Massive particles and unitarity cuts

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We present an extension of the spinor integration formalism of one loop amplitudes from the double-cut to the single-cut case. This technique can be applied for the computation of the tadpole coefficients. Moreover we describe an off-shell continuation of one loop amplitudes that allows a finite evaluation of the unitarity cuts in the channel of a single massive external fermion.

1 Introduction

Recent years have seen rapid progress in computing one-loop amplitudes, due largely to the use of generalized unitarity methods [1–16]. These methods enable the computation of loop amplitudes in terms of tree-level amplitudes, which are (relatively) easy to generate either numerically or analytically. They rely on the knowledge of the expansion of any amplitude in terms of a set of master integrals, with coefficients that are rational functions of the kinematic invariants. Indeed they operate by matching the generalized cuts of the loop amplitude and the master integrals.

Unitarity methods work most elegantly when all internal particles are massless. If massive particles are involved, there are new master integrals whose cuts are more difficult to solve. The additional master integrals are the tadpoles $A_0(m)$, and the "on-shell bubbles" $B_0(m^2; m, 0)$ and $B_0(0; m, m)$.[a] Several methods, either numerical or analytical, aim to compute these coefficients [10, 17–22].

In the following we will be focus on the tadpole integral and on the bubble $B_0(m^2; m, 0)$, presenting methods for their analytical computations. More detailed discussions can be found elsewhere [22, 23].

2 Single cut integration

The tadpole coefficient can be computed by cutting just one propagator. In this section we describe an extension of the formalism for explicit evaluation of double cuts [24–32] to the single cut case [22]. The loop momentum is parametrized in terms of spinor variables and the $(D-2)$-dimensional integral is performed algebraically by the Cauchy residue theorem. We find that the full single cut integral will typically diverge. Moreover, while the evaluation of the double cut involves the computation of residues at poles, in the evaluation of the single cut the contour integral part of the formula dominates the tadpole contribution, so the residues are not needed. Finally in the single cut case it is convenient to work at the integrand level.

[a]These bubbles are called on-shell because the momentum involved in the cut channel is the on-shell momentum of an external particle.

The starting point is the one-loop integrand,

$$I = \frac{N(\ell)}{D_0 D_1 \cdots D_k}, \tag{1}$$

where $N(\ell)$ is a polynomial in the loop momentum ℓ, $D_i = (\ell - K_i)^2 - m_i^2$, and $K_0 = 0$. The 4-dimensional single-cut operator for the propagator D_0 acts on the integrand as

$$\Delta_{D_0}[I] \equiv \int d^4\ell \; \delta^{(+)}\left(\ell^2 - m_0^2\right) \; I_0, \qquad I_0 = \frac{N(\ell)}{D_1 \cdots D_k}. \tag{2}$$

The single cut is applied to the *integrand*, because of the presence of non-vanishing contributions from the spurious terms. Working with the integrand allows us to identify the particular propagator being cut. The single cut (2) can be written as follows:

$$\Delta_{D_0}[I] = \int_0^\infty \frac{dt}{4} \int (idz \wedge d\bar{z}) \, \frac{K^2 t^2 (1 + z\bar{z})}{\sqrt{t^2(1+z\bar{z})^2 + u}} I_0, \tag{3}$$

where z, $\bar{z}$ and t are defined according to

$$\ell^\mu = t\left(p^\mu + z\bar{z}q^\mu + \frac{z}{2}\langle q|\gamma^\mu|p] - \frac{\bar{z}}{2}\langle p|\gamma^\mu|q]\right) + \frac{u}{2}\frac{1}{\sqrt{t^2(1+z\bar{z})^2 + u} + t(1+z\bar{z})} K^\mu \;, \tag{4}$$

and $u \equiv 4m_0^2/K^2$. The momenta p and q are light-like and arbitrary, while $K = p + q$. They are chosen such that $K^2 \gg m_0^2$ since it is convenient to work in the $u \to 0$ limit. The integration over z and $\bar{z}$ will be performed by the Generalized Cauchy Formula as described in [32]. For the integrand $F(z, \bar{z})$ we construct a primitive $G(z, \bar{z})$ with respect to, say, $\bar{z}$. Then

