

# Determination of the CP parity of Higgs bosons in their $\tau$ decay channels at the ILC

Stefan Berge<sup>\*1</sup>, Werner Bernreuther<sup>†2</sup> and Hubert Spiesberger<sup>\*3</sup>

<sup>\*</sup> PRISMA Cluster of Excellence, Institut für Physik (WA THEP),  
Johannes Gutenberg-Universität, 55099 Mainz, Germany

<sup>†</sup> Institut für Theoretische Physik, RWTH Aachen University, 52056 Aachen, Germany

## Abstract

We investigate a method for determining the  $CP$  nature of a neutral Higgs boson or spin-zero resonance  $\Phi$  at a future linear  $e^+e^-$  collider (ILC) in its  $\Phi \rightarrow \tau^- \tau^+$  decay channel. Our procedure is applicable if the production vertex of the Higgs boson can be measured. This will be the case, for example, for the Higgs-strahlung process  $e^+e^- \rightarrow Z + \Phi$ . We show that the method is feasible for both the leptonic and the hadronic 1-prong tau decay modes,  $\tau^\pm \rightarrow l^\pm + \nu_\tau + \nu_l$ ,  $\tau^\pm \rightarrow a_1^\pm, \rho^\pm, \pi^\pm \rightarrow \pi^\pm + X$ .

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<sup>1</sup>berge@uni-mainz.de

<sup>2</sup>breuther@physik.rwth-aachen.de

<sup>3</sup>spiesber@uni-mainz.de

## I. INTRODUCTION

Recently, the ATLAS and CMS experiments reported the discovery of a neutral boson of mass  $\sim 126$  GeV at the LHC [1, 2]. The experimental findings disfavor the option of a spin  $J = 1$  resonance. The experimental results [1, 2] are compatible with the hypothesis of identifying this resonance with the Standard Model (SM) Higgs boson; however, much more detailed investigations will be necessary to establish this conjecture. The investigations of the properties of this resonance will probably be possible at the LHC to a large extent.

A high-energy linear  $e^+e^-$  collider would be an ideal machine to investigate the properties of this resonance, i.e., its couplings, decay modes, spin, and  $CP$  parity, in great detail (and, of course, also of other, not too heavy resonances of similar type if they exist). As it is likely that the ATLAS and CMS resonance is a spin-zero (Higgs) boson, one may revert, for assessing the prospects of exploring this particle at a future linear collider, to the many existing phenomenological investigations, within the SM and many of its extensions, of Higgs-boson production and decay in  $e^+e^-$  collisions. As to the prospects of exploring the spin and  $CP$  properties of a Higgs boson, there have been a number of proposals and studies, including [3–28] that are relevant for Higgs-boson production and decay at a linear collider.

In this workshop contribution we apply a method [26, 27] for the determination of the  $CP$  properties of a neutral spin-zero (Higgs) boson  $\Phi$  in its  $\tau^+\tau^-$  decays to the production of  $\Phi$  at a future  $e^+e^-$  linear collider (ILC). For definiteness, we consider  $e^+e^- \rightarrow Z\Phi$ , but the analysis outlined below is applicable to any other  $\Phi$  production mode. In our analysis all major 1-prong  $\tau$  decays are taken into account. We demonstrate that the  $CP$  properties of  $\Phi$  can be determined with our method in an unambiguous way.

## II. CROSS SECTION AND OBSERVABLES

Here we consider the production of a neutral Higgs boson  $\Phi$  or, more general, of a spin-zero resonance of arbitrary  $CP$  nature by the Higgs-strahlung process in high energy  $e^+e^-$  collisions:

$$e^+e^- \rightarrow Z + \Phi. \quad (1)$$

For definiteness, we use  $m_\Phi = 126$  GeV in the following. The following remark is in order here. As is well known, for a pure pseudoscalar boson  $\Phi = A$ , the  $AZZ$  vertex must be loop-induced<sup>4</sup>. We assume here, for the sake of choosing a definite  $\Phi$  production mode, that (1) applies also to the production of a pure pseudoscalar.

