}$.

Allowing for angular cuts, as discussed above, the asymmetry $A_{\rm CE}$ including graviton tower exchange can for polarized beams be expressed as given by Eq. (8), with

$$\begin{aligned}
\sigma_{\rm CE}^{\rm SM}(z^*, z_{\rm cut}) =& N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,\frac{D}{4}\left\{(1 - P_{\rm eff})\left[(\mathcal{M}_{\rm LL}^{\rm SM})^2 + (\mathcal{M}_{\rm LR}^{\rm SM})^2\right] + (1 + P_{\rm eff})\left[(\mathcal{M}_{\rm RR}^{\rm SM})^2 + (\mathcal{M}_{\rm RL}^{\rm SM})^2\right]\right\}\\
&\times F^{\rm SM}(z^*, z_{\rm cut}),\\
\sigma_{\rm CE}^{\rm INT}(z^*, z_{\rm cut}) =& N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,2\,f_G\,\frac{D}{4}\left[(1 - P_{\rm eff})\left(\mathcal{M}_{\rm LL}^{\rm SM} - \mathcal{M}_{\rm LR}^{\rm SM}\right) + (1 + P_{\rm eff})\left(\mathcal{M}_{\rm RR}^{\rm SM} - \mathcal{M}_{\rm RL}^{\rm SM}\right)\right]\\
&\times F^{\rm INT}(z^*, z_{\rm cut}),\\
\sigma_{\rm CE}^{\rm NP}(z^*, z_{\rm cut}) =& N_C\,\frac{\pi\alpha_{\rm e.m.}^2}{2s}\,f_G^2\,D\,F^{\rm NP}(z^*, z_{\rm cut}).
\end{aligned} \tag{23}$$

Here, the dependences on the parameter z^* and on the angular cut $z_{\rm cut}$, are given by

$$\begin{aligned}
F^{\rm SM}(z^*, z_{\rm cut}) =& \frac{2}{3}\left[2z^*(z^{*2} + 3) - z_{\rm cut}(z_{\rm cut}^2 + 3)\right],\\
F^{\rm INT}(z^*, z_{\rm cut}) =& 2\left[2z^*(1 - z^{*2}) - z_{\rm cut}(1 - z_{\rm cut}^2)\right],\\
F^{\rm NP}(z^*, z_{\rm cut}) =& \frac{2}{5}\left[8z^{*5} + 10z^*(1 - z^{*2}) - 4z_{\rm cut}^5 - 5z_{\rm cut}(1 - z_{\rm cut}^2)\right].
\end{aligned} \tag{24}$$

The total cross sections in the denominator of Eq. (8) can be derived from Eqs. (23) and (24):

$$\sigma^{\rm SM}(z_{\rm cut}) = \sigma_{\rm CE}^{\rm SM}(z^* = z_{\rm cut}), \quad \sigma^{\rm INT}(z_{\rm cut}) = \sigma_{\rm CE}^{\rm INT}(z^* = z_{\rm cut}), \quad \sigma^{\rm NP}(z_{\rm cut}) = \sigma_{\rm CE}^{\rm NP}(z^* = z_{\rm cut}). \tag{25}$$

From Eqs. (5), (17), (18) and (20)–(24) some immediate conclusions can be drawn. First, it is clear that in the case of longitudinally polarized beams and chosen cut around the beam pipe, $|z| \leq z_{\rm cut}$, the asymmetry $A_{\rm CE}$ within the SM and in any new physics scenario with Z' exchanges, and also in the four-fermion contact interaction scenario, is given by

$$A_{\rm CE}^{\rm SM} = A_{\rm CE}^{\rm SM+CI} = 2\,\frac{z^*(z^{*2} + 3)}{z_{\rm cut}(z_{\rm cut}^2 + 3)} - 1. \tag{26}$$

Secondly, the center–edge asymmetry (26) is identical to that for unpolarized beams, see Eqs. (11) and (19), for $z_{\rm cut}=1$. Third, the asymmetry (26) is independent of energy $\sqrt{s}$, flavour of the final-state fermion f, and of the SM and NP parameters. Moreover, there is a value z_0^* for which $A_{\rm CE}^{\rm SM}$ vanishes. One obtains $z_0^*=a-a^{-1}$, where $a=[(p+\sqrt{p^2+4})/2]^{1/3}$, and $p=(3z_{\rm cut}+z_{\rm cut}^3)/2$. These zeros of $A_{\rm CE}^{\rm SM}$ are important, since the graviton exchange will there give the only contribution. Finally, $\sigma^{\rm INT}(z_{\rm cut})=0$ at $z_{\rm cut}=1$, and in this limit for the angular cut the contribution to the total polarized cross section from the graviton exchange term would be of order f_G^2, i.e., of order $(s/M_H^2)^4$, hence negligible.