$$\begin{aligned} \int_D F(z,\bar{z}) \; d\bar{z} \wedge dz &= \oint_{\partial D} dz \; G(z,\bar{z}) - 2\pi i \sum_{\text{poles } z_j} \mathrm{Res}\{G(z,\bar{z}), z_j\} \\ &= \int_0^{2\pi} d\alpha \; \Lambda e^{i\alpha} G(\Lambda e^{i\alpha}, \Lambda e^{-i\alpha}) - 2\pi i \sum_{\text{poles } z_j} \mathrm{Res}\{G(z,\bar{z}), z_j\} \;, \end{aligned} \tag{5}$$

where D is a disk of radius Λ encompassing the poles of $G(z, \bar{z})$:

$$z = re^{i\alpha}; \qquad D = \{(r,\alpha) \mid 0 \le r \le \Lambda, \; 0 \le \alpha < 2\pi\}.$$

The information we need is enclosed at the integrand level, thus the integration over t will not be carried out. It turns out that in the limit $\Lambda \to \infty$, we can ignore the residues and restrict our attention to the closed line integral term. In this limit, the leading behavior of the primitives of the master integrands is given by

$$I = \frac{1}{D_0} \implies \Lambda e^{i\alpha} G \overset{\Lambda\to\infty}{=} \Lambda^2 t \;, \qquad\qquad I = \frac{1}{D_0 \cdots D_n} \implies \Lambda e^{i\alpha} G \overset{\Lambda\to\infty}{=} \frac{\log \Lambda^2}{\Lambda^{n-2}}, \tag{6}$$

with $n = 2, 3, 4$. The higher-point integrands are suppressed by powers of Λ. Moreover tadpole primitives are purely rational, while the others are purely logarithmic. Therefore, in an algorithm targeting tadpole coefficients, we will select terms of the single cut with specific dependence on Λ^2.

Computation of tadpole coefficients

In this section we show how the single cut allows the computation of tadpole coefficients. The idea is to expand the integrand in The Ossola-Papadopoulos-Pittau (OPP) decomposition [10], in order to easily recognize the spurious contributions. The coefficients of both spurious and physical terms are treated as unknowns, and the physical terms except for the tadpole are dropped. Single cuts of all remaining terms are then evaluated. Thanks to the OPP expansion, the single-cut equation becomes a system of separate equations, which are the coefficients of independent tensors.

As a simple example, we compute the coefficient $a(0)$ of the tadpole integral $A_0(m_0^2)$ of the integrand

$$I = \frac{2\ell \cdot R}{D_0 D_1}. \tag{7}$$

We assume its Gram determinant is nonvanishing, i.e. $K_1^2 \neq 0$, and that the masses are non-degenerate.[b] The OPP decomposition of I is given by

$$I = \frac{a(0)}{D_0} + \tilde{b}_{11}(01)\,\frac{2\ell \cdot \ell_7}{D_0 D_1} + \tilde{b}_{21}(01)\,\frac{2\ell \cdot \ell_8}{D_0 D_1} + \tilde{b}_0(01)\,\frac{2\ell \cdot n}{D_0 D_1} + \cdots . \tag{8}$$

Terms whose single cut contains no $\Lambda^2 t$ contribution are included in "$\cdots$" and are neglected. The momenta n, ℓ_7 and ℓ_8 are defined [10] to satisfy the conditions

$$K_1 \cdot n = K_1 \cdot \ell_7 = K_1 \cdot \ell_8 = 0, \qquad n^2 = \ell_7 \cdot \ell_8 = -K_1^2, \qquad \ell_7^2 = \ell_8^2 = 0.$$