For  $Z$  boson decays into an electron or a muon pair, the precise reconstruction of the production vertex and of the 4-momentum of the  $Z$  boson will be possible. As to  $\Phi$ , we consider

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<sup>4</sup> The strength of the loop-induced  $AZZ$  vertex was investigated for a number of SM extensions in [29].

here the decay mode into tau pairs, with subsequent 1-prong  $\tau^\pm$  decays:

$$\Phi \rightarrow \tau^- \tau^+ \rightarrow a^- a^+ + X, \quad (2)$$

where  $a^\pm = \{e^\pm, \mu^\pm, \pi^\pm\}$  and  $X$  denotes neutrinos and, possibly, neutral pions. We assume that the tau-decay mode of the  $\Phi$  has a reasonably large branching fraction, which is the case in the Standard Model and in many of its extensions. The interaction of a Higgs boson  $\Phi$  of arbitrary  $CP$  nature ( $J^{PC} = 0^{++}$ ,  $J^{PC} = 0^{-+}$ , or  $CP$  mixture) to  $\tau$  leptons is described by the general Yukawa Lagrangian

$$\mathcal{L}_Y = -(\sqrt{2}G_F)^{1/2}m_\tau(a_\tau\bar{\tau}\tau + b_\tau\bar{\tau}i\gamma_5\tau)\Phi, \quad (3)$$

where  $G_F$  denotes the Fermi constant and  $a_\tau, b_\tau$  are the reduced  $\tau$  Yukawa coupling constants. In the following we take into account in (2) the main 1-prong  $\tau$  decay channels

$$\begin{aligned} \tau &\rightarrow l + \nu_l + \nu_\tau, \\ \tau &\rightarrow a_1 + \nu_\tau \rightarrow \pi + 2\pi^0 + \nu_\tau, \\ \tau &\rightarrow \rho + \nu_\tau \rightarrow \pi + \pi^0 + \nu_\tau, \\ \tau &\rightarrow \pi + \nu_\tau. \end{aligned} \quad (4)$$

Our method that will be applied in the following does not require the knowledge of the  $\tau$  rest frame. Therefore we can include also the leptonic  $\tau$  decays in our analysis for which the presence of two, respectively four neutrinos would preclude the reconstruction of the  $\tau^\pm$  rest frames. We do not consider here  $\tau$  decays into 3 prongs, for instance  $\tau \rightarrow a_1 \rightarrow 3$  charged pions, because in this case the reconstruction of the  $\tau$  four-momentum should always be possible. This would considerably facilitate the measurement of the tau spin correlations that will be discussed below. (A corresponding analysis for  $\Phi$  production at the LHC was made in [25].) As an aside, we remark that it will be helpful, but not essential for future experimental analyses if the different hadronic  $\tau$ -decays can be experimentally distinguished.

Our method to determine the  $CP$  properties of a spin-zero boson was first developed for the case of  $\Phi$  production in  $pp$  collisions at the Large Hadron Collider in [26] and was then applied to an analysis that included the combination of all 1-prong  $\tau$  decay channels in [27]. The method is based on the fact that the  $CP$  quantum number of a neutral spin-zero resonance  $\Phi$  can be determined in a definite way through its  $\tau^- \tau^+$  mode by measuring the two  $\tau$  spin correlations  $S = \mathbf{s}_{\tau^-} \cdot \mathbf{s}_{\tau^+}$  and  $S_{CP} = \hat{\mathbf{k}}_\tau \cdot (\mathbf{s}_{\tau^-} \times \mathbf{s}_{\tau^+})$ , where  $\hat{\mathbf{k}}_\tau = \mathbf{k}_\tau/|\mathbf{k}_\tau|$  is the normalized  $\tau^-$  momentum vector in the zero-momentum frame (ZMF) of the  $\tau^- \tau^+$ -pair [5, 16]. For a scalar  $\Phi$ , the expectation value of  $S$  is  $\langle S \rangle = 1/4$ , whereas for a pseudoscalar,  $\langle S \rangle = -3/4$ . The  $CP$ -odd and  $T$ -odd spin correlation  $S_{CP}$  probes whether or not  $\Phi$  is a mixed  $CP$  state. If  $\Phi$  is a  $CP$  mixture, i.e., if the neutral Higgs sector violates  $CP$  (that is,  $a_\tau b_\tau \neq 0$  in (3)), then a non-zero expectation value of  $S_{CP}$  is generated already at tree level [5] and can be as large as 0.5.

The spin of the  $\tau$  can not be measured directly; however it induces, in the spectrum of polarized tau decay  $\tau^\pm \rightarrow a^\pm$ , a correlation with the direction of flight of the charged particle  $a^\pm$ :

$$\frac{1}{\Gamma(\tau^\mp \rightarrow a^\mp + X)} \frac{d\Gamma(\tau^\mp(\hat{\mathbf{s}}^\mp) \rightarrow a^\mp(q^\mp) + X)}{dE_{a^\mp} d\Omega_{a^\mp} / (4\pi)} = n(E_{a^\mp}) (1 \pm b(E_{a^\mp}) \hat{\mathbf{s}}^\mp \cdot \hat{\mathbf{q}}^\mp). \quad (5)$$

