

**Dynamical Symmetry Breaking in Supersymmetric Extensions of  
Nambu–Jona-Lasinio Model**

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**Abstract :** In this paper we discuss Nambu–Jona-Lasinio model as a classical model for dynamical mass generation and symmetry breaking. In addition we discuss the possible supersymmetric extensions of this model resulting from interaction terms with four chiral superfields that may be regarded as a supersymmetric generalization of the four-fermion interactions of the Nambu–Jona-Lasinio model. A four-superfield interaction terms can be constructed as either dimension 6 or dimension 5 operators. Through analyzing solutions to the gap equations, we discuss the dynamical generation of superfield Dirac mass, including a supersymmetry breaking part. A dynamical symmetry breaking generally goes along with the dynamical mass generation, for which a bi-superfield condensate is responsible.

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# Dynamical Symmetry Breaking in Supersymmetric Extensions of Nambu–Jona-Lasinio Model

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In this paper we discuss Nambu–Jona-Lasinio model as a classical model for dynamical mass generation and symmetry breaking. In addition we discuss the possible supersymmetric extensions of this model resulting from interaction terms with four chiral superfields that may be regarded as a supersymmetric generalization of the four-fermion interactions of the Nambu–Jona-Lasinio model. A four-superfield interaction terms can be constructed as either dimension 6 or dimension 5 operators. Through analyzing solutions to the gap equations, we discuss the dynamical generation of superfield Dirac mass, including a supersymmetry breaking part. A dynamical symmetry breaking generally goes along with the dynamical mass generation, for which a bi-superfield condensate is responsible.

## 1 Introduction

Nambu adopted the idea of Cooper pairing [1] to construct a classic model of dynamical mass generation and symmetry breaking. This is the Nambu–Jona-Lasinio (NJL) model [2], with a strong attractive four-fermi interaction. After the Standard Model was generally established, the exact mechanism of electroweak symmetry breaking became a problem of paramount importance in the phenomenological domain. It is still open till today. It was pointed out by Nambu [3] that for a sufficiently heavy top quark, an NJL model of top condensate can give rise to electroweak symmetry breaking. The top quark, however, turns out to be not heavy enough [4,5].

The Lagrangian for the NJL model can be written as

$$\mathcal{L} = i\partial_m\psi_+\sigma^m\bar{\psi}_+ + i\partial_m\psi_-\sigma^m\bar{\psi}_- + g^2\psi_+\psi_-\bar{\psi}_+\bar{\psi}_- \quad (1)$$

here  $\psi_+, \psi_-$  are two-component Weyl spinors and the coupling  $g$  has mass dimension -1 which shows that the model is to be taken as an effective theory and has to be provided with a cut-off  $\Lambda$ . Clearly from eq.(1), the Lagrangian  $\mathcal{L}$  is invariant under the chiral  $U(1)$  transformations:

$$\begin{aligned} U(1)_V &: \psi_{\pm} \rightarrow e^{\pm i\alpha}\psi_{\pm} \\ U(1)_A &: \psi_{\pm} \rightarrow e^{i\beta}\psi_{\pm} \end{aligned} \quad (2)$$

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\*Gaber Faisel would like to thank Taiwan LHC Focus Group (NCTS) for their financial support to participate in the conference and present this talk.

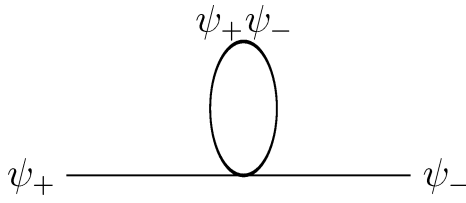


Figure 1: Diagram for proper self-energy  $\Sigma_{+-}(p)$ .