Applying the single cut operator Δ_{D_0} and selecting the $\Lambda^2 t$ terms only, we get

$$\begin{aligned} 0 \;&=\; \Big[-\left(a(0) + \alpha_1\right) K_1^\mu + \Big(\tilde{b}_{11}(01) - \alpha_3\Big)\, \ell_7^\mu \\ &\quad + \Big(\tilde{b}_{21}(01) - \alpha_4\Big)\, \ell_8^\mu + \Big(\tilde{b}_0(01) - \alpha_2\Big)\, n^\mu \Big] \, \frac{q_\mu}{K_1 \cdot q}\, \Delta_{D_0}\left[\frac{1}{D_0}\right]. \end{aligned} \tag{9}$$

The coefficients $\alpha_{i=1,\cdots,4}$ are the coordinates of R in the basis $\{K_1, n, \ell_7, \ell_8\}$. They read as follows:

$$\alpha_1 = \frac{R \cdot K_1}{K_1^2}, \quad \alpha_2 = -\frac{R \cdot n}{K_1^2}, \quad \alpha_3 = -\frac{R \cdot \ell_8}{K_1^2}, \quad \alpha_4 = -\frac{R \cdot \ell_7}{K_1^2}. \tag{10}$$

Since q is arbitrary, the expression inside the square brackets has to vanish. Therefore each of the factors multiplying the basis vectors vanishes separately, giving four equations. The tadpole coefficient is obtained from the first of these equations,

$$a(0) + \alpha_1 = 0 \quad \Longrightarrow \quad a(0) = -\frac{R \cdot K_1}{K_1^2}. \tag{11}$$

This result has been checked against the one obtained from the Passarino-Veltman decomposition.

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[b] When this is not the case, further modifications are in order [21].

3 External leg corrections in unitarity methods

The unitarity cut of the amplitude of a process with massive external particles should give information about the coefficients of the on-shell Green's function. Unfortunately the external leg correction diagrams are singular, because the propagator opposite the external leg carries the same momentum and is therefore also on shell. This problem was addressed in the context of a numerical algorithm [16, 18]. The solution proposed was to omit the problematic contributions, taking care with the associated breaking of gauge invariance.

Here, we present a different solution [23]. In the spirit of the unitarity method, we want to keep the ingredients of the cut as complete amplitudes, without discarding any contributions. We must therefore also include the corresponding counterterms. The examples given here involve 4-dimensional cuts, although the formalism is equally valid in D dimensions.
We regularize the unitarity cuts by continuing the amplitude off-shell. In particular we perform a double momentum shift modifying the massive external momentum k and one other external momentum r. If both momenta are outgoing, the shift is

$$k \to \hat{k} = k + \xi \bar{k}, \qquad r \to \hat{r} = r - \xi \bar{k}, \tag{12}$$

where the momentum $\bar{k}$ is such that $r \cdot \bar{k} = \bar{k}^2 = 0$ and $2(k \cdot \bar{k}) \neq 0$. The momenta k and $\hat{r}$ are on shell, while $\hat{k}$ is off shell. With this shift, the the full amplitude diverges as $1/\xi$. The cuts are calculated in terms of tree amplitudes. For the double cut, one has simply the off shell three-point interaction $\mathcal{M}_R$ and an $(n+1)$-point on shell tree amplitude $\mathcal{M}_L$, the latter depending on the parameter ξ. In the case of the single cut we need a single $(n+2)$-point ξ-dependent tree amplitude, $\mathcal{M}_T$, which is continued of shell. We expand $\mathcal{M}_L$ and $\mathcal{M}_T$ up to first order in ξ.

In the on-shell scheme, the external leg correction diagrams are exactly cancelled by the corresponding counterterms. The counterterm diagrams have to be constructed from the renormalization constants and the various tree-level off-shell currents. After the expansion in ξ, the divergent part is guaranteed to cancel the cut loop diagram and the on-shell limit is reached by setting $\xi = 0$. Therefore at every stage of the procedure we systematically neglect terms of $\mathcal{O}(\xi)$.

The off-shell currents are gauge-dependent. In the sum of all parts, gauge invariance is restored by construction: coefficients of master integrals are gauge invariant, and we have only added zero in the form of the external leg correction plus its counterterm.