Here,  $\hat{\mathbf{s}}^\mp$  denote the normalized spin vectors of the  $\tau^\mp$  and  $\hat{\mathbf{q}}^\mp$  the direction of flight of  $a^\mp$  in the respective  $\tau$  rest frame. The function  $b(E_a)$  encodes the  $\tau$ -spin analyzing power of particle  $a$ . The correlation of the  $\tau$ -spins,  $\hat{\mathbf{s}}^+ \cdot \hat{\mathbf{s}}^-$ , leads to a nontrivial distribution of the opening angle  $\angle(\hat{\mathbf{q}}^+, \hat{\mathbf{q}}^-)$ , whereas the  $CP$ -odd observable  $\hat{\mathbf{k}} \cdot (\hat{\mathbf{s}}^+ \times \hat{\mathbf{s}}^-)$  induces the triple correlation  $\hat{\mathbf{k}} \cdot (\hat{\mathbf{q}}^+ \times \hat{\mathbf{q}}^-)$ . The strength of these correlations depends, for a given strength of the reduced Yukawa couplings  $a_\tau, b_\tau$ , on the product  $b(E_{a'^-})b(E_{a^+})$ , while  $n(E_{a'^-})n(E_{a^+})$  is responsible for the rate of  $\tau^+ \tau^-$  decay into  $a^+ a'^-$  final states.

In order to use these observables in an experimental analysis, one has to be able to reconstruct the  $\tau^\pm$  and  $a^\pm$  momenta in the  $\tau^\pm$  and  $\Phi$  rest frames. This is, in general, not possible for the leptonic  $\tau$ -decay channel and very difficult in the case of hadronic  $\tau$  decays, because at a linear collider beamstrahlung effects can shift the initial center of mass energy by a large amount. In [26] it was shown that one can, nevertheless, construct experimentally accessible observables that have a high sensitivity to the  $CP$  quantum numbers of the  $\Phi$ . The crucial point is to employ the zero-momentum frame of the  $a^+ a'^-$  pair. The distribution of the angle

$$\varphi^* = \arccos(\hat{\mathbf{n}}_\perp^{*+} \cdot \hat{\mathbf{n}}_\perp^{*-}) \quad (6)$$

discriminates between  $CP = \pm 1$  states. Here  $\hat{\mathbf{n}}_\perp^{*\pm}$  are normalized impact parameter vectors defined in the zero-momentum frame of the  $a^+ a'^-$  pair. These vectors can be reconstructed [26] from the impact parameter vectors  $\hat{\mathbf{n}}_\mp$  measured in the laboratory frame by boosting the 4-vectors  $n_\mp^\mu = (0, \hat{\mathbf{n}}_\mp)$  into the  $a'^- a^+$  ZMF and decomposing the spatial part of the resulting 4-vectors into their components parallel and perpendicular to the respective  $\pi^\mp$  or  $l^\mp$  momentum. We emphasize that  $\varphi^*$  defined in Eq. (6) is not the true angle between the  $\tau$  decay planes, but nevertheless, it carries enough information to discriminate between  $CP$ -even and  $CP$ -odd Higgs bosons. The role of the  $CP$ -odd and  $T$ -odd triple correlation introduced above is taken over by the triple correlation  $\mathcal{O}_{CP}^* = \hat{\mathbf{p}}_-^* \cdot (\hat{\mathbf{n}}_\perp^{*+} \times \hat{\mathbf{n}}_\perp^{*-})$  between the impact parameter vectors just defined and the normalized  $a'^-$  momentum in the  $a'^- a^+$  ZMF, which is denoted by  $\hat{\mathbf{p}}_-^*$ . Equivalently, one can determine the distribution of the angle [26]

$$\psi_{CP}^* = \arccos(\hat{\mathbf{p}}_-^* \cdot (\hat{\mathbf{n}}_\perp^{*+} \times \hat{\mathbf{n}}_\perp^{*-})). \quad (7)$$

Before presenting results we would like to point out the difference of our method as compared to a previous analysis of how to determine the  $CP$  parity of a Higgs boson in its  $\tau^+ \tau^-$  decays at a linear collider. Refs. [20–22] analyzed the hadronic 1-prong decay  $\tau \rightarrow \rho \nu$ . The observable used by these authors, namely the acoplanarity angle of the  $\rho^+$  and  $\rho^-$  decay planes, requires the reconstruction of the  $\rho^+ \rho^-$  ZMF, i.e., the measurement of the  $\pi^\pm$  and the  $\pi^0$

momenta, and the reconstruction of approximate  $\tau^\pm$  rest frames. As emphasized above, our method is applicable to all 1-prong  $\tau$  decays, in particular  $\tau \rightarrow l$ .