4 Sensitivity

In order to get some feeling for the sensitivities of the processes $e^+e^-\to\mu^+\mu^-$, $b\bar{b}$ and $c\bar{c}$ to graviton exchange effects, let us consider the statistical significance defined as

$$\mathcal{S}=\frac{|\Delta A_{\rm CE}|}{\delta A_{\rm CE}}, \tag{27}$$

where $\Delta A_{\rm CE}$ is defined by Eq. (13). Here, $\delta A_{\rm CE}$ is the expected statistical uncertainty defined by Eq. (15). Fig. 3 shows the statistical significance $\mathcal{S}$ as a function of z^* for unpolarized beams for the process (1) at $M_H=2$ TeV, $\mathcal{L}_{\rm int}=50$ fb^{-1}, $\lambda=1$, and $\sqrt{s}=500$ GeV. In the sequel, we shall put $\lambda=1$; our numerical results will turn out not to depend appreciably on the choice of the sign.

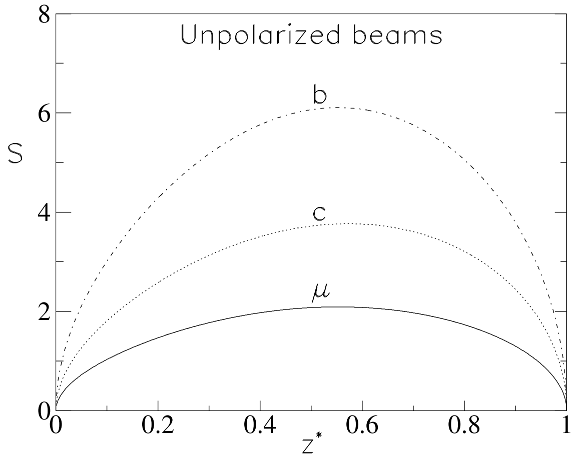

Figure 3: Statistical significance, $\mathcal{S}$, for unpolarized beams, $M_H=2$ TeV, $\mathcal{L}_{\rm int}=50{\rm fb}^{-1}$, $\lambda=1$, and $\sqrt{s}=500$ GeV. Different fermionic final states are considered: $\mu^+\mu^-$, $c\bar{c}$ and $b\bar{b}$. Here, no cut is imposed, $z_{\rm cut}=1$.

From Eqs. (27), (14), (15) and (11) one can derive the statistical significance for unpolarized initial beams limiting to the interference contribution (and for $z_{\rm cut}=1$):

$$\mathcal{S}_f = f_G\,\mathcal{S}_0\,\frac{\left|\left(\mathcal{M}^{\rm SM}_{\rm LL}-\mathcal{M}^{\rm SM}_{\rm LR}\right)+\left(\mathcal{M}^{\rm SM}_{\rm RR}-\mathcal{M}^{\rm SM}_{\rm RL}\right)\right|}{\sqrt{[(\mathcal{M}^{\rm SM}_{\rm LL})^2+(\mathcal{M}^{\rm SM}_{\rm LR})^2]+[(\mathcal{M}^{\rm SM}_{\rm RR})^2+(\mathcal{M}^{\rm SM}_{\rm RL})^2]}}, \tag{28}$$

$$\mathcal{S}_0 = \sqrt{\frac{3\pi\alpha^2_{e.m.}\,N_C\,\epsilon_f\mathcal{L}_{\rm int}}{s}\,\frac{z^*(1-z^*)}{(z^{*2}+3)(z^{*2}+z^*+4)}\,2(1+z^*)}. \tag{29}$$