We can rewrite  $\mathcal{L}$  as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I \quad (3)$$

where

$$\mathcal{L}_0 = i\partial_m \psi_+ \sigma^m \bar{\psi}_+ + i\partial_m \psi_- \sigma^m \bar{\psi}_- - m\psi_- \psi_+ \quad (4)$$

and the interaction Lagrangian  $\mathcal{L}_I$  is given by

$$\mathcal{L}_I = g^2 \psi_+ \psi_- \bar{\psi}_+ \bar{\psi}_- + m\psi_- \psi_+ \quad (5)$$

The mass  $m$  is self-consistently defined by

$$\Gamma^2(p)_{\gamma p = -m} = 0 \quad (6)$$

where  $\Gamma^2(p)$  is the proper two-point function. This yields the gap equation:

$$m = \Sigma_{+-}^{(loop)}(p) \Big|_{\text{on-shell}}, \quad (7)$$

where  $\Sigma_{+-}^{(loop)}$  denotes contributions from the proper self-energy diagram shown in Fig.(1). Upon evaluation of the diagram, the gap equation reads

$$m = mg^2 \frac{\Lambda^2}{8\pi^2} \left[ 1 - \frac{|m|^2}{\Lambda^2} \ln \frac{\Lambda^2}{|m|^2} + O(1/\Lambda^4) \right]. \quad (8)$$

which has nontrivial solution i.e.  $m \neq 0$  for coupling constant satisfying the inequality

$$g^2 > \frac{8\pi^2}{N_C \Lambda^2}, \quad (9)$$

Clearly, the nonperturbative gap equation, eq.(7), with eq.(9) show that the strong attractive four-fermi interaction induces a bi-fermion vacuum condensate of the operator  $\psi_+ \psi_-$  which serves as the source for the fermion Dirac mass. Moreover, the condensate naturally breaks the chiral symmetry that the bi-fermion carries, which was Nambu's first concern [1, 6].

## 2 Supersymmetric extensions of NJL model

A supersymmetric extension of the NJL model via dimension six four-superfield interaction (SNJL) was introduced in 1982 [7]. The gap equation analysis showed no nontrivial mass

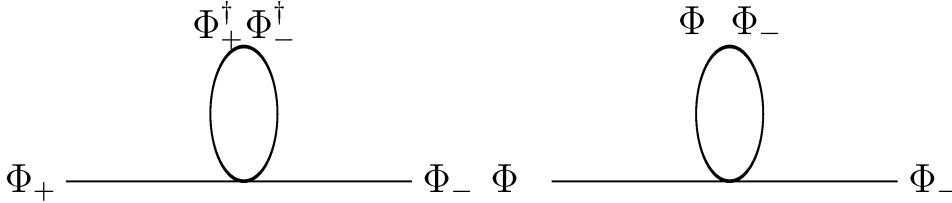


Figure 2: Supergraphs for proper self-energy  $\Sigma_{+-}(p, \theta^2)$ . Left (right) supergraph corresponds to dimension six (five) four-superfield interaction.

solution. The model can be recovered leading to dynamical mass generation upon introducing soft supersymmetric breaking mass terms [8]. However, phenomenological viability of the model has been severely unfavorable cornered with the relatively small top mass value determined and constraint on the  $\tan\beta$  parameter [9]. A natural alternative to SNJL model as elaborated in Ref. [10] was presented in Ref. [9], together with an explicit model version that can give rise to electroweak symmetry breaking. The new model has a dimension five four-superfield interaction in the superpotential and hence it is holomorphic and named as (HSNJL) model. Recently a fully detailed study for SNJL and HSNJL models based on introduction of a new perspective on the superfield gap equation using the supergraph technique has been presented in ref. [11]. The explicit illustration of dynamical symmetry breaking from HSNJL showed rich and novel features, which would be easily missed without the superfield approach developed there.

The key point in the analysis of ref. [11] is extending the gap equation for the Dirac mass  $m$ , eq.(7), to

$$-\mathcal{M} = \Sigma_{+-}^{(loop)}(p, \theta^2) \Big|_{\text{on-shell}} , \quad (10)$$

where  $\mathcal{M}$  is given in  $\mathcal{L}_I$  which is written in terms of the chiral superfields that contain  $\psi_+$  and  $\psi_-$  as one of their components.  $\mathcal{M}$  contains the usual (supersymmetric) Dirac mass  $m$  and its supersymmetry breaking counterpart  $\eta$ . The former corresponds to Dirac mass for the fermion pair  $\psi_{\pm}$  and  $|m|^2$  contributions to both  $A_{\pm}$  mass-squared, while the supersymmetry breaking part  $\eta$  gives (so-called left-right) mass mixing between the  $A_{\pm}$  pair. In eq.(10)  $\Sigma_{+-}^{(loop)}$  denotes contributions from the proper self-energy diagram involving the interaction. The interactions of interest that are expected to lead to nontrivial  $\Sigma_{+-}^{(loop)}$  can be given by the dimension six four-superfield interaction