### III. RESULTS

For predicting the distributions of the observables  $\varphi^*$  and  $\psi_{CP}^*$ , for a specific  $\Phi$ -decay mode (2), in terms of the unknown reduced Yukawa couplings  $a_\tau, b_\tau$ , one needs to know the spectral function  $n(E_a)$  and in particular  $b(E_a)$ , which determines, as mentioned, the tau-spin analyzing power of particle  $a^\pm = l^\pm, \pi^\pm$  and therefore the shapes of the  $\varphi^*$  and  $\psi_{CP}^*$  distributions. For the purpose of our analysis, the major 1-prong  $\tau$  decays (4) can be considered to be experimentally well established Standard Model physics, and the respective spectral functions are known within the SM to sufficient accuracy, cf. [25, 27] and references therein.

At a linear  $e^+e^-$  collider, a Higgs boson  $\Phi$  produced by the Higgs-strahlung process (1) will have on average a much larger transverse momentum as compared to  $\Phi$  production at the LHC by its major production mode  $gg \rightarrow \Phi$ . This calls for a study – independent of the LHC analyses [26, 27] – of the question how the  $CP$  properties of a (pseudo)scalar boson are reflected in the distributions of  $\varphi^*$  and  $\psi_{CP}^*$ . In addition, for future experimental analyses, differences between LHC and ILC can be expected from the fact that the ILC detectors will be able to measure the  $\tau$  decay products at transverse momenta as small as about 10 GeV.

As outlined above, our method is based on the reconstruction of the normalized spatial impact parameters of the  $\tau$ -decay products with respect to the production vertex of the Higgs boson. In the Higgs-strahlung process  $e^+e^- \rightarrow \Phi + Z$  at the ILC the normalized impact parameters can be reconstructed for events where the  $Z$  boson decays into electron or muon pairs,  $Z \rightarrow e^+e^-, \mu^+\mu^-$  and for  $\Phi \rightarrow \tau\tau$  decays with sufficiently long  $\tau$ -decay lengths. Here, we use this process to study the  $\varphi^*$  and  $\psi_{CP}^*$  distributions. We apply the following acceptance cuts appropriate for the ILC:

$$\begin{aligned} p_T^{l,\pi} &\geq 10 \text{ GeV}, \\ 15^\circ &< \theta_{l,\pi} < 165^\circ. \end{aligned}$$

Let us first consider the decays  $\tau^- \tau^+ \rightarrow l^- l'^+ 4\nu$ , where  $l, l' = e, \mu$ . The functions  $n(E_l)$  and  $b(E_l)$ , where  $E_l$  is the energy of  $l$  in the  $\tau$  rest frame, are shown in Fig. 1a. The function  $b(E_l)$ , which determines the correlation of the  $\tau$  spin with the momentum of  $l$ , changes sign at  $E_l = m_\tau/4$ . For a Higgs boson with specified  $CP$  parity (and specified reduced Yukawa couplings), the sign of the product  $b(E_l)b(E_{l'})$  determines the functional form of the  $\varphi^*$  distribution and in particular the sign of the associated asymmetry

$$A_{\varphi^*} = \frac{N(\varphi^* > \pi/2) - N(\varphi^* < \pi/2)}{N(\varphi^* > \pi/2) + N(\varphi^* < \pi/2)}. \quad (8)$$

As an illustration we apply the cut  $E_l > m_\tau/4$  in the  $\tau$  rest frames to allow only for contributions with  $b(E_l) < 0$ . The resulting normalized  $\varphi^*$  distributions are shown as black dotted