The extension of Eq. (28) for polarized beams is straightforward:

$$\mathcal{S}_f = f_G\,\mathcal{S}_0\,\sqrt{D}\,\frac{\left|(1-P_{\rm eff})\left(\mathcal{M}^{\rm SM}_{\rm LL}-\mathcal{M}^{\rm SM}_{\rm LR}\right)+(1+P_{\rm eff})\left(\mathcal{M}^{\rm SM}_{\rm RR}-\mathcal{M}^{\rm SM}_{\rm RL}\right)\right|}{\sqrt{(1-P_{\rm eff})\,[(\mathcal{M}^{\rm SM}_{\rm LL})^2+(\mathcal{M}^{\rm SM}_{\rm LR})^2]+(1+P_{\rm eff})\,[(\mathcal{M}^{\rm SM}_{\rm RR})^2+(\mathcal{M}^{\rm SM}_{\rm RL})^2]}}. \tag{30}$$

Note that the maximum of $\mathcal{S}$ occurs at $z^*_{\rm max}\approx 1/\sqrt{3}=0.577$ ($\theta\approx 54.7^\circ$) which is very close to z^*_0 where $A^{\rm SM}_{\rm CE}=0$.[2] The dependence of $\mathcal{S}$ in the vicinity of $z^*_{\rm max}$ is quite smooth as implied by the behaviour of the $\Delta A_{\rm CE}$ and $\delta A_{\rm CE}$ shown in Fig. 2. In other words, variation of z^* around $z^*_{\rm max}$ changes the sensitivity very little. Therefore, no stringent requirements on angular resolution are needed.

The statistical significance is expressed in terms of the SM amplitudes $\mathcal{M}^{\rm SM}_{\alpha\beta}=Q_eQ_f[1+(g^e_\alpha g^f_\beta/Q_eQ_f)\,\chi_Z]$. The factor Q_eQ_f is here extracted since it cancels in the ratios of Eqs. (28) and (30).

In order to clarify the dominant role of $q\bar{q}$-pair production over $\mu^+\mu^-$ production in searching for graviton exchange effects, as shown in Fig. 3, and also to reveal the role of polarization in such analysis, it is instructive to estimate the SM amplitudes in the limit where

$$s_W^2=0.25,\qquad M_Z^2\ll s\ll M_H^2. \tag{31}$$

With these approximations, the relations between the SM couplings can be written as

$$\frac{g^e_{\rm L}}{Q_e}=\frac{1}{2}\frac{g^c_{\rm L}}{Q_c}=\frac{1}{5}\frac{g^b_{\rm L}}{Q_b}=\frac{1}{\sqrt{3}},\qquad \frac{g^e_{\rm R}}{Q_e}=\frac{g^c_{\rm R}}{Q_c}=\frac{g^b_{\rm R}}{Q_b}=-\frac{1}{\sqrt{3}}, \tag{32}$$

and the SM amplitudes are related as ($\chi_Z\approx 1$)

$$\begin{aligned}
\frac{1}{2}\mathcal{M}^{\rm e\mu}_{\rm LL} &=\frac{1}{2}\mathcal{M}^{\rm e\mu}_{\rm RR}=\mathcal{M}^{\rm e\mu}_{\rm LR}=\mathcal{M}^{\rm e\mu}_{\rm RL}=Q_eQ_\mu\,\frac{2}{3},\\
\frac{1}{5}\mathcal{M}^{\rm ec}_{\rm LL} &=\frac{1}{4}\mathcal{M}^{\rm ec}_{\rm RR}=\frac{1}{2}\mathcal{M}^{\rm ec}_{\rm LR}=\mathcal{M}^{\rm ec}_{\rm RL}=Q_eQ_c\,\frac{1}{3},\\
\frac{1}{4}\mathcal{M}^{\rm eb}_{\rm LL} &=\frac{1}{2}\mathcal{M}^{\rm eb}_{\rm RR}=\mathcal{M}^{\rm eb}_{\rm LR}=-\mathcal{M}^{\rm eb}_{\rm RL}=Q_eQ_b\,\frac{2}{3}.
\end{aligned} \tag{33}$$

For unpolarized e^+e^- beams, we have:

$$\mathcal{S}_\mu:\mathcal{S}_c:\mathcal{S}_b=1:\sqrt{\epsilon_c\frac{135}{23}}:\sqrt{\epsilon_b\frac{135}{11}}\approx 1:1.9:3.1. \tag{34}$$

[2]Strictly, $1/\sqrt{3}$ would be the value of z^* for which $\Delta A_{\rm CE}$ in Eq. (14) is maximal. This represents to a very good approximation the location $z^*_{\rm max}$ of the maximum of the statistical significance (27).

