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Complete electroweak one loop contributions to the pair production cross section of MSSM charged and neutral Higgs bosons in e^+e^- collisions

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Abstract

In this paper, we review the production cross section for charged and neutral Higgs bosons pairs in e^+e^- collisions beyond the tree level, in the framework of the Minimal Supersymmetric Standard Model (MSSM). A complete list of formulas for all electroweak contributions at the one loop level is given. A numerical code has been developed in order to compute them accurately and, in turn, to compare the MSSM Higgs bosons pair production cross sections at tree level and at the one loop level.

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1 Introduction

One major task of a future e^+e^- linear collider will be the exploration of the Higgs sector, in the Standard Model and beyond it. Despite its success, the Standard Model suffers from the appearance of quadratically divergent contributions to the Higgs boson mass. However, this problem is solved by Supersymmetry. The theoretical framework of our study is the minimal supersymmetric extension of the Standard Model (MSSM), where the Higgs spectrum consists of three unphysical Goldstone modes (G^+, G^- and G^0) as well as five physical states. Two of these are charged (H^+ and H^-) and, among the three neutral Higgs bosons, two are CP-even states, h^0 and H^0, and one is CP-odd, A^0. Several other aspects of the MSSM Higgs boson phenomenology are reviewed in [1].

The processes $e^+e^- \to H^+H^-$, H^0A^0, h^0A^0 that will be observable at future e^+e^- linear colliders, such as ILC and/or CLIC, are among the best places where one can accurately check the Higgs structure, see references [2] to [8] for details. At tree level, e^+ and e^- annihilate through a photon and a Z boson in the case of H^+H^- production, and through only a Z boson in the case of H^0A^0 and h^0A^0 production. At this level, the amplitudes depend on the masses of the Higgs bosons and on the mixing angle α. At the one loop level, most of the MSSM parameter space is involved through self-energies, triangle and box diagrams. In a previous paper [9] it was shown that, at high energy, at the leading and sub-leading (Sudakov) logarithmic orders, a great simplification occurs. The gauge and the SUSY structures of these processes reflect directly in the coefficients of the quadratic and linear logarithmic terms. In this high energy range, they depend only on a few parameters (the Standard Model inputs, the angles α and β, as well as the SUSY scale M_{SUSY}). The next step is to study more deeply the SUSY structure by looking at sub-sub-leading effects. First, one should determine the energy range in which the above Sudakov limit is an acceptable approximation and can be accurately tested. Then, one can study the effects of the successive sub-sub-leading terms (constants, m^2/s, etc) and classify the various parameters which control each of them. We should then estimate the accuracy at which these parameters can be measured. For these purposes, we have developed a code allowing to compute numerically the complete electroweak one loop contributions to the pair production cross section of MSSM charged and neutral Higgs bosons in e^+e^- collisions. The purpose of the present paper is to write in an explicit fashion all details of the electroweak one loop contributions that are computed by this numerical code.

In Section 2, we review the tree level MSSM Higgs sector and we calculate the production cross section for H^+H^-, H^0A^0 and h^0A^0 pairs in e^+e^- collisions at tree level. In the rest of the paper, we focus on the various one loop terms. The contributions of the initial vertices and of $e^\pm$ self-energy are given in Section 3, the intermediate gauge boson self-energies are discussed in Section 4, the contributions of final vertices and of Higgs self-energies are calculated in Section 5, and the effect of box diagrams are presented in Section 6. Finally, a summary and some outlooks (in particular a more detailed description of our numerical code) are given in Section 7.

2 Tree level calculations

2.1 Tree level structure of the MSSM Higgs sector

In the MSSM, two complex scalar Higgs doublets are responsible for the breaking of the electroweak symmetry:

$$H_1 = \begin{pmatrix} (v_1 + \phi_1^0 - i\chi_1^0)/\sqrt{2} \\ -\phi_1^- \end{pmatrix}, \; H_2 = \begin{pmatrix} \phi_2^+ \\ (v_2 + \phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}. \tag{1}$$

They have opposite hypercharge ($Y_1 = -1$ and $Y_2 = +1$) and their vacuum expectation values are respectively v_1 and v_2. After diagonalization, one obtains the following states:

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \tag{2}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \tag{3}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}. \tag{4}$$

Here, G^+, G^- and G^0 describe three unphysical Goldstone modes. The five physical states are two charged bosons (H^+ and H^-), two neutral scalar bosons with CP $= +1$ (h^0 and H^0) and one pseudoscalar neutral boson with CP $= -1$ (A^0).

The quadratic part of the Higgs potential, which contains the soft breaking masses and the gauge couplings, depends on two independent parameters, which are usually chosen as the mass M_A of the A^0 boson and the ratio between the vacuum expectation values $\tan\beta = v_2/v_1$. The masses of the other physical states are expressed as follows:

$$M_H^2 = M_A^2 + M_W^2, \tag{5}$$

$$M_{H^0,h^0}^2 = \frac{1}{2}\left(M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right). \tag{6}$$

As for the mixing angle between H^0 and h^0, it is given by:

$$\tan 2\alpha = \tan 2\beta \times \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad -\frac{\pi}{2} \leq \alpha \leq 0. \tag{7}$$

Note that these results are only valid at tree level and they become slightly different when one includes radiative corrections.

2.2 Production cross section at tree level

In e^+e^- collisions, charged Higgs bosons are pair produced through virtual photon and Z boson exchange (and in top decays if M_H is small enough). As for the neutral Higgs bosons, they can be produced through several mechanisms: WW and ZZ fusion processes, Higgsstrahlung or pair production. In this paper, we only focus on this latter process (note that CP conservation forbids virtual photon exchange). The Feynman diagrams of interest are shown in Figure 1. More details about the various production mechanisms and decay modes of MSSM Higgs bosons can be found in [10]. Here, we only focus on the total pair production cross sections and we ignore the different contributions of the decay channels.

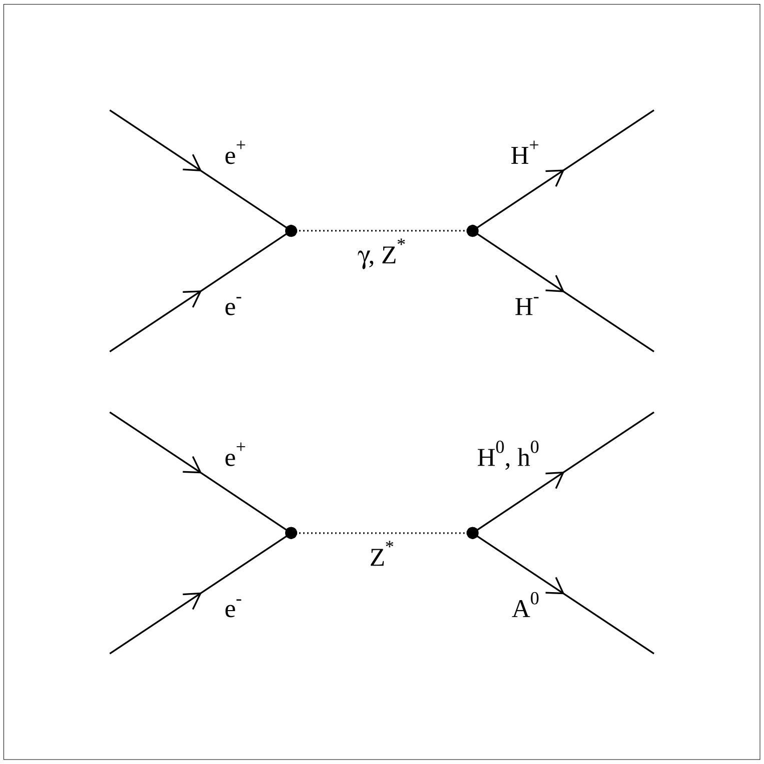

Figure 1: Feynman diagrams for the pair production of MSSM charged and neutral Higgs bosons in e^+e^- collisions.

The tree level production cross section can be easily derived using the Feynman rules. If $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$ and $\eta \equiv q^2/(q^2 - M_Z^2)$, then the Born amplitudes $a_{L,R}^{Born}$ are:

- for charged Higgs bosons:

$$a_{L,R}^{Born}(H^+H^-) = 1 - \frac{(1-2s_W^2)}{4s_W^2 c_W^2}\eta g_{L,R} \tag{8}$$

- for neutral Higgs bosons:

$$a_{L,R}^{Born}(H^0A^0/h^0A^0) = -\frac{i}{4s_W^2 c_W^2}\eta g_{L,R}\,[Z_{ab}] \tag{9}$$

where $g_L = 2s_W^2 - 1$, $g_R = 2s_W^2$ and $[Z_{ab}] = [-\sin(\beta-\alpha);\, \cos(\beta-\alpha)]$ for H^0A^0 and h^0A^0 final states, respectively.

In this paper, we use the following renormalization for the amplitude:

$$A = \frac{2e^2}{q^2}\,\bar{v}(e^+)(\not{p})(a_L P_L + a_R P_R)u(e^-),\;\; P_{L,R} = \frac{1 \mp \gamma_5}{2}. \tag{10}$$

The differential tree level cross sections are then given by:

$$\frac{d\sigma^{Born}_{L,R}}{d\cos\theta} = \frac{\pi\alpha^2_{em}\beta^3_H}{8q^2} \times (1-\cos^2\theta) \times |a^{Born}_{L,R}|^2. \tag{11}$$

Here, β_H is the velocity of the outgoing Higgs bosons. If M_1 and M_2 are the masses of the two outgoing Higgs bosons, then $\beta_H(M_1, M_2)$ is defined by:

$$\begin{aligned}\beta_H &= \frac{2|p|}{\sqrt{s}} = \frac{1}{s} \times \sqrt{(s+M_1^2-M_2^2)^2 - 4sM_1^2} \\ &= \sqrt{\left(1+\frac{M_2+M_1}{\sqrt{s}}\right)\left(1-\frac{M_2+M_1}{\sqrt{s}}\right)\left(1+\frac{M_2-M_1}{\sqrt{s}}\right)\left(1-\frac{M_2-M_1}{\sqrt{s}}\right)}.\end{aligned} \tag{12}$$

After integration over $\cos\theta$, one gets:

- for charged Higgs bosons:

$$\sigma^{Born}_{H^+H^-} = \frac{e^4}{48\pi s}\left(1-\frac{4M_H^2}{s}\right)^{3/2} \times \left(1+\frac{2c'_V c_V}{1-M_Z^2/s}+\frac{{c'_V}^2(c_V^2+c_A^2)}{(1-M_Z^2/s)^2}\right) \tag{13}$$

  with $c_V = \dfrac{-1+4s_W^2}{4s_W c_W}$, $c_A = \dfrac{-1}{4s_W c_W}$, $c'_V = \dfrac{-1+2s_W^2}{2s_W c_W}$.

- for neutral Higgs bosons:

$$\sigma^{Born}_{H^0A^0/h^0A^0} = \frac{e^4}{48\pi s} \times \left(\frac{8s_W^4-4s_W^2+1}{32s_W^4c_W^4}\right) \times [Z_{ab}]^2 \times \frac{\beta_H^3(M_{H^0/h^0}, M_A)}{(1-M_Z^2/s)^2}. \tag{14}$$

In the decoupling limit ($M_A \gg M_Z$ and $M_A \simeq M_{H^0}$), $\cos(\beta-\alpha) \to 0$ and $e^+e^- \to h^0A^0$ is strongly suppressed, i.e. only the H^0A^0 pairs can be produced in e^+e^- collisions, with a tree level cross section given by:

$$\sigma^{Born}_{H^0A^0} \to \frac{e^4}{48\pi s}\left(1-\frac{4M_A^2}{s}\right)^{3/2} \times \left(\frac{8s_W^4-4s_W^2+1}{32s_W^4c_W^4}\right) \times \frac{1}{(1-M_Z^2/s)^2}. \tag{15}$$

Figure 2 shows the pair production cross section for the MSSM charged and neutral Higgs bosons in e^+e^- collisions, at tree level, as a function of M_A and for various values of the centre-of-mass energy.

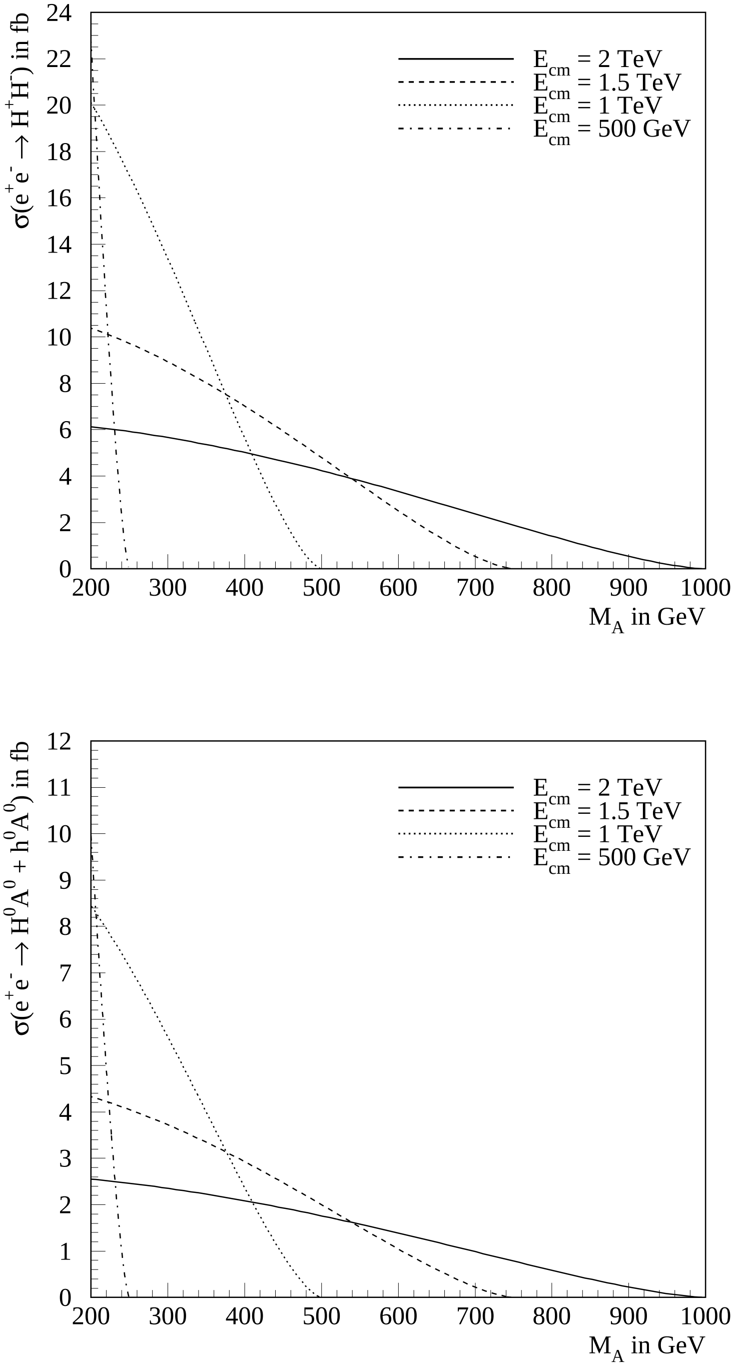

Figure 2: Tree level pair production cross section for charged (top) and neutral (bottom) Higgs bosons in e^+e^- collisions, as a function of the MSSM parameter M_A and for various centre-of-mass energies $\sqrt{s}$. For simplicity, we performed our calculations in the decoupling limit, where H, H^0 and A^0 are almost degenerate in mass.

2.3 Complete amplitude for calculations at the one loop level

The analytical expressions of all electroweak one loop contributions to the MSSM Higgs bosons pair production cross sections are given in the following: $e^{\pm}$ self-energy and initial vertices in Section 3, γ and Z self-energies with counter terms in Section 4, final vertices and Higgs self-energy in Section 5, and box diagrams in Section 6. The complete renormalized amplitude used to calculate the cross section is the sum of Born and one loop terms:

$$\begin{aligned} A(e^+e^- \to \text{Higgs pair}) &= A^{Born}(e^+e^- \to \text{Higgs pair}) \\ &+ A^{in}(e^+e^- \to \text{Higgs pair}) \\ &+ A^{RG}(e^+e^- \to \text{Higgs pair}) + A^{ct}(e^+e^- \to \text{Higgs pair}) \\ &+ A^{fin}(e^+e^- \to \text{Higgs pair}) \\ &+ A^{box}(e^+e^- \to \text{Higgs pair}). \end{aligned} \tag{16}$$

The notations used in our calculations, in particular when writing vertices in terms of real coupling constants, are described in Appendix A. In the following, we use a formalism which involves Passarino-Veltman functions, see Appendix B for details.

3 Contribution of initial vertices and $e^{\pm}$ self-energy

The amplitudes $a^{in}_{L,R}$ corresponding to initial triangles and $e^{\pm}$ self-energy are:

- for charged Higgs bosons:

$$a^{in}_{L,R}(H^+H^-) = \frac{\Gamma^{in,\gamma}}{e} + \frac{(1-2s_W^2)\eta}{2s_Wc_W} \times \frac{\Gamma^{in,Z}}{e} \tag{17}$$

- for neutral Higgs bosons:

$$a^{in}_{L,R}(H^0A^0/h^0A^0) = \frac{i\,\eta}{2s_Wc_W} \times \frac{\Gamma^{in,Z}}{e} \times [Z_{ab}] \tag{18}$$

where we write $\Gamma^{in,V} = \Gamma^{in,V}_L P_L + \Gamma^{in,V}_R P_R$ for $V = \gamma$ or Z.

$\Gamma^{in,\gamma}$ and $\Gamma^{in,Z}$ are the same in the charged and neutral sectors, since they only depend on the initial state. They are obtained by summing various contributions:

$$\begin{aligned} \Gamma^{in,V} &= \Gamma^V_{e^+e^-}(W\nu W) - \Gamma^V_{e^+e^-}(pinch) \\ &+ \Gamma^V_{e^+e^-}(\nu W\nu) + \Gamma^V_{e^+e^-}(eZe) + \Gamma^V_{e^+e^-}(e\gamma e) \\ &+ \Gamma^V_{e^+e^-}(\tilde{\chi}_j\tilde{\nu}_L\tilde{\chi}_i) + \Gamma^V_{e^+e^-}(\tilde{\chi}^0_j\tilde{e}\tilde{\chi}^0_i) \\ &+ \Gamma^V_{e^+e^-}(\tilde{\nu}_L\tilde{\chi}_i\tilde{\nu}_L) + \Gamma^V_{e^+e^-}(\tilde{e}\tilde{\chi}^0_i\tilde{e}) \\ &+ \Gamma^V_{e^+e^-}(e.s.e). \end{aligned} \tag{19}$$

The particles inside the initial triangle have internal masses m_1, m_2 and m_3. They are ordered clockwise, m_1 being the mass of the particle just after the junction involving the momentum q.

1) The contribution of the $W\nu W$ triangle is:

$$\Gamma^V_{e^+e^-}(W\nu W) = -\frac{e\alpha_{em}}{8\pi s_W^2} f^V \tilde{C}_{WW} P_L, \;\; f^V = \begin{cases} 1 \text{ for } V=\gamma \\ c_W/s_W \text{ for } V=Z \end{cases} \tag{20}$$

where $\tilde{C}_{WW} = -12C_{24}(W\nu W) + 2 - 2q^2\left[C_0(W\nu W) + C_{11}(W\nu W) + C_{23}(W\nu W)\right]$.

2) In e^+e^- annihilations, WW contributions arise in the photon and Z self-energies, as well as in the triangles connecting the photon and the Z boson to the initial e^+e^- pair or to the final Higgs pair. Therefore, it is convenient to extract a certain part (so-called pinch) from such a triangle with two W lines (in our case the $W\nu W$ triangle) and then to put it inside the photon and Z self-energies contributions, in order to have universal charge renormalization [11]:

$$-\Gamma^V(pinch) = -\frac{e\alpha_{em}}{4\pi s_W^2} f^V B_0(WW, q^2) P_L, \;\; f^V = \begin{cases} 1 \text{ for } V=\gamma \\ c_W/s_W \text{ for } V=Z \end{cases}. \tag{21}$$

3) As for the $\nu W\nu$ triangle, since neutrinos do not couple to photons, one has:

$$\Gamma^\gamma_{e^+e^-}(\nu W\nu) = 0 \tag{22}$$

while, for the Z boson, one obtains:

$$\Gamma^Z_{e^+e^-}(\nu W\nu) = -\frac{e\alpha_{em}}{16\pi s_W^3 c_W}\tilde{C}_W P_L \tag{23}$$

where $\tilde{C}_W = 4C_{24}(\nu W\nu) - 2 + 2q^2\left[C_{11}(\nu W\nu) + C_{23}(\nu W\nu)\right]$.

4) The contribution of the eZe triangle is:

$$\Gamma^\gamma_{e^+e^-}(eZe) = \frac{e\alpha_{em}}{16\pi s_W^2 c_W^2}\tilde{C}_Z\left[g_L^2 P_L + g_R^2 P_R\right] \tag{24}$$

or

$$\Gamma^Z_{e^+e^-}(eZe) = -\frac{e\alpha_{em}}{32\pi s_W^3 c_W^3}\tilde{C}_Z\left[g_L^3 P_L + g_R^3 P_R\right] \tag{25}$$

where $\tilde{C}_Z = 4C_{24}(eZe) - 2 + 2q^2\left[C_{11}(eZe) + C_{23}(eZe)\right]$.

5) The contribution of the $e\gamma e$ triangle is:

$$\Gamma^\gamma_{e^+e^-}(e\gamma e) = \frac{e\alpha_{em}}{4\pi}\tilde{C}_\gamma\left[P_L + P_R\right] \tag{26}$$

or

$$\Gamma^Z_{e^+e^-}(e\gamma e) = -\frac{e\alpha_{em}}{8\pi s_W c_W}\tilde{C}_\gamma\left[g_L P_L + g_R P_R\right] \tag{27}$$

where $\tilde{C}_\gamma = 4C_{24}(e\gamma e) - 2 + 2q^2\left[C_{11}(e\gamma e) + C_{23}(e\gamma e)\right]$.

6) The contribution of the $\tilde{\chi}_j \tilde{\nu}_L \tilde{\chi}_i$ triangles is:

$$\Gamma^{\gamma}_{e^+e^-}(\tilde{\chi}_i \tilde{\nu}_L \tilde{\chi}_i) = \sum_i \frac{e\alpha_{em}}{4\pi s_W^2} |Z^+_{1i}|^2 (2\tilde{C}^{ii}_{24} - |M_{\tilde{\chi}_i}|^2 \tilde{C}^{ii}_0) P_L \tag{28}$$

or

$$\begin{aligned}\Gamma^{Z}_{e^+e^-}(\tilde{\chi}_j \tilde{\nu}_L \tilde{\chi}_i) = & \sum_{ij} \frac{e\alpha_{em}}{8\pi s_W^3 c_W} Z^+_{1i} Z^{+*}_{1j} \times \\ & \Big\{ \left[Z^{+*}_{1i} Z^+_{1j} + \delta_{ij}(c_W^2 - s_W^2) \right] 2\tilde{C}^{ij}_{24} \\ & - \left[Z^-_{1i} Z^{-*}_{1j} + \delta_{ij}(c_W^2 - s_W^2) \right] M_{\tilde{\chi}_i} M_{\tilde{\chi}_j} \tilde{C}^{ij}_0 \Big\} P_L\end{aligned} \tag{29}$$

where $\tilde{C}^{ij}_{24} = C_{24}(\tilde{\chi}_j \tilde{\nu}_L \tilde{\chi}_i) - \frac{1}{4} + \frac{q^2}{2} \left[C_{12}(\tilde{\chi}_j \tilde{\nu}_L \tilde{\chi}_i) + C_{23}(\tilde{\chi}_j \tilde{\nu}_L \tilde{\chi}_i) \right]$ and $\tilde{C}^{ij}_0 = C_0(\tilde{\chi}_j \tilde{\nu}_L \tilde{\chi}_i)$.

7) As for the $\tilde{\chi}^0_j \tilde{e} \tilde{\chi}^0_i$ triangles, since neutralinos do not couple to photons, one has:

$$\Gamma^{\gamma}_{e^+e^-}(\tilde{\chi}^0_j \tilde{e} \tilde{\chi}^0_i) = 0 \tag{30}$$

while, for the Z boson, one obtains:

$$\Gamma^{Z}_{e^+e^-}(\tilde{\chi}^0_j \tilde{e} \tilde{\chi}^0_i) = \sum_{ij} \frac{e\alpha_{em}}{8\pi s_W^3 c_W^3} \times \left[K^{ij}_L P_L + K^{ij}_R P_R \right] \tag{31}$$

by defining K^{ij}_L and K^{ij}_R as follows:

$$\begin{aligned}K^{ij}_L = & \frac{(Z^{N*}_{1j} s_W + Z^{N*}_{2j} c_W)(Z^N_{1i} s_W + Z^N_{2i} c_W)}{2} \times \\ & \left[2(Z^N_{3j} Z^{N*}_{3i} - Z^N_{4j} Z^{N*}_{4i}) \tilde{C}^{ij}_{24} + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j} (Z^{N*}_{3j} Z^N_{3i} - Z^{N*}_{4j} Z^N_{4i}) \tilde{C}^{ij}_0 \right]\end{aligned} \tag{32}$$

$$\begin{aligned}K^{ij}_R = & 2(Z^N_{1j} Z^{N*}_{1i}) s_W^2 \times \\ & \left[2(Z^{N*}_{4j} Z^N_{4i} - Z^{N*}_{3j} Z^N_{3i}) \tilde{C}^{ij}_{24} + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j} (Z^N_{4j} Z^{N*}_{4i} - Z^N_{3j} Z^{N*}_{3i}) \tilde{C}^{ij}_0 \right]\end{aligned} \tag{33}$$

where $\tilde{C}^{ij}_{24} = C_{24}(\tilde{\chi}^0_j \tilde{e} \tilde{\chi}^0_i) - \frac{1}{4} + \frac{q^2}{2} \left[C_{12}(\tilde{\chi}^0_j \tilde{e} \tilde{\chi}^0_i) + C_{23}(\tilde{\chi}^0_j \tilde{e} \tilde{\chi}^0_i) \right]$ and $\tilde{C}^{ij}_0 = C_0(\tilde{\chi}^0_j \tilde{e} \tilde{\chi}^0_i)$.

8) As for the $\tilde{\nu}_L \tilde{\chi}_i \tilde{\nu}_L$ triangles, since sneutrinos do not couple to photons, one has:

$$\Gamma^{\gamma}_{e^+e^-}(\tilde{\nu}_L \tilde{\chi}_i \tilde{\nu}_L) = 0 \tag{34}$$

while, for the Z boson, one obtains:

$$\Gamma^{Z}_{e^+e^-}(\tilde{\nu}_L \tilde{\chi}_i \tilde{\nu}_L) = - \sum_i \frac{e\alpha_{em}}{4\pi s_W^3 c_W} |Z^+_{1i}|^2 \tilde{C}^i_{24} P_L \tag{35}$$

where $\tilde{C}^i_{24} = C_{24}(\tilde{\nu}_L \tilde{\chi}_i \tilde{\nu}_L)$.

9) The contribution of the $\tilde{e}\tilde{\chi}_i^0\tilde{e}$ triangles is:

$$\Gamma^{\gamma}_{e^+e^-}(\tilde{e}\tilde{\chi}_i^0\tilde{e}) = \sum_i \frac{e\alpha_{em}}{4\pi s_W^2 c_W^2} \times \left[|Z_{1i}^N s_W + Z_{2i}^N c_W|^2 \tilde{C}_{24}^i P_L + 4s_W^2 |Z_{1i}^N|^2 \tilde{C}_{24}^i P_R\right] \quad (36)$$

or

$$\Gamma^{Z}_{e^+e^-}(\tilde{e}\tilde{\chi}_i^0\tilde{e}) = -\sum_i \frac{e\alpha_{em}}{4\pi s_W^3 c_W^3} \times \left[(s_W^2 - \frac{1}{2})|Z_{1i}^N s_W + Z_{2i}^N c_W|^2 \tilde{C}_{24}^i P_L + 4s_W^4 |Z_{1i}^N|^2 \tilde{C}_{24}^i P_R\right] \quad (37)$$

where $\tilde{C}_{24}^i = C_{24}(\tilde{e}\tilde{\chi}_i^0\tilde{e})$.

10) The electron self-energy $(e.s.e)$ contributions are obtained as follows:

$$\Gamma^{\gamma}_{e^+e^-}(e.s.e) = -e\left[\delta_L P_L + \delta_R P_R\right] \quad (38)$$

or

$$\Gamma^{Z}_{e^+e^-}(e.s.e) = \frac{e}{2s_W c_W}\left[\delta_L g_L P_L + \delta_R g_R P_R\right] \quad (39)$$

where the following loops are taken into account: $(W\nu)$, (Ze), (γe), $(\tilde{\chi}\tilde{\nu})$, $(\tilde{\chi}^0\tilde{e})$.