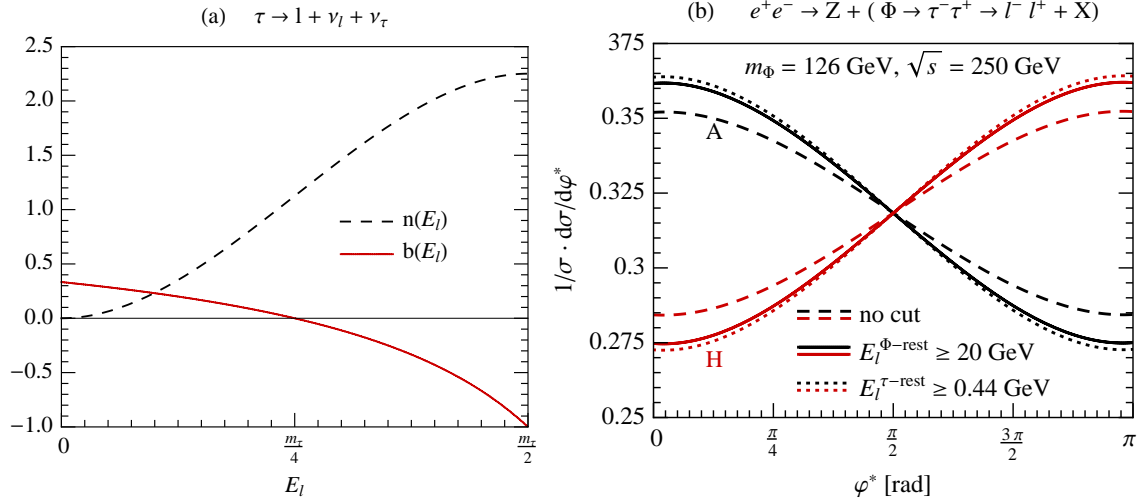


Figure 1: (a) The spectral functions  $n(E_l)$  and  $b(E_l)$ , Eq. (5), for the leptonic  $\tau$  decays. The function  $n(E_l)$  is given in units of  $\text{GeV}^{-1}$ . (b) Normalized  $\varphi^*$  distribution for a  $l^+l^-$  final state for a Higgs boson with mass of 126 GeV produced at  $\sqrt{s} = 250$  GeV. Scalar  $\phi = H$ , pseudoscalar  $\phi = A$ . The dashed lines show the distribution if no cuts are applied, the solid lines refer to the case where a cut  $E_l^{\Phi\text{-rest}} \geq 20$  GeV was applied on both lepton energies in the Higgs rest frame. The dotted lines show the results for the ideal cut  $E_l^{\tau\text{-rest}} \geq m_\tau/4$ .

lines in Fig. 1b. For a  $CP$ -odd boson it has its maximum at  $\varphi^* = 0$  and its minimum at  $\varphi^* = \pi$ , while for a  $CP$ -even boson (red dotted line) the distribution is flipped,  $\varphi^* \leftrightarrow \pi - \varphi^*$ . If one applies instead, either for  $\tau^+$  or  $\tau^-$  decay (but not for both), the cut  $E_l < m_\tau/4$  which leads to  $b(E_l) > 0$ , the behavior of the  $H$  and  $A$  distributions with increasing  $\varphi^*$  will be interchanged. The magnitude of the resulting asymmetry (8) becomes smaller because the maximum of  $|b(E_l)|$  is smaller for  $E_l < m_\tau/4$  than the maximum of  $|b(E_l)|$  for  $E_l > m_\tau/4$ . In addition, with the cut  $E_l < m_\tau/4$  the total decay rate is smaller than for  $E_l > m_\tau/4$ . This would make a measurement more difficult. Without a cut on the lepton energy, the asymmetry of the normalized  $\varphi^*$ -distributions is reduced, but remains non-zero because the averaging over the different signs of  $b(E_l)$  is weighted by the spectral function  $n(E_l)$  displayed in Fig. 1a. The result is shown in Fig. 1b (dashed lines, black for a pseudoscalar, red for a scalar boson).

Obviously, it would be an advantage if one could apply a cut on  $E_l$  to separate the contributions that involve different signs of  $b(E_l)$ . However, this would require to reconstruct the full  $\tau$  4-momentum in order to perform the necessary boost into the  $\tau$  rest frame. On the other hand, the energy of the lepton  $l$  in the Higgs rest frame,  $E_l^\Phi$ , is correlated with the energy  $E_l$  in the  $\tau$  rest frame and a cut on the former can enrich the event sample with events in the desired range of the latter. The Higgs rest frame can, in fact, be reconstructed for the production process  $e^+e^- \rightarrow \Phi + Z$ , because the  $Z$ -boson 4-momentum is known for  $Z \rightarrow e, \mu$  decays, provided that initial state radiation is negligible or can be corrected for. This should be the case at least for the TESLA design [30]. In Fig. 1b we show the resulting  $\varphi^*$  distributions

with a cut  $E_l^\Phi \geq 20$  GeV applied to both leptons from  $\tau^\pm$  decay. The solid black line (solid red line) shows the distribution for a  $CP$ -odd ( $CP$ -even) Higgs boson. The sensitivity of the distributions to the  $CP$  parity of  $\Phi$  is clearly enhanced compared to the case where no cut is applied (dashed lines). These distributions are only slightly less sensitive than those with the ideal cut  $E_l^{\tau\text{-rest}} > m_\tau/4$  (dotted lines).