Comparison of the ratios (34) obtained in the adopted approximation (31) with those presented in Fig. 3 and derived from the full expression of Eq. (27) shows that this approximation is quite reasonable. With fully polarized beams, $e^+_{\rm L}e^-_{\rm R}$ ($P_{\rm eff}=1$) and $e^+_{\rm R}e^-_{\rm L}$ ($P_{\rm eff}=-1$), we find

$$\mathcal{S}_\mu(P_{\rm eff}=0):\mathcal{S}_\mu(P_{\rm eff}=1):\mathcal{S}_\mu(P_{\rm eff}=-1)=1:\sqrt{2}:\sqrt{2}=1:1.4:1.4, \qquad (35)$$

$$\mathcal{S}_c(P_{\rm eff}=0):\mathcal{S}_c(P_{\rm eff}=1):\mathcal{S}_c(P_{\rm eff}=-1)=1:\sqrt{\frac{46}{17}}:\sqrt{\frac{46}{29}}=1:1.6:1.3, \qquad (36)$$

$$\mathcal{S}_b(P_{\rm eff}=0):\mathcal{S}_b(P_{\rm eff}=1):\mathcal{S}_b(P_{\rm eff}=-1)=1:\sqrt{\frac{22}{5}}:\sqrt{\frac{22}{17}}=1:2.1:1.1. \qquad (37)$$

Note that the $b\bar{b}$ channel becomes more sensitive to graviton exchange effects both for unpolarized and polarized beams and would carry large statistical weight in the analysis. The advantage of polarization is lessened by the fact that the signal behaves as $(\sqrt{s}/M_H)^4$ compared to, e.g., the case of four-fermion *contact interactions*. This high power reduces the considerable gain in sensitivity to a less dramatic 20% gain in reach on M_H for the $b\bar{b}$ case, see Eq. (37).

The sign of the SM–NP interference term in the $A_{\rm CE}$ asymmetry for the process $e^+e^-\to c\bar{c}$ is opposite to those of $e^+e^-\to\mu^+\mu^-$ and $e^+e^-\to b\bar{b}$. This sign correlation might yield additional information to identify graviton exchange effects.

5 Identification reach

To assess a realistic reach on the mass scale M_H we can consider a χ^2-function made of the deviation of the asymmetry $A_{\rm CE}$ from its SM value. For a fixed integrated luminosity this can be done using the statistical errors as well as the systematic errors. We find that, to a very large extent, the systematic errors associated with the uncertainties expected on the luminosity measurements cancel out, and the same is true for the systematic errors induced by the uncertainty on beam polarizations. Accordingly, the errors on $A_{\rm CE}$ are largely dominated by statistics. In this estimation we assume the values $\delta\mathcal{L}_{\rm int}/\mathcal{L}_{\rm int}=\delta P/P=\delta\bar{P}/\bar{P}=0.5\%$. We take the beam polarization to be 80% and 60% for electrons and positrons, respectively, and employ a 10° angular cut around the beam pipe, i.e., $z_{\rm cut}=0.98$. Since most of the error is statistical in origin, we expect the bound on M_H to scale as $\sim(\mathcal{L}_{\rm int}s^3)^{1/8}$. The dependence of the reach on M_H on $z_{\rm cut}$ varying in a reasonable range close to 1 is, for the chosen values of energy, luminosity and polarization, quite smooth. For example, in the range $z_{\rm cut}=0.96-1$, the bound on M_H is found to vary by only a few percent.