For the $(W\nu)$ loop, one has:

$$\delta_L(W\nu) = -\frac{\alpha_{em}}{4\pi s_W^2}\left(B_1(W\nu,0) + \frac{1}{2}\right), \quad (40)$$

$$\delta_R(W\nu) = 0. \quad (41)$$

For the (Ze) loop, one has:

$$\delta_L(Ze) = -\frac{\alpha_{em} g_L^2}{8\pi s_W^2 c_W^2}\left(B_1(Ze,0) + \frac{1}{2}\right), \quad (42)$$

$$\delta_R(Ze) = -\frac{\alpha_{em} g_R^2}{8\pi s_W^2 c_W^2}\left(B_1(Ze,0) + \frac{1}{2}\right). \quad (43)$$

For the (γe) loop, one has:

$$\delta_L(\gamma e) = \delta_R(\gamma e) = -\frac{\alpha_{em}}{2\pi}\left(B_1(\gamma e,0) + \frac{1}{2}\right). \quad (44)$$

For each $(\tilde{\chi}_i\tilde{\nu})$ loop, one has:

$$\delta_L(\tilde{\chi}_i\tilde{\nu}) = -\frac{\alpha_{em}}{4\pi s_W^2}|Z_{1i}^+|^2 B_1(\tilde{\chi}_i\tilde{\nu}_L,0), \quad (45)$$

$$\delta_R(\tilde{\chi}_i\tilde{\nu}) = 0. \quad (46)$$

For each $(\tilde{\chi}_i^0\tilde{e})$ loop, one has:

$$\delta_L(\tilde{\chi}_i^0\tilde{e}) = -\frac{\alpha_{em}}{8\pi s_W^2 c_W^2}|Z_{1i}^N s_W + Z_{2i}^N c_W|^2 B_1(\tilde{\chi}_i^0\tilde{e}_L,0), \quad (47)$$

$$\delta_R(\tilde{\chi}_i^0\tilde{e}) = -\frac{\alpha_{em}}{2\pi c_W^2}|Z_{1i}^{N*}|^2 B_1(\tilde{\chi}_i^0\tilde{e}_R,0). \quad (48)$$

4 Contribution of γ and Z self-energies

4.1 Definition of gauge self-energy functions

The on-shell renormalization procedure [12, 13] that allows full determination of the MSSM Higgs sector at one loop, as well as of the corresponding counter terms, makes use of several gauge self-energy functions, which are detailed in the following of this section [13, 14, 15]. Let us first define several useful expressions:

$$PV1(XY,q^2) = \frac{M_X^2+M_Y^2}{2} - 2B_{22}(XY,q^2) - \frac{q^2}{6} - q^2\left[B_1(XY,q^2)+B_{21}(XY,q^2)\right], \quad (49)$$

$$\begin{aligned} PV2(XY,q^2) &= 10B_{22}(XY,q^2) + (4q^2+M_X^2+M_Y^2)B_0(XY,q^2) \\ &+ A(M_X^2)+A(M_Y^2) - 2\left(M_X^2+M_Y^2-\frac{q^2}{3}\right), \end{aligned} \quad (50)$$

$$PV3(XY,q^2) = 2B_{22}(XY,q^2) - \frac{A(M_X^2)+A(M_Y^2)}{2} + \frac{(q^2-M_X^2-M_Y^2)}{2}B_0(XY,q^2). \quad (51)$$

a) Photon self-energies:

The photon self-energy is defined as:

$$A_{\gamma\gamma}(q^2) = \Sigma_{\gamma\gamma}(q^2) + A_{\gamma\gamma}(pinch). \quad (52)$$

The pinch term is given by:

$$A_{\gamma\gamma}(pinch) = -\frac{\alpha_{em}}{\pi}q^2B_0(WW,q^2). \quad (53)$$

The self-energy term without pinch $\Sigma_{\gamma\gamma}(q^2)$ is the sum of various contributions:

$$\begin{aligned} \Sigma_{\gamma\gamma}(q^2) &= \Sigma_{\gamma\gamma}(\text{g+H}) + \Sigma_{\gamma\gamma}(ff) \\ &+ \Sigma_{\gamma\gamma}(\tilde{\chi}\tilde{\chi}) + \Sigma_{\gamma\gamma}(\tilde{f}\tilde{f}). \end{aligned} \quad (54)$$

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned} \Sigma_{\gamma\gamma}(\text{g+H}) &= -\frac{\alpha_{em}}{2\pi}\Big\{2B_{22}(HH,q^2) - A(M_H^2) + 6B_{22}(WW,q^2) \\ &\quad -3A(M_W^2) + 2q^2B_0(WW,q^2) + \frac{q^2}{3}\Big\}. \end{aligned} \quad (55)$$

The contribution of the fermion pairs is:

$$\Sigma_{\gamma\gamma}(f\bar{f}) = \sum_f \frac{\alpha_{em}N_c^fQ_f^2}{\pi}\Big\{PV1(ff,q^2) + M_f^2B_0(ff,q^2)\Big\}. \quad (56)$$

The contribution of the chargino pairs is:

$$\Sigma_{\gamma\gamma}(\tilde{\chi}\tilde{\chi}) = \sum_i \frac{\alpha_{em}}{\pi}\Big\{PV1(\tilde{\chi}_i\tilde{\chi}_i,q^2) + M_{\tilde{\chi}_i}^2B_0(\tilde{\chi}_i\tilde{\chi}_i,q^2)\Big\}. \quad (57)$$

The contribution of the sfermion pairs is:

$$\Sigma_{\gamma\gamma}(\tilde{f}\tilde{f}) \;=\; -\sum_{\tilde{f}} \frac{\alpha_{em} N_c^f Q_f^2}{2\pi} \sum_{i=1,2} \Big\{ 2B_{22}(\tilde{f}_i\tilde{f}_i, q^2) - A(M^2_{\tilde{f}_i}) \Big\}. \tag{58}$$

Here, $\tilde{f}_1$, $\tilde{f}_2$ account for $\tilde{f}_L$, $\tilde{f}_R$ in the case of unmixed sfermions, or for the physical states obtained after mixing (i.e. $\tilde{t}_1$, $\tilde{t}_2$, $\tilde{b}_1$, $\tilde{b}_2$ in the case of third generation squarks). The coupling between a photon and a sfermion pair is the same with and without mixing.

b) Z self-energies:

The Z boson self-energy is defined as:

$$A_{ZZ}(q^2) = \Sigma_{ZZ}(q^2) + A_{ZZ}(pinch). \tag{59}$$

The pinch term is given by:

$$A_{ZZ}(pinch) = -\frac{\alpha_{em} c_W^2}{\pi s_W^2}(q^2 - M_Z^2) B_0(WW, q^2). \tag{60}$$

The self-energy term without pinch $\Sigma_{ZZ}(q^2)$ is the sum of various contributions:

$$\begin{aligned} \Sigma_{ZZ}(q^2) \;&=\; \Sigma_{ZZ}(\text{g+H}) + \Sigma_{ZZ}(ff) \\ &+\; \Sigma_{ZZ}(\tilde{\chi}\tilde{\chi}) + \Sigma_{ZZ}(\tilde{\chi}^0\tilde{\chi}^0) + \Sigma_{ZZ}(\tilde{f}\tilde{f}). \end{aligned} \tag{61}$$

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned} \Sigma_{ZZ}(\text{g+H}) \;=\; \frac{\alpha_{em}}{4\pi s_W^2 c_W^2} \;& \Big\{ \frac{1}{4}\Big[A(M^2_{h^0}) + A(M^2_{H^0}) + A(M^2_A) + A(M^2_Z)\Big] \\ & + \sin^2(\beta-\alpha)\Big[M_Z^2 B_0(Zh^0, q^2) - B_{22}(Zh^0, q^2) - B_{22}(A^0H^0, q^2)\Big] \\ & + \cos^2(\beta-\alpha)\Big[M_Z^2 B_0(ZH^0, q^2) - B_{22}(ZH^0, q^2) - B_{22}(A^0h^0, q^2)\Big] \\ & - \frac{1}{2}\cos^2(2\theta_W)\Big[2B_{22}(HH, q^2) - A(M_H^2)\Big] \\ & - \Big[8c_W^4 + \cos^2(2\theta_W)\Big] B_{22}(WW, q^2) \\ & - \Big[4c_W^4 q^2 + 2M_W^2\cos(2\theta_W)\Big] B_0(WW, q^2) \\ & + \frac{1}{2}\Big[12c_W^4 - 4c_W^2 + 1\Big] A(M_W^2) - \frac{2}{3}c_W^4 q^2 \Big\}. \end{aligned} \tag{62}$$

The contribution of the fermion pairs is:

$$\Sigma_{ZZ}(f\bar{f}) \;=\; \sum_f \frac{\alpha_{em} N_c^f}{4\pi s_W^2 c_W^2} \Big\{ (g_{Vf}^2 + g_{Af}^2) PV1(M_f^2, q^2) + (g_{Vf}^2 - g_{Af}^2) M_f^2 B_0(f\bar{f}, q^2) \Big\} \tag{63}$$

where $g_{Vf} = T_f^3(1 - 4|Q_f|s_W^2)$ and $g_{Af} = T_f^3$.

The contribution of the chargino pairs is:

$$\Sigma_{ZZ}(\tilde{\chi}\tilde{\chi}) = \frac{2}{16\pi^2}\sum_{ij} \Big\{PV1(\tilde{\chi}_i\tilde{\chi}_j,q^2)\left[\mathcal{O}^{ZL*}_{ij}\mathcal{O}^{ZL}_{ij}+\mathcal{O}^{ZR*}_{ij}\mathcal{O}^{ZR}_{ij}\right] + M_{\tilde{\chi}_i}M_{\tilde{\chi}_j}\left[\mathcal{O}^{ZL*}_{ij}\mathcal{O}^{ZR}_{ij}+\mathcal{O}^{ZL}_{ij}\mathcal{O}^{ZR*}_{ij}\right]B_0(\tilde{\chi}_i\tilde{\chi}_j,q^2)\Big\}. \quad (64)$$

The contribution of the neutralino pairs is:

$$\Sigma_{ZZ}(\tilde{\chi}^0\tilde{\chi}^0) = \frac{1}{16\pi^2}\sum_{ij} \Big\{PV1(\tilde{\chi}^0_i\tilde{\chi}^0_j,q^2)\left[\mathcal{O}^{0L*}_{ij}\mathcal{O}^{0L}_{ij}+\mathcal{O}^{0R*}_{ij}\mathcal{O}^{0R}_{ij}\right] + M_{\tilde{\chi}^0_i}M_{\tilde{\chi}^0_j}\left[\mathcal{O}^{0L*}_{ij}\mathcal{O}^{0R}_{ij}+\mathcal{O}^{0L}_{ij}\mathcal{O}^{0R*}_{ij}\right]B_0(\tilde{\chi}^0_i\tilde{\chi}^0_j,q^2)\Big\}. \quad (65)$$

The contribution of sfermion pairs is:

- for unmixed sfermions:

$$\Sigma^{light}_{ZZ}(\tilde{f}\tilde{f}) = -\sum_{\tilde{f}_{L,R}} \frac{\alpha_{em}N^f_c}{2\pi}\left(\frac{g^0_{Z\tilde{f}\tilde{f}}}{e}\right)^2 \Big\{2B_{22}(\tilde{f}\tilde{f},q^2)-A(M^2_{\tilde{f}})\Big\}, \quad (66)$$

- with sfermion mixing (third generation squarks):

$$\begin{aligned}\Sigma^{heavy}_{ZZ}(\tilde{f}\tilde{f}) = -\frac{3\alpha_{em}}{2\pi s^2_W c^2_W}\sum_{\tilde{f}=\tilde{t},\tilde{b}} \Big\{&\frac{c^2_{\tilde{f}}s^2_{\tilde{f}}}{2}\left[B_{22}(\tilde{f}_1\tilde{f}_2,q^2)+B_{22}(\tilde{f}_2\tilde{f}_1,q^2)\right]\\ &+2(T^3_{f_L}c^2_{\tilde{f}}-s^2_WQ_f)^2B_{22}(\tilde{f}_1\tilde{f}_1,q^2)\\ &-\left[c^2_{\tilde{f}}(T^3_{f_L}-s^2_WQ_f)^2+s^2_{\tilde{f}}Q^2_fs^4_W\right]A(M^2_{\tilde{f}_1})\\ &+2(T^3_{f_L}s^2_{\tilde{f}}-s^2_WQ_f)^2B_{22}(\tilde{f}_2\tilde{f}_2,q^2)\\ &-\left[s^2_{\tilde{f}}(T^3_{f_L}-s^2_WQ_f)^2+c^2_{\tilde{f}}Q^2_fs^4_W\right]A(M^2_{\tilde{f}_2})\Big\}. \quad (67)\end{aligned}$$

c) Mixed γZ self-energies:

The mixed γZ self-energy is defined as:

$$A_{\gamma Z}(q^2) = \Sigma_{\gamma Z}(q^2) + A_{\gamma Z}(pinch). \quad (68)$$

The pinch term is given by:

$$A_{\gamma Z}(pinch) = -\frac{\alpha_{em}c_W}{\pi s_W}(q^2-\frac{M^2_Z}{2})B_0(WW,q^2). \quad (69)$$

The self-energy term without pinch $\Sigma_{\gamma Z}(q^2)$ is the sum of various contributions:

$$\begin{aligned}\Sigma_{\gamma Z}(q^2) &= \Sigma_{\gamma Z}(\text{g+H}) + \Sigma_{\gamma Z}(ff)\\ &+ \Sigma_{\gamma Z}(\tilde{\chi}\tilde{\chi}) + \Sigma_{\gamma Z}(\tilde{f}\tilde{f}). \quad (70)\end{aligned}$$

The contribution of the gauge and Higgs sectors can be expressed in several ways. Here, we choose the definition given in [15]:

$$\begin{aligned}\Sigma_{\gamma Z}(\text{g+H}) &= \frac{\alpha_{em}}{4\pi}\Big\{-2\frac{c_W^2-s_W^2}{s_W c_W}\Big[B_{22}(HH,q^2)+B_{22}(WW,q^2)\Big]\\ &+\frac{c_W^2-s_W^2}{s_W c_W}\Big[A(M_H^2)+A(M_W^2)\Big]+\frac{c_W}{s_W}\Big[6A(M_W^2)-4M_W^2\Big]\\ &+2\frac{c_W}{s_W}B_{22}(WW,q^2)-2s_W c_W M_Z^2 B_0(WW,q^2)\\ &-\frac{c_W}{s_W}PV2(WW,q^2)\Big\}. \end{aligned} \tag{71}$$

The contribution of the fermion pairs is:

$$\Sigma_{\gamma Z}(f\bar{f}) = \sum_f \frac{\alpha_{em}N_c^f Q_f g_{Vf}}{2\pi s_W c_W}\Big\{PV1(ff,q^2)+M_f^2 B_0(ff,q^2)\Big\}. \tag{72}$$

The contribution of the chargino pairs is:

$$\begin{aligned}\Sigma_{\gamma Z}(\tilde{\chi}\tilde{\chi}) \quad \frac{2}{16\pi^2}\sum_{ij} &\Big\{PV1(\tilde{\chi}_i\tilde{\chi}_j,q^2)\Big[\mathcal{O}_{ij}^{\gamma L*}\mathcal{O}_{ij}^{ZL}+\mathcal{O}_{ij}^{\gamma R*}\mathcal{O}_{ij}^{ZR}\Big]\\ &+M_{\tilde{\chi}_i}M_{\tilde{\chi}_j}\Big[\mathcal{O}_{ij}^{\gamma L*}\mathcal{O}_{ij}^{ZR}+\mathcal{O}_{ij}^{ZL}\mathcal{O}_{ij}^{\gamma R*}\Big]B_0(\tilde{\chi}_i\tilde{\chi}_j,q^2)\Big\}. \end{aligned}\tag{73}$$

The contribution of the sfermion pairs is:

- for unmixed sfermions:

$$\Sigma_{\gamma Z}^{light}(\tilde{f}\tilde{f}) = -\frac{\alpha_{em}}{2\pi}\sum_{\tilde{f}_{L,R}}N_c^f Q_f\left(-\frac{g^0_{Z\tilde{f}\tilde{f}}}{e}\right)\times\Big\{2B_{22}(\tilde{f}\tilde{f},q^2)-A(M_{\tilde{f}}^2)\Big\}, \tag{74}$$

- with sfermion mixing (third generation squarks):

$$\begin{aligned}\Sigma_{\gamma Z}^{heavy}(\tilde{f}\tilde{f}) &= -\frac{3\alpha_{em}}{2\pi s_W c_W}\sum_{\tilde{f}=\tilde{t},\tilde{b}}Q_f \quad \Big\{(T^3_{f_L}c^2_{\tilde{f}}-s_W^2 Q_f)\Big[2B_{22}(\tilde{f}_1\tilde{f}_1,q^2)-A(M^2_{\tilde{f}_1})\Big]\\ &+(T^3_{f_L}s^2_{\tilde{f}}-s_W^2 Q_f)\Big[2B_{22}(\tilde{f}_2\tilde{f}_2,q^2)-A(M^2_{\tilde{f}_2})\Big]\Big\}. \end{aligned}\tag{75}$$

d) W self-energies:

The self-energy term without pinch $\Sigma_{WW}(q^2)$ is the sum of various contributions:

$$\begin{aligned}\Sigma_{WW}(q^2) &= \Sigma_{WW}(\text{g+H})+\Sigma_{WW}(ff')\\ &+ \Sigma_{WW}(\tilde{\chi}\tilde{\chi}^0)+\Sigma_{WW}(\tilde{f}\tilde{f}'). \end{aligned}\tag{76}$$

The contribution of the gauge and Higgs sectors can be expressed in several ways. Here, we choose the definition given in [15]:

$$\begin{aligned}
\Sigma_{WW}(\text{g+H}) &= \frac{\alpha_{em}}{4\pi s_W^2} \Big\{ -\sin^2(\beta-\alpha)\left[B_{22}(HH^0,q^2)+B_{22}(Wh^0,q^2)\right] \\
&\quad -\cos^2(\beta-\alpha)\left[B_{22}(Hh^0,q^2)+B_{22}(WH^0,q^2)\right] \\
&\quad -B_{22}(WZ,q^2)-B_{22}(HA^0,q^2) \\
&\quad +2s_W^2 B_{22}(\gamma W,q^2)+2c_W^2 B_{22}(WZ,q^2) \\
&\quad +\frac{1}{4}\left[A(M_{H^0}^2)+A(M_{h^0}^2)+A(M_Z^2)+A(M_A^2)\right] \\
&\quad +\frac{1}{2}\left[A(M_W^2)+A(M_H^2)\right] \\
&\quad +M_W^2\left[\sin^2(\beta-\alpha)B_0(h^0W,q^2)+\cos^2(\beta-\alpha)B_0(H^0W,q^2)\right] \\
&\quad +M_W^2\left[s_W^2 B_0(W\gamma,q^2)+\frac{s_W^4}{c_W^2}B_0(WZ,q^2)\right] \\
&\quad +\left[3A(M_W^2)-2M_W^2\right]+c_W^2\left[3A(M_Z^2)-2M_Z^2\right] \\
&\quad -c_W^2 PV2(ZW,q^2)-s_W^2 PV2(\gamma W,q^2)\Big\}. \qquad (77)
\end{aligned}$$

The contribution of the fermion pairs is:

$$\Sigma_{WW}(ff') = \sum_{(ff')} \frac{\alpha_{em} N_c^f}{4\pi s_W^2} PV_3(ff',q^2). \qquad (78)$$

The contribution of the gaugino pairs is:

$$\begin{aligned}
\Sigma_{WW}(\tilde{\chi}\tilde{\chi}^0) &= \frac{\alpha_{em}}{2\pi s_W^2}\sum_{ij} \Big\{ \left(\mathcal{O}_{ij}^{WL}\mathcal{O}_{ij}^{WL*}+\mathcal{O}_{ij}^{WR}\mathcal{O}_{ij}^{WR*}\right) PV_3(\tilde{\chi}_i\tilde{\chi}_j^0,q^2) \\
&\quad + \left(\mathcal{O}_{ij}^{WL}\mathcal{O}_{ij}^{WR*}+\mathcal{O}_{ij}^{WL*}\mathcal{O}_{ij}^{WR}\right) M_{\tilde{\chi}_i} M_{\tilde{\chi}_j^0} B_0(\tilde{\chi}_i\tilde{\chi}_j^0,q^2)\Big\}. \qquad (79)
\end{aligned}$$

The contribution of the sfermion pairs is:

- for unmixed sfermions:

$$\Sigma_{WW}^{light}(\tilde{f}\tilde{f}') = -\frac{\alpha_{em}}{2\pi s_W^2}\sum_{(ff')} N_c^f \left[B_{22}(\tilde{f}\tilde{f}',q^2) - \frac{A(M_{\tilde{f}}^2)+A(M_{\tilde{f}'}^2)}{4}\right], \qquad (80)$$

- with sfermion mixing (third generation squarks):

$$\begin{aligned}
\Sigma_{WW}^{heavy}(\tilde{f}\tilde{f}') &= -\frac{3\alpha_{em}}{2\pi s_W^2} \Big\{ c_{\tilde{t}}^2 c_{\tilde{b}}^2 B_{22}(\tilde{t}_1\tilde{b}_1,q^2)+c_{\tilde{t}}^2 s_{\tilde{b}}^2 B_{22}(\tilde{t}_1\tilde{b}_2,q^2) \\
&\quad + s_{\tilde{t}}^2 c_{\tilde{b}}^2 B_{22}(\tilde{t}_2\tilde{b}_1,q^2)+s_{\tilde{t}}^2 s_{\tilde{b}}^2 B_{22}(\tilde{t}_2\tilde{b}_2,q^2) \\
&\quad -\frac{1}{4}\left[c_{\tilde{t}}^2 A(M_{\tilde{t}_1}^2)+s_{\tilde{t}}^2 A(M_{\tilde{t}_2}^2)+c_{\tilde{b}}^2 A(M_{\tilde{b}_1}^2)+s_{\tilde{b}}^2 A(M_{\tilde{b}_2}^2)\right]\Big\}. \qquad (81)
\end{aligned}$$

4.2 Charged Higgs sector

For $e^+e^- \to H^+H^-$, the on-shell renormalization procedure leads to the following RG terms:

$$a^{RG}_{L,R}(H^+H^-) = \left[\frac{\eta^2(1-2s_W^2)g_{L,R}}{4s_W^2c_W^2}\right]\frac{A_{ZZ}(q^2)}{q^2} - \left[\frac{\eta(1-2s_W^2-g_{L,R})}{2s_Wc_W}\right]\frac{A_{\gamma Z}(q^2)}{q^2} - \frac{A_{\gamma\gamma}(q^2)}{q^2} \quad (82)$$

with the corresponding counter terms:

$$\begin{aligned} a^{ct}_{L,R}(H^+H^-) &= \left[\Pi_{\gamma\gamma}(0) + \frac{2s_W\Sigma_{\gamma Z}(0)}{c_WM_Z^2}\right] \times \left[1+\eta g_{L,R}\frac{2s_W^2-1}{4s_W^2c_W^2}\right] \\ &+ \frac{\eta g_{L,R}(2s_W^2-1)}{4s_W^2c_W^2} \times \left[\frac{\Sigma_{ZZ}(M_Z^2)}{q^2-M_Z^2}\right] \\ &+ \frac{\eta}{4s_W^2c_W^2} \times \left[\frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Sigma_{WW}(M_W^2)}{M_W^2})\right] \times \left[\frac{g_L}{s_W^2}P_L + g_RP_R\right]. \quad (83) \end{aligned}$$

Here, $\Pi_{\gamma\gamma}(q^2) \equiv \dfrac{\Sigma_{\gamma\gamma}(q^2)}{q^2}$ (no pinch term) and $\Pi_{\gamma\gamma}(0)$ is thus simply obtained as follows:

$$\Pi_{\gamma\gamma}(0) = \left(\frac{d\Sigma_{\gamma\gamma}}{dq^2}\right)_{q^2=0}. \quad (84)$$

4.3 Neutral Higgs sector

For $e^+e^- \to H^0A^0/h^0A^0$, the on-shell renormalization procedure leads to the following RG terms:

$$a^{RG}_{L,R}(H^0A^0/h^0A^0) = \frac{i}{q^2}\left[Z_{ab}\right] \times \left(\frac{\eta^2 g_{L,R}}{4s_W^2c_W^2}A_{ZZ}(q^2) - \frac{\eta}{2s_Wc_W}A_{\gamma Z}(q^2)\right). \quad (85)$$

As for the counter terms, we only consider those corresponding to electroweak couplings and gauge boson masses here (the counter terms corresponding to H^0A^0 or h^0A^0 final states will be calculated in Section 5.3.9):

$$\begin{aligned} a^{ct}_{L,R}(H^0A^0/h^0A^0) &= i\eta\left[Z_{ab}\right]\left[\frac{1-2s_W^2+2s_W^4}{4s_W^4c_W^2}P_L + \frac{1}{2c_W^2}P_R\right]\left[\frac{\Sigma_{WW}(M_W^2)}{M_W^2} - \frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2}\right] \\ &- i\eta\left[Z_{ab}\right]\frac{g_{L,R}}{4s_W^2c_W^2}\left[\Pi_{\gamma\gamma}(0) + \frac{\Sigma_{ZZ}(M_Z^2)}{q^2-M_Z^2} + \frac{2s_W\Sigma_{\gamma Z}(0)}{c_WM_Z^2}\right]. \quad (86) \end{aligned}$$

5 Contribution of final vertices and Higgs self-energies

5.1 Diagram structures for final triangles

Several useful expressions are needed when estimating the contributions of the final vertices. The particles inside the final triangle have internal masses m_1, m_2 and m_3. They are ordered clockwise, m_1 being the mass of the particle just after the junction involving the momentum q.

Let P_{f1} and P_{f2} (respectively M_1 and M_2) be the momenta (respectively the masses) of the two outgoing Higgs bosons (i.e. H^+H^- or H^0A^0 or h^0A^0), then one has:

$$P_{f1}^2 = M_1^2, \tag{87}$$
$$P_{f2}^2 = M_2^2, \tag{88}$$
$$P_{f1}P_{f2} = \frac{q^2-(M_1^2+M_2^2)}{2}. \tag{89}$$

a) <u>Tri1-type triangles:</u>

$$\begin{aligned}
\mathcal{C}_1 &= \frac{1}{6}+6(C_{001}-C_{002})+P_{f1}^2C_{111}-P_{f2}^2C_{222}\\
&+ (2P_{f1}P_{f2}-P_{f1}^2)C_{112}+(P_{f2}^2-2P_{f1}P_{f2})C_{122}\\
&+ 2\left[P_{f1}P_{f2}C_{21}-P_{f2}^2C_{22}+(P_{f2}^2-P_{f1}P_{f2})C_{23}-C_{24}\right]\\
&- (2P_{f1}P_{f2}+P_{f1}^2)(C_{11}-C_{12}).
\end{aligned} \tag{90}$$

b) <u>Tri2-type triangles:</u>

$$\begin{aligned}
\mathcal{C}_2 &= (8P_{f1}P_{f2}+6P_{f1}^2+2P_{f2}^2)C_0+(8P_{f1}P_{f2}+7P_{f1}^2+P_{f2}^2)C_{11}\\
&+ (P_{f1}^2-P_{f2}^2)C_{12}+(2P_{f1}P_{f2}+2P_{f1}^2)C_{21}+(2P_{f1}P_{f2}+2P_{f2}^2)C_{22}\\
&+ (4P_{f1}P_{f2}+2P_{f1}^2+2P_{f2}^2)C_{23}+12C_{24}-2.
\end{aligned} \tag{91}$$

c) <u>Tri3-type triangles:</u>

$$\begin{aligned}
\mathcal{C}_3 &= \frac{1}{6}+6(C_{001}-C_{002})+P_{f1}^2C_{111}-P_{f2}^2C_{222}+(2P_{f1}P_{f2}-P_{f1}^2)C_{112}\\
&+ (P_{f2}^2-2P_{f1}P_{f2})C_{122}+P_{f1}^2C_{21}-(2P_{f1}P_{f2}+P_{f2}^2)C_{22}\\
&- 2P_{f1}^2C_{23}-q^2C_{12}-2C_{24}+\frac{1}{2},
\end{aligned} \tag{92}$$

$$\mathcal{C}_3' = C_{11}-C_{12}, \tag{93}$$

$$\mathcal{C}_3'' = C_0+C_{11}-C_{12}. \tag{94}$$

d) <u>Tri4-type triangles:</u>

$$\mathcal{C}_4 = C_{12}-C_{11}-2C_0. \tag{95}$$

e) <u>Tri5-type triangles:</u>

$$\mathcal{C}_5 = C_{11}-C_{12}-C_0. \tag{96}$$

f) <u>Tri6-type triangles:</u>

$$\mathcal{C}_6 = C_{11}-C_{12}. \tag{97}$$

5.2 Charged Higgs sector

The amplitudes $a^{fin}_{L,R}$ corresponding to final vertices with H^+H^- states are:

$$a^{fin}_{L,R}(H^+H^-) = \frac{\Gamma^{fin,\gamma}(H^+H^-)}{2e} - \frac{\eta g_{L,R}}{2s_W c_W} \times \frac{\Gamma^{fin,Z}(H^+H^-)}{2e}. \qquad (98)$$

Here, $\Gamma^{fin,\gamma}(H^+H^-)$ and $\Gamma^{fin,Z}(H^+H^-)$ are obtained by summing the contributions of various triangles and of charged Higgs self-energy terms, as detailed in the following.