In the 2-body decay  $\tau \rightarrow \pi + \nu_\tau$ , the  $\pi$  is monochromatic in the  $\tau$  rest frame. (Its  $\tau$ -spin analyzing power is maximal.) A cut on the energy of the charged prong, i.e., of the charged pion, is very important for the hadronic 1-prong decays  $\tau^\pm \rightarrow \rho^\pm \nu \rightarrow \pi^\pm + \pi^0 \nu$  and  $\tau^\pm \rightarrow a_1^\pm \nu \rightarrow \pi^\pm + 2\pi^0 \nu$ . For example, for the decay  $\rho^\pm \rightarrow \pi^\pm + \pi^0$ , the function  $b(E_\pi)$  changes sign within the range of  $E_\pi$  (see, e.g., Fig. 4a in [27]) and the  $\varphi^*$  distributions for both a scalar and a pseudoscalar boson turn out to be flat if no cut was applied. The same is true for the  $\tau \rightarrow a_1$  decay mode. As in the case of  $\tau \rightarrow l$ , a cut on the energy of the charged pion in the Higgs-boson rest frame such that  $b(E_\pi)$  is either positive or negative for the selected events significantly enhances the discriminating power of the  $\varphi^*$  distribution. Provided that the event rate is large enough, a value for  $E_{\pi,\text{cut}}^\Phi$  may be chosen such as to optimize the separation of events with positive and negative  $b(E_\pi)$ , and both sets of events could be used to determine the  $CP$  nature of  $\Phi$ .

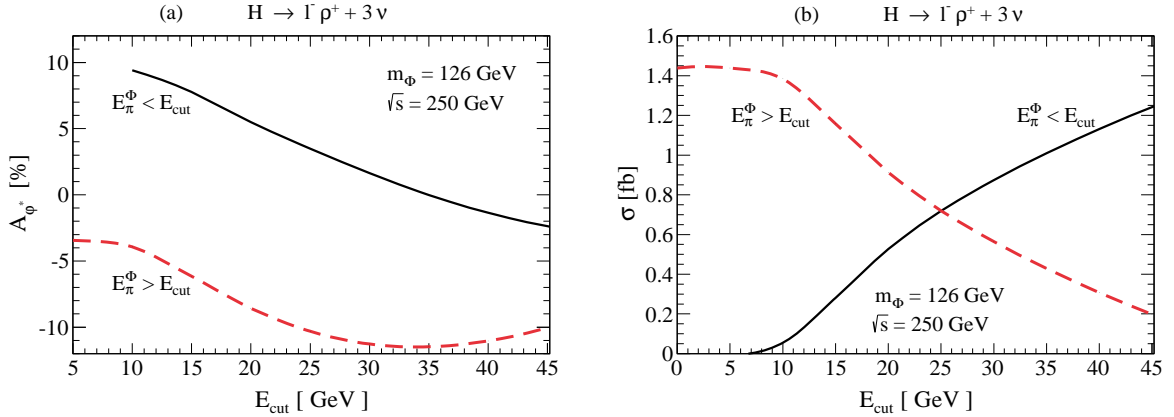


Figure 2: Influence of a cut on the energy of the charged pion in  $l + \rho$  final states for a scalar boson  $\phi = H$  with a mass of 126 GeV produced at  $\sqrt{s} = 250$  GeV: (a) on the  $\varphi^*$  asymmetry, (b) on the cross section. The black full lines correspond to applying the cut  $E_\pi^\Phi < E_{\pi,\text{cut}}^\Phi$ , while the red dashed lines are for  $E_\pi^\Phi > E_{\pi,\text{cut}}^\Phi$ .

We illustrate this in Fig. 2 for the decay  $\Phi \rightarrow l^- \rho^+ + 3\nu$  of a scalar boson. The effect can be quantified by calculating the associated asymmetry (8). The dashed red line in Fig. 2a shows the asymmetry for events with  $E_\pi^\Phi > E_{\pi,\text{cut}}^\Phi$ . The value  $E_{\pi,\text{cut}}^\Phi = 0$ , not shown in the figure, corresponds to the case without cut. In this case the asymmetry is rather small; applying a cut, the asymmetry increases to almost  $-12\%$  for  $E_{\pi,\text{cut}}^\Phi = 35$  GeV. However, increasing  $E_{\pi,\text{cut}}^\Phi$  will decrease the cross section as shown in Fig. 2b. (The cross section was computed for the Standard Model Higgs boson.) Without any cut the cross section is about 1.5 fb; it