In the present analysis we also take into account the radiative corrections. Among the complete $\mathcal{O}(\alpha)$ corrections to the process (1), the numerically largest QED corrections are the effects of initial state radiation, which in general are of major importance for new physics searches. The initial state corrections have been calculated in the flux function approach (see, *e.g.*, ref. [4]). The structure of the corrected differential cross section in terms of $z_{\rm c.m.} \equiv \cos\theta$ (where θ now refers to the final-state $f\bar{f}$ c.m. frame) is [24]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_{\rm c.m.}} \propto (1+z_{\rm c.m.}^2)\,\sigma_s + 2z_{\rm c.m.}\,\sigma_a, \tag{38}$$

The symmetric and antisymmetric parts of the cross section are given by convolutions of the non-radiative cross section with the flux functions $H_A^e(v)$, with v the energy of the emitted photon in units of the beam energy. Due to the radiative return to the Z resonance for $\sqrt{s} > M_Z$ the energy spectrum of the radiated photons is peaked around $E_\gamma/E_{\rm beam} \approx 1 - M_Z^2/s$. In order to increase a possible new physics signal, events with hard photons should be removed by a cut on the photon energy, $\Delta = E_\gamma/E_{\rm beam} < 1 - M_Z^2/s$, with $\Delta = 0.9$. We also take into account electroweak corrections to the propagators and vertices amounting essentially to effective (momentum-dependent) coupling constants (effective Born approximation [25] with $m_{\rm top} = 175$ GeV and $m_{\rm higgs} = 300$ GeV). Concerning the other QED corrections, the final state ones and the initial-final state interference, they can be checked to be numerically unimportant for the chosen kinematical cuts, in particular that on Δ, using the existing codes, e.g., ZFITTER [26]. In addition z_0^*, the zero of $A_{\rm CE}^{\rm SM}$, is shifted by these corrections by a little amount from the effective Born approximation value. The box-diagram contributions, which introduce a different angular dependence, are found to be very small.

Since the form of the corrected cross section, Eq. (38), is the same as that of Eq. (2), it follows that the radiatively-corrected zero of $A_{\rm CE}^{\rm SM}$, z_0^*, can again be defined by:

$$\left[\int_{-z_0^*}^{z_0^*} - \left(\int_{-1}^{-z_0^*} + \int_{z_0^*}^{1}\right)\right](1+z^2)\mathrm{d}z = 0, \tag{39}$$

and one finds the same value for z_0^* as given by Eq. (12). Moreover, in both the SM and SM+CI cases the radiatively corrected asymmetry $A_{\rm CE}$ is still determined by Eq. (19).

Summing over $\mu^+\mu^-$, $\tau^+\tau^-$, $b\bar{b}$ and $c\bar{c}$ final states (the top quark is excluded as its mass effects would alter the angular distribution (38)) one can perform a conventional χ^2 analysis:

$$\chi^2 = \sum_{f=\mu,\tau,c,b} \frac{(\Delta A_{\rm CE}^f)^2}{(\delta A_{\rm CE}^f)^2}, \tag{40}$$

keeping $z^* = z_0^*$ fixed (recall from Fig. 3 that the sensitivities for the various final states are rather smooth in an interval around $z_0^* \simeq z_{\rm max}^*$). This leads to the 5σ identification reach as a function of integrated luminosity with energy $\sqrt{s} = 0.5$, 1, 3 and 5 TeV shown in Fig. 4. The chosen range of energy corresponds to TESLA, NLC [16] and CLIC [17]. Specifically, for $\sqrt{s} = 0.5-1$ TeV and $3-5$ TeV machines with integrated luminosity 1 ab^{-1} the identification reach with double beam polarization is found to be $(7-6)\times\sqrt{s}$ and $(4.5-4)\times\sqrt{s}$, respectively. The effects of spin-2 graviton exchange can be distinguished from the other forms of contact-interaction-like effects considered in Table 1 for $M_H \leq$ 3.5, 6, 13.6 and 20 TeV at $\sqrt{s} = 0.5$, 1, 3 and 5 TeV, respectively.

