Track Parameters in LCIO

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Abstract

A precise definition of the track parameters used for detector concept studies at the International Linear Collider [2] (ILC) is presented.

1 Introduction

Even though the four different detector concept studies for the International Linear Collider [2] (ILC) are still ongoing at the time this document is prepared, it is important to have a unique and generally accepted way of describing tracks of charged particles travelling through the detector. Together with the definition of a coordinate system for the LDC [3] this has various advantages:

- simulation tools can be used without adaptation,
- data can be exchanged without conversion, and
- results can be compared without reinterpretation.

The following document offers a definition of the track parameters in a concise, yet general way.

The basis for the parametrisation of tracks is the LDC coordinate system as defined in [3]. It is Cartesian and right-handed with its origin located at the nominal point of interaction. Let $\hat{\boldsymbol{p}}_{\mathrm{e}^-}$ ($\hat{\boldsymbol{p}}_{\mathrm{e}^+}$) be the direction of the three-momentum of the incoming electrons (positrons). The z axis lies along the mean beam direction which is the bisecting line of the (smaller) angle between $\hat{\boldsymbol{p}}_{\mathrm{e}^-}$ and $\hat{\boldsymbol{p}}_{\mathrm{e}^+}$. In general the z axis is parallel to $\hat{\boldsymbol{p}}_{\mathrm{e}^-} - \hat{\boldsymbol{p}}_{\mathrm{e}^+}$. In case of a head-on geometry, the z axis is parallel to $\hat{\boldsymbol{p}}_{\mathrm{e}^-}$. The y axis lies along the vertical direction, pointing upwards. In the following this coordinate system is referred to as the reference system. An arbitrary vector $\boldsymbol{v} = (v_x, v_y, v_z)$ in the reference system can also be expressed in spherical coordinates $|\boldsymbol{v}|$, θ, and ϕ with the transformation:

$$\boldsymbol{v} = \begin{pmatrix} |\boldsymbol{v}| & \sin\theta\ \cos\phi \\ |\boldsymbol{v}| & \sin\theta\ \sin\phi \\ |\boldsymbol{v}| & \cos\theta \end{pmatrix} \qquad \text{with} \qquad \begin{array}{ccc} \theta & \in & [0,\pi] \\ \phi & \in &]-\pi,\pi] \end{array} \tag{1}$$

where θ is the polar and ϕ the azimuthal angle.

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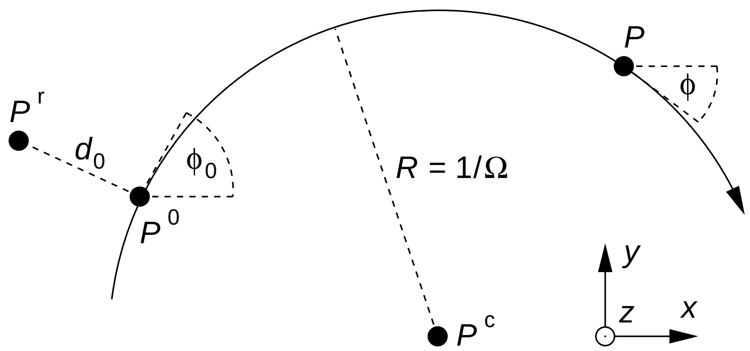

Figure 1: The projection of a helix segment in the xy plane is a part of an arc with centre $\boldsymbol{P}^{\mathrm{c}}$ and radius R. The direction of the particle is shown with the arrow at the arc. All track parameters are given relative to the reference point $\boldsymbol{P}^{\mathrm{r}}$.

2 Parametrisation of a Track

Whenever a charged particle is affected by a constant magnetic field it moves on a helicoidal trajectory, where here and in the following both energy loss and multiple scattering are neglected. It is assumed that this magnetic field is homogeneous and parallel to the z axis of the reference system. In this case the trajectory of a charged particle is a segment of a circle in the xy projection and the z displacement is a linear function of the length s of the arc that is described in the xy plane. This results in a straight line in the sz plane.