$$g^2 \int d^4\theta \Phi_+^\dagger \Phi_-^\dagger \Phi_+ \Phi_- (1 - \tilde{m}_c^2 \theta^2 \bar{\theta}^2) \quad (11)$$

coming with a supersymmetry breaking part which gives the SNJL model, here extended to include the supersymmetry breaking  $\tilde{m}_c^2$  part. The HSNJL model alternative has rather a dimension five four-superfield interaction which is given by

$$-\frac{G}{2} \int d^4\theta \Phi_+ \Phi_- \Phi_+ \Phi_- (1 + B\theta^2) \delta^2(\bar{\theta}) . \quad (12)$$

It is really a superpotential term, as indicated by the  $\delta^2(\bar{\theta})$ , hence holomorphic.

## 2.1 Dimension six interaction

The  $g^2$  vertex gives at one-loop level the proper self-energy diagram shown in Fig.(2) left. The gap equation, eq.(10) with  $\Sigma^{(fig2)}$ , reads [11]

$$\begin{aligned} m &= 2mg^2 I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2), \\ \eta &= -\eta g^2 \tilde{m}_c^2 I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2). \end{aligned} \quad (13)$$

where

$$\begin{aligned} I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) &= \frac{1}{16\pi^2} \left[ \frac{1}{2} (|m|^2 + \tilde{m}^2) \ln \frac{(|m|^2 + \tilde{m}^2 + \Lambda^2)^2 - |\eta|^2}{(|m|^2 + \tilde{m}^2)^2 - |\eta|^2} - |m|^2 \ln \frac{(|m|^2 + \Lambda^2)}{|m|^2} \right. \\ &\quad \left. + |\eta| \left( \tanh^{-1} \frac{|m|^2 + \tilde{m}^2 + \Lambda^2}{|\eta|} - \tanh^{-1} \frac{|m|^2 + \tilde{m}^2}{|\eta|} \right) \right], \\ I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) &= \frac{1}{16\pi^2} \left[ \frac{1}{2} \ln \frac{(|m|^2 + \tilde{m}^2 + \Lambda^2)^2 - |\eta|^2}{(|m|^2 + \tilde{m}^2)^2 - |\eta|^2} \right. \\ &\quad \left. + \frac{|m|^2 + \tilde{m}^2}{|\eta|} \left( \tanh^{-1} \frac{|m|^2 + \tilde{m}^2 + \Lambda^2}{|\eta|} - \tanh^{-1} \frac{|m|^2 + \tilde{m}^2}{|\eta|} \right) \right]. \end{aligned} \quad (14)$$

Note that the case with both  $\tilde{m}_c^2$  and  $\tilde{m}^2$  being zero corresponds to the SNJL model with an exactly supersymmetric Lagrangian [7]. In that case, a supergraph analysis has been performed going to two-loop evaluation of  $\Sigma_{+-}^{(loop)}(p, \theta)$  [7]. No nontrivial solution for  $m$  exists. It should be noted that when our result of eq.(13) is applied to the case, nontrivial solution for  $\eta$  will imply spontaneous supersymmetry breaking exist.

On the other hand, taking the limit  $\tilde{m} \rightarrow \infty$  where the scalar particles of  $\Phi_{\pm}$  become heavy and decoupled,  $m$  becomes the simple Dirac fermion/quark mass which then satisfies eq.(7) after including a factor  $N_c$  to account for the number of the colors of the quarks. A nontrivial solution for  $m$  exists for the coupling constant satisfying the inequality [8]

$$g^2 > \frac{8\pi^2}{\tilde{m}^2 \ln \left( 1 + \frac{\Lambda^2}{\tilde{m}^2} \right)}, \quad (15)$$

generating a mass for the Dirac fermion pair.