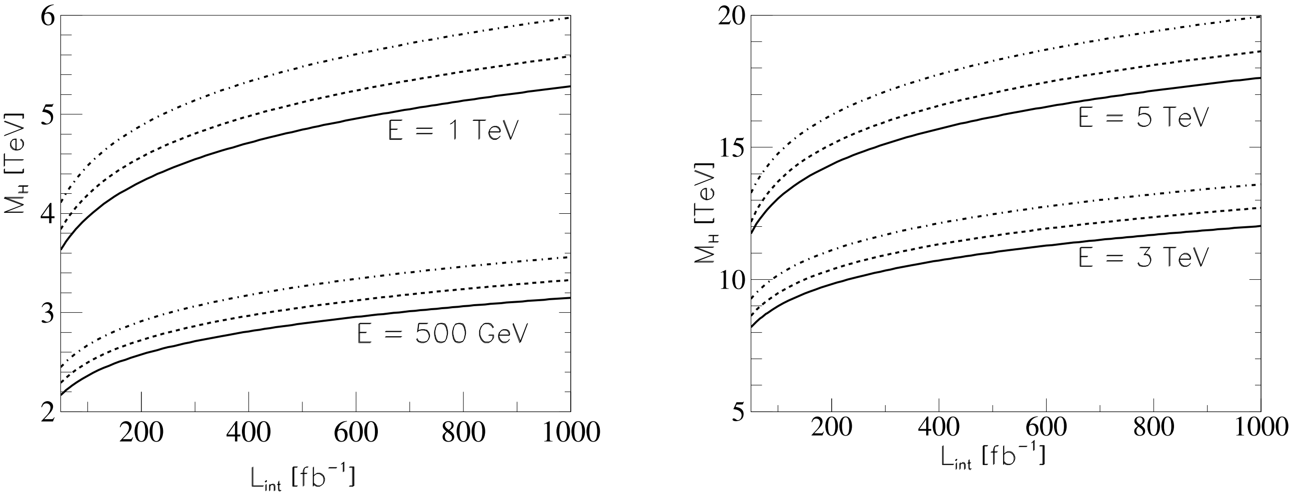

Figure 4: 5σ reach on the mass scale M_H *vs.* integrated luminosity from the process $e^+e^- \to f\bar{f}$, with f summed over μ, τ, b, c, and for a range of energies from 0.5 to 5 TeV. Solid: unpolarized; dashed: electrons polarized, $P = 0.8$; dash-dotted: both beams polarized, $P = 0.8$, $\bar{P} = -0.6$.

It turns out that under the assumption of no observation of $\Delta A_{\rm CE}$ within the expected experimental uncertainty, in which case only bounds on f_G can be derived, the 95% CL lower limits on $M_{\rm H}$ would be represented by the values shown in Fig. 4 essentially multiplied by a factor of the order of 1.3.

Finally, we consider a scenario that would most closely mimic massive graviton exchange, namely the exchange of a scalar field in the s- and t-channels, limiting ourselves to the production of lepton pairs. To be specific, we can concentrate on the example R-parity breaking SUSY interactions mediated by sneutrino exchange [6, 7]. First, we consider the t-channel $\tilde{\nu}$ contribution to $e^+e^- \to \mu^+\mu^-$ or $\tau^+\tau^-$. In this case the helicity cross sections are given by Eq. (3) with an additional contribution to the helicity amplitudes caused by $\tilde{\nu}$ exchange:

$$\Delta_{\rm LL} = \Delta_{\rm RR} = 0, \qquad \Delta_{\rm LR} = \Delta_{\rm RL} = \frac{1}{2} C_{\tilde{\nu}} P^t_{\tilde{\nu}}, \tag{41}$$

where $P^t_{\tilde{\nu}} = s/(t - m^2_{\tilde{\nu}})$ and $t = -s(1-z)/2$, $C_{\tilde{\nu}} = \lambda^2/4\pi\alpha_{\rm e.m.}$, with λ in this case the Yukawa coupling [7]. It is clear that in the contact interaction limit, i.e. $|t| \ll m^2_{\tilde{\nu}}$, these two new physics effects, graviton exchange and $\tilde{\nu}$ exchange, are easily separable by the previous analysis based on the asymmetry $A_{\rm CE}$. If we are not in the contact interaction limit, $\mathcal{M}_{\rm LR}$ and $\mathcal{M}_{\rm RL}$ pick up an additional z dependence resulting in a z^* dependence of $\Delta A_{\rm CE}$ different from the one in Eq. (14) determined by graviton exchange. We find that polarization will also help to distinguish these two new physics effects. For this purpose one can define the polarized observable, the absolute center-edge left-right asymmetry:

$$\sigma A_{\rm CE,LR} \equiv \sigma_{\rm CE,LR} = \left[\int_{-z_0^*}^{z_0^*} - \left(\int_{-1}^{-z_0^*} + \int_{z_0^*}^{1}\right)\right] \left(\frac{{\rm d}\sigma_{\rm L}}{{\rm d}z} - \frac{{\rm d}\sigma_{\rm R}}{{\rm d}z}\right) {\rm d}z. \tag{42}$$