The parametrisation of the movement of a charged particle is defined by a reference point, $\boldsymbol{P}^{\mathrm{r}} = (P^{\mathrm{r}}_x, P^{\mathrm{r}}_y, P^{\mathrm{r}}_z)$, and five so-called 'track parameters' (Ω, ϕ_0, d_0, z_0 and $\tan\lambda$). In general the reference point can be any[1] point in space. The five track parameters refer to a specific point $\boldsymbol{P}^0 = (P^0_x, P^0_y, P^0_z)$ along the helix. In this text $\boldsymbol{P}^0$ is the point of closest approach (p. c. a.) to the reference point in the xy plane (see Figure 1).

2.1 xy Plane

In the xy plane the movement of a charged particle is defined by a reference point, $\boldsymbol{P}^{\mathrm{r}} = (P^{\mathrm{r}}_x, P^{\mathrm{r}}_y)$, and three parameters, Ω, ϕ_0, and d_0 (see Figure 1):

- ϕ_0 is the azimuthal angle of the momentum of the particle (track tangent) at the p. c. a.

- $\Omega \in \,]-\infty, \infty[$ describes the curvature of the track with

$$|\Omega| = \frac{1}{R} \tag{2}$$

[1]For convenience, often the reference point is choose to be $\boldsymbol{P}^{\mathrm{r}} = (0, 0, 0)$. To get the track parameters of a given point of the track, it is required, that $\boldsymbol{P}^{\mathrm{r}}$ can be chosen to be arbitrary.

where R is the radius of curvature of the track. The sign of Ω is defined by moving along the track, following the direction of the particle's momentum (see Section 3). Passing through the arc in (anti)clock-wise direction defines positive (negative) curvature. In case of an axial magnetic field parallel to the z axis ($\boldsymbol{B} = (0, 0, B_z)$, $B_z > 0$), $\Omega > 0$ ($\Omega < 0$) corresponds to a particle with positive (negative) electric charge.

- $d_0 \in]-\infty, \infty[$ is the signed impact parameter in the xy plane. Let $\boldsymbol{d} = (d_x, d_y)$ be the vector in the xy plane pointing from the reference point $\boldsymbol{P}^{\mathrm{r}}$ to the p. c. a. $\boldsymbol{P}^0$

$$\boldsymbol{d} = \boldsymbol{P}^0 - \boldsymbol{P}^{\mathrm{r}}$$

and

$$\boldsymbol{n}_{\mathrm{pca}} = \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}$$

be the normal vector of the track in the xy plane at the p. c. a., then d_0 is the projection of $\boldsymbol{d}$ onto $\boldsymbol{n}_{\mathrm{pca}}$:

$$d_0 = \boldsymbol{n}_{\mathrm{pca}} \cdot \boldsymbol{d} = -(P^{\mathrm{r}}_x - P^0_x)\sin\phi_0 + (P^{\mathrm{r}}_y - P^0_y)\cos\phi_0 \quad . \tag{3}$$

Since $|\boldsymbol{n}_{\mathrm{pca}}| = 1$, $|d_0|$ is the distance between $\boldsymbol{P}^{\mathrm{r}}$ and $\boldsymbol{P}^0$ in the xy plane. The signing convention is defined as follows: Looking from the reference point to the p. c. a., then $d_0 > 0$ ($d_0 < 0$) if the particle travels from left to right (right to left). This results in $\mathrm{sgn}(\Omega) = \mathrm{sgn}(d_0)$ if $\boldsymbol{P}^{\mathrm{r}}$ is inside and conversely in $\mathrm{sgn}(\Omega) = -\mathrm{sgn}(d_0)$ if $\boldsymbol{P}^{\mathrm{r}}$ is outside the arc.

The centre point $\boldsymbol{P}^{\mathrm{c}} = (P^{\mathrm{c}}_x, P^{\mathrm{c}}_y)$ of the circle in the xy plane is usually different from the reference point $\boldsymbol{P}^{\mathrm{r}}$. If ϕ is defined as the azimuthal angle of the momentum vector at a given point $\boldsymbol{P} = (P_x, P_y, P_z)$ on the track, the coordinates of $\boldsymbol{P}^{\mathrm{c}}$ can be calculated as:

$$P^{\mathrm{c}}_x = P_x + \frac{\sin\phi}{\Omega} \tag{4}$$

$$P^{\mathrm{c}}_y = P_y - \frac{\cos\phi}{\Omega} \tag{5}$$

With $P^0_x - P^{\mathrm{r}}_x = -d_0 \sin\phi_0$ $P^0_y - P^{\mathrm{r}}_y = d_0 \cos\phi_0$, Eq. 4, Eq. 5 and the definition of Ω, the coordinates of $\boldsymbol{P}^{\mathrm{c}}$ can be expressed using only the track parameters at $\boldsymbol{P}^0$:

$$P^{\mathrm{c}}_x = P^{\mathrm{r}}_x + \left(\frac{1}{\Omega} - d_0\right) \cdot \sin\phi_0 \tag{6}$$

$$P^{\mathrm{c}}_y = P^{\mathrm{r}}_y - \left(\frac{1}{\Omega} - d_0\right) \cdot \cos\phi_0 \tag{7}$$

In this case $\boldsymbol{P}$ turns into $\boldsymbol{P}^0$, which can be expressed using d_0 and ϕ_0.

2.2 sz Plane

In the sz plane a charged particle moves along a straight line, which is described by two parameters, $\tan\lambda$ and z_0 (see Figure 2):

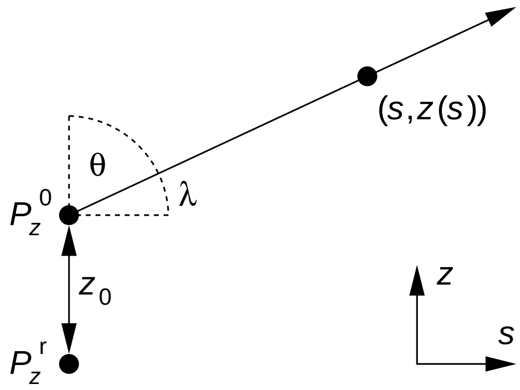

Figure 2: The projection of a helix in the sz plane is a straight line (see Eq. 10). The variable s at a point $\boldsymbol{P}$ is the arc length in the xy plane from $\boldsymbol{P}^0$ to $\boldsymbol{P}$. This also implies that $s = 0$, if $z = z_0$.

- $\tan\lambda$ is the slope $\mathrm{d}z/\mathrm{d}s$ of the straight line in the sz plane. This parameter is constant for a given track and it is directly related to the polar angle θ of the momentum vector $\boldsymbol{p} = (p_x, p_y, p_z)$:

$$\tan\lambda = \frac{p_z}{\sqrt{p_x^2 + p_y^2}} = \cot\theta \tag{8}$$

- $z_0 \in \,]-\infty, \infty[$ is the z position of the track at the p. c. a. with respect to the z coordinate of the reference point P_z^{r}:

$$z_0 = P_z^0 - P_z^{\mathrm{r}} \tag{9}$$

The equation of the trajectory of a helicoidal track in the sz projection is then

$$z = (z_0 + P_z^{\mathrm{r}}) + s \cdot \tan\lambda \tag{10}$$

where s is the path integral (i.e. the arc length) in the xy projection when a particle travels from $\boldsymbol{P}^0$ to $\boldsymbol{P}$. s is positive (negative) if $\boldsymbol{P}$ is located in the direction (against the direction) of the momentum with respect to the p. c. a. $\boldsymbol{P}^0$.

3 Remarks

1. During the track reconstruction, the direction of the particle momentum is not known, therefore it is expected to go along the track from the origin towards the outside of the detector. This hypothesis is used in the early stages of the reconstruction to determine the sign of Ω. At this time, particles travelling in the opposite direction (curling or backscattered particles) get the wrong sign of Ω and therefore seem to have incorrect charge. This is corrected at a later stage, when the flight direction of

the particle is determined. Sometimes it is not clear in which direction a particle moves along the track. In these cases a track has two equivalent representations (ϕ_0, Ω, d_0, $\tan\lambda$, z_0 and ϕ'_0, Ω', d'_0, $\tan\lambda'$, z'_0), which are related to each other by:

$$\begin{aligned} \phi'_0 &= (\phi_0 + \pi \bmod 2\pi) \\ \Omega' &= -\Omega \\ d'_0 &= -d_0 \\ \tan\lambda' &= -\tan\lambda = -\cot\theta \text{ with } \quad \lambda' = -\lambda, \theta' = \pi - \theta \\ z'_0 &= z_0 \end{aligned} \tag{11}$$

Both describe the same helix, with the particle travelling in opposite directions.