Considering the scenario  $m = 0$  but  $\eta \neq 0$  solution for Eq.(13). Naively, one enforces zero  $m$  in the the  $I_2$  integral of the equation for  $\eta$ . Nontrivial solution for the latter exists under the condition

$$\frac{1}{16\pi^2} \left[ \ln \left( 1 + \frac{\Lambda^2}{\tilde{m}^2} \right) - \frac{\Lambda^2}{\Lambda^2 + \tilde{m}^2} \right] \leq \frac{1}{-g^2 \tilde{m}_c^2} < \frac{1}{16\pi^2} \ln \left( 1 + \frac{\Lambda^2}{2\tilde{m}^2} \right), \quad (16)$$

The last part of the inequality comes from an analysis similar to that of the condition for nontrivial  $m$  under  $\eta = 0$ . The magnitude of the responsible coupling,  $g^2 \tilde{m}_c^2$  here, has to be big enough. The other part of the inequality is actually from  $|\eta| \leq (m^2 + \tilde{m}^2)$  beyond which there will be a tachyonic scalar mass eigenvalue. Note that one always needs a negative  $\tilde{m}_c^2$  for nontrivial  $\eta$  solution.

## 2.2 Dimension five interaction

Turning now to the dimension five interaction case in which the  $G$  vertex gives at one-loop level a diagram only slightly different from the previous case, as shown in Fig.(2) right. The gap equation, eq.(10) with  $\Sigma^{(fig2)}$ , reads [11]

$$\begin{aligned} m &= \frac{\bar{\eta}G}{2} I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2), \\ \eta &= \bar{m}G I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) - \frac{\bar{\eta}GB}{2} I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2). \end{aligned} \quad (17)$$

The first thing to note in the gap equation result is the important fact that the equations for  $m$  and  $\eta$  are completely coupled. If one naively drop  $\eta$  from consideration, for instance, one will not see any nontrivial expression and completely miss the possible dynamical mass generation. The two parameters will either both have nontrivial solutions or both vanishing.

Considering only the case of real values for  $m$  and  $\eta$  under the assumption of a real and small  $B$  value, we find that nontrivial solution exists for large enough  $G$  (taken as real and positive here by convention) satisfying

$$G > \sqrt{G_0^2 + b^2} + b \sim G_0 + b, \quad (18)$$

where

$$G_0^2 = \frac{512\pi^2}{\tilde{m}^2 \ln\left(1 + \frac{\Lambda^2}{\tilde{m}^2}\right) \left[\ln\left(1 + \frac{\Lambda^2}{\tilde{m}^2}\right) - \frac{\Lambda^2}{\tilde{m}^2 + \Lambda^2}\right]} \quad (19)$$

gives the critical  $G^2$  for  $B = 0$ , and

$$b = B \frac{8\pi^2}{\tilde{m}^2 \ln\left(1 + \frac{\Lambda^2}{\tilde{m}^2}\right)}. \quad (20)$$

Details can be found in ref. [11]. Solution condition for more general cases is to be further investigated.

## 3 Conclusion

In this talk we have discussed NJL model with its possible supersymmetric extensions that can be constructed from either dimension 6 or dimension 5 four-superfield interactions. The two kinds of four-superfield interactions may be considered alternative supersymmetrization of the four-fermion interaction in the NJL model of dynamical mass generation and symmetry breaking. They could each be used as a mechanism for dynamical electroweak symmetry breaking. The two kinds of models (SNJL and HSNJL models) have otherwise very different theoretical mass generation features, with phenomenological implications. In addition, we presented the superfield gap equations for both dimension six and dimension five four-superfield interactions and discussed some interesting cases for nontrivial solution.

We have shown also that dimension five four-superfield interaction can induce the dynamical mass generation for the prototype HSNJL model. The model has actually no four-fermion interaction and has a bi-scalar condensate, instead of bi-fermion condensate, as

the source of Dirac fermion mass. It has otherwise theoretical features that look like a direct supersymmetric version of the NJL model. It is expected to provide an alternative paradigm for dynamical mass generation and symmetry breaking, at least for superfield theories. The explicit symmetry breaking picture of the simplest HSNJL model maybe considered as  $Z_4 \rightarrow Z_2$ . A version of the HSNJL with the basics superfields being (gauge) multiplets gives a simple application to the breaking of a continuous symmetry. The model can be also extended with more than two basic superfield multiplets that can achieve a rich spectrum of dynamical symmetry breaking. A case example, which was also the original target for the idea of the HSNJL model is the one for electroweak symmetry breaking which we refer to ref. [11] for more details.

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