In the following we will show the cancellation of divergences in the cut between loop Feynman diagrams and the counterterms.

Bubbles from double cut

We consider the double cut of the external leg correction diagram for a massive fermion, as shown in Figure 1. The tree-level amplitudes $\mathcal{M}_L$ and $\mathcal{M}_R$ depicted in Figure 1 read as follows:

$$\begin{aligned} \mathcal{M}_L &= \frac{g\, T^A_{c'c''}}{(k+\xi\bar{k})^2 - m^2} \left(\bar{u}_{k+\xi\bar{k}-\ell}\, \not{\epsilon}^*_\ell \,(m + \not{k} + \xi \not{\bar{k}})\, \widehat{\mathcal{A}}_{c'' c_{\rm ext}} \right), \\ \mathcal{M}_R &= -i\, g\, T^A_{cc'} \left(\bar{u}_k \not{\epsilon}_\ell \, u_{k+\xi\bar{k}-\ell} \right), \end{aligned} \tag{13}$$

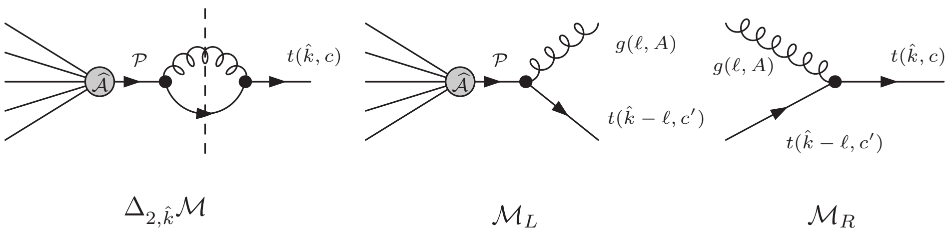

Figure 1: Double cut of the external leg correction diagram, and the left and right tree-level amplitudes. The momentum of the external massive fermion, k, is outgoing. The cut momenta are ℓ and $k-\ell$. Color information is indicated by c and c'. The massive propagator giving the on-shell divergence is denoted by $\mathcal{P}$.

where $\widehat{\mathcal{A}}_{c''c_{\text{ext}}}$ is the remaining parts of the diagram, including the color-flow information. It is worth noticing that the external massive spinor u_k is *not* being shifted.[c] In the Feynman gauge, the double cut including the sum over the polarization states is

$$-\frac{2ig^2C_F}{\xi\gamma}\frac{1}{(2\pi)^4}\int d^4\ell\,\delta(\ell^2)\,\delta\left((\ell-\hat{k})-m^2\right)$$
$$\left[(2m^2-\xi\gamma)\left(\bar{u}_k\,\widehat{\mathcal{A}}_{cc_{\text{ext}}}\right)+\left(\bar{u}_k\,\not{\ell}\,(m+\not{k}+\xi\not{\bar{k}})\,\widehat{\mathcal{A}}_{cc_{\text{ext}}}\right)+m\xi\left(\bar{u}_k\,\not{\bar{k}}\,\widehat{\mathcal{A}}_{cc_{\text{ext}}}\right)\right].$$

The Dynkin index of the fundamental representation of $SU(N)$ is denoted by C_F, while $\gamma\equiv 2k\cdot\bar{k}$.

Computing the double cut and performing the ξ-expansion, we get the bubble part of the diagram:

$$\begin{aligned}\mathcal{M}_B &= \frac{g^2C_F}{16\pi^2\xi\gamma}\Bigg\{4m^2\xi\gamma\left(\bar{u}_k\,\mathcal{A}_{cc_{\text{ext}}}\right)B_0'(m^2;m,0)\\ &+\left[4m^2\left(\bar{u}_k\,\mathcal{A}_{cc_{\text{ext}}}\right)+4m^2\xi\left(\bar{u}_k\,\mathcal{A}'_{cc_{\text{ext}}}\right)+2m\xi\left(\bar{u}_k\,\not{\bar{k}}\,\mathcal{A}_{cc_{\text{ext}}}\right)\right]B_0(m^2;m,0)\Bigg\}.\end{aligned}\tag{14}$$

