

Thrust distribution resummation in e^+e^- collisions.

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In this talk [1] we report on the recent progresses on IR logarithms resummation for the Thrust distribution in e^+e^- collisions. Using renormalisation group (RG) evolution in Laplace space, the resummation of logarithmically enhanced corrections is performed to next-to-next-to-leading logarithmic (NNLL) accuracy. To combine the resummed expressions with the fixed-order results, we derive the $\log(R)$ -matching and R -matching of the NNLL approximation to the fixed-order NNLO distribution.

1 Introduction

Event-shapes are observables which measure the geometrical properties of energy-momentum flow in a hadronic final state. They have been precisely measured over a broad range in energies at electron-positron colliders. The event-shape distributions allow for a detailed probe of the dynamics of QCD and especially for a precise determination of the strong coupling constant α_s . Owing to their infrared and collinear safety, they can be computed systematically in perturbation theory. The fixed-order description, based on a power series expansion of the distribution in the strong coupling constant, is reliable over most of the kinematical range of the event-shape. In the dijet limit, which is attained for the thrust variable [2] as $T \rightarrow 1$, the convergence of the fixed-order expansion is spoiled by large logarithmic terms $\log(1 - T)$ at each order in the strong coupling constant, thus it necessitates a resummed description. During LEP times, precision studies of a standard set of six event-shapes were based on the combination of fixed-order NLO calculations [3–10] with NLL resummation [11–13]. To avoid the double counting of terms, both expansions need to be matched to each other according to matching procedures such as the R and $\log(R)$ schemes [14]. In the recent past, substantial progress was made both on the fixed-order and the resummed description of event-shapes. Following the development of new methods for calculations of QCD jet observables at NNLO [15], the NNLO corrections to $e^+e^- \rightarrow 3$ jets and related event-shape observables were computed [16–21]. More recently, in the context of Soft-Collinear-Effective-Theory, the resummation for thrust [24, 25] and the heavy jet mass [26] beyond NLL has been performed and applied for a precise determination of α_s , and the framework for the resummation of the jet broadening distributions has been outlined [27, 28]. In these calculations, the $\mathcal{O}(\alpha_s^2)$ soft corrections were determined only up to a constant term by exploiting the renormalisation group invariance of the cross section. Such term is also needed to unambiguously match the resummed distribution to the NNLO result in the R scheme. In this talk we report on the direct computation of these corrections and we provide a new resummed formula. Finally we match the latter to the existing NNLO prediction comparing two different matching schemes.

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2 Fixed-order and resummed distributions

The differential thrust distribution in perturbation theory is numerically known at NNLO [17, 20]. At a centre-of-mass energy Q and for a renormalisation scale μ it reads

$$\begin{aligned} \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau, Q) &= \bar{\alpha}_s(\mu) \frac{dA}{d\tau}(\tau) + \bar{\alpha}_s^2(\mu) \frac{dB}{d\tau}(\tau, x_\mu) \\ &\quad + \bar{\alpha}_s^3(\mu) \frac{dC}{d\tau}(\tau, x_\mu) + \mathcal{O}(\bar{\alpha}_s^4), \end{aligned} \quad (1)$$

where we defined

$$\bar{\alpha}_s = \frac{\alpha_s}{2\pi}, \quad x_\mu = \frac{\mu}{Q}, \quad (2)$$

and where σ is the total perturbative hadronic cross-section for $e^+e^- \rightarrow \text{hadrons}$. The explicit dependence on the renormalisation scale is given by

$$\frac{dB}{d\tau}(\tau, x_\mu) = \frac{dB}{d\tau}(\tau) + 2\beta_0 \log(x_\mu^2) \frac{dA}{d\tau}(\tau), \quad (3)$$

$$\begin{aligned} \frac{dC}{d\tau}(\tau, x_\mu) &= \frac{dC}{d\tau}(\tau) + 2 \log(x_\mu^2) (2\beta_0 \frac{dB}{d\tau}(\tau) \\ &\quad + 2\beta_1 \frac{dA}{d\tau}(\tau)) + (2\beta_0 \log(x_\mu^2))^2 \frac{dA}{d\tau}(\tau). \end{aligned} \quad (4)$$