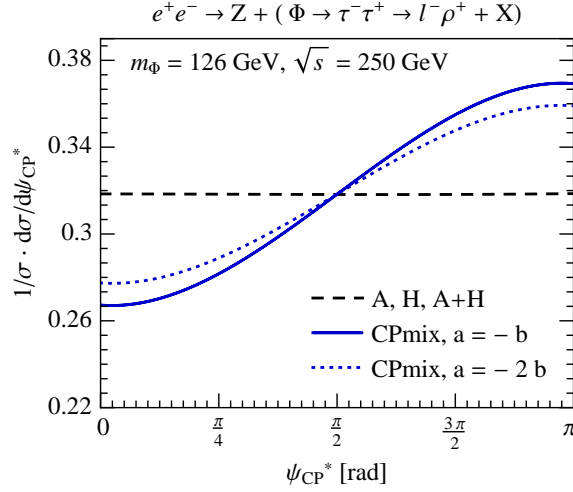


Figure 3: Normalized  $\psi_{CP}^*$  distribution for  $l\rho$  final states for different types of Higgs bosons with mass of 126 GeV produced at  $\sqrt{s} = 250$  GeV. A minimum cut on the  $\pi^+$  energy of 25 GeV in the Higgs rest frame was applied.

decreases to 0.2 fb for the cut  $E_{\pi,\text{cut}}^\Phi = 45$  GeV.

On the other hand, the complementary region  $E_\pi^\Phi < E_{\pi,\text{cut}}^\Phi$  leads to a positive asymmetry  $A_{\varphi^*}$  for cut values  $\lesssim 35$  GeV due to the fact that  $b(E_\pi)$  has changed sign. For small values of  $E_\pi$ , one finds  $A_{\varphi^*} \sim +9\%$ , but the cross section is tiny, about 0.05 fb. The asymmetry decreases to almost  $-2\%$  for  $E_{\pi,\text{cut}}^\Phi = 45$  GeV. It is clear that a judicious choice of this cut, taking into account experimental conditions and the available luminosity, is required to reach an optimal discrimination between a  $CP$ -even and  $CP$ -odd boson.

The  $\varphi^*$  distribution is well suited to distinguish between  $CP$ -even and  $CP$ -odd states. However, if the Higgs boson is a  $CP$ -mixture, or if there would exist two (almost) mass-degenerate bosons that escape experimental resolution, one of which has  $CP$  parity  $+1$  and the other one  $-1$ , the  $\varphi^*$  distribution would be flat, assuming the cross sections are of comparable magnitude. The distribution with respect to the angle  $\psi_{CP}^*$  defined in Eq. (7) would be appropriate to resolve these scenarios. A typical result is shown in Fig. 3 which applies to the decay chain  $\Phi \rightarrow \tau^+\tau^- \rightarrow l^-\pi^+ + 3\nu$  via hadronic  $\tau \rightarrow \rho$  decay. The solid blue line shows the normalized  $\psi^*$  distribution of a maximally  $CP$ -mixed boson ( $|a_\tau| = |b_\tau| > 0$ ) and we have chosen  $a_\tau = -b_\tau$ . For the scenario of mass degenerate bosons with opposite  $CP$  parities the  $\psi_{CP}^*$  distribution is shown by the horizontal dashed black line. The dotted blue line corresponds to the case of a non-maximal mixture with reduced Yukawa couplings  $a_\tau = -2b_\tau$ . If  $a_\tau$  and  $b_\tau$  have the same sign, the distribution is flipped,  $\psi_{CP}^* \leftrightarrow \pi - \psi_{CP}^*$ . The resulting asymmetry of the  $\psi_{CP}^*$  distribution will clearly be observable, provided the event rates are large enough.

We have also performed a Monte Carlo study to estimate the uncertainty of the measurement of the impact parameters. We applied a simple Gaussian smearing with  $\sigma_{\text{impact}} = 25^\circ$  [21] on the direction of the normalized impact parameter vectors. We found that the  $\varphi^*$  and  $\psi_{CP}^*$  distributions are only mildly affected by such an uncertainty.



## IV. CONCLUSIONS

Using the method of [26, 27] we have shown that the CP nature of a neutral Higgs boson  $\Phi$  produced at a future linear  $e^+e^-$  collider can be determined in a definite way in the  $\Phi \rightarrow \tau^+\tau^-$  decay channel with subsequent 1-prong  $\tau$  decays. We have considered the production of  $\Phi$  with mass  $m_\Phi = 126$  GeV by the Higgs-strahlung process, but our method can also be applied to Higgs bosons with other masses and to any other  $\Phi$  production mode. Our approach does not require the knowledge of the  $\tau$  rest frames; therefore, all 1-prong  $\tau$  decays, including  $\tau \rightarrow l$  can be used. The joint measurement of the distributions and associated asymmetries of the angles  $\varphi^*$  and  $\psi_{CP}^*$  allows to discriminate between a number of scenarios, some of which were discussed above. For a statistically significant determination of the CP parity of  $\Phi$ , only very few 1-prong events are required. We will elaborate on this and on other related issues in future work [31].