2. The previous remark directly implies that the sign of Ω does not give the chirality (handedness) of the helix.

3. As stated in Section 2, it is common to define the track parameters in terms of the origin of the reference system. Here they would then be given with respect to the nominal point of interaction. For analysis it is better to work with track parameters which are determined with respect to the primary event vertex or to a secondary vertex from which they come. Converting the track parameters to the so called 'corrected track parameters', which refer to $\boldsymbol{P}^{\mathrm{v}} = (P^{\mathrm{v}}_x, P^{\mathrm{v}}_y, P^{\mathrm{v}}_z)$ changes the parameters ϕ_0, d_0, and z_0 to ϕ^{v}_0, d^{v}_0, and z^{v}_0.

4. The free choice of the reference point, $\boldsymbol{P}^{\mathrm{r}}$ allows for the determination of the track parameters at any point on the track. Usually it is convenient to chose the reference point to be a point on the track (e.g. the point a particle enters or leaves a subdetector).

4 Physics Track Variables

The five geometrical track parameters (Ω, ϕ_0, d_0, z_0 and $\tan\lambda$), as defined above, are used to get the physics track variables (momentum $\boldsymbol{p}$ and charge q). In the presence of an axial magnetic field, the curvature Ω of the helix only contains information about the transverse momentum p_{T} with respect to the beam direction:

$$p_{\mathrm{T}} = a \left| \frac{B_z}{\Omega} \right| \tag{12}$$

If p_{T} is measured in GeV/c, B_z in T, Ω in mm^{-1}, and c in mm/s, then:

$$a = c \times 10^{-15} \simeq 3 \times 10^{-4}$$

Now p_{T} is used to calculate the absolute value of the momentum $p = |\boldsymbol{p}|$:

$$p = \frac{p_{\mathrm{T}}}{\cos\lambda} = p_{\mathrm{T}}\sqrt{1 + \tan^2\lambda} \tag{13}$$

This results in the momentum vector $\boldsymbol{p} = (p_x, p_y, p_z)$ with the components:

$$p_x = p_{\mathrm{T}} \cos\phi_0 = p \cos\phi_0 \sin\theta$$

$$
\begin{aligned}
p_y &= p_{\mathrm{T}} \sin\phi_0 = p \sin\phi_0 \sin\theta \\
p_z &= p_{\mathrm{T}} \tan\lambda = p \cos\theta
\end{aligned}
$$

The electric charge q is obtained from the sign of the curvature Ω and the direction (sign of) of the magnetic field B_z:

$$q = \mathrm{sgn}\left(\frac{B_z}{\Omega}\right) \tag{14}$$

5 Concluding Remarks

The track parametrisation as presented in this document has already been used by the L3 collaboration [5] at LEP. LCIO [4] already offers methods in the `Track` class to store and retrieve these parameters and the reference point they refer to. Furthermore LCIO also proposes the units (θ and ϕ as in Eq. 1, d_0 [mm], z_0 [mm], and Ω [1/mm], as well as P_x^{r} [mm], P_y^{r} [mm], P_z^{r} [mm]), in which the parameters should be given to ensure compatibility. In addition the covariance matrix

$$\mathbf{Cov} = \mathrm{Cov}(i, j) \quad \text{with} \quad i, j \in \{d_0, \phi_0, \Omega, z_0, \tan\lambda\} \tag{15}$$

can be accessed. The order of the parameters in the rows and columns of the matrix equals the order of the parameters in Eq. 15. Only the lower triangle of the matrix is stored in a vector where the order of the elements is: $\mathrm{Cov}(d_0, d_0)$, $\mathrm{Cov}(\phi_0, d_0)$, $\mathrm{Cov}(\phi_0, \phi_0)$, $\mathrm{Cov}(\Omega, d_0)$, ...

References

[1] LDC Web Site, `www.ilcldc.org`

[2] ILC Web Site, `www.linearcollider.org`

[3] A. Vogel: The Coordinate System for LDC Detector Studies, LC-DET-2005-009 (2005)

[4] LCIO Web Site, `lcio.desy.de`

[5] J. Alcaraz: Helicoidal Tracks, L3-Note 1666 (1995)