Here, z_0^* is the zero of $A_{\rm CE}^{\rm SM}$, see Eq. (39), and ${\rm d}\sigma_{\rm L}/{\rm d}z$ and ${\rm d}\sigma_{\rm R}/{\rm d}z$ are the differential cross sections defined by Eq. (20) with specific choices of electron and positron beam polarizations, for example $(P,\bar{P})=(-P_1,P_2)$ and $(P_1,-P_2)$, respectively, with P_1 and P_2 positive. The deviation from the SM prediction of the differential cross section difference involved in Eq. (42) and caused by $\tilde{\nu}$ exchange is given by

$$\Delta\frac{{\rm d}\sigma_{\rm LR}}{{\rm d}z}\equiv\left(\frac{{\rm d}\sigma_{\rm L}}{{\rm d}z}-\frac{{\rm d}\sigma_{\rm R}}{{\rm d}z}\right)-\left(\frac{{\rm d}\sigma_{\rm L}^{\rm SM}}{{\rm d}z}-\frac{{\rm d}\sigma_{\rm R}^{\rm SM}}{{\rm d}z}\right)\propto P_{\rm eff}\,(\mathcal{M}_{\rm LR}^{\rm SM}-\mathcal{M}_{\rm RL}^{\rm SM})C_{\tilde{\nu}}P_{\tilde{\nu}}^t=0, \qquad (43)$$

because $\mathcal{M}_{\rm LR}^{\rm SM}=\mathcal{M}_{\rm RL}^{\rm SM}$ for the process (1) with $f=\mu,\tau$. Notice that this property, easily checked in the tree approximation of the SM, continues to hold also in the effective Born approximation. Accordingly, $\sigma_{\rm CE,LR}$ is unaltered by sneutrino exchange in the leptonic processes $e^+e^-\to\mu^+\mu^-$ and $e^+e^-\to\tau^+\tau^-$, i.e., $\Delta\sigma_{\rm CE,LR}^{\tilde{\nu}}=0$, whereas it is modified by graviton exchange, $\Delta\sigma_{\rm CE,LR}^{G}\neq 0$. The choice of z_0^* as integration limits in (42) assures that the contribution of the SM as well as that of any conventional contact interaction vanish, leaving room only for graviton and sneutrino exchanges. The role of polarization is that, in the combination (42), the sneutrino contributions cancel as explicitly seen in (43), so that only the signal of the graviton exchange term can survive. One can notice that this kind of analysis is allowed also in the case of only electron beam longitudinal polarization and unpolarized positron, namely, $P_1\neq 0$ and $P_2=0$. Also, the quadratic term in the differential cross sections, proportional to $(C_{\tilde{\nu}}P_{\tilde{\nu}}^t)^2$, cancels in Eq. (42), so that Eq. (43), linear in $(C_{\tilde{\nu}}P_{\tilde{\nu}}^t)$ is the exact representation of the deviation from the SM.

Concerning $\tilde{\nu}$ exchange in the s channel, the polarized differential cross section (20) picks up an additional, z-independent, term:

$$\frac{{\rm d}\sigma^s}{{\rm d}z}\propto(1+P\bar{P})(C_{\tilde{\nu}}P_{\tilde{\nu}}^s)^2, \qquad (44)$$

with $P_{\tilde{\nu}}^s\simeq s/(s-m_{\tilde{\nu}}^2)$. Indeed, the s-channel scalar exchange diagram does not interfere with the electroweak SM amplitudes mediated by the γ and Z boson and the resulting effects are of quadratic order, $(C_{\tilde{\nu}}P_{\tilde{\nu}}^s)^2$. As is easily seen from Eq. (44), either the electron beam polarization or both electron and positron polarizations allow to remove the sneutrino s channel exchange contribution to Eq. (42), i.e., $\Delta\sigma_{\rm CE,LR}^{\tilde{\nu}}=0$ also in this case.