For the photon, one has:

$$\begin{aligned}\Gamma^{fin,\gamma}(H^+H^-) &= \Gamma^{\gamma}_{H^+H^-}(1\mathrm{ch}) + \Gamma^{\gamma}_{H^+H^-}(2) - \Gamma^{\gamma}_{H^+H^-}(2, pinch)\\ &+ \Gamma^{\gamma}_{H^+H^-}(3f) + \Gamma^{\gamma}_{H^+H^-}(\tilde{\chi}\tilde{\chi}^0\tilde{\chi}) + \Gamma^{\gamma}_{H^+H^-}(6\mathrm{ch}) + \Gamma^{\gamma}_{H^+H^-}(6\tilde{f})\\ &+ \Gamma^{\gamma}_{H^+H^-}(4\text{-leg}) + \Gamma^{\gamma}_{H^+H^-}(H.s.e). \qquad (99)\end{aligned}$$

For the Z boson, one has:

$$\begin{aligned}\Gamma^{fin,Z}(H^+H^-) &= \Gamma^{Z}_{H^+H^-}(1\mathrm{ch}) + \Gamma^{Z}_{H^+H^-}(1\mathrm{n}) + \Gamma^{Z}_{H^+H^-}(2) - \Gamma^{Z}_{H^+H^-}(2, pinch)\\ &+ \Gamma^{Z}_{H^+H^-}(3f) + \Gamma^{Z}_{H^+H^-}(\tilde{\chi}\tilde{\chi}^0\tilde{\chi}) + \Gamma^{Z}_{H^+H^-}(\tilde{\chi}^0\tilde{\chi}\tilde{\chi}^0) + \Gamma^{Z}_{H^+H^-}(4)\\ &+ \Gamma^{Z}_{H^+H^-}(6\mathrm{ch}) + \Gamma^{Z}_{H^+H^-}(6\mathrm{n}) + \Gamma^{Z}_{H^+H^-}(6\tilde{f})\\ &+ \Gamma^{Z}_{H^+H^-}(4\text{-leg}) + \Gamma^{Z}_{H^+H^-}(H.s.e). \qquad (100)\end{aligned}$$

5.2.1 Tri1-type triangles

The Tri1-type triangles contribute to $\Gamma^{fin,\gamma}(H^+H^-)$ with:

$$\Gamma^{\gamma}_{H^+H^-}(1\mathrm{ch}) = -\frac{e^3}{8\pi^2}\left[\mathcal{C}_1(H\gamma H) + \left(\frac{1-2s_W^2}{2s_W c_W}\right)^2 \mathcal{C}_1(HZH)\right]. \qquad (101)$$

The Tri1-type triangles contribute to $\Gamma^{fin,Z}(H^+H^-)$ with:

$$\Gamma^{Z}_{H^+H^-}(1\mathrm{ch}) = -\frac{e^3}{8\pi^2}\left(\frac{1-2s_W^2}{2s_W c_W}\right)\left[\mathcal{C}_1(H\gamma H) + \left(\frac{1-2s_W^2}{2s_W c_W}\right)^2 \mathcal{C}_1(HZH)\right], \qquad (102)$$

$$\begin{aligned}\Gamma^{Z}_{H^+H^-}(1\mathrm{n}) &= \frac{e^3}{64\pi^2 s_W^3 c_W} \times \Big\{\sin^2(\beta-\alpha)\left[\mathcal{C}_1(H^0WA^0) + \mathcal{C}_1(A^0WH^0)\right]\\ &\quad + \cos^2(\beta-\alpha)\left[\mathcal{C}_1(h^0WA^0) + \mathcal{C}_1(A^0Wh^0)\right]\Big\}. \qquad (103)\end{aligned}$$

5.2.2 Tri2-type triangles

With $f^V = \begin{cases} 1 \text{ for } V=\gamma \\ c_W/s_W \text{ for } V=Z \end{cases}$, the contribution of the Tri2-type triangles is:

$$\begin{aligned}\Gamma^{V}_{H^+H^-}(2) &= \frac{e^3 f^V}{64\pi^2 s_W^2} \times \Big\{\sin^2(\beta-\alpha)\,\mathcal{C}_2(WH^0W)\\ &\quad + \cos^2(\beta-\alpha)\,\mathcal{C}_2(Wh^0W)\\ &\quad + \mathcal{C}_2(WA^0W)\Big\}. \qquad (104)\end{aligned}$$

However, one must also take into account the pinch term:

$$-\Gamma^V_{H^+H^-}(2, pinch) \;=\; -\frac{e^3 f^V}{8\pi^2 s_W^2} \times B_0(WW, q^2). \tag{105}$$

5.2.3 Tri3-type triangles

The Tri3-type triangles contribute to both $\Gamma^{fin,\gamma}(H^+H^-)$ and $\Gamma^{fin,Z}(H^+H^-)$ with:

$$\begin{aligned}
\Gamma^V_{H^+H^-}(3f) \;=\; & -\frac{1}{8\pi^2}\sum_{(ff')}\frac{N_c^f e^3}{2s_W^2 M_W^2} \times \\
& \Big\{ \Big[g_{VRf} M_f^2 \cot^2\beta + g_{VLf} M_{f'}^2 \tan^2\beta\Big]\, \mathcal{C}_3(ff'f) \\
& - \Big[g_{VRf'} M_{f'}^2 \tan^2\beta + g_{VLf'} M_f^2 \cot^2\beta\Big]\, \mathcal{C}_3(f'ff') \\
& + 2M_f^2 M_{f'}^2 (g_{VRf} + g_{VLf})\, \mathcal{C}'_3(ff'f) \\
& - 2M_f^2 M_{f'}^2 (g_{VRf'} + g_{VLf'})\, \mathcal{C}'_3(f'ff') \\
& + M_f^2 \Big[g_{VLf} M_f^2 \cot^2\beta + g_{VRf} M_{f'}^2 \tan^2\beta\Big]\, \mathcal{C}''_3(ff'f) \\
& - M_{f'}^2 \Big[g_{VLf'} M_{f'}^2 \tan^2\beta + g_{VRf'} M_f^2 \cot^2\beta\Big]\, \mathcal{C}''_3(f'ff')\Big\}
\end{aligned} \tag{106}$$

$$\begin{aligned}
\Gamma^V_{H^+H^-}(\tilde\chi\tilde\chi^0\tilde\chi) \;=\; & \frac{1}{8\pi^2}\sum_{ijk} \Big\{ \Big[\mathcal{O}^{VL}_{kj} c^R_{Hji} c^{R*}_{Hki} + \mathcal{O}^{VR}_{kj} c^L_{Hji} c^{L*}_{Hki}\Big]\, \mathcal{C}_3(\tilde\chi_k \tilde\chi^0_i \tilde\chi_j) \\
& + M_{\tilde\chi_j} M_{\tilde\chi^0_i} \Big[\mathcal{O}^{VL}_{kj} c^L_{Hji} c^{R*}_{Hki} + \mathcal{O}^{VR}_{kj} c^R_{Hji} c^{L*}_{Hki}\Big]\, \mathcal{C}'_3(\tilde\chi_k \tilde\chi^0_i \tilde\chi_j) \\
& + M_{\tilde\chi^0_i} M_{\tilde\chi_k} \Big[\mathcal{O}^{VL}_{kj} c^R_{Hji} c^{L*}_{Hki} + \mathcal{O}^{VR}_{kj} c^L_{Hji} c^{R*}_{Hki}\Big]\, \mathcal{C}'_3(\tilde\chi_k \tilde\chi^0_i \tilde\chi_j) \\
& + M_{\tilde\chi_j} M_{\tilde\chi_k} \Big[\mathcal{O}^{VL}_{kj} c^L_{Hji} c^{L*}_{Hki} + \mathcal{O}^{VR}_{kj} c^R_{Hji} c^{R*}_{Hki}\Big]\, \mathcal{C}''_3(\tilde\chi_k \tilde\chi^0_i \tilde\chi_j)\Big\},
\end{aligned} \tag{107}$$

where $V = \gamma$ or Z, and where (f, f') is either (q_u, q_d) or (ν_ℓ, ℓ) for each fermion generation.

In addition, Tri3-type triangles contribute to $\Gamma^{fin,Z}(H^+H^-)$ with the following term:

$$\begin{aligned}
\Gamma^Z_{H^+H^-}(\tilde\chi^0\tilde\chi\tilde\chi^0) \;=\; & -\frac{1}{8\pi^2}\sum_{ijk} \Big\{ \Big[\mathcal{O}^{0L}_{jk} c^L_{Hij} c^{L*}_{Hik} + \mathcal{O}^{0R}_{jk} c^R_{Hij} c^{R*}_{Hik}\Big]\, \mathcal{C}_3(\tilde\chi^0_j \tilde\chi_i \tilde\chi^0_k) \\
& + M_{\tilde\chi^0_k} M_{\tilde\chi_i} \Big[\mathcal{O}^{0L}_{jk} c^L_{Hij} c^{R*}_{Hik} + \mathcal{O}^{0R}_{jk} c^R_{Hij} c^{L*}_{Hik}\Big]\, \mathcal{C}'_3(\tilde\chi^0_j \tilde\chi_i \tilde\chi^0_k) \\
& + M_{\tilde\chi_i} M_{\tilde\chi^0_j} \Big[\mathcal{O}^{0L}_{jk} c^R_{Hij} c^{L*}_{Hik} + \mathcal{O}^{0R}_{jk} c^L_{Hij} c^{R*}_{Hik}\Big]\, \mathcal{C}'_3(\tilde\chi^0_j \tilde\chi_i \tilde\chi^0_k) \\
& + M_{\tilde\chi^0_k} M_{\tilde\chi^0_j} \Big[\mathcal{O}^{0L}_{jk} c^R_{Hij} c^{R*}_{Hik} + \mathcal{O}^{0R}_{jk} c^L_{Hij} c^{L*}_{Hik}\Big]\, \mathcal{C}''_3(\tilde\chi^0_j \tilde\chi_i \tilde\chi^0_k)\Big\}.
\end{aligned} \tag{108}$$

5.2.4 Tri4-type triangles

The Tri4-type triangles contribute to $\Gamma^{fin,Z}(H^+H^-)$ with the following term:

$$\begin{aligned}
\Gamma^Z_{H^+H^-}(4) \;=\; & -\frac{1}{16\pi^2}\left(\frac{e^2 M_W (1 - 2s_W^2)}{2s_W^2 c_W^3}\right) \times \\
& \Big\{ g_{H^0HH}\cos(\beta-\alpha)\Big[\mathcal{C}_4(H^0HZ) + \mathcal{C}_4(ZHH^0)\Big] \\
& + g_{h^0HH}\sin(\beta-\alpha)\Big[\mathcal{C}_4(h^0HZ) + \mathcal{C}_4(ZHh^0)\Big]\Big\}.
\end{aligned} \tag{109}$$

5.2.5 Tri5-type triangles

There is no contribution from Tri5-type triangles in the production of H^+H^- pairs.

5.2.6 Tri6-type triangles

The Tri6-type triangles contribute to both $\Gamma^{fin,\gamma}(H^+H^-)$ and $\Gamma^{fin,Z}(H^+H^-)$ with:

$$\begin{aligned}\Gamma^V_{H^+H^-}(6\text{ch}) &= -\frac{1}{8\pi^2}\Big\{g_{VGG}g^2_{A^0GH}\mathcal{C}_6(GA^0G) \\ &\quad +g_{VGG}g^2_{H^0GH}\mathcal{C}_6(GH^0G)+g_{VGG}g^2_{h^0GH}\mathcal{C}_6(Gh^0G) \\ &\quad +g_{VHH}g^2_{H^0HH}\mathcal{C}_6(HH^0H)+g_{VHH}g^2_{h^0HH}\mathcal{C}_6(Hh^0H)\Big\} \qquad (110)\end{aligned}$$

and

$$\Gamma^V_{H^+H^-}(6\tilde{f}) = \Gamma^{V,\,heavy}_{H^+H^-}(6\tilde{f}) + \Gamma^{V,\,light}_{H^+H^-}(6\tilde{f}). \qquad (111)$$

The third generation squark contribution, with sfermion mixing, is:

$$\begin{aligned}\Gamma^{V,\,heavy}_{H^+H^-}(6\tilde{f}) &= \frac{3}{8\pi^2}\sum_{ijk=1,2}\Big\{g_{V\tilde{b}_i\tilde{b}_k}g_{H\tilde{t}_j\tilde{b}_i}g_{H\tilde{t}_j\tilde{b}_k}\mathcal{C}_6(\tilde{b}_i\tilde{t}_j\tilde{b}_k) \\ &\quad -g_{V\tilde{t}_i\tilde{t}_k}g_{H\tilde{t}_i\tilde{b}_j}g_{H\tilde{t}_k\tilde{b}_j}\mathcal{C}_6(\tilde{t}_i\tilde{b}_j\tilde{t}_k)\Big\}. \qquad (112)\end{aligned}$$

The coupling of L-sfermions to the charged Higgs boson does not vanish like the fermion mass, so they also contribute to $\Gamma^V(6\tilde{f})$. With $(f,f')=(u,d),(c,s)$ or $3\times(\nu_\ell,\ell)$, one has:

$$\Gamma^{V,\,light}_{H^+H^-}(6\tilde{f}) = \frac{1}{8\pi^2}\sum_{(\tilde{f}\tilde{f}')}N^f_c g^2_{H\tilde{f}_L\tilde{f}'_L}\Big\{g^0_{V\tilde{f}'_L\tilde{f}'_L}\mathcal{C}_6(\tilde{f}'_L\tilde{f}_L\tilde{f}'_L) - g^0_{V\tilde{f}_L\tilde{f}_L}\mathcal{C}_6(\tilde{f}_L\tilde{f}'_L\tilde{f}_L)\Big\}. \qquad (113)$$

In addition, Tri6-type triangles contribute to $\Gamma^{fin,Z}(H^+H^-)$ with the following term:

$$\begin{aligned}\Gamma^Z_{H^+H^-}(6\text{n}) &= \frac{1}{8\pi^2}\left(\frac{e^2M_W}{4s^2_Wc_W}\right)\times \\ &\quad \Big\{g_{H^0GH}\sin(\beta-\alpha)\Big[\mathcal{C}_6(H^0GA^0)+\mathcal{C}_6(A^0GH^0)\Big] \\ &\quad -g_{h^0GH}\cos(\beta-\alpha)\Big[\mathcal{C}_6(h^0GA^0)+\mathcal{C}_6(A^0Gh^0)\Big]\Big\}. \qquad (114)\end{aligned}$$

5.2.7 4-leg diagrams

The 4-leg diagrams contribute to $\Gamma^{fin,\gamma}(H^+H^-)$ with the following term:

$$\begin{aligned}\Gamma^\gamma_{H^+H^-}(4\text{-leg}) &= -\frac{1}{8\pi^2}\Big\{2e^3\Big[B_0(H\gamma,M_H^2)-B_1(H\gamma,M_H^2)\Big] \\ &\quad +\frac{e^3(1-2s^2_W)^2}{2s^2_Wc^2_W}\Big[B_0(HZ,M_H^2)-B_1(HZ,M_H^2)\Big] \\ &\quad +\frac{e^3\sin^2(\beta-\alpha)}{4s^2_W}\Big[B_0(H^0W,M_H^2)-B_1(H^0W,M_H^2)\Big] \\ &\quad +\frac{e^3\cos^2(\beta-\alpha)}{4s^2_W}\Big[B_0(h^0W,M_H^2)-B_1(h^0W,M_H^2)\Big] \\ &\quad +\frac{e^3}{4s^2_W}\Big[B_0(A^0W,M_H^2)-B_1(A^0W,M_H^2)\Big]\Big\}. \qquad (115)\end{aligned}$$

The 4-leg diagrams contribute to $\Gamma^{fin,\,Z}(H^+H^-)$ with the following term:

$$\begin{aligned}\Gamma^Z_{H^+H^-}(\text{4-leg}) \;=\; -\frac{1}{8\pi^2}\;\Big\{&\frac{2e^3(1-2s_W^2)}{2s_Wc_W}\Big[B_0(H\gamma,M_H^2)-B_1(H\gamma,M_H^2)\Big]\\ &+\frac{e^3(1-2s_W^2)^3}{4s_W^3c_W^3}\Big[B_0(HZ,M_H^2)-B_1(HZ,M_H^2)\Big]\\ &-\frac{e^3\sin^2(\beta-\alpha)}{4s_Wc_W}\Big[B_0(H^0W,M_H^2)-B_1(H^0W,M_H^2)\Big]\\ &-\frac{e^3\cos^2(\beta-\alpha)}{4s_Wc_W}\Big[B_0(h^0W,M_H^2)-B_1(h^0W,M_H^2)\Big]\\ &-\frac{e^3}{4s_Wc_W}\Big[B_0(A^0W,M_H^2)-B_1(A^0W,M_H^2)\Big]\Big\}. \end{aligned} \tag{116}$$

5.2.8 Charged Higgs self-energies

The charged Higgs self-energies contribute to $\Gamma^{fin,\,\gamma}(H^+H^-)$ and $\Gamma^{fin,\,Z}(H^+H^-)$ with:

$$\Gamma^\gamma_{H^+H^-}(H.s.e) = 2e \times \left(\frac{d\Sigma_{H^+H^-}}{dq^2}\right)_{q^2=M_H^2} \tag{117}$$

and

$$\Gamma^Z_{H^+H^-}(H.s.e) = \frac{2e(1-2s_W^2)}{2s_Wc_W} \times \left(\frac{d\Sigma_{H^+H^-}}{dq^2}\right)_{q^2=M_H^2} \tag{118}$$

where $\Sigma_{H^+H^-}(q^2)$ is the sum of various bubble terms.

These terms contain some combinations of Passarino-Veltman functions, such as:

$$SE_1^\pm(XY,q^2) \;=\; 4B_{22}(XY,q^2)+q^2\Big[B_0(XY,q^2)+B_{21}(XY,q^2)-2B_1(XY,q^2)\Big], \tag{119}$$

$$SE_2^\pm(XY,q^2) \;=\; 4B_{22}(XY,q^2)+q^2\Big[B_1(XY,q^2)+B_{21}(XY,q^2)\Big]. \tag{120}$$

Here, we consider only the contributions which depend on q^2, because $\Gamma^{fin,\,\gamma}(H^+H^-)$ and $\Gamma^{fin,\,Z}(H^+H^-)$ depend on the derivate of $\Sigma_{H^+H^-}(q^2)$. Four types of bubbles are taken into account when calculating this "reduced" self-energy, which we refer to as $\tilde{\Sigma}_{H^+H^-}(q^2)$:

$$\tilde{\Sigma}_{H^+H^-}(q^2) = \tilde{\Sigma}_{H^+H^-}(VS,q^2)+\tilde{\Sigma}_{H^+H^-}(SS',q^2)+\tilde{\Sigma}_{H^+H^-}(ff',q^2)+\tilde{\Sigma}_{H^+H^-}(\tilde{\chi}\tilde{\chi}^0,q^2). \tag{121}$$

The VS bubbles contribute to $\tilde{\Sigma}_{H^+H^-}(q^2)$ with:

$$\begin{aligned}\tilde{\Sigma}_{H^+H^-}(VS,q^2) \;=\; \frac{1}{16\pi^2}\;\Big\{&e^2SE_1^\pm(H\gamma,q^2)+\frac{e^2(1-2s_W^2)^2}{4s_W^2c_W^2}SE_1^\pm(HZ,q^2)\\ &+\frac{e^2\sin^2(\beta-\alpha)}{4s_W^2}SE_1^\pm(H^0W,q^2)+\frac{e^2\cos^2(\beta-\alpha)}{4s_W^2}SE_1^\pm(h^0W,q^2)\\ &+\frac{e^2}{4s_W^2}SE_1^\pm(A^0W,q^2)\Big\}. \end{aligned} \tag{122}$$

The SS' bubbles contribute to $\tilde{\Sigma}_{H^+H^-}(q^2)$ with:

$$\begin{aligned}\tilde{\Sigma}_{H^+H^-}(SS',q^2) &= -\frac{1}{16\pi^2} \Big\{ \sum_{light\ (\tilde{f}\tilde{f}')} N_c^f g^2_{H\tilde{f}_L\tilde{f}'_L} B_0(\tilde{f}_L\tilde{f}'_L,q^2) + \sum_{ij=1,2} 3g^2_{H\tilde{t}_i\tilde{b}_j} B_0(\tilde{t}_i\tilde{b}_j,q^2) \\ &\quad + g^2_{H^0HH} B_0(HH^0,q^2) + g^2_{h^0HH} B_0(Hh^0,q^2) \\ &\quad + g^2_{H^0GH} B_0(GH^0,q^2) + g^2_{h^0GH} B_0(Gh^0,q^2) \\ &\quad + g^2_{A^0GH} B_0(GA^0,q^2)\Big\}. \end{aligned} \tag{123}$$

The fermion and gaugino bubbles contribute to $\tilde{\Sigma}_{H^+H^-}(q^2)$ with respectively:

$$\begin{aligned}\tilde{\Sigma}_{H^+H^-}(ff',q^2) &= \frac{e^2}{16\pi^2 s_W^2 M_W^2} \sum_{(ff')} N_c^f \Big\{(M^2_{f'}\tan^2\beta + M_f^2\cot^2\beta)\, SE_2^{\pm}(ff',q^2) \\ &\quad +2M_f^2 M_{f'}^2 B_0(ff',q^2)\Big\} \end{aligned} \tag{124}$$

and

$$\begin{aligned}\tilde{\Sigma}_{H^+H^-}(\tilde{\chi}\tilde{\chi}^0,q^2) &= \frac{1}{8\pi^2}\sum_{ij} \Big\{ \left[c^{R*}_{Hij}c^R_{Hij} + c^{L*}_{Hij}c^L_{Hij}\right] SE_2^{\pm}(\tilde{\chi}_i\tilde{\chi}^0_j,q^2) \\ &\quad + M_{\tilde{\chi}_i} M_{\tilde{\chi}^0_j}\left[c^{R*}_{Hij}c^L_{Hij} + c^{L*}_{Hij}c^R_{Hij}\right] B_0(\tilde{\chi}_i\tilde{\chi}^0_j,q^2)\Big\}. \end{aligned} \tag{125}$$

After having computed the full expression for $\tilde{\Sigma}_{H^+H^-}(q^2)$, one simply needs to calculate its derivative at $q^2 = M_H^2$ and then insert it into equations (117) and (118).

5.3 Neutral Higgs sector

The amplitudes $a^{fin}_{L,R}$ corresponding to final vertices with H^0A^0 or h^0A^0 are:

$$a^{fin}_{L,R}(H^0A^0/h^0A^0) = \frac{i\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)}{2e} - \frac{\eta g_{L,R}}{2s_W c_W} \times \frac{i\Gamma^{fin,Z}(H^0A^0/h^0A^0)}{2e}. \tag{126}$$

$\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)$ and $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$ depend on the final state. They are obtained by summing the contributions of various triangles and of neutral Higgs self-energy terms.

For the photon, one has:

$$\begin{aligned}\Gamma^{fin,\gamma}(H^0A^0/h^0A^0) &= \Gamma^\gamma_{H^0A^0/h^0A^0}(1\text{ch}) + \Gamma^\gamma_{H^0A^0/h^0A^0}(2) - \Gamma^\gamma_{H^0A^0/h^0A^0}(2,pinch) \\ &+ \Gamma^\gamma_{H^0A^0/h^0A^0}(3f) + \Gamma^\gamma_{H^0A^0/h^0A^0}(\tilde{\chi}\tilde{\chi}\tilde{\chi}) + \Gamma^\gamma_{H^0A^0/h^0A^0}(4\text{ch}) \\ &+ \Gamma^\gamma_{H^0A^0/h^0A^0}(6\tilde{f}) + \Gamma^\gamma_{H^0A^0/h^0A^0}(6\text{ch}) + \Gamma^\gamma_{H^0A^0/h^0A^0}(4\text{-leg}). \end{aligned} \tag{127}$$

For the Z boson, one has:

$$\begin{aligned}\Gamma^{fin,Z}(H^0A^0/h^0A^0) &= \Gamma^Z_{H^0A^0/h^0A^0}(1\text{ch}) + \Gamma^Z_{H^0A^0/h^0A^0}(1\text{n}) \\ &+ \Gamma^Z_{H^0A^0/h^0A^0}(2) - \Gamma^Z_{H^0A^0/h^0A^0}(2,pinch) + \Gamma^Z_{H^0A^0/h^0A^0}(3f) + \\ &+ \Gamma^Z_{H^0A^0/h^0A^0}(\tilde{\chi}\tilde{\chi}\tilde{\chi}) + \Gamma^Z_{H^0A^0/h^0A^0}(\tilde{\chi}^0\tilde{\chi}^0\tilde{\chi}^0) + \Gamma^Z_{H^0A^0/h^0A^0}(4\text{n}) \\ &+ \Gamma^Z_{H^0A^0/h^0A^0}(4\text{ch}) + \Gamma^Z_{H^0A^0/h^0A^0}(5) + \Gamma^Z_{H^0A^0/h^0A^0}(6\tilde{f}) \\ &+ \Gamma^Z_{H^0A^0/h^0A^0}(6\text{ch}) + \Gamma^Z_{H^0A^0/h^0A^0}(6\text{n}) + \Gamma^Z_{H^0A^0/h^0A^0}(4\text{-leg}) + \\ &+ \Gamma^Z_{H^0A^0/h^0A^0}(H.s.e) + \Gamma^Z_{H^0A^0/h^0A^0}(H.c.t). \end{aligned} \tag{128}$$

5.3.1 Tri1-type triangles

The Tri1-type triangles contribute to $\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)$ with charged terms only:

$$\Gamma^{\gamma}_{H^0A^0/h^0A^0}(1\mathrm{ch}) = \frac{1}{8\pi^2} \times \frac{e^3}{2s_W^2}[Z_{ab}] \times \mathcal{C}_1(HWH). \quad (129)$$

The Tri1-type triangles contribute to $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$ with the following charged terms:

$$\Gamma^{Z}_{H^0A^0/h^0A^0}(1\mathrm{ch}) = \frac{1}{8\pi^2} \times \frac{e^3(1-2s_W^2)}{4s_W^3 c_W}[Z_{ab}] \times \mathcal{C}_1(HWH) \quad (130)$$

and with the following neutral terms:

$$\Gamma^{Z}_{H^0A^0}(1\mathrm{n}) = \frac{e^3\sin(\beta-\alpha)}{64\pi^2 s_W^3 c_W^3}\Big\{\sin^2(\beta-\alpha)\,\mathcal{C}_1(A^0ZH^0) + \cos^2(\beta-\alpha)\,\mathcal{C}_1(G^0ZH^0)$$
$$+\cos^2(\beta-\alpha)\,\mathcal{C}_1(A^0Zh^0) - \cos^2(\beta-\alpha)\,\mathcal{C}_1(G^0Zh^0)\Big\}, \quad (131)$$

$$\Gamma^{Z}_{h^0A^0}(1\mathrm{n}) = -\frac{e^3\cos(\beta-\alpha)}{64\pi^2 s_W^3 c_W^3}\Big\{\sin^2(\beta-\alpha)\,\mathcal{C}_1(A^0ZH^0) - \sin^2(\beta-\alpha)\,\mathcal{C}_1(G^0ZH^0)$$
$$+\cos^2(\beta-\alpha)\,\mathcal{C}_1(A^0Zh^0) + \sin^2(\beta-\alpha)\,\mathcal{C}_1(G^0Zh^0)\Big\}. \quad (132)$$

5.3.2 Tri2-type triangles

With $f^V = \begin{cases} 1 \text{ for } V=\gamma \\ c_W/s_W \text{ for } V=Z \end{cases}$, the contribution of the Tri2-type triangles is:

$$\Gamma^{V}_{H^0A^0/h^0A^0}(2) = \frac{e^3 f^V}{32\pi^2 s_W^2}[Z_{ab}] \times \mathcal{C}_2(WHW). \quad (133)$$

However, one must also take into account the pinch term:

$$-\Gamma^{V}_{H^0A^0/h^0A^0}(2,pinch) = -\frac{e^3 f^V}{8\pi^2 s_W^2}[Z_{ab}] \times B_0(WW,q^2). \quad (134)$$

5.3.3 Tri3-type triangles

Let V be either the photon or the Z boson. The Tri3-type triangles contribute to both $\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)$ and $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$ with two different terms.

The first term, with fermion triangles at the final vertex, is given by:

$$\Gamma^{V}_{H^0A^0/h^0A^0}(3f) = -\frac{e^3}{16\pi^2 s_W^2 M_W^2}\sum_f N_c^f M_f^2 y_f(g_{VLf}-g_{VRf})\left[\mathcal{C}_3(fff) - M_f^2\,\mathcal{C}_3''(fff)\right]. \quad (135)$$

In the previous equation, the term y_f depends both on the fermion in the triangle and on the final state. More explicitely, for (q_u, q_d) or (ν_ℓ, ℓ) doublets, one writes y_f as follows:

$$y_f = \left(\frac{\sin\alpha\cot\beta}{\sin\beta}, \frac{\cos\alpha\tan\beta}{\cos\beta}\right) \text{ for } H^0 A^0, \tag{136}$$

$$y_f = \left(\frac{\cos\alpha\cot\beta}{\sin\beta}, -\frac{\sin\alpha\tan\beta}{\cos\beta}\right) \text{ for } h^0 A^0. \tag{137}$$

The second term corresponds to chargino triangles at the final vertex.