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- [1] G. Aad *et al.* [The ATLAS Collaboration], “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” arXiv:1207.7214 [hep-ex].
  - [2] S. Chatrchyan *et al.* [The CMS Collaboration], “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” arXiv:1207.7235 [hep-ex].
  - [3] J. R. Dell’Aquila and C. A. Nelson, Phys. Rev. D **33** (1986) 93.
  - [4] J. R. Dell’Aquila and C. A. Nelson, Nucl. Phys. B **320** (1989) 61.
  - [5] W. Bernreuther and A. Brandenburg, Phys. Lett. B **314** (1993) 104.
  - [6] A. Soni and R. M. Xu, Phys. Rev. D **48** (1993) 5259 [hep-ph/9301225].
  - [7] V. D. Barger, K. Cheung, A. Djouadi, B. A. Kniehl and P. M. Zerwas, Phys. Rev. D **49** (1994) 79 [hep-ph/9306270].
  - [8] K. Hagiwara and M. L. Stong, Z. Phys. C **62** (1994) 99 [hep-ph/9309248].
  - [9] T. Arens, U. D. J. Gieseler and L. M. Sehgal, Phys. Lett. B **339** (1994) 127 [hep-ph/9408316].
  - [10] A. Skjold and P. Osland, Phys. Lett. B **329** (1994) 305 [hep-ph/9402358].
  - [11] T. Arens and L. M. Sehgal, Z. Phys. C **66** (1995) 89 [hep-ph/9409396].
  - [12] A. Skjold and P. Osland, Nucl. Phys. B **453** (1995) 3 [hep-ph/9502283].
  - [13] S. Bar-Shalom, D. Atwood, G. Eilam, R. R. Mendel and A. Soni, Phys. Rev. D **53** (1996) 1162

- [hep-ph/9508314].
- [14] B. Grzadkowski and J. F. Gunion, Phys. Lett. B **350** (1995) 218 [hep-ph/9501339].
- [15] J. F. Gunion, B. Grzadkowski and X. -G. He, Phys. Rev. Lett. **77** (1996) 5172 [hep-ph/9605326].
- [16] W. Bernreuther, A. Brandenburg and M. Flesch, Phys. Rev. D **56** (1997) 90 [hep-ph/9701347].
- [17] S. Bar-Shalom, D. Atwood and A. Soni, Phys. Lett. B **419** (1998) 340 [hep-ph/9707284].
- [18] B. Grzadkowski, J. F. Gunion and J. Kalinowski, Phys. Rev. D **60** (1999) 075011 [hep-ph/9902308].
- [19] S. Y. Choi, D. J. Miller, M. M. Muhlleitner and P. M. Zerwas, Phys. Lett. B **553** (2003) 61 [hep-ph/0210077].
- [20] G. R. Bower, T. Pierzchala, Z. Was and M. Worek, Phys. Lett. B **543** (2002) 227 [hep-ph/0204292].
- [21] K. Desch, Z. Was and M. Worek, Eur. Phys. J. C **29** (2003) 491 [hep-ph/0302046].
- [22] K. Desch, A. Imhof, Z. Was and M. Worek, Phys. Lett. B **579** (2004) 157 [hep-ph/0307331].
- [23] E. Accomando *et al.*, hep-ph/0608079.
- [24] P. S. Bhupal Dev, A. Djouadi, R. M. Godbole, M. M. Muhlleitner and S. D. Rindani, Phys. Rev. Lett. **100** (2008) 051801 [arXiv:0707.2878 [hep-ph]].
- [25] S. Berge, W. Bernreuther and J. Ziethe, Phys. Rev. Lett. **100** (2008) 171605 [arXiv:0801.2297 [hep-ph]].
- [26] S. Berge and W. Bernreuther, Phys. Lett. B **671** (2009) 470 [arXiv:0812.1910 [hep-ph]].
- [27] S. Berge, W. Bernreuther, B. Niepelt and H. Spiesberger, Phys. Rev. D **84** (2011) 116003 [arXiv:1108.0670 [hep-ph]].
- [28] R. M. Godbole, C. Hangst, M. Muhlleitner, S. D. Rindani and P. Sharma, Eur. Phys. J. C **71** (2011) 1681 [arXiv:1103.5404 [hep-ph]].
- [29] W. Bernreuther, P. Gonzalez and M. Wiebusch, Eur. Phys. J. C **69** (2010) 31 [arXiv:1003.5585 [hep-ph]].
- [30] R. Brinkmann, (ed.), K. Flottmann, (ed.), J. Rossbach, (ed.), P. Schmueser, (ed.), N. Walker, (ed.) and H. Weise, (ed.), “TESLA: The superconducting electron positron linear collider with an integrated X-ray laser laboratory. Technical design report. Pt. 2: The accelerator,” DESY-01-011.
- [31] S. Berge, W. Bernreuther, and H. Spiesberger, in preparation.