In obtaining eq. (14) we have used the expansions

$$B_0(m^2+\xi\gamma;m,0)=B_0(m^2;m,0)+\xi\gamma B_0'(m^2;m,0)\ ,$$
$$\widehat{\mathcal{A}}_{cc_{\text{ext}}}=\mathcal{A}_{cc_{\text{ext}}}+\xi\mathcal{A}'_{cc_{\text{ext}}}\ .$$

When the amplitudes are written in the spinor-helicity formalism the completeness relation for polarization vectors is that of a light-like axial gauge rather than Feynman gauge. Therefore the procedure described above must be modified. In particular the bubble part of the diagram, eq. (14), gets an extra $\mathcal{O}(\xi^0)$ contribution

$$\frac{g^2C_F}{16\pi^2\gamma}\frac{\bar{u}_k\,\not{\bar{k}}\,\not{k}\,\not{q}(m+\not{k})\,\mathcal{A}_{cc_{\text{ext}}}}{q\cdot k}B_0(m^2;m,0),\tag{15}$$

[c] As explained in [23], the shift of u_k is possible but unnecessary.

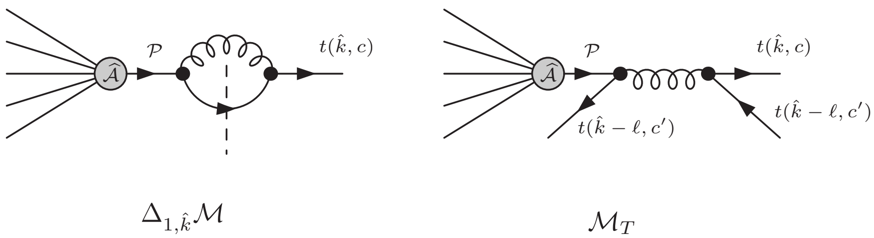

Figure 2: Single cut of the external leg correction diagram.

where q is an arbitrary light-like "reference" momentum. This new contribution does not affect the divergent part of the bubble coefficient.

Tadpole from single cut

The single cut of the massive propagator of the external leg correction diagram is depicted in Figure 2. The tree-level amplitude $\mathcal{M}_T$ is

$$\mathcal{M}_T = -\frac{g^2 C_F}{\ell^2 \xi\gamma}\left(\bar{u}_k\,\gamma^\mu\,u_{k+\xi\bar{k}-\ell}\right)\left(\bar{u}_{k+\xi\bar{k}-\ell}\,\gamma_\mu\,(m+\not{k}+\xi\not{\bar{k}})\,\widehat{\mathcal{A}}_{c'c_{\rm ext}}\right). \tag{16}$$

The single cut reads as follows

$$-\frac{2\,g^2 C_F}{\xi\gamma}\frac{i}{(2\pi)^4}\int d^4\ell\;\delta(\ell^2)$$
$$\left[\frac{(2m^2-\xi\gamma)\,(\bar{u}_k\,\mathcal{A}_{cc_{\rm ext}}) + \left(\bar{u}_k\,\not{\ell}\,(m+\not{k}+\xi\not{\bar{k}})\,\mathcal{A}_{cc_{\rm ext}}\right) + m\xi\left(\bar{u}_k\,\not{\bar{k}}\,\widehat{\mathcal{A}}_{cc_{\rm ext}}\right)}{\ell^2}\right],$$

and can be computed using the method described in section 2. Expanding around $\xi = 0$ the tadpole portion of the external leg correction, we get

$$\mathcal{M}_A = \frac{g^2 C_F}{16\pi^2}\left[\left(\frac{2}{\xi\gamma}-\frac{1}{m^2}\right)(\bar{u}_k\,\mathcal{A}_{cc_{\rm ext}}) + \frac{1}{\gamma m}\left(\bar{u}_k\,\not{\bar{k}}\,\mathcal{A}_{cc_{\rm ext}}\right) + \frac{2}{\gamma}\left(\bar{u}_k\,\mathcal{A}'_{cc_{\rm ext}}\right)\right] A_0(m). \tag{17}$$