For $H^0 A^0$ final states:

$$\begin{aligned}
\Gamma^V_{H^0A^0}(\tilde{\chi}\tilde{\chi}\tilde{\chi}) = \frac{1}{8\pi^2}\sum_{ijk} & \Big\{ \left[\mathcal{O}^{VL}_{ik} c^R_{A^0kj} c^L_{H^0ji} + \mathcal{O}^{VR}_{ik} c^L_{A^0kj} c^R_{H^0ji}\right] \mathcal{C}_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_j} M_{\tilde{\chi}_k} \left[\mathcal{O}^{VL}_{ik} c^L_{A^0kj} c^L_{H^0ji} + \mathcal{O}^{VR}_{ik} c^R_{A^0kj} c^R_{H^0ji}\right] \mathcal{C}'_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_i} M_{\tilde{\chi}_j} \left[\mathcal{O}^{VL}_{ik} c^R_{A^0kj} c^R_{H^0ji} + \mathcal{O}^{VR}_{ik} c^L_{A^0kj} c^L_{H^0ji}\right] \mathcal{C}'_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_i} M_{\tilde{\chi}_k} \left[\mathcal{O}^{VL}_{ik} c^L_{A^0kj} c^R_{H^0ji} + \mathcal{O}^{VR}_{ik} c^R_{A^0kj} c^L_{H^0ji}\right] \mathcal{C}''_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \Big\} \\
- \frac{1}{8\pi^2}\sum_{ijk} & \Big\{ \left[\mathcal{O}^{VR}_{ki} c^R_{A^0jk} c^L_{H^0ij} + \mathcal{O}^{VL}_{ki} c^L_{A^0jk} c^R_{H^0ij}\right] \mathcal{C}_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_j} M_{\tilde{\chi}_k} \left[\mathcal{O}^{VR}_{ki} c^L_{A^0jk} c^L_{H^0ij} + \mathcal{O}^{VL}_{ki} c^R_{A^0jk} c^R_{H^0ij}\right] \mathcal{C}'_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_i} M_{\tilde{\chi}_j} \left[\mathcal{O}^{VR}_{ki} c^R_{A^0jk} c^R_{H^0ij} + \mathcal{O}^{VL}_{ki} c^L_{A^0jk} c^L_{H^0ij}\right] \mathcal{C}'_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_i} M_{\tilde{\chi}_k} \left[\mathcal{O}^{VR}_{ki} c^L_{A^0jk} c^R_{H^0ij} + \mathcal{O}^{VL}_{ki} c^R_{A^0jk} c^L_{H^0ij}\right] \mathcal{C}''_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \Big\},
\end{aligned} \tag{138}$$

For $h^0 A^0$ final states:

$$\begin{aligned}
\Gamma^V_{h^0A^0}(\tilde{\chi}\tilde{\chi}\tilde{\chi}) = \frac{1}{8\pi^2}\sum_{ijk} & \Big\{ \left[\mathcal{O}^{VL}_{ik} c^R_{A^0kj} c^L_{h^0ji} + \mathcal{O}^{VR}_{ik} c^L_{A^0kj} c^R_{h^0ji}\right] \mathcal{C}_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_j} M_{\tilde{\chi}_k} \left[\mathcal{O}^{VL}_{ik} c^L_{A^0kj} c^L_{h^0ji} + \mathcal{O}^{VR}_{ik} c^R_{A^0kj} c^R_{h^0ji}\right] \mathcal{C}'_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_i} M_{\tilde{\chi}_j} \left[\mathcal{O}^{VL}_{ik} c^R_{A^0kj} c^R_{h^0ji} + \mathcal{O}^{VR}_{ik} c^L_{A^0kj} c^L_{h^0ji}\right] \mathcal{C}'_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_i} M_{\tilde{\chi}_k} \left[\mathcal{O}^{VL}_{ik} c^L_{A^0kj} c^R_{h^0ji} + \mathcal{O}^{VR}_{ik} c^R_{A^0kj} c^L_{h^0ji}\right] \mathcal{C}''_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \Big\} \\
- \frac{1}{8\pi^2}\sum_{ijk} & \Big\{ \left[\mathcal{O}^{VR}_{ki} c^R_{A^0jk} c^L_{h^0ij} + \mathcal{O}^{VL}_{ki} c^L_{A^0jk} c^R_{h^0ij}\right] \mathcal{C}_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_j} M_{\tilde{\chi}_k} \left[\mathcal{O}^{VR}_{ki} c^L_{A^0jk} c^L_{h^0ij} + \mathcal{O}^{VL}_{ki} c^R_{A^0jk} c^R_{h^0ij}\right] \mathcal{C}'_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_i} M_{\tilde{\chi}_j} \left[\mathcal{O}^{VR}_{ki} c^R_{A^0jk} c^R_{h^0ij} + \mathcal{O}^{VL}_{ki} c^L_{A^0jk} c^L_{h^0ij}\right] \mathcal{C}'_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
& + M_{\tilde{\chi}_i} M_{\tilde{\chi}_k} \left[\mathcal{O}^{VR}_{ki} c^L_{A^0jk} c^R_{h^0ij} + \mathcal{O}^{VL}_{ki} c^R_{A^0jk} c^L_{h^0ij}\right] \mathcal{C}''_3(\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \Big\}.
\end{aligned} \tag{139}$$

In addition, neutralino triangles at the final vertex give no contribution to $\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)$ but they enter into the expression of $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$.

For H^0A^0 final states:

$$
\begin{aligned}
\Gamma^V_{H^0A^0}(\tilde{\chi}^0\tilde{\chi}^0\tilde{\chi}^0) \; &= \frac{1}{16\pi^2}\sum_{ijk}\Big\{ \Big[\mathcal{O}^{0L}_{ik} n^R_{A^0kj} n^L_{H^0ji} + \mathcal{O}^{0R}_{ik} n^L_{A^0kj} n^R_{H^0ji}\Big]\, \mathcal{C}_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_j} M_{\tilde{\chi}^0_k}\Big[\mathcal{O}^{0L}_{ik} n^L_{A^0kj} n^L_{H^0ji} + \mathcal{O}^{0R}_{ik} n^R_{A^0kj} n^R_{H^0ji}\Big]\, \mathcal{C}'_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j}\Big[\mathcal{O}^{0L}_{ik} n^R_{A^0kj} n^R_{H^0ji} + \mathcal{O}^{0R}_{ik} n^L_{A^0kj} n^L_{H^0ji}\Big]\, \mathcal{C}'_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_k}\Big[\mathcal{O}^{0L}_{ik} n^L_{A^0kj} n^R_{H^0ji} + \mathcal{O}^{0R}_{ik} n^R_{A^0kj} n^L_{H^0ji}\Big]\, \mathcal{C}''_3(\chi^0_i\chi^0_j\chi^0_k)\Big\} \\
&- \frac{1}{16\pi^2}\sum_{ijk}\Big\{ \Big[\mathcal{O}^{0R}_{ki} n^R_{A^0jk} n^L_{H^0ij} + \mathcal{O}^{0L}_{ki} n^L_{A^0jk} n^R_{H^0ij}\Big]\, \mathcal{C}_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_j} M_{\tilde{\chi}^0_k}\Big[\mathcal{O}^{0R}_{ki} n^L_{A^0jk} n^L_{H^0ij} + \mathcal{O}^{0L}_{ki} n^R_{A^0jk} n^R_{H^0ij}\Big]\, \mathcal{C}'_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j}\Big[\mathcal{O}^{0R}_{ki} n^R_{A^0jk} n^R_{H^0ij} + \mathcal{O}^{0L}_{ki} n^L_{A^0jk} n^L_{H^0ij}\Big]\, \mathcal{C}'_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_k}\Big[\mathcal{O}^{0R}_{ki} n^L_{A^0jk} n^R_{H^0ij} + \mathcal{O}^{0L}_{ki} n^R_{A^0jk} n^L_{H^0ij}\Big]\, \mathcal{C}''_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k)\Big\}. \qquad (140)
\end{aligned}
$$

For the h^0A^0 final states:

$$
\begin{aligned}
\Gamma^V_{h^0A^0}(\tilde{\chi}^0\tilde{\chi}^0\tilde{\chi}^0) \; &= \frac{1}{16\pi^2}\sum_{ijk}\Big\{ \Big[\mathcal{O}^{0L}_{ik} n^R_{A^0kj} n^L_{h^0ji} + \mathcal{O}^{0R}_{ik} n^L_{A^0kj} n^R_{h^0ji}\Big]\, \mathcal{C}_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_j} M_{\tilde{\chi}^0_k}\Big[\mathcal{O}^{0L}_{ik} n^L_{A^0kj} n^L_{h^0ji} + \mathcal{O}^{0R}_{ik} n^R_{A^0kj} n^R_{h^0ji}\Big]\, \mathcal{C}'_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j}\Big[\mathcal{O}^{0L}_{ik} n^R_{A^0kj} n^R_{h^0ji} + \mathcal{O}^{0R}_{ik} n^L_{A^0kj} n^L_{h^0ji}\Big]\, \mathcal{C}'_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_k}\Big[\mathcal{O}^{0L}_{ik} n^L_{A^0kj} n^R_{h^0ji} + \mathcal{O}^{0R}_{ik} n^R_{A^0kj} n^L_{h^0ji}\Big]\, \mathcal{C}''_3(\chi^0_i\chi^0_j\chi^0_k)\Big\} \\
&- \frac{1}{16\pi^2}\sum_{ijk}\Big\{ \Big[\mathcal{O}^{0R}_{ki} n^R_{A^0jk} n^L_{h^0ij} + \mathcal{O}^{0L}_{ki} n^L_{A^0jk} n^R_{h^0ij}\Big]\, \mathcal{C}_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_j} M_{\tilde{\chi}^0_k}\Big[\mathcal{O}^{0R}_{ki} n^L_{A^0jk} n^L_{h^0ij} + \mathcal{O}^{0L}_{ki} n^R_{A^0jk} n^R_{h^0ij}\Big]\, \mathcal{C}'_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j}\Big[\mathcal{O}^{0R}_{ki} n^R_{A^0jk} n^R_{h^0ij} + \mathcal{O}^{0L}_{ki} n^L_{A^0jk} n^L_{h^0ij}\Big]\, \mathcal{C}'_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k) \\
&\qquad + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_k}\Big[\mathcal{O}^{0R}_{ki} n^L_{A^0jk} n^R_{h^0ij} + \mathcal{O}^{0L}_{ki} n^R_{A^0jk} n^L_{h^0ij}\Big]\, \mathcal{C}''_3(\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{\chi}^0_k)\Big\}. \qquad (141)
\end{aligned}
$$

5.3.4 Tri4-type triangles

Let V denote either the photon or the Z boson. The Tri4-type triangles contribute to both $\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)$ and $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$ with charged terms, which are given by:

$$
\Gamma^V_{H^0A^0}(4\text{ch}) = \frac{1}{8\pi^2}\, g_{VWG} \times \Big\{ g_{WHA^0} g_{H^0GH}\, \mathcal{C}_4(GHW) + g_{A^0GH} g_{WHH^0}\, \mathcal{C}_4(WHG) \Big\} \qquad (142)
$$

and

$$
\Gamma^V_{h^0A^0}(4\text{ch}) = \frac{1}{8\pi^2}\, g_{VWG} \times \Big\{ g_{WHA^0} g_{h^0GH}\, \mathcal{C}_4(GHW) + g_{A^0GH} g_{WHh^0}\, \mathcal{C}_4(WHG) \Big\}. \qquad (143)
$$

The coupling constants g_{WHH^0}, g_{WHh^0} and g_{A^0GH} depend on the charge of the particles at the vertex. Note that, in the triangles considered here, W and G carry the same charge.

Tri4-type triangles also contribute to $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$ with neutral terms:

$$\begin{aligned}\Gamma^Z_{H^0A^0}(4\mathrm{n}) &= -\frac{1}{16\pi^2}\Big\{g_{ZZH^0}\Big[g_{H^0H^0H^0}g_{ZH^0A^0}\,\mathcal{C}_4(H^0H^0Z)+g_{h^0H^0H^0}g_{Zh^0A^0}\,\mathcal{C}_4(H^0h^0Z)\Big]\\ &+g_{ZZh^0}\Big[g_{h^0H^0H^0}g_{ZH^0A^0}\,\mathcal{C}_4(h^0H^0Z)+g_{H^0h^0h^0}g_{Zh^0A^0}\,\mathcal{C}_4(h^0h^0Z)\Big]\\ &+g_{ZZH^0}\Big[g_{H^0A^0A^0}g_{ZH^0A^0}\,\mathcal{C}_4(H^0A^0Z)+g_{H^0A^0G^0}g_{ZH^0G^0}\,\mathcal{C}_4(H^0G^0Z)\Big]\\ &+g_{ZZh^0}\Big[g_{h^0A^0A^0}g_{ZH^0A^0}\,\mathcal{C}_4(h^0A^0Z)+g_{h^0A^0G^0}g_{ZH^0G^0}\,\mathcal{C}_4(h^0G^0Z)\Big]\Big\},\end{aligned} \tag{144}$$

$$\begin{aligned}\Gamma^Z_{h^0A^0}(4\mathrm{n}) &= -\frac{1}{16\pi^2}\Big\{g_{ZZH^0}\Big[g_{h^0H^0H^0}g_{ZH^0A^0}\,\mathcal{C}_4(H^0H^0Z)+g_{H^0h^0h^0}g_{Zh^0A^0}\,\mathcal{C}_4(H^0h^0Z)\Big]\\ &+g_{ZZh^0}\Big[g_{H^0h^0h^0}g_{ZH^0A^0}\,\mathcal{C}_4(h^0H^0Z)+g_{h^0h^0h^0}g_{Zh^0A^0}\,\mathcal{C}_4(h^0h^0Z)\Big]\\ &+g_{ZZH^0}\Big[g_{H^0A^0A^0}g_{Zh^0A^0}\,\mathcal{C}_4(H^0A^0Z)+g_{H^0A^0G^0}g_{Zh^0G^0}\,\mathcal{C}_4(H^0G^0Z)\Big]\\ &+g_{ZZh^0}\Big[g_{h^0A^0A^0}g_{Zh^0A^0}\,\mathcal{C}_4(h^0A^0Z)+g_{h^0A^0G^0}g_{Zh^0G^0}\,\mathcal{C}_4(h^0G^0Z)\Big]\Big\}.\end{aligned} \tag{145}$$

5.3.5 Tri5-type triangles

The Tri5-type triangles contribute only to $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$.

$$\Gamma^Z_{H^0A^0}(5) = \frac{1}{16\pi^2}\Big[g^2_{ZZH^0}g_{ZH^0A^0}\,\mathcal{C}_5(ZZH^0)+g_{ZZh^0}g_{ZZH^0}g_{Zh^0A^0}\,\mathcal{C}_5(ZZh^0)\Big], \tag{146}$$

$$\Gamma^Z_{h^0A^0}(5) = \frac{1}{16\pi^2}\Big[g_{ZZH^0}g_{ZZh^0}g_{ZH^0A^0}\,\mathcal{C}_5(ZZH^0)+g^2_{ZZh^0}g_{Zh^0A^0}\,\mathcal{C}_5(ZZh^0)\Big]. \tag{147}$$

5.3.6 Tri6-type triangles

Two Tri6-type triangles contribute to both $\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)$ and $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$.

Let V denote either the photon or the Z boson, we focus on $\Gamma^V_{H^0A^0/h^0A^0}(6\tilde{f})$ first. The coupling of A^0 to sfermions is proportional to the corresponding fermion mass and it is thus negligible, except in the case of third generation squarks, with sfermion mixing.

For H^0A^0 final states, one has:

$$\begin{aligned}\Gamma^V_{H^0A^0}(6\tilde{f}) &= \frac{3}{8\pi^2}\sum_{ijk=1,2}\Big\{g_{V\tilde{b}_i\tilde{b}_k}g_{H^0\tilde{b}_i\tilde{b}_j}g_{A^0\tilde{b}_j\tilde{b}_k}\mathcal{C}_6(\tilde{b}_i\tilde{b}_j\tilde{b}_k)+g_{V\tilde{b}_i\tilde{b}_k}g_{A^0\tilde{b}_i\tilde{b}_j}g_{H^0\tilde{b}_j\tilde{b}_k}\mathcal{C}_6(\tilde{b}_i\tilde{b}_j\tilde{b}_k)\Big\}\\ &+\frac{3}{8\pi^2}\sum_{ijk=1,2}\Big\{g_{V\tilde{t}_i\tilde{t}_k}g_{H^0\tilde{t}_i\tilde{t}_j}g_{A^0\tilde{t}_j\tilde{t}_k}\mathcal{C}_6(\tilde{t}_i\tilde{t}_j\tilde{t}_k)+g_{V\tilde{t}_i\tilde{t}_k}g_{A^0\tilde{t}_i\tilde{t}_j}g_{H^0\tilde{t}_j\tilde{t}_k}\mathcal{C}_6(\tilde{t}_i\tilde{t}_j\tilde{t}_k)\Big\}.\end{aligned} \tag{148}$$

For h^0A^0 final states, one has:

$$\begin{aligned}\Gamma^V_{h^0A^0}(6\tilde{f}) &= \frac{3}{8\pi^2}\sum_{ijk=1,2}\Big\{g_{V\tilde{b}_i\tilde{b}_k}g_{h^0\tilde{b}_i\tilde{b}_j}g_{A^0\tilde{b}_j\tilde{b}_k}\mathcal{C}_6(\tilde{b}_i\tilde{b}_j\tilde{b}_k)+g_{V\tilde{b}_i\tilde{b}_k}g_{A^0\tilde{b}_i\tilde{b}_j}g_{h^0\tilde{b}_j\tilde{b}_k}\mathcal{C}_6(\tilde{b}_i\tilde{b}_j\tilde{b}_k)\Big\}\\ &+\frac{3}{8\pi^2}\sum_{ijk=1,2}\Big\{g_{V\tilde{t}_i\tilde{t}_k}g_{h^0\tilde{t}_i\tilde{t}_j}g_{A^0\tilde{t}_j\tilde{t}_k}\mathcal{C}_6(\tilde{t}_i\tilde{t}_j\tilde{t}_k)+g_{V\tilde{t}_i\tilde{t}_k}g_{A^0\tilde{t}_i\tilde{t}_j}g_{h^0\tilde{t}_j\tilde{t}_k}\mathcal{C}_6(\tilde{t}_i\tilde{t}_j\tilde{t}_k)\Big\}.\end{aligned} \tag{149}$$

The 6ch triangles also contribute to both $\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)$ and $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$:

$$\Gamma^V_{H^0A^0}(\text{6ch}) = \frac{eM_W}{8\pi^2 s_W} g_{H^0GH}\left[g_{VHH}\,\mathcal{C}_6(HGH) - g_{VGG}\,\mathcal{C}_6(GHG)\right], \tag{150}$$

$$\Gamma^V_{h^0A^0}(\text{6ch}) = \frac{eM_W}{8\pi^2 s_W} g_{h^0GH}\left[g_{VHH}\,\mathcal{C}_6(HGH) - g_{VGG}\,\mathcal{C}_6(GHG)\right]. \tag{151}$$

Since neutral Higgs or Goldstone bosons do not couple to a photon, the 6n triangles contribute to $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$ only.

For H^0A^0 final states, one has:

$$\begin{aligned}
\Gamma^Z_{H^0A^0}(\text{6n}) = \frac{1}{8\pi^2}\Big\{ & g_{Zh^0G^0}\left[g_{h^0H^0H^0}g_{H^0A^0G^0}\mathcal{C}_6(h^0H^0G^0) + g_{H^0h^0h^0}g_{h^0A^0G^0}\mathcal{C}_6(h^0h^0G^0)\right] \\
& + g_{ZH^0G^0}\left[g_{H^0H^0H^0}g_{H^0A^0G^0}\mathcal{C}_6(H^0H^0G^0) + g_{h^0H^0H^0}g_{h^0A^0G^0}\mathcal{C}_6(H^0h^0G^0)\right] \\
& + g_{Zh^0A^0}\left[g_{h^0H^0H^0}g_{H^0A^0A^0}\mathcal{C}_6(h^0H^0A^0) + g_{H^0h^0h^0}g_{h^0A^0A^0}\mathcal{C}_6(h^0h^0A^0)\right] \\
& + g_{ZH^0A^0}\left[g_{H^0H^0H^0}g_{H^0A^0A^0}\mathcal{C}_6(H^0H^0A^0) + g_{h^0H^0H^0}g_{h^0A^0A^0}\mathcal{C}_6(H^0h^0A^0)\right]\Big\} \\
-\frac{1}{8\pi^2}\Big\{ & g_{Zh^0G^0}\left[g_{H^0G^0G^0}g_{h^0A^0G^0}\mathcal{C}_6(G^0G^0h^0) + g_{H^0A^0G^0}g_{h^0A^0A^0}\mathcal{C}_6(G^0A^0h^0)\right] \\
& + g_{ZH^0G^0}\left[g_{H^0G^0G^0}g_{H^0A^0G^0}\mathcal{C}_6(G^0G^0H^0) + g_{H^0A^0G^0}g_{H^0A^0A^0}\mathcal{C}_6(G^0A^0H^0)\right] \\
& + g_{Zh^0A^0}\left[g_{H^0A^0A^0}g_{h^0A^0A^0}\mathcal{C}_6(A^0A^0h^0) + g_{H^0A^0G^0}g_{h^0A^0G^0}\mathcal{C}_6(A^0G^0h^0)\right] \\
& + g_{ZH^0A^0}\left[g^2_{H^0A^0A^0}\mathcal{C}_6(A^0A^0H^0) + g^2_{H^0A^0G^0}\mathcal{C}_6(A^0G^0H^0)\right]\Big\}.
\end{aligned} \tag{152}$$

For h^0A^0 final states, one has:

$$\begin{aligned}
\Gamma^Z_{h^0A^0}(\text{6n}) = \frac{1}{8\pi^2}\Big\{ & g_{ZH^0G^0}\left[g_{h^0H^0H^0}g_{H^0A^0G^0}\mathcal{C}_6(H^0H^0G^0) + g_{H^0h^0h^0}g_{h^0A^0G^0}\mathcal{C}_6(H^0h^0G^0)\right] \\
& + g_{Zh^0G^0}\left[g_{H^0h^0h^0}g_{H^0A^0G^0}\mathcal{C}_6(h^0H^0G^0) + g_{h^0h^0h^0}g_{h^0A^0G^0}\mathcal{C}_6(h^0h^0G^0)\right] \\
& + g_{ZH^0A^0}\left[g_{h^0H^0H^0}g_{H^0A^0A^0}\mathcal{C}_6(H^0H^0A^0) + g_{H^0h^0h^0}g_{h^0A^0A^0}\mathcal{C}_6(H^0h^0A^0)\right] \\
& + g_{Zh^0A^0}\left[g_{H^0h^0h^0}g_{H^0A^0A^0}\mathcal{C}_6(h^0H^0A^0) + g_{h^0h^0h^0}g_{h^0A^0A^0}\mathcal{C}_6(h^0h^0A^0)\right]\Big\} \\
-\frac{1}{8\pi^2}\Big\{ & g_{ZH^0G^0}\left[g_{h^0G^0G^0}g_{H^0A^0G^0}\mathcal{C}_6(G^0G^0H^0) + g_{h^0A^0G^0}g_{H^0A^0A^0}\mathcal{C}_6(G^0A^0H^0)\right] \\
& + g_{Zh^0G^0}\left[g_{h^0G^0G^0}g_{h^0A^0G^0}\mathcal{C}_6(G^0G^0h^0) + g_{h^0A^0G^0}g_{h^0A^0A^0}\mathcal{C}_6(G^0A^0h^0)\right] \\
& + g_{ZH^0A^0}\left[g_{h^0A^0A^0}g_{H^0A^0A^0}\mathcal{C}_6(A^0A^0H^0) + g_{H^0A^0G^0}g_{h^0A^0G^0}\mathcal{C}_6(A^0G^0H^0)\right] \\
& + g_{Zh^0A^0}\left[g^2_{h^0A^0A^0}\mathcal{C}_6(A^0A^0h^0) + g^2_{h^0A^0G^0}\mathcal{C}_6(A^0G^0h^0)\right]\Big\}.
\end{aligned} \tag{153}$$

5.3.7 4-leg diagrams

For H^0A^0 final states, the 4-leg diagrams give the following contributions:

$$\Gamma^{\gamma}_{H^0A^0}(\text{4-leg}) = -\frac{\sin(\beta-\alpha)}{16\pi^2}\left(\frac{e^3}{2s_W^2}\right)\times\Big\{\Big[B_0(HW,M_{H^0}^2)-B_1(HW,M_{H^0}^2)\Big] + \Big[B_0(HW,M_A^2)-B_1(HW,M_A^2)\Big]\Big\} \quad (154)$$

$$\Gamma^{Z}_{H^0A^0}(\text{4-leg}) = -\frac{s_W}{c_W}\Gamma^{\gamma}(\text{4-leg}) + \frac{\sin(\beta-\alpha)}{16\pi^2}\left(\frac{e^3}{4s_W^3c_W^3}\right)\times \Big\{\Big[B_0(A^0Z,M_{H^0}^2)-B_1(A^0Z,M_{H^0}^2)\Big] + \Big[B_0(H^0Z,M_A^2)-B_1(H^0Z,M_A^2)\Big]\Big\}. \quad (155)$$

For h^0A^0 final states, the 4-leg diagrams give the following contributions:

$$\Gamma^{\gamma}_{h^0A^0}(\text{4-leg}) = \frac{\cos(\beta-\alpha)}{16\pi^2}\left(\frac{e^3}{2s_W^2}\right)\times\Big\{\Big[B_0(HW,M_{h^0}^2)-B_1(HW,M_{h^0}^2)\Big] + \Big[B_0(HW,M_A^2)-B_1(HW,M_A^2)\Big]\Big\} \quad (156)$$

$$\Gamma^{Z}_{h^0A^0}(\text{4-leg}) = -\frac{s_W}{c_W}\Gamma^{\gamma}(\text{4-leg}) - \frac{\cos(\beta-\alpha)}{16\pi^2}\left(\frac{e^3}{4s_W^3c_W^3}\right)\times \Big\{\Big[B_0(A^0Z,M_{h^0}^2)-B_1(A^0Z,M_{h^0}^2)\Big] + \Big[B_0(H^0Z,M_A^2)-B_1(H^0Z,M_A^2)\Big]\Big\}. \quad (157)$$

5.3.8 Neutral Higgs self-energies

Before estimating the self-energy terms, let us first define several useful expressions:

$$v_1 = \sin 2\alpha \sin 2\beta - \frac{s_W^2}{c_W^2}\cos 2\alpha\cos 2\beta. \quad (158)$$

$$v_2 = \cos 2\alpha \sin 2\beta + \frac{s_W^2}{c_W^2}\sin 2\alpha\cos 2\beta. \quad (159)$$

$$SE_1^0(XY,q^2) = 2q^2B_1(XY,q^2) - A(M_Y^2) - (q^2+M_X^2)B_0(XY,q^2), \quad (160)$$

$$SE_2^0(ff,q^2) = 2M_f^2B_0(ff,q^2) + A(M_f^2) + q^2B_1(ff,q^2), \quad (161)$$

$$SE_3^0(XY,q^2,a,b,c,d) = 8\Big\{(ad+bc)\Big[q^2B_1(XY,q^2) + A(M_X^2) + M_Y^2B_0(XY,q^2)\Big] + (ac+bd)M_XM_YB_0(XY,q^2)\Big\}, \quad (162)$$

$$SE_4^0(XY,q^2) = B_1(XY,q^2) - B_0(XY,q^2), \quad (163)$$

$$SE_5^0(XY,q^2) = 2B_1(XY,q^2) + B_0(XY,q^2), \quad (164)$$

$$SE_6^0(XY,q^2,a,b,c,d) = -8\Big\{(ad+bc)M_XB_1(XY,q^2) + (ac+bd)M_Y\Big[B_0(XY,q^2)+B_1(XY,q^2)\Big]\Big\}, \quad (165)$$

In the following, all neutral Higgs self-energies $\Sigma(q^2)$ and the Higgs tadpoles T_{H^0/h^0} are written as the sum of various contributions coming from the gauge and Higgs sectors, fermion pairs, gaugino pairs and sfermion pairs (where we consider separately the unmixed case and the third generation squarks with mixing):

$$\Sigma(q^2) = \Sigma(\mathrm{g+H}) + \Sigma(ff) + \Sigma(\tilde{\chi}\tilde{\chi}) + \Sigma(\tilde{\chi}^0\tilde{\chi}^0) + \Sigma(\tilde{f}\tilde{f}), \tag{166}$$

$$T_{H^0/h^0} = T_{H^0/h^0}(\mathrm{g+H}) + T_{H^0/h^0}(ff) + T_{H^0/h^0}(\tilde{\chi}\tilde{\chi}) + T_{H^0/h^0}(\tilde{\chi}^0\tilde{\chi}^0) + T_{H^0/h^0}(\tilde{f}\tilde{f}). \tag{167}$$

a) H^0 self-energies:

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned}
\Sigma_{H^0H^0}(\mathrm{g+H}) = \frac{1}{16\pi^2} & \Big\{ g^2_{ZH^0A^0} SE^0_1(A^0Z, q^2) + g^2_{ZH^0G^0} SE^0_1(ZZ, q^2) \\
& + 2g^2_{WHH^0} SE^0_1(HW, q^2) + 2g^2_{WGH^0} SE^0_1(WW, q^2) \\
& + 2g^2_{WWH^0}\left[2B_0(WW, q^2) - 1\right] + g^2_{ZZH^0}\left[2B_0(ZZ, q^2) - 1\right] \\
& + g^2_{H^0HH} B_0(HH, q^2) + g^2_{H^0GG} B_0(WW, q^2) + 2g^2_{H^0GH} B_0(WH, q^2) \\
& + \frac{1}{2}\left[g^2_{H^0h^0h^0} B_0(h^0h^0, q^2) + g^2_{H^0H^0H^0} B_0(H^0H^0, q^2)\right] \\
& + \frac{1}{2}\left[g^2_{H^0A^0A^0} B_0(A^0A^0, q^2) + g^2_{H^0G^0G^0} B_0(ZZ, q^2)\right] \\
& + g^2_{h^0H^0H^0} B_0(H^0h^0, q^2) + g^2_{H^0A^0G^0} B_0(A^0Z, q^2) \\
& - \frac{e^2 M_W^2 \cos^2(\beta-\alpha)}{2s_W^2}\left[B_0(WW, q^2) + \frac{1}{2c_W^4} B_0(ZZ, q^2)\right] \\
& + \frac{e^2}{s_W^2}\left(\left[2A(M_W^2) - M_W^2\right] + \frac{1}{2c_W^2}\left[2A(M_Z^2) - M_Z^2\right]\right) \\
& + \frac{e^2}{8s_W^2c_W^2}\left[(3\sin^2 2\alpha - 1)A(M_{h^0}^2) + 3\cos^2 2\alpha A(M_{H^0}^2)\right] \\
& - \frac{e^2}{8s_W^2c_W^2}\cos 2\beta\cos 2\alpha\left[A(M_A^2) - A(M_Z^2)\right] \\
& + \frac{e^2}{4s_W^2}\left[(1-v_1)A(M_W^2) + (1+v_1)A(M_H^2)\right]\Big\}.
\end{aligned} \tag{168}$$

The contribution of the fermion pairs is:

$$\Sigma_{H^0H^0}(ff) = -\frac{1}{4\pi^2}\sum_f N_c^f \times (c^L_{H^0f})^2 \times SE^0_2(ff, q^2). \tag{169}$$

The contributions of the gaugino pairs are:

$$\Sigma_{H^0H^0}(\tilde{\chi}\tilde{\chi}) = -\frac{1}{64\pi^2}\sum_{ij} SE^0_3(\tilde{\chi}_i\tilde{\chi}_j, q^2, c^L_{H^0ji}, c^L_{H^0ij}, c^L_{H^0ij}, c^L_{H^0ji}), \tag{170}$$

$$\Sigma_{H^0H^0}(\tilde{\chi}^0\tilde{\chi}^0) = -\frac{1}{128\pi^2}\sum_{ij} SE^0_3(\tilde{\chi}^0_i\tilde{\chi}^0_j, q^2, n^L_{H^0ji}, n^L_{H^0ij}, n^L_{H^0ij}, n^L_{H^0ji}). \tag{171}$$