The QCD β -function is defined by the renormalisation group equation for the QCD coupling constant

$$\frac{d\alpha_s(\mu)}{d \log \mu^2} = -\alpha_s(\mu) \left(\frac{\alpha_s(\mu)}{\pi} \beta_0 + \frac{\alpha_s^2(\mu)}{\pi^2} \beta_1 + \dots \right). \quad (5)$$

The normalised thrust cross-section is then defined as

$$R_T(\tau) \equiv \frac{1}{\sigma} \int_0^1 \frac{d\sigma(\tau', Q)}{d\tau'} \Theta(\tau - \tau') d\tau', \quad (6)$$

where σ is the total cross section for $e^+e^- \rightarrow \text{hadrons}$. In the two-jet region the fixed-order thrust distribution is enhanced by large infrared logarithms which spoil the convergence of the perturbative series. The convergence can be restored by resumming the logarithms to all orders in the coupling constant. The matched cross section can in general be written as

$$R_T(\tau) = C(\alpha_s) \Sigma(\tau, \alpha_s) + D(\tau, \alpha_s), \quad (7)$$

where

$$C(\alpha_s) = 1 + \sum_{k=1}^{\infty} C_k \bar{\alpha}_s^k, \quad (8)$$

$$\begin{aligned} \log \Sigma(\tau, \alpha_s) &= \sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \bar{\alpha}_s^n L^m \\ &= Lg_1(\alpha_s L) + g_2(\alpha_s L) + \frac{\alpha_s}{\pi} \beta_0 g_3(\alpha_s L) + \dots \end{aligned} \quad (9)$$

where $L \equiv \log(1/\tau)$. The function g_1 encodes all the leading logarithms, the function g_2 resums all next-to-leading logarithms and so on. The constant terms C_i are required to achieve a full N^{1+i} LL accuracy. $D(\tau, \alpha_s)$ is a remainder function that vanishes order-by-order in perturbation theory in the dijet limit $\tau \rightarrow 0$.

In view of matching the NNLL resummed distribution to the NNLO fixed order prediction using the R -matching scheme, we need to include the logarithmically subleading terms C_2 , C_3 and G_{31} in the expansions (8),(9).

The resummation of the thrust distribution beyond NLL was first achieved in [24] using an effective-theory approach and revisited in [29], where the full analytic expressions for the $\mathcal{O}(\bar{\alpha}_s^2)$ constant term C_2 and the coefficient G_{31} were derived. The $\mathcal{O}(\bar{\alpha}_s^3)$ constant term C_3 is currently unknown, and a numerical estimate is given in [29] together with the full analytic expressions of the functions $g_i(\alpha_s L)$.

2.1 Factorisation and Resummation

Factorisation properties of event-shapes have been widely studied in the literature [34–36]. Referring to Fig. 1 we recast the cross section (6) as

$$R_T(\tau) = H\left(\frac{Q}{\mu}, \alpha_s(\mu)\right) \int dk^2 d\bar{k}^2 \mathcal{J}\left(\frac{k}{\mu}, \alpha_s(\mu)\right) \bar{\mathcal{J}}\left(\frac{\bar{k}}{\mu}, \alpha_s(\mu)\right) \times \int dw \mathcal{S}\left(\frac{w}{\mu}, \alpha_s(\mu)\right) \Theta(Q^2\tau - \bar{k}^2 - k^2 - wQ) + \mathcal{O}(\tau), \quad (10)$$

where we neglected terms of order $\mathcal{O}(\tau)$ which are absorbed in the remainder function $D(\tau, \alpha_s)$. We use the integral representation of the Θ -function

$$\Theta(Q^2\tau - \bar{k}^2 - k^2 - wQ) = \frac{1}{2\pi i} \int_C \frac{d\nu}{\nu} e^{\nu\tau Q^2} e^{-\nu k^2} e^{-\nu\bar{k}^2} e^{-\nu wQ}, \quad (11)$$

and the Laplace transform to recast Eq. (10) as

$$R_T(\tau) = H\left(\frac{Q}{\mu}, \alpha_s(\mu)\right) \frac{1}{2\pi i} \int_C \frac{dN}{N} e^{\tau N} \tilde{\mathcal{J}}^2\left(\sqrt{\frac{N_0}{N}} \frac{Q}{\mu}, \alpha_s(\mu)\right) \tilde{\mathcal{S}}\left(\frac{N_0}{N} \frac{Q}{\mu}, \alpha_s(\mu)\right) \quad (12)$$