Cancellation against the counterterm

The external leg counterterm, $\mathcal{M}^{\rm ct}$, depicted in Figure 3 is

$$\mathcal{M}^{\rm ct} = -\frac{1}{\xi\gamma}\left(\bar{u}_k\,\left(\not{k}\delta Z_\psi + \xi\not{\bar{k}}\delta Z_\psi - m\delta Z_\psi - m\delta Z_m\right)\,(\not{k}+\xi\not{\bar{k}}+m)\,\widehat{\mathcal{A}}_{cc_{\rm ext}}\right). \tag{18}$$

In the on-shell scheme, the renormalization constants δZ_m and δZ_ψ read as follows:

$$\begin{aligned}\delta Z_m &= -\frac{g^2 C_F}{16\pi^2}\left[\frac{A_0(m)}{m^2} + 2B_0(m^2;m,0)\right],\\ \delta Z_\psi &= -\frac{g^2 C_F}{16\pi^2}\left[\frac{A_0(m)}{m^2} - 4m^2 B_0'(m^2;m,0)\right].\end{aligned}\tag{19}$$

Expanding around $\xi = 0$ at $\mathcal{O}(\xi^0)$, we get

$$\begin{aligned}\mathcal{M}^{\rm ct} = -\frac{g^2 C_F}{16\pi^2}\Bigg\{&\left[\frac{2}{\xi\gamma}A_0(m) + \frac{4m^2}{\xi\gamma}B_0(m^2;m,0)\right](\bar{u}_k\,\mathcal{A}_{cc_{\rm ext}})\\ &+\left[\frac{2}{\gamma}A_0(m) + \frac{4m^2}{\gamma}B_0(m^2;m,0)\right]\left(\bar{u}_k\,\mathcal{A}'_{cc_{\rm ext}}\right)\\ &+\left[\frac{1}{\gamma m}A_0(m) + \frac{2m}{\gamma}B_0(m^2;m,0)\right]\left(\bar{u}_k\,\not{\bar{k}}\,\mathcal{A}_{cc_{\rm ext}}\right)\\ &+\left[\frac{1}{m^2}A_0(m) - 4m^2 B_0'(m^2;m,0)\right](\bar{u}_k\,\mathcal{A}_{cc_{\rm ext}})\Bigg\}.\end{aligned}\tag{20}$$

When the spinor-helicity formalism is used, the extra contribution (15) is accounted for by adding the following term to eq. (18):

$$\mathcal{M}^k = -\frac{1}{\xi\gamma}\bar{u}_k\left[(\not{k} + \xi\not{\bar{k}} - m)\,(\not{k} + \xi\not{\bar{k}})\not{q}\,\delta Z_k'\right](\not{k} + \xi\not{\bar{k}} + m)\widehat{\mathcal{A}}_{cc_{\rm ext}},\tag{21}$$

where

$$\delta Z_k' = \frac{g^2 C_F}{16\pi^2}\frac{B_0(m^2;m,0)}{q\cdot k}.\tag{22}$$

Once eq. (14), eq. (17), and eq. (20) are added together, the terms proportional to $1/\xi$ and the ones which depend on $\bar{k}$ cancel. The sum is identically zero, since in the on-shell scheme the external leg correction diagram and the external leg counterterm cancel exactly. The actual contribution to the tadpole and bubble coefficients comes from the other diagrams in the full amplitude. Since they are finite in ξ, no $\bar{k}$ dependence arises in the $\xi \to 0$ limit.

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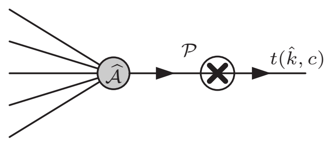

$\mathcal{M}^{\mathrm{ct}}$

Figure 3: Counterterm diagram.

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