The contribution of sfermion pairs consists of two terms:

$$\Sigma^{light}_{H^0H^0}(\tilde{f}\tilde{f}) = \frac{1}{16\pi^2}\sum_{\tilde{f}} N_c^f \quad \Big\{ \Big[g^2_{H^0\tilde{f}_L\tilde{f}_L} + g^2_{H^0\tilde{f}_R\tilde{f}_R}\Big] B_0(\tilde{f}\tilde{f}, q^2) - \Big[g_{H^0H^0\tilde{f}_L\tilde{f}_L} + g_{H^0H^0\tilde{f}_R\tilde{f}_R}\Big] A(M^2_{\tilde{f}})\Big\}, \tag{172}$$

$$\begin{aligned}\Sigma^{heavy}_{H^0H^0}(\tilde{f}\tilde{f}) = & \frac{3}{16\pi^2}\sum_{ij=1,2}\Big\{g^2_{H^0\tilde{t}_i\tilde{t}_j}B_0(\tilde{t}_i\tilde{t}_j, q^2) + g^2_{H^0\tilde{b}_i\tilde{b}_j}B_0(\tilde{b}_i\tilde{b}_j, q^2)\Big\} \\ & -\frac{3}{16\pi^2}\sum_{\tilde{f}=\tilde{t},\tilde{b}}\Big\{c^2_{\tilde{f}}\Big[g_{H^0H^0\tilde{f}_L\tilde{f}_L}A(M^2_{\tilde{f}_1}) + g_{H^0H^0\tilde{f}_R\tilde{f}_R}A(M^2_{\tilde{f}_2})\Big]\Big\} \\ & -\frac{3}{16\pi^2}\sum_{\tilde{f}=\tilde{t},\tilde{b}}\Big\{s^2_{\tilde{f}}\Big[g_{H^0H^0\tilde{f}_R\tilde{f}_R}A(M^2_{\tilde{f}_1}) + g_{H^0H^0\tilde{f}_L\tilde{f}_L}A(M^2_{\tilde{f}_2})\Big]\Big\}.\end{aligned} \tag{173}$$

b) h^0 self-energies:

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned}\Sigma_{h^0h^0}(\mathrm{g+H}) = \frac{1}{16\pi^2} & \Big\{g^2_{Zh^0A^0}SE^0_1(A^0Z, q^2) + g^2_{Zh^0G^0}SE^0_1(ZZ, q^2) \\ & +2g^2_{WHh^0}SE^0_1(HW, q^2) + 2g^2_{WGh^0}SE^0_1(WW, q^2) \\ & +2g^2_{WWh^0}\Big[2B_0(WW, q^2) - 1\Big] + g^2_{ZZh^0}\Big[2B_0(ZZ, q^2) - 1\Big] \\ & +g^2_{h^0HH}B_0(HH, q^2) + g^2_{h^0GG}B_0(WW, q^2) + 2g^2_{h^0GH}B_0(WH, q^2) \\ & +\frac{1}{2}\Big[g^2_{h^0H^0H^0}B_0(H^0H^0, q^2) + g^2_{h^0h^0h^0}B_0(h^0h^0, q^2)\Big] \\ & +\frac{1}{2}\Big[g^2_{h^0A^0A^0}B_0(A^0A^0, q^2) + g^2_{h^0G^0G^0}B_0(ZZ, q^2)\Big] \\ & +g^2_{H^0h^0h^0}B_0(H^0h^0, q^2) + g^2_{h^0A^0G^0}B_0(A^0Z, q^2) \\ & -\frac{e^2M^2_W\sin^2(\beta-\alpha)}{2s^2_W}\Big[B_0(WW, q^2) + \frac{1}{2c^4_W}B_0(ZZ, q^2)\Big] \\ & +\frac{e^2}{s^2_W}\left(\Big[2A(M^2_W) - M^2_W\Big] + \frac{1}{2c^2_W}\Big[2A(M^2_Z) - M^2_Z\Big]\right) \\ & +\frac{e^2}{8s^2_Wc^2_W}\Big[(3\sin^2 2\alpha - 1)A(M^2_{H^0}) + 3\cos^2 2\alpha A(M^2_{h^0})\Big] \\ & +\frac{e^2}{8s^2_Wc^2_W}\cos 2\beta\cos 2\alpha\Big[A(M^2_A) - A(M^2_Z)\Big] \\ & +\frac{e^2}{4s^2_W}\Big[(1+v_1)A(M^2_W) + (1-v_1)A(M^2_H)\Big]\Big\}.\end{aligned} \tag{174}$$

The contribution of the fermion pairs is:

$$\Sigma_{h^0h^0}(ff) = -\frac{1}{4\pi^2}\sum_f N_c^f \times (c^L_{h^0f})^2 \times SE^0_2(ff, q^2). \tag{175}$$

The contributions of the gaugino pairs are:

$$\Sigma_{h^0h^0}(\tilde{\chi}\tilde{\chi}) = -\frac{1}{64\pi^2}\sum_{ij} SE_3^0(\tilde{\chi}_i\tilde{\chi}_j, q^2, c^L_{h^0ji}, c^L_{h^0ij}, c^L_{h^0ij}, c^L_{h^0ji}), \tag{176}$$

$$\Sigma_{h^0h^0}(\tilde{\chi}^0\tilde{\chi}^0) = -\frac{1}{128\pi^2}\sum_{ij} SE_3^0(\tilde{\chi}^0_i\tilde{\chi}^0_j, q^2, n^L_{h^0ji}, n^L_{h^0ij}, n^L_{h^0ij}, n^L_{h^0ji}). \tag{177}$$

The contribution of sfermion pairs consists of two terms:

$$\begin{aligned}\Sigma^{light}_{h^0h^0}(\tilde{f}\tilde{f})_{\text{light}} &= \frac{1}{16\pi^2}\sum_{\tilde{f}} N_c^f \quad \Big\{ \Big[g^2_{h^0\tilde{f}_L\tilde{f}_L} + g^2_{h^0\tilde{f}_R\tilde{f}_R}\Big] B_0(\tilde{f}\tilde{f}, q^2) \\ &\quad - \Big[g_{h^0h^0\tilde{f}_L\tilde{f}_L} + g_{h^0h^0\tilde{f}_R\tilde{f}_R}\Big] A(M^2_{\tilde{f}})\Big\},\end{aligned} \tag{178}$$

$$\begin{aligned}\Sigma^{heavy}_{h^0h^0}(\tilde{f}\tilde{f}) &= \frac{3}{16\pi^2}\sum_{ij=1,2}\Big\{g^2_{h^0\tilde{t}_i\tilde{t}_j} B_0(\tilde{t}_i\tilde{t}_j, q^2) + g^2_{h^0\tilde{b}_i\tilde{b}_j} B_0(\tilde{b}_i\tilde{b}_j, q^2)\Big\} \\ &\quad -\frac{3}{16\pi^2}\sum_{\tilde{f}=\tilde{t},\tilde{b}}\Big\{c^2_{\tilde{f}}\Big[g_{h^0h^0\tilde{f}_L\tilde{f}_L} A(M^2_{\tilde{f}_1}) + g_{h^0h^0\tilde{f}_R\tilde{f}_R} A(M^2_{\tilde{f}_2})\Big]\Big\} \\ &\quad -\frac{3}{16\pi^2}\sum_{\tilde{f}=\tilde{t},\tilde{b}}\Big\{s^2_{\tilde{f}}\Big[g_{h^0h^0\tilde{f}_R\tilde{f}_R} A(M^2_{\tilde{f}_1}) + g_{h^0h^0\tilde{f}_L\tilde{f}_L} A(M^2_{\tilde{f}_2})\Big]\Big\}.\end{aligned} \tag{179}$$

c) Mixed H^0h^0 self-energies:

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned}\Sigma_{H^0h^0}(\text{g+H}) &= \frac{1}{16\pi^2}\quad \Big\{g_{ZH^0A^0}g_{Zh^0A^0}SE_1^0(A^0Z, q^2) + g_{ZH^0G^0}g_{Zh^0G^0}SE_1^0(ZZ, q^2) \\ &\quad +2g_{WHH^0}g_{WHh^0}SE_1^0(HW, q^2) + 2g_{WGH^0}g_{WGh^0}SE_1^0(WW, q^2) \\ &\quad +2g_{WWH^0}g_{WWh^0}\Big[2B_0(WW, q^2) - 1\Big] \\ &\quad +g_{ZZH^0}g_{ZZh^0}\Big[2B_0(ZZ, q^2) - 1\Big] \\ &\quad +g_{H^0HH}g_{h^0HH}B_0(HH, q^2) + g_{H^0GG}g_{h^0GG}B_0(WW, q^2) \\ &\quad +2g_{H^0GH}g_{h^0GH}B_0(WH, q^2) \\ &\quad +\frac{1}{2}\Big[g_{H^0H^0H^0}g_{h^0H^0H^0}B_0(H^0H^0, q^2) + g_{H^0h^0h^0}g_{h^0h^0h^0}B_0(h^0h^0, q^2)\Big] \\ &\quad +\frac{1}{2}\Big[g_{H^0A^0A^0}g_{h^0A^0A^0}B_0(A^0A^0, q^2) + g_{H^0G^0G^0}g_{h^0G^0G^0}B_0(ZZ, q^2)\Big] \\ &\quad +g_{h^0H^0H^0}g_{H^0h^0h^0}B_0(H^0h^0, q^2) + g_{H^0A^0G^0}g_{h^0A^0G^0}B_0(A^0Z, q^2) \\ &\quad -\frac{e^2M_W^2\sin(\beta-\alpha)\cos(\beta-\alpha)}{2s_W^2}\Big[B_0(WW, q^2) + \frac{1}{2c_W^4}B_0(ZZ, q^2)\Big] \\ &\quad +\frac{3e^2\sin 2\alpha\cos 2\alpha}{8s_W^2c_W^2}\Big[A(M^2_{h^0}) - A(M^2_{H^0})\Big] \\ &\quad +\frac{e^2}{8s_W^2c_W^2}\cos 2\beta\sin 2\alpha\Big[A(M_A^2) - A(M_Z^2)\Big] \\ &\quad +\frac{e^2v_2}{4s_W^2}\Big[A(M_H^2) - A(M_W^2)\Big]\Big\}.\end{aligned} \tag{180}$$

The contribution of the fermion pairs is:

$$\Sigma_{H^0h^0}(ff) = -\frac{1}{4\pi^2}\sum_f N_c^f \times c^L_{H^0f}c^L_{h^0f} \times SE_2^0(ff,q^2). \tag{181}$$

The contributions of the gaugino pairs are:

$$\Sigma_{H^0h^0}(\tilde{\chi}\tilde{\chi}) = -\frac{1}{64\pi^2}\sum_{ij} SE_3^0(\tilde{\chi}_i\tilde{\chi}_j, q^2, c^L_{h^0ji}, c^L_{h^0ij}, c^L_{H^0ij}, c^L_{H^0ji}), \tag{182}$$

$$\Sigma_{H^0h^0}(\tilde{\chi}^0\tilde{\chi}^0) = -\frac{1}{128\pi^2}\sum_{ij} SE_3^0(\tilde{\chi}_i^0\tilde{\chi}_j^0, q^2, n^L_{h^0ji}, n^L_{h^0ij}, n^L_{H^0ij}, n^L_{H^0ji}). \tag{183}$$

The contribution of the sfermion pairs consists of two terms:

$$\begin{aligned}\Sigma^{light}_{H^0h^0}(\tilde{f}\tilde{f}) &= \frac{1}{16\pi^2}\sum_{\tilde{f}} N_c^f \quad \Big\{ \left[g_{H^0\tilde{f}_L\tilde{f}_L}g_{h^0\tilde{f}_L\tilde{f}_L} + g_{H^0\tilde{f}_R\tilde{f}_R}g_{h^0\tilde{f}_R\tilde{f}_R}\right] B_0(\tilde{f}\tilde{f}, q^2) \\ &\qquad - \left[g_{H^0h^0\tilde{f}_L\tilde{f}_L} + g_{H^0h^0\tilde{f}_R\tilde{f}_R}\right] A(M^2_{\tilde{f}})\Big\}.\end{aligned} \tag{184}$$

$$\begin{aligned}\Sigma^{heavy}_{H^0h^0}(\tilde{f}\tilde{f}) &= \frac{3}{16\pi^2} \sum_{ij=1,2} \Big\{g_{H^0\tilde{t}_i\tilde{t}_j}g_{h^0\tilde{t}_i\tilde{t}_j}B_0(\tilde{t}_i\tilde{t}_j, q^2) + g_{H^0\tilde{b}_i\tilde{b}_j}g_{h^0\tilde{b}_i\tilde{b}_j}B_0(\tilde{b}_i\tilde{b}_j, q^2)\Big\} \\ &- \frac{3}{16\pi^2} \sum_{\tilde{f}=\tilde{t},\tilde{b}} \Big\{c^2_{\tilde{f}}\left[g_{H^0h^0\tilde{f}_L\tilde{f}_L}A(M^2_{\tilde{f}_1}) + g_{H^0h^0\tilde{f}_R\tilde{f}_R}A(M^2_{\tilde{f}_2})\right]\Big\} \\ &- \frac{3}{16\pi^2} \sum_{\tilde{f}=\tilde{t},\tilde{b}} \Big\{s^2_{\tilde{f}}\left[g_{H^0h^0\tilde{f}_R\tilde{f}_R}A(M^2_{\tilde{f}_1}) + g_{H^0h^0\tilde{f}_L\tilde{f}_L}A(M^2_{\tilde{f}_2})\right]\Big\}.\end{aligned} \tag{185}$$

d) A^0 self-energies:

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned}\Sigma_{A^0A^0}(\mathrm{g+H}) &= \frac{1}{16\pi^2} \Big\{g^2_{Zh^0A^0}SE_1^0(h^0Z,q^2) + g^2_{ZH^0A^0}SE_1^0(H^0Z,q^2) \\ &+2g^2_{WHA^0}SE_1^0(HW,q^2) + 2g^2_{A^0GH}B_0(WH,q^2) \\ &+g^2_{H^0A^0A^0}B_0(A^0H^0,q^2) + g^2_{h^0A^0A^0}B_0(A^0h^0,q^2) \\ &+g^2_{H^0A^0G^0}B_0(ZH^0,q^2) + g^2_{h^0A^0G^0}B_0(Zh^0,q^2) \\ &+\frac{e^2}{s^2_W}\left(\left[2A(M^2_W) - M^2_W\right] + \frac{1}{2c^2_W}\left[2A(M^2_Z) - M^2_Z\right]\right) \\ &+\frac{e^2\cos^2 2\beta}{4s^2_Wc^2_W}A(M^2_H) + \frac{e^2}{4s^2_W}\left[1 + \sin^2 2\beta - \frac{s^2_W}{c^2_W}\cos^2 2\beta\right]A(M^2_W) \\ &+\frac{e^2}{8s^2_Wc^2_W}\left[(3\sin^2 2\beta - 1)A(M^2_Z) + 3\cos^2 2\beta A(M^2_A)\right] \\ &+\frac{e^2}{8s^2_Wc^2_W}\cos 2\beta\cos 2\alpha\left[A(M^2_{h^0}) - A(M^2_{H^0})\right]\Big\}.\end{aligned} \tag{186}$$

The contribution of the fermion pairs is:

$$\Sigma_{A^0A^0}(ff) = \frac{1}{64\pi^2} \sum_f N_c^f SE_3^0(ff, q^2, c^L_{A^0f}, -c^L_{A^0f}, c^L_{A^0f}, -c^L_{A^0f}). \quad (187)$$

The contributions of the gaugino pairs are:

$$\Sigma_{A^0A^0}(\tilde{\chi}\tilde{\chi}) = \frac{1}{64\pi^2} \sum_{ij} SE_3^0(\tilde{\chi}_i\tilde{\chi}_j, q^2, c^L_{A^0ji}, -c^L_{A^0ij}, c^L_{A^0ij}, -c^L_{A^0ji}), \quad (188)$$

$$\Sigma_{A^0A^0}(\tilde{\chi}^0\tilde{\chi}^0) = \frac{1}{128\pi^2} \sum_{ij} SE_3^0(\tilde{\chi}^0_i\tilde{\chi}^0_j, q^2, n^L_{A^0ji}, -n^L_{A^0ij}, n^L_{A^0ij}, -n^L_{A^0ji}). \quad (189)$$

The contribution of the sfermion pairs consists of two terms:

$$\Sigma^{light}_{A^0A^0}(\tilde{f}\tilde{f}) = -\frac{1}{16\pi^2} \sum_{\tilde{f}} N_c^f \Big\{ \left[g_{A^0A^0\tilde{f}_L\tilde{f}_L} + g_{A^0A^0\tilde{f}_R\tilde{f}_R} \right] A(M^2_{\tilde{f}}) \Big\}, \quad (190)$$

$$\begin{aligned}
\Sigma^{heavy}_{A^0A^0}(\tilde{f}\tilde{f}) &= \frac{3}{16\pi^2} \sum_{ij=1,2} \Big\{ g^2_{A^0\tilde{t}_i\tilde{t}_j} B_0(\tilde{t}_i\tilde{t}_j, q^2) + g^2_{A^0\tilde{b}_i\tilde{b}_j} B_0(\tilde{b}_i\tilde{b}_j, q^2) \Big\} \\
&- \frac{3}{16\pi^2} \sum_{\tilde{f}=\tilde{t},\tilde{b}} \Big\{ c^2_{\tilde{f}} \left[g_{A^0A^0\tilde{f}_L\tilde{f}_L} A(M^2_{\tilde{f}_1}) + g_{A^0A^0\tilde{f}_R\tilde{f}_R} A(M^2_{\tilde{f}_2}) \right] \Big\} \\
&- \frac{3}{16\pi^2} \sum_{\tilde{f}=\tilde{t},\tilde{b}} \Big\{ s^2_{\tilde{f}} \left[g_{A^0A^0\tilde{f}_R\tilde{f}_R} A(M^2_{\tilde{f}_1}) + g_{A^0A^0\tilde{f}_L\tilde{f}_L} A(M^2_{\tilde{f}_2}) \right] \Big\}. \quad (191)
\end{aligned}$$

e) Mixed A^0Z self-energies:

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned}
\Sigma_{A^0Z}(\text{g+H}) &= \frac{e^2}{32\pi^2 s_W^2 c_W^2} \Big\{ M_Z \cos(\beta-\alpha)\sin(\beta-\alpha) SE_4^0(H^0Z, q^2) \\
&- M_Z \cos(\beta-\alpha)\sin(\beta-\alpha) SE_4^0(h^0Z, q^2) \\
&+ \frac{1}{2} M_Z \cos 2\beta \cos(\beta+\alpha)\sin(\beta-\alpha) SE_5^0(A^0H^0, q^2) \\
&- \frac{1}{2} M_Z \cos 2\beta \sin(\beta+\alpha)\cos(\beta-\alpha) SE_5^0(A^0h^0, q^2) \\
&- \frac{1}{2} M_Z \sin 2\beta \cos(\beta+\alpha)\cos(\beta-\alpha) SE_5^0(ZH^0, q^2) \\
&+ \frac{1}{2} M_Z \sin 2\beta \sin(\beta+\alpha)\sin(\beta-\alpha) SE_5^0(Zh^0, q^2) \Big\}. \quad (192)
\end{aligned}$$

The contribution of the fermion pairs is:

$$\Sigma_{A^0Z}(ff) = \frac{1}{64\pi^2} \sum_f N_c^f SE_6^0(ff, q^2, v_f + a_f, v_f - a_f, c^L_{A^0f}, -c^L_{A^0f}). \quad (193)$$

Here, v_f and a_f are defined as follows:

$$v_f = \frac{1}{2s_W c_W}\left[+\frac{1}{2}-\frac{4}{3}s_W^2\right], \quad a_f = +\frac{1}{4s_W c_W} \quad \text{if } f = q_u \text{ or } \nu_\ell, \tag{194}$$

$$v_f = \frac{1}{2s_W c_W}\left[-\frac{1}{2}+\frac{2}{3}s_W^2\right], \quad a_f = -\frac{1}{4s_W c_W} \quad \text{if } f = q_d \text{ or } \ell. \tag{195}$$

The contributions of the gaugino pairs are:

$$\begin{aligned}\Sigma_{A^0Z}(\tilde{\chi}\tilde{\chi}) &= \frac{1}{8\pi^2}\sum_{ij} \Big\{M_{\tilde{\chi}_i}\left(c^L_{A^0ji}\mathcal{O}^{ZR}_{ij}+c^R_{A^0ji}\mathcal{O}^{ZL}_{ij}\right)\left[B_0(\tilde{\chi}_i\tilde{\chi}_j,q^2)+B_1(\tilde{\chi}_i\tilde{\chi}_j,q^2)\right] \\ &\quad + M_{\tilde{\chi}_j}\left[c^L_{A^0ji}\mathcal{O}^{ZL}_{ij}+c^R_{A^0ji}\mathcal{O}^{ZR}_{ij}\right]B_1(\tilde{\chi}_i\tilde{\chi}_j,q^2)\Big\}, \qquad (196)\\ \Sigma_{A^0Z}(\tilde{\chi}^0\tilde{\chi}^0) &= \frac{1}{16\pi^2}\sum_{ij} \Big\{M_{\tilde{\chi}^0_i}\left(n^L_{A^0ji}\mathcal{O}^{ZR}_{ij}+n^R_{A^0ji}\mathcal{O}^{ZL}_{ij}\right)\left[B_0(\tilde{\chi}^0_i\tilde{\chi}^0_j,q^2)+B_1(\tilde{\chi}^0_i\tilde{\chi}^0_j,q^2)\right] \\ &\quad + M_{\tilde{\chi}^0_j}\left[n^L_{A^0ji}\mathcal{O}^{ZL}_{ij}+n^R_{A^0ji}\mathcal{O}^{ZR}_{ij}\right]B_1(\tilde{\chi}^0_i\tilde{\chi}^0_j,q^2)\Big\}. \qquad (197)\end{aligned}$$

Note that there is no contribution from sfermion pairs to $\Sigma_{A^0Z}(q^2)$.

f) H^0 tadpole:

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned}T_{H^0}(\text{g+H}) &= \frac{1}{16\pi^2}\Big\{g_{H^0HH}A(M_H^2)+g_{H^0GG}A(M_W^2) \\ &\quad +\frac{1}{2}g_{H^0H^0H^0}A(M_{H^0}^2)+\frac{1}{2}g_{H^0h^0h^0}A(M_{h^0}^2) \\ &\quad +\frac{1}{2}g_{H^0A^0A^0}A(M_{A^0}^2)+\frac{1}{2}g_{H^0G^0G^0}A(M_Z^2) \\ &\quad -g_{WWH^0}\left[4A(M_W^2)-2M_W^2\right]-\frac{1}{2}g_{ZZH^0}\left[4A(M_Z^2)-2M_Z^2\right] \\ &\quad +\frac{eM_W\cos(\beta-\alpha)}{s_W}\left[A(M_W^2)+\frac{1}{2c_W^2}A(M_Z^2)\right]\Big\}. \qquad (198)\end{aligned}$$

The contribution of the fermion pairs is:

$$T_{H^0}(ff) = -\frac{1}{4\pi^2}\sum_f N_c^f c^L_{H^0f}M_f A(M_f^2). \tag{199}$$

The contributions of the gaugino pairs are:

$$T_{H^0}(\tilde{\chi}\tilde{\chi}) = -\frac{1}{4\pi^2}\sum_i c^L_{H^0ii}M_{\tilde{\chi}_i}A(M^2_{\tilde{\chi}_i}), \tag{200}$$

$$T_{H^0}(\tilde{\chi}^0\tilde{\chi}^0) = -\frac{1}{8\pi^2}\sum_i n^L_{H^0ii}M_{\tilde{\chi}^0_i}A(M^2_{\tilde{\chi}^0_i}). \tag{201}$$

The contribution of the sfermion pairs is the sum of two terms:

$$T_{H^0}^{light}(\tilde{f}\tilde{f}) = \frac{1}{16\pi^2}\sum_f N_c^f\Big\{\left[g_{H^0\tilde{f}_L\tilde{f}_L} + g_{H^0\tilde{f}_R\tilde{f}_R}\right]A(M_{\tilde{f}}^2)\Big\}, \tag{202}$$

$$T_{H^0}^{heavy}(\tilde{f}\tilde{f}) = \frac{3}{16\pi^2}\sum_{i=1,2}\Big\{g_{H^0\tilde{t}_i\tilde{t}_i}A(M_{\tilde{t}_i}^2) + g_{H^0\tilde{b}_i\tilde{b}_i}A(M_{\tilde{b}_i}^2)\Big\}. \tag{203}$$

g) h^0 tadpole:

The contribution of the gauge and Higgs sectors is:

$$\begin{aligned}
T_{h^0}(\text{g+H}) = \frac{1}{16\pi^2} & \Big\{g_{h^0HH}A(M_H^2) + g_{h^0GG}A(M_W^2) \\
& + \frac{1}{2}g_{h^0H^0H^0}A(M_{H^0}^2) + \frac{1}{2}g_{h^0h^0h^0}A(M_{h^0}^2) \\
& + \frac{1}{2}g_{h^0A^0A^0}A(M_{A^0}^2) + \frac{1}{2}g_{h^0G^0G^0}A(M_Z^2) \\
& - g_{WWh^0}\left[4A(M_W^2) - 2M_W^2\right] - \frac{1}{2}g_{ZZh^0}\left[4A(M_Z^2) - 2M_Z^2\right] \\
& + \frac{eM_W\sin(\beta-\alpha)}{s_W}\left[A(M_W^2) + \frac{1}{2c_W^2}A(M_Z^2)\right]\Big\}.
\end{aligned} \tag{204}$$

The contribution of the fermion pairs is:

$$T_{h^0}(ff) = -\frac{1}{4\pi^2}\sum_f N_c^f c_{h^0f}^L M_f A(M_f^2). \tag{205}$$

The contributions of the gaugino pairs are:

$$T_{h^0}(\tilde{\chi}\tilde{\chi}) = -\frac{1}{4\pi^2}\sum_i c_{h^0ii}^L M_{\tilde{\chi}_i}A(M_{\tilde{\chi}_i}^2), \tag{206}$$

$$T_{h^0}(\tilde{\chi}^0\tilde{\chi}^0) = -\frac{1}{8\pi^2}\sum_i n_{h^0ii}^L M_{\tilde{\chi}_i^0}A(M_{\tilde{\chi}_i^0}^2). \tag{207}$$

The contribution of the sfermion pairs is the sum of two terms:

$$T_{h^0}^{light}(\tilde{f}\tilde{f}) = \frac{1}{16\pi^2}\sum_f N_c^f\Big\{\left[g_{h^0\tilde{f}_L\tilde{f}_L} + g_{h^0\tilde{f}_R\tilde{f}_R}\right]A(M_{\tilde{f}}^2)\Big\}, \tag{208}$$

$$T_{h^0}^{heavy}(\tilde{f}\tilde{f}) = \frac{3}{16\pi^2}\sum_{i=1,2}\Big\{g_{h^0\tilde{t}_i\tilde{t}_i}A(M_{\tilde{t}_i}^2) + g_{h^0\tilde{b}_i\tilde{b}_i}A(M_{\tilde{b}_i}^2)\Big\}. \tag{209}$$

h) Expressions of the renormalized self-energies:

When calculating the effective contribution of the neutral Higgs self-energies to the pair production cross section at the one loop level, we must consider the renormalized self-energy terms $\hat{\Sigma}$, obtained by adding various counter terms to the unnormalized self-energies Σ.