Conversely, it is possible to define an observable 'orthogonal' to $\sigma_{\rm CE,LR}$ which is sensitive to $\tilde{\nu}$ exchange in the s channel and independent of the effects of *graviton exchange*, *contact interactions*, and Z' exchange. This is the double beam polarization asymmetry defined as [7]

$$A_{\rm double}=\frac{\sigma(P_1,-P_2)+\sigma(-P_1,P_2)-\sigma(P_1,P_2)-\sigma(-P_1,-P_2)}{\sigma(P_1,-P_2)+\sigma(-P_1,P_2)+\sigma(P_1,P_2)+\sigma(-P_1,-P_2)}. \qquad (45)$$

Here, σ are the cross sections integrated over z in the indicated polarization configurations. One can see immediately that for the case of the SM, *contact interactions*, Z' exchange, $\tilde{\nu}$ exchange in the t channel and for *graviton exchange* one obtains $A_{\rm double}=P_1P_2$ since these exchanges contribute to the same amplitudes, whereas $\tilde{\nu}$ exchange in the s channel will force this observable to smaller values as $\Delta A_{\rm double} \propto -P_1P_2\,(C_{\tilde\nu}P^s_{\tilde\nu})^2 < 0$. A value of $A_{\rm double}$ smaller than P_1P_2 can provide a signature of scalar exchange in the s channel.

In conclusion, we have seen that one can define a set of observables using cross sections integrated within appropriate angular limits that can discriminate among deviations from the SM prediction related either to graviton or to scalar exchange in the s channel.

6 Summary and observations

We conclude with a summary of the main points and some observations. We have developed a specific approach based on an integrated observable, the center-edge asymmetry $A_{\rm CE}$, to search for and identify spin-2 graviton exchange with uniquely distinct signature. Indeed, the spin-2 graviton KK exchanges contribute to the asymmetry $A_{\rm CE}$, whereas no deviation from the SM is induced by other kinds of new physics such as the composite-like contact interactions, a heavy vector boson Z', gauge boson KK excitations listed in Table 1. Both in the SM and in any new physics scenario described by effective current–current interactions, the asymmetry $A_{\rm CE}$ is identical for any value of the parameter z^*.

Particularly convenient is the range of z^* values around the zero of $A_{\rm CE}$ (z^*_0) for the SM. In this range, the sensitivity of $A_{\rm CE}$ to the graviton coupling f_G is maximal and rather smooth in z^*, so that one can obtain not only the discovery but the real unambiguous identification of this new physics effect. This kind of analysis based on $A_{\rm CE}$ can be applied also to the case where a cut is imposed on the full angular range covered by the experiment, and its nice distinctive features continue to hold to a very good approximation.

Initial electron and positron beam polarization appears to increase the sensitivity to graviton exchange, but their impact on the mass scale parameter $M_{\rm H}$ is not dramatic due to the large power $(\sqrt{s}/M_{\rm H})^4$ that parametrizes the graviton coupling. In particular, for an e^+e^- linear collider with energy $\sqrt{s} = 0.5$, 1, 3 and 5 TeV, with integrated luminosity 1 ab^{-1}, double beam polarization and a 10° angular cut, the 5σ identification reach is found to be $M_H \leq 3.5,\ 6,\ 13.6$ and 20 TeV, respectively.

Instead, initial polarization can play a key role in distinguishing graviton exchange from competing effects, such as those originating from exchange of scalar particles, for which appropriate polarization asymmetries can be defined.

An approach aiming to isolate graviton-exchange effects has recently been proposed in Ref. [18], based on the differential cross section convoluted with Legendre polynomials and integrated over the angular range. Alternatively, our method directly uses the integrated cross sections to construct the center-edge asymmetry $A_{\rm CE}$. It has the main advantage of a mild dependence of $A_{\rm CE}$ on the kinematical cut, systematics, and on the number of angular bins, and in particular it depends on the total luminosity and not on the statistics available in each bin. These features may lead to some improvement in the 5-σ discovery reach on the mass scale M_H.

Finally, we note that an analysis based on asymmetries analogous to $A_{\rm CE}$ might be useful in the context of hadronic collisions, in the Drell–Yan process.

Acknowledgement
This research has been partially supported by MIUR (Italian Ministry of University and Research), by funds of the University of Trieste, and by the Research Council of Norway.

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