Let us first define the Higgs field renormalization constants:

$$\delta Z_{H_1} = -\left(\frac{d\Sigma_{A^0A^0}}{dq^2}\right)_{q^2=M_A^2} - \frac{\cot\beta}{M_Z}\Sigma_{A^0Z}(M_A^2), \tag{210}$$

$$\delta Z_{H_2} = -\left(\frac{d\Sigma_{A^0A^0}}{dq^2}\right)_{q^2=M_A^2} + \frac{\tan\beta}{M_Z}\Sigma_{A^0Z}(M_A^2). \tag{211}$$

They are used in the calculation of the various mass counter terms, together with the Higgs tadpoles and the parameter $\delta M^2_{A^0A^0}$ defined as:

$$\delta M^2_{A^0A^0} = \Sigma_{A^0A^0}(M_A^2) - M_A^2\left(\frac{d\Sigma_{A^0A^0}}{dq^2}\right)_{q^2=M_A^2}. \tag{212}$$

Indeed, one has the following expressions for the mass counter terms:

$$\begin{aligned}\delta M^2_{H^0H^0} &= \sin^2(\beta-\alpha)\,\delta M^2_{A^0A^0} + \cos^2(\beta+\alpha)\Sigma_{ZZ}(M_Z^2)\\ &- \frac{e\cos(\beta-\alpha)}{2s_WM_W}\Big\{\left[1+\sin^2(\beta-\alpha)\right]T_{H^0} - \cos(\beta-\alpha)\sin(\beta-\alpha)T_{h^0}\Big\}\\ &+ M_Z^2\cos(\beta+\alpha)\cos(\beta-\alpha)\left[\delta Z_{H_1}-\delta Z_{H_2}\right]\\ &+ M_Z^2\cos^2(\beta+\alpha)\left[\delta Z_{H_1}\sin^2\beta+\delta Z_{H_2}\cos^2\beta\right],\end{aligned} \tag{213}$$

$$\begin{aligned}\delta M^2_{h^0h^0} &= \cos^2(\beta-\alpha)\,\delta M^2_{A^0A^0} + \sin^2(\beta+\alpha)\Sigma_{ZZ}(M_Z^2)\\ &+ \frac{e\sin(\beta-\alpha)}{2s_WM_W}\Big\{\sin(\beta-\alpha)\cos(\beta-\alpha)T_{H^0} - \left[1+\cos^2(\beta-\alpha)\right]T_{h^0}\Big\}\\ &- M_Z^2\sin(\beta+\alpha)\sin(\beta-\alpha)\left[\delta Z_{H_1}-\delta Z_{H_2}\right]\\ &+ M_Z^2\sin^2(\beta+\alpha)\left[\delta Z_{H_1}\sin^2\beta+\delta Z_{H_2}\cos^2\beta\right],\end{aligned} \tag{214}$$

$$\begin{aligned}\delta M^2_{H^0h^0} &= -\sin(\beta-\alpha)\cos(\beta-\alpha)\delta M^2_{A^0A^0} - \cos(\beta+\alpha)\sin(\beta+\alpha)\Sigma_{ZZ}(M_Z^2)\\ &- \frac{e}{2s_WM_W}\Big\{\sin^3(\beta-\alpha)T_{H^0} + \cos^3(\beta-\alpha)T_{h^0}\Big\}\\ &- M_Z^2\sin\alpha\cos\alpha\left[\delta Z_{H_1}-\delta Z_{H_2}\right]\\ &- M_Z^2\cos(\beta+\alpha)\sin(\beta+\alpha)\left[\delta Z_{H_1}\sin^2\beta+\delta Z_{H_2}\cos^2\beta\right].\end{aligned} \tag{215}$$

There is no contribution from the neutral Higgs self-energies to $\Gamma^{fin,\gamma}(H^0A^0/h^0A^0)$. As for their contributions to $\Gamma^{fin,Z}(H^0A^0/h^0A^0)$, they can be derived using the renormalized self-energies, which are expressed as follows:

$$\hat{\Sigma}_{H^0H^0}(q^2) = \Sigma_{H^0H^0}(q^2) + q^2\left[\delta Z_{H_1}\cos^2\alpha+\delta Z_{H_2}\sin^2\alpha\right] - \delta M^2_{H^0H^0}, \tag{216}$$

$$\hat{\Sigma}_{h^0h^0}(q^2) = \Sigma_{h^0h^0}(q^2) + q^2\left[\delta Z_{H_1}\sin^2\alpha+\delta Z_{H_2}\cos^2\alpha\right] - \delta M^2_{h^0h^0}, \tag{217}$$

$$\hat{\Sigma}_{H^0h^0}(q^2) = \Sigma_{H^0h^0}(q^2) + q^2\sin\alpha\cos\alpha\left[\delta Z_{H_2}-\delta Z_{H_1}\right] - \delta M^2_{H^0h^0}. \tag{218}$$

For H^0A^0 final states, one has:

$$\Gamma^Z_{H^0A^0}(H.s.e) = -\frac{e\sin(\beta-\alpha)}{s_W c_W}\left[\cot(\beta-\alpha)\frac{\hat{\Sigma}_{H^0h^0}(M^2_{H^0})}{M^2_{H^0}-M^2_{h^0}} - \frac{1}{2}\left(\frac{d\hat{\Sigma}_{H^0H^0}}{dq^2}\right)_{q^2=M^2_{H^0}}\right]. \quad (219)$$

For h^0A^0 final states, one has:

$$\Gamma^Z_{h^0A^0}(H.s.e) = +\frac{e\cos(\beta-\alpha)}{s_W c_W}\left[\tan(\beta-\alpha)\frac{\hat{\Sigma}_{H^0h^0}(M^2_{h^0})}{M^2_{h^0}-M^2_{H^0}} - \frac{1}{2}\left(\frac{d\hat{\Sigma}_{h^0h^0}}{dq^2}\right)_{q^2=M^2_{h^0}}\right]. \quad (220)$$

5.3.9 Neutral Higgs counter terms

For H^0A^0 final states, the neutral Higgs counter terms give the following contribution:

$$\begin{aligned}\Gamma^Z_{H^0A^0}(H.c.t) &= \frac{e\cos(\beta-\alpha)}{2s_W c_W}\left[\cos\beta\sin\beta+\cos\alpha\sin\alpha\right](\delta Z_{H_2}-\delta Z_{H_1}) \\ &- \frac{e\sin(\beta-\alpha)}{2s_W c_W}\left[(\cos^2\alpha+\sin^2\beta)\delta Z_{H_1}+(\sin^2\alpha+\cos^2\beta)\delta Z_{H_2}\right]. \quad (221)\end{aligned}$$

For h^0A^0 final states, the neutral Higgs counter terms give the following contribution:

$$\begin{aligned}\Gamma^Z_{h^0A^0}(H.c.t) &= \frac{e\sin(\beta-\alpha)}{2s_W c_W}\left[\cos\beta\sin\beta-\cos\alpha\sin\alpha\right](\delta Z_{H_2}-\delta Z_{H_1}) \\ &+ \frac{e\cos(\beta-\alpha)}{2s_W c_W}\left[(\sin^2\alpha+\sin^2\beta)\delta Z_{H_1}+(\cos^2\alpha+\cos^2\beta)\delta Z_{H_2}\right]. \quad (222)\end{aligned}$$

6 Contribution of box diagrams

6.1 Diagram structures for boxes

Several useful expressions are needed when estimating the contributions of the box diagrams. The particles inside the box are ordered clockwise and have internal masses m_1, m_2, m_3, m_4 starting with m_1 between the e^- and e^+ junctions. In the following, we will use the Mandelstam variables s, t and u, which can be expressed as a function of q^2 and of the masses M_1 and M_2 of the two outgoing Higgs bosons (i.e. H^+H^- or H^0A^0 or h^0A^0):

$$s = q^2, \quad (223)$$

$$\begin{aligned}t &= \frac{1}{2}(M_2^2+M_1^2-s) \\ &+ \frac{s\cos\theta}{2}\sqrt{\left(1+\frac{M_2+M_1}{\sqrt{s}}\right)\left(1-\frac{M_2+M_1}{\sqrt{s}}\right)\left(1+\frac{M_2-M_1}{\sqrt{s}}\right)\left(1-\frac{M_2-M_1}{\sqrt{s}}\right)}, \quad (224)\end{aligned}$$

$$\begin{aligned}u &= \frac{1}{2}(M_2^2+M_1^2-s) \\ &- \frac{s\cos\theta}{2}\sqrt{\left(1+\frac{M_2+M_1}{\sqrt{s}}\right)\left(1-\frac{M_2+M_1}{\sqrt{s}}\right)\left(1+\frac{M_2-M_1}{\sqrt{s}}\right)\left(1-\frac{M_2-M_1}{\sqrt{s}}\right)}. \quad (225)\end{aligned}$$

Note that u and t have an angular dependence and, as a result, so does the contribution of the diagram boxes to the one loop cross sections.

Let ℓ and ℓ' be the momenta of the incoming electron and positron, respectively. Let P_{f1} and P_{f2} be the momenta of the two outgoing Higgs bosons. When not otherwise specified, ℓ', P_{f1}, P_{f2} and ℓ are oriented clockwise around the box. Then, one has:

$$P_{f1}^2 = M_1^2, \tag{226}$$

$$P_{f2}^2 = M_2^2, \tag{227}$$

$$P_{f1}P_{f2} = \frac{s-(M_1^2+M_2^2)}{2}, \tag{228}$$

$$\ell' P_{f1} = \frac{t-M_1^2}{2}, \tag{229}$$

$$\ell' P_{f2} = \frac{u-M_2^2}{2}. \tag{230}$$

a) Box7 structures:

$$\begin{aligned}\mathcal{D}_7 &= 6(D_{002}-D_{003}) + P_{f1}^2(D_{222}-D_{223}) + P_{f2}^2(D_{332}-D_{333}) \\ &- 2P_{f1}P_{f2}(D_{323}-D_{322}) - 2\ell' P_{f2}(D_{133}-D_{132}) - 2\ell' P_{f1}(D_{123}-D_{122}) \\ &+ 4\left[P_{f1}^2 D_{22} + P_{f2}^2 D_{23} + 2\ell' P_{f1} D_{24} + 2\ell' P_{f2} D_{25} + 2P_{f1}P_{f2} D_{26} + 4D_{27}\right] \\ &- 4\left[P_{f2}^2 D_{13} - P_{f1}^2 D_{12} - 2\ell' P_{f1} D_{11}\right].\end{aligned} \tag{231}$$

b) Box8 structures:

$$\begin{aligned}\mathcal{D}_8 &= 6(D_{002}-D_{003}) + P_{f1}^2(D_{222}-D_{223}) + P_{f2}^2(D_{332}-D_{333}) \\ &- 2P_{f1}P_{f2}(D_{323}-D_{322}) - 2\ell' P_{f2}(D_{133}-D_{132}) - 2\ell' P_{f1}(D_{123}-D_{122}) \\ &+ (2\ell' P_{f1} + P_{f1}^2)D_{22} - (2\ell' P_{f2} + 2P_{f1}P_{f2} + P_{f2}^2)D_{23} - (2\ell' P_{f2} + 2\ell' P_{f1})D_{25} \\ &+ (2\ell' P_{f2} - 2\ell' P_{f1} - 2P_{f1}^2)D_{26} - 2D_{27},\end{aligned} \tag{232}$$

$$\mathcal{D}'_8 = D_{12} - D_{13}, \tag{233}$$

$$D''_8 = D_0 + D_{12} - D_{13}. \tag{234}$$

c) Box9 structures:

$$\mathcal{D}_9 = D_{12} - D_{13}. \tag{235}$$

d) Box10 structures:

$$\mathcal{D}_{10} = D_{12} + D_0, \tag{236}$$

$$D'_{10} = D_{12}. \tag{237}$$

Finally, the crossed functions $\bar{\mathcal{D}}_j$ are obtained from the $\mathcal{D}_j$ functions by making the following changes: $P_{f1} \to P_{f2}$, i.e. $t \to u$ and $M_1^2 \to M_2^2$.

6.2 Charged Higgs sector

For $e^+e^- \to H^+H^-$, the box diagrams give the following one loop contribution:

$$a^{box}_{L,R}(H^+H^-) = \frac{q^2}{2e^2}\times \Big\{A^{box7}_{L,R}(H^+H^-) + A^{box8}_{L,R}(H^+H^-) + A^{box9}_{L,R}(H^+H^-) + A^{box10}_{L,R}(H^+H^-)\Big\}. \quad (238)$$

6.2.1 Box7 diagrams

The amplitude $A^{box7}_{L,R}(H^+H^-)$ is obtained by summing various contributions with gauge and Higgs bosons inside the box:

$$\begin{aligned} A^{box7}_{L,R}(H^+H^-) &= \frac{\alpha^2_{em}}{8s^4_W}\sin^2(\beta-\alpha)P_L\mathcal{D}_7(\nu W H^0 W) \\ &+ \frac{\alpha^2_{em}}{8s^4_W}\cos^2(\beta-\alpha)P_L\mathcal{D}_7(\nu W h^0 W) \\ &+ \frac{\alpha^2_{em}}{8s^4_W}P_L\mathcal{D}_7(\nu W A^0 W) \\ &- \left(\frac{1-2s^2_W}{2s_Wc_W}\right)\times\left[\frac{g_LP_L+g_RP_R}{2s_Wc_W}\right]\times\alpha^2_{em}\Big\{\mathcal{D}_7(e\gamma HZ)-\bar{\mathcal{D}}_7(e\gamma HZ)\Big\} \\ &- \left(\frac{1-2s^2_W}{2s_Wc_W}\right)\times\left[\frac{g_LP_L+g_RP_R}{2s_Wc_W}\right]\times\alpha^2_{em}\Big\{\mathcal{D}_7(eZH\gamma)-\bar{\mathcal{D}}_7(eZH\gamma)\Big\} \\ &+ \left(\frac{1-2s^2_W}{2s_Wc_W}\right)^2\times\left[\frac{g^2_LP_L+g^2_RP_R}{4s^2_Wc^2_W}\right]\times\alpha^2_{em}\Big\{\mathcal{D}_7(eZHZ)-\bar{\mathcal{D}}_7(eZHZ)\Big\} \\ &+ [P_L+P_R]\times\alpha^2_{em}\Big\{\mathcal{D}_7(e\gamma H\gamma)-\bar{\mathcal{D}}_7(e\gamma H\gamma)\Big\}. \end{aligned} \quad (239)$$

6.2.2 Box8 diagrams

The amplitude $A^{box8}_{L,R}(H^+H^-)$ is obtained by summing two types of box diagrams, which have gauginos inside the box, $\tilde{\nu}\tilde{\chi}\tilde{\chi}^0\tilde{\chi}$ and $\tilde{e}\tilde{\chi}^0\tilde{\chi}\tilde{\chi}^0$:

$$A^{box8}_{L,R}(H^+H^-) = P_{L,R}\left[A^{box8}_{H^+H^-}(\tilde{\nu}\tilde{\chi}\tilde{\chi}^0\tilde{\chi}) + A^{box8}_{H^+H^-}(\tilde{e}\tilde{\chi}^0\tilde{\chi}\tilde{\chi}^0)\right]. \quad (240)$$

For the $\tilde{\nu}\tilde{\chi}\tilde{\chi}^0\tilde{\chi}$ boxes, since there is no right-handed sneutrino in the MSSM, only a left-handed term is considered:

$$\begin{aligned} A^{box8}_{H^+H^-}(\tilde{\nu}_L\tilde{\chi}\tilde{\chi}^0\tilde{\chi}) = -\frac{e^2}{16\pi^2s^2_W}\sum_{ijk}Z^{+*}_{1i}Z^+_{1k}\times &\Big\{M_{\tilde{\chi}_i}M_{\tilde{\chi}_k}c^{L*}_{Hij}c^L_{Hkj}\mathcal{D}''_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}^0_j\tilde{\chi}_k) \\ &+M_{\tilde{\chi}_i}M_{\tilde{\chi}^0_j}c^{L*}_{Hij}c^R_{Hkj}\mathcal{D}'_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}^0_j\tilde{\chi}_k) \\ &+M_{\tilde{\chi}^0_j}M_{\tilde{\chi}_k}c^{R*}_{Hij}c^L_{Hkj}\mathcal{D}'_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}^0_j\tilde{\chi}_k) \\ &+c^{R*}_{Hij}c^R_{Hkj}\mathcal{D}_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}^0_j\tilde{\chi}_k)\Big\}. \end{aligned} \quad (241)$$

For the $\tilde{e}\tilde{\chi}^0\tilde{\chi}\tilde{\chi}^0$ boxes, one has both left-handed and right-handed terms. The left-handed term is given by:

$$
\begin{aligned}
A^{box8}_{H^+H^-}(\tilde{e}_L\tilde{\chi}^0\tilde{\chi}\tilde{\chi}^0) &= \frac{e^2}{32\pi^2 s_W^2 c_W^2} \sum_{ijk}(Z^{N*}_{1i}s_W + Z^{N*}_{2i}c_W)(Z^N_{1k}s_W + Z^N_{2k}c_W) \times \\
&\Big\{M_{\tilde{\chi}^0_i}M_{\tilde{\chi}^0_k}c^{R*}_{Hji}c^R_{Hjk}\bar{\mathcal{D}}''_8(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+M_{\tilde{\chi}^0_i}M_{\tilde{\chi}_j}c^{R*}_{Hji}c^L_{Hjk}\bar{\mathcal{D}}'_8(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+M_{\tilde{\chi}_j}M_{\tilde{\chi}^0_k}c^{L*}_{Hji}c^R_{Hjk}\bar{\mathcal{D}}'_8(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+c^{L*}_{Hji}c^L_{Hjk}\bar{\mathcal{D}}_8(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k)\Big\} \\
&-\frac{e^2}{32\pi^2 s_W^2 c_W^2} \sum_{ijk}(Z^{N*}_{1i}s_W + Z^{N*}_{2i}c_W)(Z^N_{1k}s_W + Z^N_{2k}c_W) \times \\
&\Big\{M_{\tilde{\chi}^0_i}M_{\tilde{\chi}^0_k}c^{L*}_{Hji}c^L_{Hjk}\mathcal{D}''_8(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+M_{\tilde{\chi}^0_i}M_{\tilde{\chi}_j}c^{L*}_{Hji}c^R_{Hjk}\mathcal{D}'_8(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+M_{\tilde{\chi}_j}M_{\tilde{\chi}^0_k}c^{R*}_{Hji}c^L_{Hjk}\mathcal{D}'_8(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+c^{R*}_{Hji}c^R_{Hjk}\mathcal{D}_8(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k)\Big\} \qquad (242)
\end{aligned}
$$

while the right-handed term is:

$$
\begin{aligned}
A^{box8}_{H^+H^-}(\tilde{e}_R\tilde{\chi}^0\tilde{\chi}\tilde{\chi}^0) &= \frac{e^2}{8\pi^2 c_W^2} \sum_{ijk} Z^N_{1i}Z^{N*}_{1k}\times \Big\{M_{\tilde{\chi}^0_i}M_{\tilde{\chi}^0_k}c^{L*}_{Hji}c^L_{Hjk}\bar{\mathcal{D}}''_8(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&M_{\tilde{\chi}^0_i}M_{\tilde{\chi}_j}c^{L*}_{Hji}c^R_{Hjk}\bar{\mathcal{D}}'_8(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+M_{\tilde{\chi}_j}M_{\tilde{\chi}^0_k}c^{R*}_{Hji}c^L_{Hjk}\bar{\mathcal{D}}'_8(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+c^{R*}_{Hji}c^R_{Hjk}\bar{\mathcal{D}}_8(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k)\Big\} \\
&-\frac{e^2}{8\pi^2 c_W^2} \sum_{ijk} Z^N_{1i}Z^{N*}_{1k}\times \Big\{M_{\tilde{\chi}^0_i}M_{\tilde{\chi}^0_k}c^{R*}_{Hji}c^R_{Hjk}\mathcal{D}''_8(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+M_{\tilde{\chi}^0_i}M_{\tilde{\chi}_j}c^{R*}_{Hji}c^L_{Hjk}\mathcal{D}'_8(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+M_{\tilde{\chi}_j}M_{\tilde{\chi}^0_k}c^{L*}_{Hji}c^R_{Hjk}\mathcal{D}'_8(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k) \\
&+c^{L*}_{Hji}c^L_{Hjk}\mathcal{D}_8(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}_j\tilde{\chi}^0_k)\Big\}. \qquad (243)
\end{aligned}
$$

6.2.3 Box9 diagrams

There is no right-handed contribution from box9 diagrams. As for the left-handed amplitude $A^{box9}_L(H^+H^-)$, it is obtained as follows:

$$
\begin{aligned}
A^{box9}_L(H^+H^-) &= \frac{e^2}{32\pi^2 s_W^2 c_W^2}\sum_i |Z^N_{1i}s_W + Z^N_{2i}c_W|^2 g^2_{H\tilde{e}_L\tilde{\nu}_L}\mathcal{D}_9(\tilde{\chi}^0_i\tilde{e}_L\tilde{\nu}_L\tilde{e}_L) \\
&- \frac{e^2}{16\pi^2 s_W^2}\sum_i |Z^+_{1i}|^2 g^2_{H\tilde{e}_L\tilde{\nu}_L}\bar{\mathcal{D}}_9(\tilde{\chi}_i\tilde{\nu}_L\tilde{e}_L\tilde{\nu}_L). \qquad (244)
\end{aligned}
$$

6.2.4 Twisted box10 diagrams

Twisted box10 diagrams contribute only with a left-handed term:

$$
\begin{aligned}
A_L^{box10}(H^+H^-) &= \frac{e^2}{16\sqrt{2}\pi^2 s_W^2 c_W}\sum_{ij} Z_{1i}^{+*}(Z_{1j}^N s_W + Z_{2j}^N c_W) \times g_{H\tilde{e}_L\tilde{\nu}_L} \times \\
&\quad \left[M_{\tilde{\chi}_i}\, c_{Hij}^{L*}\mathcal{D}_{10}(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j^0\tilde{e}_L) + M_{\tilde{\chi}_j^0}\, c_{Hij}^{R*}\mathcal{D}'_{10}(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j^0\tilde{e}_L)\right] \\
&- \frac{e^2}{16\sqrt{2}\pi^2 s_W^2 c_W}\sum_{ij} Z_{1i}^{+}(Z_{1j}^{N*} s_W + Z_{2j}^{N*} c_W) \times g_{H\tilde{e}_L\tilde{\nu}_L} \times \\
&\quad \left[M_{\tilde{\chi}_j^0}\, c_{Hij}^{R}\bar{\mathcal{D}}_{10}(\tilde{e}_L\tilde{\chi}_j^0\tilde{\chi}_i\tilde{\nu}_L) + M_{\tilde{\chi}_i}\, c_{Hij}^{L}\bar{\mathcal{D}}'_{10}(\tilde{e}_L\tilde{\chi}_j^0\tilde{\chi}_i\tilde{\nu}_L)\right]. \quad (245)
\end{aligned}
$$

6.3 Neutral Higgs sector

For $e^+e^- \to H^0A^0/h^0A^0$, there is no box9 diagram, so one has:

$$
\begin{aligned}
a_{L,R}^{box}(H^0A^0/h^0A^0) &= \frac{iq^2}{2e^2}\ \Big\{A_{L,R}^{box7}(H^0A^0/h^0A^0) + A_{L,R}^{box8}(H^0A^0/h^0A^0) \\
&\quad + A_{L,R}^{box10}(H^0A^0/h^0A^0)\Big\}. \quad (246)
\end{aligned}
$$

6.3.1 Box7 diagrams

In the neutral Higgs sector, there is no right-handed contribution from box7 diagrams. The left-handed amplitude $A_L^{box7}(H^0A^0/h^0A^0)$ is obtained as follows:

$$
A_L^{box7}(H^0A^0/h^0A^0) = \frac{e^4}{128\pi^2 s_W^4} \times [Z_{ab}] \times \Big\{\mathcal{D}_7(\nu WHW) + \bar{\mathcal{D}}_7(\nu WHW)\Big\}. \quad (247)
$$

6.3.2 Box8 diagrams

Both $\tilde{\nu}\tilde{\chi}\tilde{\chi}\tilde{\chi}$ and $\tilde{e}\tilde{\chi}^0\tilde{\chi}^0\tilde{\chi}^0$ box diagrams contribute to the amplitude $A_{L,R}^{box8}(H^0A^0/h^0A^0)$:

$$
A_{L,R}^{box8}(H^0A^0/h^0A^0) = P_{L,R}\left[A_{H^0A^0/h^0A^0}^{box8}(\tilde{\nu}\tilde{\chi}\tilde{\chi}\tilde{\chi}) + A_{H^0A^0/h^0A^0}^{box8}(\tilde{e}\tilde{\chi}^0\tilde{\chi}^0\tilde{\chi}^0)\right]. \quad (248)
$$

The $\tilde{\nu}\tilde{\chi}\tilde{\chi}\tilde{\chi}$ boxes contribute only with left-handed terms:

$$
\begin{aligned}
A_{H^0A^0}^{box8}(\tilde{\nu}_L\tilde{\chi}\tilde{\chi}\tilde{\chi}) &= \frac{e^2}{16\pi^2 s_W^2}\sum_{ijk} Z_{1i}^{+*}Z_{1k}^{+}\times\ \Big\{M_{\tilde{\chi}_i}M_{\tilde{\chi}_k}c_{A^0ji}^{R}c_{H^0kj}^{L}\mathcal{D}''_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
&\quad + M_{\tilde{\chi}_i}M_{\tilde{\chi}_j}c_{A^0ji}^{R}c_{H^0kj}^{R}\mathcal{D}'_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
&\quad + M_{\tilde{\chi}_j}M_{\tilde{\chi}_k}c_{A^0ji}^{L}c_{H^0kj}^{L}\mathcal{D}'_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
&\quad + c_{A^0ji}^{L}c_{H^0kj}^{R}\mathcal{D}_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k)\Big\} \\
&- \frac{e^2}{16\pi^2 s_W^2}\sum_{ijk} Z_{1i}^{+*}Z_{1k}^{+}\times\ \Big\{M_{\tilde{\chi}_i}M_{\tilde{\chi}_k}c_{H^0ji}^{R}c_{A^0kj}^{L}\bar{\mathcal{D}}''_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
&\quad + M_{\tilde{\chi}_i}M_{\tilde{\chi}_j}c_{H^0ji}^{R}c_{A^0kj}^{R}\bar{\mathcal{D}}'_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
&\quad + M_{\tilde{\chi}_j}M_{\tilde{\chi}_k}c_{H^0ji}^{L}c_{A^0kj}^{L}\bar{\mathcal{D}}'_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k) \\
&\quad + c_{H^0ji}^{L}c_{A^0kj}^{R}\bar{\mathcal{D}}_8(\tilde{\nu}_L\tilde{\chi}_i\tilde{\chi}_j\tilde{\chi}_k)\Big\}, \quad (249)
\end{aligned}
$$

$$
\begin{aligned}
A^{box8}_{h^0A^0}(\tilde\nu_L\tilde\chi\tilde\chi\tilde\chi) \;=\; & \frac{e^2}{16\pi^2 s_W^2}\sum_{ijk} Z^{+*}_{1i}Z^{+}_{1k}\times \quad \Big\{M_{\tilde\chi_i}M_{\tilde\chi_k}c^R_{A^0ji}c^L_{h^0kj}\mathcal{D}''_8(\tilde\nu_L\tilde\chi_i\tilde\chi_j\tilde\chi_k)\\
& +M_{\tilde\chi_i}M_{\tilde\chi_j}c^R_{A^0ji}c^R_{h^0kj}\mathcal{D}'_8(\tilde\nu_L\tilde\chi_i\tilde\chi_j\tilde\chi_k)\\
& +M_{\tilde\chi_j}M_{\tilde\chi_k}c^L_{A^0ji}c^L_{h^0kj}\mathcal{D}'_8(\tilde\nu_L\tilde\chi_i\tilde\chi_j\tilde\chi_k)\\
& +c^L_{A^0ji}c^R_{h^0kj}\mathcal{D}_8(\tilde\nu_L\tilde\chi_i\tilde\chi_j\tilde\chi_k)\Big\}\\
& -\frac{e^2}{16\pi^2 s_W^2}\sum_{ijk} Z^{+*}_{1i}Z^{+}_{1k}\times \quad \Big\{M_{\tilde\chi_i}M_{\tilde\chi_k}c^R_{h^0ji}c^L_{A^0kj}\bar{\mathcal{D}}''_8(\tilde\nu_L\tilde\chi_i\tilde\chi_j\tilde\chi_k)\\
& +M_{\tilde\chi_i}M_{\tilde\chi_j}c^R_{h^0ji}c^R_{A^0kj}\bar{\mathcal{D}}'_8(\tilde\nu_L\tilde\chi_i\tilde\chi_j\tilde\chi_k)\\
& +M_{\tilde\chi_j}M_{\tilde\chi_k}c^L_{h^0ji}c^L_{A^0kj}\bar{\mathcal{D}}'_8(\tilde\nu_L\tilde\chi_i\tilde\chi_j\tilde\chi_k)\\
& +c^L_{h^0ji}c^R_{A^0kj}\bar{\mathcal{D}}_8(\tilde\nu_L\tilde\chi_i\tilde\chi_j\tilde\chi_k)\Big\}. \qquad (250)
\end{aligned}
$$

The $\tilde e\tilde\chi^0\tilde\chi^0\tilde\chi^0$ boxes contribute with both left-handed and right-handed terms:

$$
\begin{aligned}
A^{box8}_{H^0A^0}(\tilde e\tilde\chi^0\tilde\chi^0\tilde\chi^0) \;=\; & \frac{e^2}{32\pi^2 s_W^2 c_W^2}P_L\sum_{ijk}(Z^{N*}_{1i}s_W+Z^{N*}_{2i}c_W)(Z^N_{1k}s_W+Z^N_{2k}c_W)\times\\
& \Big\{M_{\tilde\chi^0_i}M_{\tilde\chi^0_k}n^R_{A^0ij}n^L_{H^0jk}\mathcal{D}''_8(\tilde e_L\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +M_{\tilde\chi^0_i}M_{\tilde\chi^0_j}n^R_{A^0ij}n^R_{H^0jk}\mathcal{D}'_8(\tilde e_L\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +M_{\tilde\chi^0_j}M_{\tilde\chi^0_k}n^L_{A^0ij}n^L_{H^0jk}\mathcal{D}'_8(\tilde e_L\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +n^L_{A^0ij}n^R_{H^0jk}\mathcal{D}_8(\tilde e_L\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\Big\}\\
& -\frac{e^2}{32\pi^2 s_W^2 c_W^2}P_L\sum_{ijk}(Z^{N*}_{1i}s_W+Z^{N*}_{2i}c_W)(Z^N_{1k}s_W+Z^N_{2k}c_W)\times\\
& \Big\{M_{\tilde\chi^0_i}M_{\tilde\chi^0_k}n^R_{H^0ij}n^L_{A^0jk}\bar{\mathcal{D}}''_8(\tilde e_L\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +M_{\tilde\chi^0_i}M_{\tilde\chi^0_j}n^R_{H^0ij}n^R_{A^0jk}\bar{\mathcal{D}}'_8(\tilde e_L\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +M_{\tilde\chi^0_j}M_{\tilde\chi^0_k}n^L_{H^0ij}n^L_{A^0jk}\bar{\mathcal{D}}'_8(\tilde e_L\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +n^L_{H^0ij}n^R_{A^0jk}\bar{\mathcal{D}}_8(\tilde e_L\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\Big\}\\
& -\frac{e^2}{8\pi^2 c_W^2}P_R\sum_{ijk}Z^N_{1i}Z^{N*}_{1k}\times\ \Big\{M_{\tilde\chi^0_i}M_{\tilde\chi^0_k}n^L_{A^0ij}n^R_{H^0jk}\mathcal{D}''_8(\tilde e_R\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +M_{\tilde\chi^0_i}M_{\tilde\chi^0_j}n^L_{A^0ij}n^L_{H^0jk}\mathcal{D}'_8(\tilde e_R\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +M_{\tilde\chi^0_j}M_{\tilde\chi^0_k}n^R_{A^0ij}n^R_{H^0jk}\mathcal{D}'_8(\tilde e_R\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +n^R_{A^0ij}n^L_{H^0jk}\mathcal{D}_8(\tilde e_R\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\Big\}\\
& +\frac{e^2}{8\pi^2 c_W^2}P_R\sum_{ijk}Z^N_{1i}Z^{N*}_{1k}\times\ \Big\{M_{\tilde\chi^0_i}M_{\tilde\chi^0_k}n^L_{H^0ij}n^R_{A^0jk}\bar{\mathcal{D}}''_8(\tilde e_R\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +M_{\tilde\chi^0_i}M_{\tilde\chi^0_j}n^L_{H^0ij}n^L_{A^0jk}\bar{\mathcal{D}}'_8(\tilde e_R\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +M_{\tilde\chi^0_j}M_{\tilde\chi^0_k}n^R_{H^0ij}n^R_{A^0jk}\bar{\mathcal{D}}'_8(\tilde e_R\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\\
& +n^R_{H^0ij}n^L_{A^0jk}\bar{\mathcal{D}}_8(\tilde e_R\tilde\chi^0_i\tilde\chi^0_j\tilde\chi^0_k)\Big\}, \qquad (251)
\end{aligned}
$$

$$
\begin{aligned}
A^{box8}_{h^0A^0}(\tilde{e}\tilde{\chi}^0\tilde{\chi}^0\tilde{\chi}^0) = {} & \frac{e^2}{32\pi^2 s_W^2 c_W^2} P_L \sum_{ijk} (Z^{N*}_{1i} s_W + Z^{N*}_{2i} c_W)(Z^N_{1k} s_W + Z^N_{2k} c_W) \times \\
& \Big\{ M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_k} n^R_{A^0ij} n^L_{h^0jk} \mathcal{D}''_8(\tilde{e}_L \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j} n^R_{A^0ij} n^R_{h^0jk} \mathcal{D}'_8(\tilde{e}_L \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + M_{\tilde{\chi}^0_j} M_{\tilde{\chi}^0_k} n^L_{A^0ij} n^L_{h^0jk} \mathcal{D}'_8(\tilde{e}_L \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + n^L_{A^0ij} n^R_{h^0jk} \mathcal{D}_8(\tilde{e}_L \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \Big\} \\
& - \frac{e^2}{32\pi^2 s_W^2 c_W^2} P_L \sum_{ijk} (Z^{N*}_{1i} s_W + Z^{N*}_{2i} c_W)(Z^N_{1k} s_W + Z^N_{2k} c_W) \times \\
& \Big\{ M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_k} n^R_{h^0ij} n^L_{A^0jk} \bar{\mathcal{D}}''_8(\tilde{e}_L \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j} n^R_{h^0ij} n^R_{A^0jk} \bar{\mathcal{D}}'_8(\tilde{e}_L \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + M_{\tilde{\chi}^0_j} M_{\tilde{\chi}^0_k} n^L_{h^0ij} n^L_{A^0jk} \bar{\mathcal{D}}'_8(\tilde{e}_L \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + n^L_{h^0ij} n^R_{A^0jk} \bar{\mathcal{D}}_8(\tilde{e}_L \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \Big\} \\
& - \frac{e^2}{8\pi^2 c_W^2} P_R \sum_{ijk} Z^N_{1i} Z^{N*}_{1k} \times \Big\{ M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_k} n^L_{A^0ij} n^R_{h^0jk} \mathcal{D}''_8(\tilde{e}_R \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j} n^L_{A^0ij} n^L_{h^0jk} \mathcal{D}'_8(\tilde{e}_R \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + M_{\tilde{\chi}^0_j} M_{\tilde{\chi}^0_k} n^R_{A^0ij} n^R_{h^0jk} \mathcal{D}'_8(\tilde{e}_R \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + n^R_{A^0ij} n^L_{h^0jk} \mathcal{D}_8(\tilde{e}_R \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \Big\} \\
& + \frac{e^2}{8\pi^2 c_W^2} P_R \sum_{ijk} Z^N_{1i} Z^{N*}_{1k} \times \Big\{ M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_k} n^L_{h^0ij} n^R_{A^0jk} \bar{\mathcal{D}}''_8(\tilde{e}_R \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + M_{\tilde{\chi}^0_i} M_{\tilde{\chi}^0_j} n^L_{h^0ij} n^L_{A^0jk} \bar{\mathcal{D}}'_8(\tilde{e}_R \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + M_{\tilde{\chi}^0_j} M_{\tilde{\chi}^0_k} n^R_{h^0ij} n^R_{A^0jk} \bar{\mathcal{D}}'_8(\tilde{e}_R \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \\
& + n^R_{h^0ij} n^L_{A^0jk} \bar{\mathcal{D}}_8(\tilde{e}_R \tilde{\chi}^0_i \tilde{\chi}^0_j \tilde{\chi}^0_k) \Big\}. \qquad (252)
\end{aligned}
$$

6.3.3 Twisted box10 diagrams

Two types of twisted box10 diagrams must be considered: $\tilde{\nu}\tilde{\chi}\tilde{\chi}\tilde{\nu}$ and $\tilde{e}\tilde{\chi}^0\tilde{\chi}^0\tilde{e}$.

The $\tilde{\nu}\tilde{\chi}\tilde{\chi}\tilde{\nu}$ boxes contribute with left-handed terms only:

$$
\begin{aligned}
A^{box10}_{H^0A^0}(\tilde{\nu}_L\tilde{\chi}\tilde{\chi}\tilde{\nu}_L) = {} & \frac{e^2}{16\pi^2 s_W^2} \sum_{ij} Z^{+*}_{1i} Z^+_{1j} \times g_{H^0\tilde{\nu}_L\tilde{\nu}_L} \times \\
& \Big[M_{\tilde{\chi}_i} c^R_{A^0ji} \mathcal{D}_{10}(\tilde{\nu}_L \tilde{\chi}_i \tilde{\chi}_j \tilde{\nu}_L) + M_{\tilde{\chi}_j} c^L_{A^0ji} \mathcal{D}'_{10}(\tilde{\nu}_L \tilde{\chi}_i \tilde{\chi}_j \tilde{\nu}_L) \Big], \qquad (253)
\end{aligned}
$$

$$
\begin{aligned}
A^{box10}_{h^0A^0}(\tilde{\nu}_L\tilde{\chi}\tilde{\chi}\tilde{\nu}_L) = {} & \frac{e^2}{16\pi^2 s_W^2} \sum_{ij} Z^{+*}_{1i} Z^+_{1j} \times g_{h^0\tilde{\nu}_L\tilde{\nu}_L} \times \\
& \Big[M_{\tilde{\chi}_i} c^R_{A^0ji} \mathcal{D}_{10}(\tilde{\nu}_L \tilde{\chi}_i \tilde{\chi}_j \tilde{\nu}_L) + M_{\tilde{\chi}_j} c^L_{A^0ji} \mathcal{D}'_{10}(\tilde{\nu}_L \tilde{\chi}_i \tilde{\chi}_j \tilde{\nu}_L) \Big]. \qquad (254)
\end{aligned}
$$

The $\tilde{e}\tilde{\chi}^0\tilde{\chi}^0\tilde{e}$ boxes contribute with both left-handed and right-handed terms. Writing all these terms into one single expression leads to:

$$\begin{aligned}
A^{box10}_{H^0A^0}(\tilde{e}\tilde{\chi}^0\tilde{\chi}^0\tilde{e}) &= \frac{e^2}{32\pi^2 s_W^2 c_W^2} P_L \sum_{ij} (Z^{N*}_{1i} s_W + Z^{N*}_{2i} c_W)(Z^N_{1j} s_W + Z^N_{2j} c_W) \times g_{H^0\tilde{e}_L\tilde{e}_L} \times \\
&\qquad \left[M_{\tilde{\chi}^0_i}\, n^R_{A^0ji} \mathcal{D}_{10}(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{e}_L) + M_{\tilde{\chi}^0_j}\, n^L_{A^0ji} \mathcal{D}'_{10}(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{e}_L) \right] \\
&+ \frac{e^2}{8\pi^2 c_W^2} P_R \sum_{ij} Z^{N*}_{1i} Z^N_{1j} \times g_{H^0\tilde{e}_R\tilde{e}_R} \times \\
&\qquad \left[M_{\tilde{\chi}^0_i}\, n^L_{A^0ji} \mathcal{D}_{10}(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{e}_R) + M_{\tilde{\chi}^0_j}\, n^R_{A^0ji} \mathcal{D}'_{10}(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{e}_R) \right], \quad (255)
\end{aligned}$$

$$\begin{aligned}
A^{box10}_{h^0A^0}(\tilde{e}\tilde{\chi}^0\tilde{\chi}^0\tilde{e}) &= \frac{e^2}{32\pi^2 s_W^2 c_W^2} P_L \sum_{ij} (Z^{N*}_{1i} s_W + Z^{N*}_{2i} c_W)(Z^N_{1j} s_W + Z^N_{2j} c_W) \times g_{h^0\tilde{e}_L\tilde{e}_L} \times \\
&\qquad \left[M_{\tilde{\chi}^0_i}\, n^R_{A^0ji} \mathcal{D}_{10}(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{e}_L) + M_{\tilde{\chi}^0_j}\, n^L_{A^0ji} \mathcal{D}'_{10}(\tilde{e}_L\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{e}_L) \right] \\
&+ \frac{e^2}{8\pi^2 c_W^2} P_R \sum_{ij} Z^{N*}_{1i} Z^N_{1j} \times g_{h^0\tilde{e}_R\tilde{e}_R} \times \\
&\qquad \left[M_{\tilde{\chi}^0_i}\, n^L_{A^0ji} \mathcal{D}_{10}(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{e}_R) + M_{\tilde{\chi}^0_j}\, n^R_{A^0ji} \mathcal{D}'_{10}(\tilde{e}_R\tilde{\chi}^0_i\tilde{\chi}^0_j\tilde{e}_R) \right]. \quad (256)
\end{aligned}$$

7 Conclusion and outlooks

In this paper, we discussed all electroweak one loop contributions to the pair production cross section for charged and neutral Higgs bosons in e^+e^- collisions, in the theoretical framework of the MSSM. The one loop amplitudes of initial vertices and $e^\pm$ self-energy, of γ and Z boson self-energies, of the corresponding counter terms, of final vertices and Higgs self-energies and of box diagrams are respectively given by equations (17), (82), (83), (98) and (238) for the charged Higgs sector, and by equations (18), (85), (86), (126) and (246) for the neutral Higgs sector. The left-handed and right-handed amplitudes of all these electroweak one loop contributions are:

- in the charged Higgs sector:

$$\begin{aligned}
a^{1\,loop}_{L,R}(H^+H^-) &= a^{in}_{L,R}(H^+H^-) \\
&+ a^{RG}_{L,R}(H^+H^-) + a^{ct}_{L,R}(H^+H^-) \\
&+ a^{fin}_{L,R}(H^+H^-) \\
&+ a^{box}_{L,R}(H^+H^-), \qquad (257)
\end{aligned}$$

- in the neutral Higgs sector:

$$\begin{aligned}
a^{1\,loop}_{L,R}(H^0A^0/h^0A^0) &= a^{in}_{L,R}(H^0A^0/h^0A^0) \\
&+ a^{RG}_{L,R}(H^0A^0/h^0A^0) + a^{ct}_{L,R}(H^0A^0/h^0A^0) \\
&+ a^{fin}_{L,R}(H^0A^0/h^0A^0) \\
&+ a^{box}_{L,R}(H^0A^0/h^0A^0). \qquad (258)
\end{aligned}$$

The differential production cross section for charged or neutral Higgs bosons at the one loop level can then be calculated as follows:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha_{em}^2\beta_H^3}{8q^2}(1-\cos^2\theta)\times\Big[|a_L^{Born}|^2 + 2|a_L^{Born}a_L^{1\,loop}| + |a_R^{Born}|^2 + 2|a_R^{Born}a_R^{1\,loop}|\Big]. \quad (259)$$

In the previous equation, $a_{L,R}^{Born}$ is the Born amplitude of equation (8) or (9). As for β_H, it stands for the velocity of the Higgs bosons, see equation (12). After integration over $\cos\theta$ (which appears in the contributions of the box diagrams), one obtains the total pair production cross section at the one loop level. Note that, in the case of the tree level cross sections for H^+H^- and $H^0A^0 + h^0A^0$, there is a direct dependence on M_A only and not on the other MSSM parameters. However, after having taken into account all electroweak one loop contributions, this is not true anymore. Indeed, $a_{L,R}^{1\,loop}$ depends on other MSSM parameters than just M_A.

A C++ numerical code has been developed in order to calculate accurately all one loop electroweak contributions and, in turn, to compare the pair production cross sections for MSSM charged and neutral Higgs bosons at tree level and at the one loop level. The relevant Feynman diagrams are computed by calling the suitable functions in the LoopTools 2.1 library [16]. The input of the code, in standard notation, is the following: $\tan\beta$, μ, M_A (the mass of the A^0 Higgs boson), the gaugino parameters M_1 and M_2, the scalar mass scale M_S, the sfermion mixing matrix parameters A_u and A_d. A possible reference for these parameters is [15]. This input requires a preliminary pre-processing using the FeynHiggs 2.1 [17] code. A subset of these parameters is then fed into FeynHiggs in order to compute the masses of the Higgs bosons h^0, H^0, $H^\pm$, as well as the mixing angle in the neutral sector α. These additional parameters do indeed appear in the analytical expressions described in the text. The output of the code is the cross section for the various processes under consideration. We have successfully checked that the MSSM Higgs bosons pair production cross section computed by our code at the one loop level remains stable against UV divergences, both in the charged and neutral Higgs sectors. Also, we have checked that the variation of the computed one loop cross section with q^2 agrees with our expectations.

Note that, in this code, we have included all virtual contributions involving particles having electroweak interactions in the MSSM, but we did not treat pure QED effects, such as Initial State Radiation (ISR) and Final State Radiation (FSR). The reason is that these effects may depend on the characteristics of the detectors (for instance, they need specific kinematical cuts) and some specific codes exist in order to treat them. Nevertheless, in order to be able to test electroweak symmetry properties at high energy, in particular those of supersymmetric nature, which is indeed the purpose of this work, we have included the virtual photon effects, but with a photon mass set to M_Z in order to keep these effects finite. In order to have the complete (observable) contribution including QED effects, one should compute the following combination: *Our contribution + ISR + FSR + virtual soft photon with zero mass − virtual soft photon with* M_Z. This combination should be calculated at the level of the codes that include the ISR and FSR effects.

Appendix A: Vertices and couplings

Gauge - Fermion - Fermion

Let Q_f and T_f^3 be respectively the charge and the third isospin component of the fermion f, then the couplings of gauge bosons to left-handed and right-handed fermions are:

$$P_L\, g_{\gamma ff} = Q_f \quad \text{and} \quad P_R\, g_{\gamma ff} = Q_f,$$
$$P_L\, g_{Zff} = \frac{T_f^3 - Q_f s_W^2}{s_W c_W} \quad \text{and} \quad P_R\, g_{Zff} = -Q_f \frac{s_W}{c_W},$$
$$P_L\, g_{Wff'} = \frac{1}{s_W\sqrt{2}} \quad \text{and} \quad P_R\, g_{Wff'} = 0.$$

Note that, in this paper, we have also used a simplified notation for the couplings of γ or Z to fermion pairs:

$$g_{VLf} \equiv P_L\, g_{Vff},$$
$$g_{VRf} \equiv P_R\, g_{Vff},$$

where V stands for either γ or Z.

Gauge - Gaugino - Gaugino

$$\mathcal{O}_{ij}^{\gamma L} = -e\delta_{ij} \quad \text{and} \quad \mathcal{O}_{ij}^{\gamma R} = -e\delta_{ij},$$
$$\mathcal{O}_{ij}^{ZL} = -\frac{e\left[Z_{1i}^{+*}Z_{1j}^{+} + \delta_{ij}(c_W^2 - s_W^2)\right]}{2s_W c_W} \quad \text{and} \quad \mathcal{O}_{ij}^{ZR} = -\frac{e\left[Z_{1i}^{-}Z_{1j}^{-*} + \delta_{ij}(c_W^2 - s_W^2)\right]}{2s_W c_W},$$
$$\mathcal{O}_{ij}^{0L} = \frac{e(Z_{4i}^{N*}Z_{4j}^{N} - Z_{3i}^{N*}Z_{3j}^{N})}{2s_W c_W} \quad \text{and} \quad \mathcal{O}_{ij}^{0R} = -\frac{e(Z_{4i}^{N}Z_{4j}^{N*} - Z_{3i}^{N}Z_{3j}^{N*})}{2s_W c_W},$$
$$\mathcal{O}_{ij}^{WL} = \frac{e}{s_W}\left(Z_{2j}^{N}Z_{1i}^{+*} - \frac{1}{\sqrt{2}}Z_{4k}^{N}Z_{2i}^{+*}\right) \quad \text{and} \quad \mathcal{O}_{ij}^{WR} = \frac{e}{s_W}\left(Z_{2j}^{N*}Z_{1i}^{-} + \frac{1}{\sqrt{2}}Z_{3j}^{N*}Z_{2i}^{-}\right).$$

Here, the Z_{ij} terms correspond to the various elements of the unitary mixing matrices of the charginos and neutralinos. They are derived from the diagonalization of the MSSM gaugino mass matrix, see for instance reference [18] for details.

Gauge - Sfermion - Sfermion

The couplings of gauge bosons to unmixed left-handed and right-handed sfermions are:

$$g^0_{\gamma\tilde{f}_L\tilde{f}_L} = -eQ_f \quad \text{and} \quad g^0_{\gamma\tilde{f}_R\tilde{f}_R} = -eQ_f,$$
$$g^0_{Z\tilde{f}_L\tilde{f}_L} = -\frac{e(T_f^3 - Q_f s_W^2)}{s_W c_W} \quad \text{and} \quad g^0_{Z\tilde{f}_R\tilde{f}_R} = eQ_f\frac{s_W}{c_W},$$
$$g^0_{W\tilde{f}_L\tilde{f}'_L} = -\frac{e}{s_W\sqrt{2}} \quad \text{and} \quad g^0_{W\tilde{f}_R\tilde{f}'_R} = 0.$$

Let $\theta_{\tilde{f}}$ be the mixing angle of the sfermion $\tilde{f}$ (generally a third generation squark). Let us also define $c_{\tilde{f}} \equiv \cos\theta_{\tilde{f}}$ and $s_{\tilde{f}} \equiv \sin\theta_{\tilde{f}}$. The coupling between a gauge boson and two sfermions with mixing is then given by:

$$g_{\gamma\tilde{f}_1\tilde{f}_1} = g_{\gamma\tilde{f}_2\tilde{f}_2} = g^0_{\gamma\tilde{f}_L\tilde{f}_L} = g^0_{\gamma\tilde{f}_R\tilde{f}_R} = -eQ_f,$$

$$\begin{aligned}
g_{Z\tilde{f}_1\tilde{f}_1} &= c^2_{\tilde{f}} g^0_{Z\tilde{f}_L\tilde{f}_L} + s^2_{\tilde{f}} g^0_{Z\tilde{f}_R\tilde{f}_R}, \\
g_{Z\tilde{f}_2\tilde{f}_2} &= s^2_{\tilde{f}} g^0_{Z\tilde{f}_L\tilde{f}_L} + c^2_{\tilde{f}} g^0_{Z\tilde{f}_R\tilde{f}_R}, \\
g_{Z\tilde{f}_1\tilde{f}_2} &= g_{Z\tilde{f}_2\tilde{f}_1} = c_{\tilde{f}} s_{\tilde{f}} (g^0_{Z\tilde{f}_R\tilde{f}_R} - g^0_{Z\tilde{f}_L\tilde{f}_L}),
\end{aligned}$$

$$\begin{aligned}
g_{W\tilde{f}_1\tilde{f}'_1} &= c_{\tilde{f}} c_{\tilde{f}'} g^0_{W\tilde{f}_L\tilde{f}'_L}, \\
g_{W\tilde{f}_2\tilde{f}'_2} &= s_{\tilde{f}} s_{\tilde{f}'} g^0_{W\tilde{f}_L\tilde{f}'_L}, \\
g_{W\tilde{f}_1\tilde{f}'_2} &= -c_{\tilde{f}} s_{\tilde{f}'} g^0_{W\tilde{f}_L\tilde{f}'_L}, \\
g_{W\tilde{f}_2\tilde{f}'_1} &= -c_{\tilde{f}'} s_{\tilde{f}} g^0_{W\tilde{f}_L\tilde{f}'_L}.
\end{aligned}$$

Gauge - Gauge - Higgs

$$\begin{aligned}
g_{ZZH^0} &= \frac{eM_W}{s_W c_W^2}\cos(\beta-\alpha) \quad \text{and} \quad g_{ZZh^0} = \frac{eM_W}{s_W c_W^2}\sin(\beta-\alpha), \\
g_{WWH^0} &= \frac{eM_W}{s_W}\cos(\beta-\alpha) \quad \text{and} \quad g_{WWh^0} = \frac{eM_W}{s_W}\sin(\beta-\alpha), \\
g_{\gamma WG} &= eM_W \quad \text{and} \quad g_{ZWG} = -eM_W\frac{s_W}{c_W}.
\end{aligned}$$

Gauge - Higgs - Higgs

$$\begin{aligned}
g_{ZH^0A^0} &= -\frac{e}{2s_W c_W}\sin(\beta-\alpha) \quad \text{and} \quad g_{Zh^0A^0} = +\frac{e}{2s_W c_W}\cos(\beta-\alpha), \\
g_{ZH^0G^0} &= +\frac{e}{2s_W c_W}\cos(\beta-\alpha) \quad \text{and} \quad g_{Zh^0G^0} = +\frac{e}{2s_W c_W}\sin(\beta-\alpha), \\
g_{W^\pm H^\pm H^0} &= +Q_W \times \frac{e}{2s_W}\sin(\beta-\alpha) \quad \text{and} \quad g_{W^\pm H^\pm h^0} = -Q_W \times \frac{e}{2s_W}\cos(\beta-\alpha), \\
g_{W^\pm G^\pm H^0} &= -Q_W \times \frac{e}{2s_W}\cos(\beta-\alpha) \quad \text{and} \quad g_{W^\pm G^\pm h^0} = -Q_W \times \frac{e}{2s_W}\sin(\beta-\alpha), \\
g_{WHA^0} &= +\frac{e}{2s_W} \quad \text{and} \quad g_{WHG^0} = 0, \\
g_{WGA^0} &= 0 \quad \text{and} \quad g_{WGG^0} = +\frac{e}{2s_W}, \\
g_{\gamma HH} &= -e \quad \text{and} \quad g_{ZHH} = -e\frac{1-2s_W^2}{2s_W c_W}, \\
g_{\gamma GG} &= -e \quad \text{and} \quad g_{ZGG} = -e\frac{1-2s_W^2}{2s_W c_W}.
\end{aligned}$$

Higgs - Fermion - Fermion

The coupling constant between a Higgs boson and a fermion pair is proportional to the mass of the fermion(s). Thus, only the third generation quarks are usually considered.

In the charged Higgs sector:

- for $b \to t\ H^-$: $c^L_{b\to tH^-} = \frac{e}{\sqrt{2}s_W M_W} M_t \cot\beta$ and $c^R_{b\to tH^-} = \frac{e}{\sqrt{2}s_W M_W} M_b \tan\beta$,
- for $t \to b\ H^+$: $c^L_{t\to bH^+} = \frac{e}{\sqrt{2}s_W M_W} M_b \tan\beta$ and $c^R_{t\to bH^+} = \frac{e}{\sqrt{2}s_W M_W} M_t \cot\beta$.

In the neutral Higgs sector, the left-handed coupling constants are:

- $c^L_{H^0 t} = -\frac{eM_t}{2s_W M_W} \times \frac{\sin\alpha}{\sin\beta}$ and $c^L_{H^0 b} = -\frac{eM_b}{2s_W M_W} \times \frac{\cos\alpha}{\cos\beta}$,
- $c^L_{h^0 t} = -\frac{eM_t}{2s_W M_W} \times \frac{\cos\alpha}{\sin\beta}$ and $c^L_{H^0 b} = +\frac{eM_b}{2s_W M_W} \times \frac{\sin\alpha}{\cos\beta}$,
- $c^L_{A^0 t} = \frac{eM_t}{2s_W M_W} \times \cot\beta$ and $c^L_{A^0 b} = \frac{eM_b}{2s_W M_W} \times \tan\beta$.

As for the right-handed couplings constants, one simply has:

- $c^R_{H^0 f} = c^L_{H^0 f}$,
- $c^R_{h^0 f} = c^L_{h^0 f}$,
- $c^R_{A^0 f} = -c^L_{A^0 f}$.

Higgs - Gaugino - Gaugino

In the charged Higgs sector, there are two types of vertex to consider: $\tilde{\chi}^0_j \to \tilde{\chi}^+_i\ H^-$ and $\tilde{\chi}^+_j \to \tilde{\chi}^0_i\ H^+$. Let us choose the case where i and j label respectively a chargino and a neutralino, then the corresponding left-handed and right-handed coupling constants are:

- $c^L_{Hij} = \frac{e\sin\beta}{s_W c_W}\left[\frac{1}{\sqrt{2}} Z^-_{2i}(Z^N_{1j}s_W + Z^N_{2j}c_W) - Z^-_{1i}Z^N_{3j}c_W\right]$,
- $c^R_{Hij} = -\frac{e\cos\beta}{s_W c_W}\left[\frac{1}{\sqrt{2}} Z^{+*}_{2i}(Z^{N*}_{1j}s_W + Z^{N*}_{2j}c_W) + Z^{+*}_{1i}Z^{N*}_{4j}c_W\right]$.

For the other vertex (where i and j label respectively a neutralino and a chargino), one should instead use the left-handed and right-handed coupling constants c^{R*}_{Hji} and c^{L*}_{Hji}, respectively.

In the neutral Higgs sector, one must consider the coupling between a neutral Higgs boson and either two charginos or two neutralinos.

For the coupling between a neutral Higgs boson and two charginos:

$$c^L_{H^0ij} = -\frac{e}{\sqrt{2}s_W}\left[\cos\alpha Z^-_{2i}Z^+_{1j} + \sin\alpha Z^-_{1i}Z^+_{2j}\right] \quad \text{and} \quad c^R_{H^0ij} = c^{L*}_{H^0ji},$$
$$c^L_{h^0ij} = -\frac{e}{\sqrt{2}s_W}\left[-\sin\alpha Z^-_{2i}Z^+_{1j} + \cos\alpha Z^-_{1i}Z^+_{2j}\right] \quad \text{and} \quad c^R_{h^0ij} = c^{L*}_{h^0ji},$$
$$c^L_{A^0ij} = -\frac{e}{\sqrt{2}s_W}\left[\sin\beta Z^-_{2i}Z^+_{1j} + \cos\beta Z^-_{1i}Z^+_{2j}\right] \quad \text{and} \quad c^R_{A^0ij} = -c^{L*}_{A^0ji}.$$

Let us now consider the coupling between a neutral Higgs boson and two neutralinos. For the left-handed components, one has:

$$\begin{aligned}
n^L_{H^0ij} &= \frac{e}{2s_Wc_W}\times \quad \Big\{(\cos\alpha Z^N_{3j} - \sin\alpha Z^N_{4j})(Z^N_{1i}s_W - Z^N_{2i}c_W) \\
&\qquad +(\cos\alpha Z^N_{3i} - \sin\alpha Z^N_{4i})(Z^N_{1j}s_W - Z^N_{2j}c_W)\Big\}, \\
n^L_{h^0ij} &= -\frac{e}{2s_Wc_W}\times \quad \Big\{(\sin\alpha Z^N_{3j} + \cos\alpha Z^N_{4j})(Z^N_{1i}s_W - Z^N_{2i}c_W) \\
&\qquad +(\sin\alpha Z^N_{3i} + \cos\alpha Z^N_{4i})(Z^N_{1j}s_W - Z^N_{2j}c_W)\Big\}, \\
n^L_{A^0ij} &= \frac{e}{2s_Wc_W}\times \quad \Big\{(\sin\beta Z^N_{3j} - \cos\beta Z^N_{4j})(Z^N_{1i}s_W - Z^N_{2i}c_W) \\
&\qquad +(\sin\beta Z^N_{3i} - \cos\beta Z^N_{4i})(Z^N_{1j}s_W - Z^N_{2j}c_W)\Big\}.
\end{aligned}$$

As for the right-handed components, one simply has:

$$\begin{aligned}
n^R_{H^0ij} &= n^{L*}_{H^0ji}, \\
n^R_{h^0ij} &= n^{L*}_{h^0ji}, \\
n^R_{A^0ij} &= -n^{L*}_{A^0ji}.
\end{aligned}$$

Higgs - Sfermion - Sfermion

Let us first consider the light unmixed sfermions. Their coupling constant to the charged and neutral Higgs bosons is not proportional to the mass of the corresponding fermion(s) and it can thus not be neglected.

In the charged Higgs sector, if $\tilde{f}$ and $\tilde{f}'$ represent respectively up-squarks and down-squarks of the first and second generations, or sneutrinos and charged sleptons, one has:

$$g_{H\tilde{f}_L\tilde{f}'_L} = -\frac{eM_W}{s_W\sqrt{2}}\sin 2\beta \quad \text{and} \quad g_{H\tilde{f}_R\tilde{f}'_R} = 0.$$

In the neutral Higgs sector, one has:

$$g_{H^0\tilde{f}_L\tilde{f}_L} = -\frac{eM_W}{s_Wc^2_W}(T^3_f - Q_fs^2_W)\cos(\alpha+\beta) \quad \text{and} \quad g_{H^0\tilde{f}_R\tilde{f}_R} = -\frac{eM_W}{s_Wc^2_W}Q_fs^2_W\cos(\alpha+\beta),$$
$$g_{h^0\tilde{f}_L\tilde{f}_L} = \frac{eM_W}{s_Wc^2_W}(T^3_f - Q_fs^2_W)\sin(\alpha+\beta) \quad \text{and} \quad g_{h^0\tilde{f}_R\tilde{f}_R} = \frac{eM_W}{s_Wc^2_W}Q_fs^2_W\sin(\alpha+\beta).$$

Note that the couplings between A^0 and a pair of light unmixed sfermions are proportional to the fermion mass and are thus negligible.

Let us now consider the heavy sfermions, i.e. the third generation squarks, and, in a first step, let us assume that there is no mixing. The coupling constants between the MSSM Higgs boson and a pair of unmixed heavy sfermions are given below.

For the charged Higgs bosons H, one has:

$$\begin{aligned}
g^0_{H\tilde{t}_L\tilde{b}_L} &= -\frac{eM_W}{s_W\sqrt{2}}\left[\sin 2\beta - \frac{M_b^2\tan\beta + M_t^2\cot\beta}{M_W^2}\right], \\
g^0_{H\tilde{t}_R\tilde{b}_R} &= \frac{eM_tM_b}{s_WM_W\sqrt{2}}\left[\tan\beta + \cot\beta\right], \\
g^0_{H\tilde{t}_L\tilde{b}_R} &= -\frac{eM_b}{s_WM_W\sqrt{2}}\left[\mu - A_b\tan\beta\right]. \\
g^0_{H\tilde{t}_R\tilde{b}_L} &= -\frac{eM_t}{s_WM_W\sqrt{2}}\left[\mu - A_t\cot\beta\right].
\end{aligned}$$

For the neutral Higgs boson A^0, there are only off-diagonal terms:

$$\begin{aligned}
g^0_{A^0\tilde{t}_L\tilde{t}_L} = g^0_{A^0\tilde{t}_R\tilde{t}_R} &= 0 \text{ and } g^0_{A^0\tilde{b}_L\tilde{b}_L} = g^0_{A^0\tilde{b}_R\tilde{b}_R} = 0, \\
g^0_{A^0\tilde{t}_L\tilde{t}_R} = g^0_{A^0\tilde{t}_R\tilde{t}_L} &= -\frac{eM_t}{2s_WM_W}\left[-\mu - A_t\cot\beta\right], \\
g^0_{A^0\tilde{b}_L\tilde{b}_R} = g^0_{A^0\tilde{b}_R\tilde{b}_L} &= -\frac{eM_b}{2s_WM_W}\left[-\mu - A_b\tan\beta\right].
\end{aligned}$$

For the neutral Higgs boson H^0, one has:

$$\begin{aligned}
g^0_{H^0\tilde{t}_L\tilde{t}_L} &= -\frac{eM_W}{s_Wc_W^2}\left(\frac{1}{2} - \frac{2}{3}s_W^2\right)\cos(\alpha+\beta) - \frac{eM_t^2}{s_WM_W}\frac{\sin\alpha}{\sin\beta}, \\
g^0_{H^0\tilde{t}_R\tilde{t}_R} &= -\frac{eM_W}{s_Wc_W^2}\left(\frac{2}{3}s_W^2\right)\cos(\alpha+\beta) - \frac{eM_t^2}{s_WM_W}\frac{\sin\alpha}{\sin\beta}, \\
g^0_{H^0\tilde{t}_L\tilde{t}_R} &= g^0_{H^0\tilde{t}_R\tilde{t}_L} = -\frac{eM_t}{2s_WM_W}\left(\frac{-\mu\cos\alpha + A_t\sin\alpha}{\sin\beta}\right).
\end{aligned}$$

$$\begin{aligned}
g^0_{H^0\tilde{b}_L\tilde{b}_L} &= \frac{eM_W}{s_Wc_W^2}\left(\frac{1}{2} - \frac{1}{3}s_W^2\right)\cos(\alpha+\beta) - \frac{eM_b^2}{s_WM_W}\frac{\cos\alpha}{\cos\beta}, \\
g^0_{H^0\tilde{b}_R\tilde{b}_R} &= \frac{eM_W}{s_Wc_W^2}\left(\frac{1}{3}s_W^2\right)\cos(\alpha+\beta) - \frac{eM_b^2}{s_WM_W}\frac{\cos\alpha}{\cos\beta}, \\
g^0_{H^0\tilde{b}_L\tilde{b}_R} &= g^0_{H^0\tilde{b}_R\tilde{b}_L} = -\frac{eM_b}{2s_WM_W}\left(\frac{-\mu\sin\alpha + A_b\cos\alpha}{\cos\beta}\right).
\end{aligned}$$

For the neutral Higgs boson h^0, one has:

$$\begin{aligned}
g^0_{h^0\tilde{t}_L\tilde{t}_L} &= \frac{eM_W}{s_W c_W^2}\left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\sin(\alpha+\beta)-\frac{eM_t^2}{s_W M_W}\frac{\cos\alpha}{\sin\beta},\\
g^0_{h^0\tilde{t}_R\tilde{t}_R} &= \frac{eM_W}{s_W c_W^2}\left(\frac{2}{3}s_W^2\right)\sin(\alpha+\beta)-\frac{eM_t^2}{s_W M_W}\frac{\cos\alpha}{\sin\beta},\\
g^0_{h^0\tilde{t}_L\tilde{t}_R} &= g^0_{h^0\tilde{t}_R\tilde{t}_L}=\frac{eM_t}{2s_W M_W}\left(\frac{-\mu\sin\alpha-A_t\cos\alpha}{\sin\beta}\right).
\end{aligned}$$

$$\begin{aligned}
g^0_{h^0\tilde{b}_L\tilde{b}_L} &= -\frac{eM_W}{s_W c_W^2}\left(\frac{1}{2}-\frac{1}{3}s_W^2\right)\sin(\alpha+\beta)+\frac{eM_b^2}{s_W M_W}\frac{\sin\alpha}{\cos\beta},\\
g^0_{h^0\tilde{b}_R\tilde{b}_R} &= -\frac{eM_W}{s_W c_W^2}\left(\frac{1}{3}s_W^2\right)\sin(\alpha+\beta)+\frac{eM_b^2}{s_W M_W}\frac{\sin\alpha}{\cos\beta},\\
g^0_{h^0\tilde{b}_L\tilde{b}_R} &= g^0_{h^0\tilde{b}_R\tilde{b}_L}=-\frac{eM_b}{2s_W M_W}\left(\frac{-\mu\cos\alpha-A_b\sin\alpha}{\cos\beta}\right).
\end{aligned}$$

Let us know take the sfermion mixing into account and let $R^{\tilde{t}}$ and $R^{\tilde{b}}$ be the rotation matrices for $\tilde{t}$ and $\tilde{b}$ squarks, respectively. If $\tilde{f}^0_{1,2}\equiv\tilde{f}_{L,R}$, then:

$$\tilde{f}_i = R^{\tilde{f}}_{ij}\tilde{f}^0_j \text{ with } R^{\tilde{f}}_{ij}=\begin{pmatrix} c_{\tilde{f}} & s_{\tilde{f}} \\ -s_{\tilde{f}} & c_{\tilde{f}}\end{pmatrix}.$$

In the charged Higgs sector, one has:

$$g_{H\tilde{t}_i\tilde{b}_j} = \sum_{i'j'} R^{\tilde{t}}_{ii'}R^{\tilde{b}}_{jj'}g^0_{H\tilde{t}_{i'}\tilde{b}_{j'}}.$$

Similarly, in the neutral Higgs sector, for $\tilde{f}$ stands for either $\tilde{t}$ or $\tilde{b}$, then one obtains the following coupling constants:

$$g_{A^0\tilde{f}_i\tilde{f}_j} = \sum_{i'j'} R^{\tilde{f}}_{ii'}R^{\tilde{f}}_{jj'}g^0_{A^0\tilde{f}_{i'}\tilde{f}_{j'}}$$

$$g_{H^0\tilde{f}_i\tilde{f}_j} = \sum_{i'j'} R^{\tilde{f}}_{ii'}R^{\tilde{f}}_{jj'}g^0_{H^0\tilde{f}_{i'}\tilde{f}_{j'}}$$

$$g_{h^0\tilde{f}_i\tilde{f}_j} = \sum_{i'j'} R^{\tilde{f}}_{ii'}R^{\tilde{f}}_{jj'}g^0_{h^0\tilde{f}_{i'}\tilde{f}_{j'}}.$$

Note that, in the case of the neutral boson A^0, the coupling to a pair of sfermions is the same with or without mixing.

Higgs - Higgs - Higgs

$$g_{H^0HH} = \frac{eM_W}{s_W}\left[\frac{\cos 2\beta\cos(\beta+\alpha)}{2c_W^2} - \cos(\beta-\alpha)\right],$$

$$g_{h^0HH} = -\frac{eM_W}{s_W}\left[\frac{\cos 2\beta\sin(\beta+\alpha)}{2c_W^2} + \sin(\beta-\alpha)\right],$$

$$g_{H^0GG} = -\frac{eM_W}{2s_W c_W^2}\left[\cos 2\beta\cos(\beta+\alpha)\right],$$

$$g_{h^0GG} = \frac{eM_W}{2s_W c_W^2}\left[\cos 2\beta\sin(\beta+\alpha)\right],$$

$$g_{H^0GH} = -\frac{eM_W}{2s_W}\left[\sin(\beta-\alpha) - \frac{\sin 2\beta\cos(\alpha+\beta)}{c_W^2}\right],$$

$$g_{h^0GH} = \frac{eM_W}{2s_W}\left[\cos(\beta-\alpha) - \frac{\sin 2\beta\sin(\alpha+\beta)}{c_W^2}\right],$$

$$g_{A^0G^\pm H^\pm} = Q_G \times \frac{eM_W}{2s_W},$$

$$g_{H^0H^0H^0} = -\frac{3eM_W}{2s_W c_W^2}\cos 2\alpha\cos(\beta+\alpha),$$

$$g_{h^0h^0h^0} = -\frac{3eM_W}{2s_W c_W^2}\cos 2\alpha\sin(\beta+\alpha),$$

$$g_{h^0H^0H^0} = \frac{eM_W}{2s_W c_W^2}\left[2\sin 2\alpha\cos(\beta+\alpha) + \cos 2\alpha\sin(\beta+\alpha)\right],$$

$$g_{H^0h^0h^0} = -\frac{eM_W}{2s_W c_W^2}\left[2\sin 2\alpha\sin(\beta+\alpha) - \cos 2\alpha\cos(\beta+\alpha)\right],$$

$$g_{H^0G^0G^0} = -\frac{eM_W}{2s_W c_W^2}\cos 2\beta\cos(\beta+\alpha),$$

$$g_{h^0G^0G^0} = \frac{eM_W}{2s_W c_W^2}\cos 2\beta\sin(\beta+\alpha),$$

$$g_{H^0A^0A^0} = \frac{eM_W}{2s_W c_W^2}\cos 2\beta\cos(\beta+\alpha),$$

$$g_{h^0A^0A^0} = -\frac{eM_W}{2s_W c_W^2}\cos 2\beta\sin(\beta+\alpha),$$

$$g_{H^0A^0G^0} = \frac{eM_W}{2s_W c_W^2}\sin 2\beta\cos(\beta+\alpha),$$

$$g_{h^0A^0G^0} = -\frac{eM_W}{2s_W c_W^2}\sin 2\beta\sin(\beta+\alpha).$$

Higgs - Higgs - Sfermion - Sfermion

For light unmixed sfermions, the coupling constants are not proportional to the mass of the fermion(s) and can not be neglected. The coupling constants for the heavy sfermions are then obtained by adding a term proportional to the mass of the corresponding fermion(s).

If $\tilde{f}$ is a slepton ($\tilde{\ell}$ or $\tilde{\nu}$) or a squark from the first or second generation ($\tilde{q}$), one has:

$$
\begin{aligned}
g_{H^0H^0\tilde{f}_L\tilde{f}_L} &= \frac{e^2}{2s_W^2}\left[-\frac{\cos 2\alpha}{c_W^2}\left(T_f^3 - Q_f s_W^2\right)\right], \\
g_{H^0H^0\tilde{f}_R\tilde{f}_R} &= \frac{e^2}{2s_W^2}\left[-\frac{\cos 2\alpha}{c_W^2}\left(Q_f s_W^2\right)\right], \\
g_{h^0h^0\tilde{f}_L\tilde{f}_L} &= \frac{e^2}{2s_W^2}\left[\frac{\cos 2\alpha}{c_W^2}\left(T_f^3 - Q_f s_W^2\right)\right], \\
g_{h^0h^0\tilde{f}_R\tilde{f}_R} &= \frac{e^2}{2s_W^2}\left[\frac{\cos 2\alpha}{c_W^2}\left(Q_f s_W^2\right)\right], \\
g_{H^0h^0\tilde{f}_L\tilde{f}_L} &= \frac{e^2}{2s_W^2}\left[\frac{\sin 2\alpha}{c_W^2}\left(T_f^3 - Q_f s_W^2\right)\right], \\
g_{H^0h^0\tilde{f}_R\tilde{f}_R} &= \frac{e^2}{2s_W^2}\left[\frac{\sin 2\alpha}{c_W^2}\left(Q_f s_W^2\right)\right], \\
g_{A^0A^0\tilde{f}_L\tilde{f}_L} &= \frac{e^2}{2s_W^2}\left[\frac{\cos 2\beta}{c_W^2}\left(T_f^3 - Q_f s_W^2\right)\right], \\
g_{A^0A^0\tilde{f}_R\tilde{f}_R} &= \frac{e^2}{2s_W^2}\left[\frac{\cos 2\beta}{c_W^2}\left(Q_f s_W^2\right)\right].
\end{aligned}
$$

For the third generation squarks (stop and sbottom), one has:

$$
\begin{aligned}
g_{H^0H^0\tilde{t}_{L,R}\tilde{t}_{L,R}} &= g_{H^0H^0\tilde{u}_{L,R}\tilde{u}_{L,R}} - \frac{e^2}{2s_W^2}\left(\frac{M_t \sin\alpha}{M_W \sin\beta}\right)^2, \\
g_{h^0h^0\tilde{t}_{L,R}\tilde{t}_{L,R}} &= g_{h^0h^0\tilde{u}_{L,R}\tilde{u}_{L,R}} - \frac{e^2}{2s_W^2}\left(\frac{M_t \cos\alpha}{M_W \sin\beta}\right)^2, \\
g_{H^0h^0\tilde{t}_{L,R}\tilde{t}_{L,R}} &= g_{H^0h^0\tilde{u}_{L,R}\tilde{u}_{L,R}} - \frac{e^2 \sin 2\alpha}{4s_W^2}\left(\frac{M_t}{M_W \sin\beta}\right)^2, \\
g_{A^0A^0\tilde{t}_{L,R}\tilde{t}_{L,R}} &= g_{A^0A^0\tilde{u}_{L,R}\tilde{u}_{L,R}} - \frac{e^2}{2s_W^2}\left(\frac{M_t \cos\beta}{M_W \sin\beta}\right)^2
\end{aligned}
$$

and

$$
\begin{aligned}
g_{H^0H^0\tilde{b}_{L,R}\tilde{b}_{L,R}} &= g_{H^0H^0\tilde{d}_{L,R}\tilde{d}_{L,R}} - \frac{e^2}{2s_W^2}\left(\frac{M_b \cos\alpha}{M_W \cos\beta}\right)^2, \\
g_{h^0h^0\tilde{b}_{L,R}\tilde{b}_{L,R}} &= g_{h^0h^0\tilde{d}_{L,R}\tilde{d}_{L,R}} - \frac{e^2}{2s_W^2}\left(\frac{M_b \sin\alpha}{M_W \cos\beta}\right)^2, \\
g_{H^0h^0\tilde{b}_{L,R}\tilde{b}_{L,R}} &= g_{H^0h^0\tilde{d}_{L,R}\tilde{d}_{L,R}} - \frac{e^2 \sin 2\alpha}{4s_W^2}\left(\frac{M_b}{M_W \cos\beta}\right)^2, \\
g_{A^0A^0\tilde{b}_{L,R}\tilde{b}_{L,R}} &= g_{A^0A^0\tilde{d}_{L,R}\tilde{d}_{L,R}} - \frac{e^2}{2s_W^2}\left(\frac{M_b \sin\beta}{M_W \cos\beta}\right)^2.
\end{aligned}
$$

Appendix B: Passarino-Veltman functions

The calculation of one loop Feynman diagrams can be performed by combining propagators using the following formula:

$$\frac{1}{A_1^{\alpha_1}\cdots A_n^{\alpha_n}} = \frac{\Gamma(\alpha_1+\cdots+\alpha_n)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_n)}\int_0^1 dx_1\cdots dx_n\delta(x_1+\cdots+x_n-1)\frac{x_1^{\alpha_1-1}\cdots x_n^{\alpha_n-1}}{(A_1x_1+\cdots+A_nx_n)^{\alpha_1+\cdots+\alpha_n}}.$$

However, it is often convenient to reduce each one loop diagram to the sum of standard contributions, the so-called Passarino-Veltman (PV) functions.

B.1 Standard definitions

Let us define the 1, 2, 3 and 4 point functions according to:

$$\begin{aligned}
A_0(a) &= \int \frac{d^Dk}{i\pi^2}\frac{1}{N_a},\\
\{B_0, B^\mu, B^{\mu\nu}\}(ab) &= \int \frac{d^Dk}{i\pi^2}\frac{\{1, k^\mu, k^\mu k^\nu\}}{N_aN_b},\\
\{C_0, C^\mu, C^{\mu\nu}\}(abc) &= \int \frac{d^Dk}{i\pi^2}\frac{\{1, k^\mu, k^\mu k^\nu\}}{N_aN_bN_c},\\
\{D_0, D^\mu, D^{\mu\nu}, D^{\mu\nu\rho}\}(abcd) &= \int \frac{d^Dk}{i\pi^2}\frac{\{1, k^\mu, k^\mu k^\nu, k^\mu k^\nu k^\rho\}}{N_aN_bN_cN_d},
\end{aligned}$$

where the denominators are:

$$\begin{aligned}
N_1 &= k^2 - m_1^2 + i\epsilon,\\
N_2 &= (k+p_1)^2 - m_2^2 + i\epsilon,\\
N_3 &= (k+p_1+p_2)^2 - m_3^2 + i\epsilon,\\
N_4 &= (k+p_1+p_2+p_3)^2 - m_4^2 + i\epsilon.
\end{aligned}$$

Here, all integrals are kept D-dimensional. However, the rest of the calculations will be performed in four dimensions.

In one loop diagrams, the following conventions are used:

- the external momenta $p_{1\ldots N}$ are oriented clockwise,
- the internal masses $m_{1\ldots N}$ are oriented clockwise as well, with m_1 between p_N and p_1.

Let K be a multi-index such that:

$$\begin{aligned}
B_K(12) &= B_K(p_1^2, m_1^2, m_2^2),\\
C_K(123) &= C_K(p_1^2, p_2^2, (p_1+p_2)^2, m_1^2, m_2^2, m_3^2),\\
D_K(1234) &= D_K(p_1^2, p_2^2, p_3^2, (p_1+p_2+p_3)^2, (p_1+p_2)^2, (p_2+p_3)^2, m_1^2, m_2^2, m_3^2, m_4^2).
\end{aligned}$$

The reduction of tensorial functions into scalar functions can then be done according to the following standard notations:

$$
\begin{aligned}
B^{\mu}(12) &= p_1^{\mu} B_1(12), \\
B^{\mu\nu}(12) &= p_1^{\mu} p_1^{\nu} B_{21}(12) + g^{\mu\nu} B_{22}(12),
\end{aligned}
$$

$$
\begin{aligned}
C^{\mu}(123) &= p_1^{\mu} C_{11}(123) + p_2^{\mu} C_{12}(123), \\
C^{\mu\nu}(123) &= p_1^{\mu} p_1^{\nu} C_{21}(123) + p_2^{\mu} p_2^{\nu} C_{22}(123) + p_1^{\{\mu} p_2^{\nu\}} C_{23}(123) + g^{\mu\nu} C_{24}(123), \\
C^{\mu\nu\rho}(123) &= (g^{\mu\nu} p_1^{\rho} + g^{\mu\rho} p_1^{\nu} + g^{\nu\rho} p_1^{\mu}) C_{001}(123) \\
&+ (g^{\mu\nu} p_2^{\rho} + g^{\mu\rho} p_2^{\nu} + g^{\nu\rho} p_2^{\mu}) C_{002}(123) \\
&+ p_1^{\mu} p_1^{\nu} p_1^{\rho} C_{111}(123) + p_2^{\mu} p_2^{\nu} p_2^{\rho} C_{222}(123) \\
&+ (p_1^{\mu} p_1^{\nu} p_2^{\rho} + p_1^{\mu} p_2^{\nu} p_1^{\rho} + p_2^{\mu} p_1^{\nu} p_1^{\rho}) C_{112}(123) \\
&+ (p_2^{\mu} p_2^{\nu} p_1^{\rho} + p_2^{\mu} p_1^{\nu} p_2^{\rho} + p_1^{\mu} p_2^{\nu} p_2^{\rho}) C_{122}(123),
\end{aligned}
$$

$$
\begin{aligned}
D^{\mu}(1234) &= p_1^{\mu} D_{11}(1234) + p_2^{\mu} D_{12}(1234) + p_3^{\mu} D_{13}(1234), \\
D^{\mu\nu}(1234) &= p_1^{\mu} p_1^{\nu} D_{21}(1234) + p_2^{\mu} p_2^{\nu} D_{22}(1234) + p_3^{\mu} p_3^{\nu} D_{23}(1234) \\
&+ p_1^{\{\mu} p_2^{\nu\}} D_{24}(1234) + p_1^{\{\mu} p_3^{\nu\}} D_{25}(1234) + p_2^{\{\mu} p_3^{\nu\}} D_{26}(1234) + g^{\mu\nu} D_{27}(1234), \\
D^{\mu\nu\rho}(1234) &= (g^{\mu\nu} p_1^{\rho} + g^{\nu\rho} p_1^{\mu} + g^{\mu\rho} p_1^{\nu}) D_{001}(1234) \\
&+ (g^{\mu\nu} p_2^{\rho} + g^{\nu\rho} p_2^{\mu} + g^{\mu\rho} p_2^{\nu}) D_{002}(1234) \\
&+ (g^{\mu\nu} p_3^{\rho} + g^{\nu\rho} p_3^{\mu} + g^{\mu\rho} p_3^{\nu}) D_{003}(1234) \\
&+ \sum_{1 \leq ijk \leq 3} p_i^{\mu} p_j^{\nu} p_k^{\rho} D_{ijk}(1234).
\end{aligned}
$$

In the reduction of $D^{\mu\nu\rho}$, the sum is over all triplets (i, j, k) with repetitions, i.e. $3^3 = 27$ terms. By construction, the coefficients D_{ijk} are invariant under index permutations.

More details about this approach and about the reduction of the PV tensorial integrals into scalar ones can be found in [19].

B.2 `LoopTools` definitions

Sometimes, as for instance in the `LoopTools` library [16], it is convenient to use another notation and to introduce momenta k_i given by:

$$
\begin{aligned}
k_1 &= p_1, \\
k_2 &= p_1 + p_2, \\
k_3 &= p_1 + p_2 + p_3 \ldots \\
k_N &= \sum_{i=1}^{N} p_i
\end{aligned}
$$

In that case, the tensorial coefficients are:

$$
\begin{aligned}
B^{\mu} &= k_1^{\mu} B_1^L, \\
B^{\mu\nu} &= k_1^{\mu} k_1^{\nu} B_{11}^L + g^{\mu\nu} B_{00}^L.
\end{aligned}
$$

$$
\begin{aligned}
C^\mu &= k_1^\mu C_1^L + k_2^\mu C_2^L, \\
C^{\mu\nu} &= \sum_{ij=1,2} k_i^\mu k_j^\nu C_{ij}^L + g^{\mu\nu} C_{00}^L, \\
C^{\mu\nu\rho} &= \sum_{ijl=1,2} k_i^\mu k_j^\nu k_l^\rho C_{ijl}^L + \sum_{i=1,2} (g^{\mu\nu} k_i^\rho + g^{\mu\rho} k_i^\nu + g^{\nu\rho} k_i^\mu) C_{00i}^L,
\end{aligned}
$$

$$
\begin{aligned}
D^\mu &= k_1^\mu D_1^L + k_2^\mu D_2^L + k_3^\mu D_3^L, \\
D^{\mu\nu} &= \sum_{1 \le ijk \le 3} k_i^\mu k_j^\nu D_{ij}^L + g^{\mu\nu} D_{00}^L,
\end{aligned}
$$

where C_{ij}^L, C_{ijl}^L and D_{ij}^L are completely symmetric.

By expanding these equations and by then comparing all their terms to those arising from the reduction of standard tensorial functions, one can find the relations that exist between the standard PV functions and those which are computed in the `LoopTools` library.

For the 2 point functions, one obtains:

$$
\begin{aligned}
B_1 &= B_1^L \\
&\bullet \\
B_{21} &= B_{11}^L \\
B_{22} &= B_{00}^L
\end{aligned}
$$

For the 3 point functions, one obtains:

$$
\begin{aligned}
C_{11} &= C_1^L + C_2^L \\
C_{12} &= C_2^L \\
&\bullet \\
C_{21} &= C_{11}^L + 2C_{12}^L + C_{22}^L \\
C_{22} &= C_{22}^L \\
C_{23} &= C_{12}^L + C_{22}^L \\
C_{24} &= C_{00}^L \\
&\bullet \\
C_{001} &= C_{001}^L + C_{002}^L \\
C_{002} &= C_{002}^L \\
&\bullet \\
C_{111} &= C_{111}^L + 3C_{112}^L + 3C_{122}^L + C_{222}^L \\
C_{222} &= C_{222}^L \\
C_{112} &= C_{112}^L + 2C_{122}^L + C_{222}^L \\
C_{122} &= C_{122}^L + C_{222}^L
\end{aligned}
$$

For the 4 point functions, one obtains:

$$
\begin{aligned}
D_{11} &= D_1^L + D_2^L + D_3^L \\
D_{12} &= D_2^L + D_3^L \\
D_{13} &= D_3^L \\
&\bullet \\
D_{21} &= D_{11}^L + D_{22}^L + D_{33}^L + 2(D_{12}^L + D_{13}^L + D_{23}^L) \\
D_{22} &= D_{22}^L + 2D_{23}^L + D_{33}^L \\
D_{23} &= D_{33}^L \\
D_{24} &= D_{12}^L + D_{13}^L + D_{22}^L + 2D_{23}^L + D_{33}^L \\
D_{25} &= D_{13}^L + D_{23}^L + D_{33}^L \\
D_{26} &= D_{23}^L + D_{33}^L \\
D_{27} &= D_{00}^L \\
&\bullet \\
D_{001} &= D_{001}^L + D_{002}^L + D_{003}^L \\
D_{002} &= D_{002}^L + D_{003}^L \\
D_{003} &= D_{003}^L \\
&\bullet \\
D_{111} &= D_{111}^L + 3D_{112}^L + 3D_{113}^L + 3D_{122}^L + 6D_{123}^L + 3D_{133}^L + D_{222}^L + 3D_{223}^L + 3D_{233}^L + D_{333}^L \\
D_{112} &= D_{112}^L + D_{113}^L + 2D_{122}^L + 4D_{123}^L + 2D_{133}^L + D_{222}^L + 3D_{223}^L + 3D_{233}^L + D_{333}^L \\
D_{113} &= D_{113}^L + 2D_{123}^L + 2D_{133}^L + D_{223}^L + 2D_{233}^L + D_{333}^L \\
D_{122} &= D_{122}^L + 2D_{123}^L + D_{133}^L + D_{222}^L + 3D_{223}^L + 3D_{233}^L + D_{333}^L \\
D_{133} &= D_{133}^L + D_{233}^L + D_{333}^L \\
D_{123} &= D_{123}^L + D_{133}^L + D_{223}^L + 2D_{233}^L + D_{333}^L \\
&\bullet \\
D_{222} &= D_{222}^L + 3D_{223}^L + 3D_{233}^L + D_{333}^L \\
D_{223} &= D_{223}^L + 2D_{233}^L + D_{333}^L \\
D_{233} &= D_{233}^L + D_{333}^L \\
&\bullet \\
D_{333} &= D_{333}^L
\end{aligned}
$$

References

[1] S. Heinemeyer, CERN-PH-TH/2004-139; LMU 09/04; hep-ph/0407244.

[2] P.H. Chankowski, S. Pokorski and J. Rosiek, Nucl. Phys. B423 (1994) 497.

[3] V. Driesen and W. Hollik, Z. Phys. C68 (1995) 485; hep-ph/9504335.

[4] V. Driesen, W. Hollik and J. Rosiek, Z. Phys. C71 (1996) 259; hep-ph/9512441.

[5] S. Heinemeyer, W. Hollik, J. Rosiek and G. Weiglein, Eur. Phys. J. C19 (2001) 535; hep-ph/0102081.

[6] S. Heinemeyer and G. Weiglein, Nucl. Phys. Proc. Suppl. 89 (2000) 210; hep-ph/0102177.

[7] A. Arhrib and G. Moultaka, Nucl. Phys. B558 (1999) 3; hep-ph/9808317.

[8] J. Guasch, W. Hollik and A. Kraft, Nucl. Phys. B596 (2001) 66; hep-ph/9911452.

[9] M. Beccaria, M. Melles, F.M. Renard and C. Verzegnassi, Phys. Rev. D65 (2002) 093007.

[10] A. Djouadi, J. Kalinowski, P. Ohmann and P.M. Zerwas, Z. Phys. C74 (1997) 93.

[11] G. Degrassi and A. Sirlin, Nucl. Phys. B383 (1992) 73, Phys. Rev. D46 (1992) 3104.

[12] M. Böhm, W. Hollik and H. Spiesberger, Fortschr. Phys. 34 (1986) 687; W. Hollik, Fortschr. Phys. 38 (1990) 165.

[13] A. Dabelstein, hep-ph/9409375.

[14] G.J. Gounaris, J. Layssac and F.M. Renard, hep-ph/0207273; LC-TH-2002-011, Phys. Rev. D67 (2003) 013012.

[15] W. Hollik and C. Schappacher, Nucl. Phys. B545 (1999) 98; hep-ph/9807427.

[16] T. Hahn, M. Perez-Victoria, Comput. Phys. Commun. 118 (1999) 153; hep-ph/9807565.

[17] S. Heinemeyer, hep-ph/0407244; M. Frank, S. Heinemeyer, W. Hollik and G. Weiglein, hep-ph/0212037; S. Heinemeyer, hep-ph/0108059.

[18] J. Rosiek, Phys. Rev. D41 (1990) 3464; hep-ph/9511250.

[19] G. Passarino and M.J.G. Veltman, Nucl. Phys. B160 (1979) 151.