where we set $N = \nu Q^2$ and $N_0 = e^{-\gamma_E}$. The soft subprocess $\tilde{\mathcal{S}}(N_0/NQ/\mu, \alpha_s(\mu))$ describes the interaction between the two jets of hard collinear particles through soft gluon exchange. It can be therefore defined in a gauge invariant way as a correlator of Wilson lines

$$\tilde{\mathcal{S}}\left(\frac{N_0}{N} \frac{Q}{\mu}, \alpha_s(\mu)\right) = \frac{Q}{N_c} \int d\tau_s e^{-\tau_s N} \sum_{k_{eik}} \langle 0 | W_n^\dagger(0) W_n^\dagger(0) | k_{eik} \rangle \mathcal{J}_{cut}(\tau_s Q) \langle k_{eik} | W_n(0) W_{\bar{n}}(0) | 0 \rangle, \quad (13)$$

where we defined $\tau_s = w/Q$. W_n and $W_{\bar{n}}$ are Wilson lines

$$W_n(y) = \mathbf{P} \exp\left(ig \int_0^\infty ds n \cdot A(ns + y)\right), \quad (14)$$

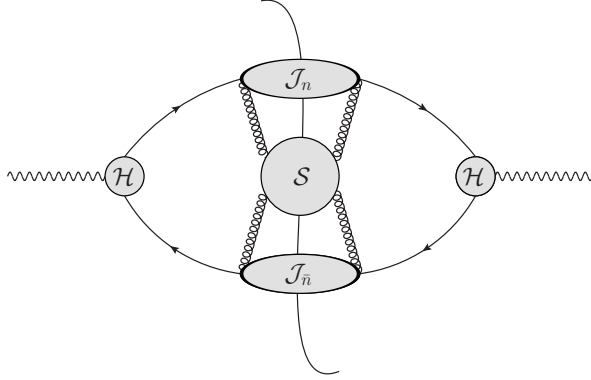


Figure 1: Leading regions in dijet factorisation.

describing the eikonal interaction of soft gluons with the fast moving quarks along the light-like directions n^μ and \bar{n}^μ respectively. $A(ns+y)$ in eq. (14) denotes the gluon field in QCD. The sum runs over the final states $|k_{eik}\rangle$ involving k soft particles whose phase space is constrained according to the thrust trigger function $\mathcal{J}_{cut}(\tau Q^2)$. Both soft and soft-collinear contributions are encoded into the soft subprocess. The two-loop expression was computed analytically in [29] by performing direct phase-space cuts. The results are in agreement with those presented in [32,33]. The collinear subprocess \mathcal{J} ($\bar{\mathcal{J}}$) describes the decay of the jet-initiating hard quark (antiquark) into a jet of collinear particles moving along the n^μ (\bar{n}^μ) direction. It is therefore an inclusive quantity which can be found in many other relevant QCD processes such as deep inelastic scattering and heavy quarks decay [22,30,37]. The short-distance hard function $H(Q/\mu, \alpha_s(\mu)) = |\mathcal{H}(Q/\mu, \alpha_s(\mu))|^2$ takes into account the hard virtual corrections to the quark-antiquark production subprocess. It is free of large logarithms and it can be generally defined such that Eq. (10) reproduces the fixed-order cross section up to power suppressed terms.

Using the Renormalisation Group evolution of the soft and collinear subprocesses [29,30,37], we can recast eq. (12) as

$$R_T(\tau) = H\left(\frac{Q}{\mu}, \alpha_s(\mu)\right) \frac{1}{2\pi i} \int_C \frac{dN}{N} e^{\tau N} \tilde{J}^2\left(1, \alpha_s\left(\sqrt{\frac{N_0}{N}}Q\right)\right) \tilde{S}\left(1, \alpha_s\left(\frac{N_0 Q}{N}\right)\right) \times \exp\left\{-2 \int_{\frac{N_0}{N}}^1 \frac{du}{u} \left[\int_{u^2 Q^2}^{u Q^2} \frac{dk^2}{k^2} \mathcal{A}(\alpha_s(k^2)) + \mathcal{B}(\alpha_s(u Q^2)) \right]\right\}, \quad (15)$$

where the two coefficients $\mathcal{A}(\alpha_s)$ and $\mathcal{B}(\alpha_s)$ can be computed in perturbation theory. The coefficient $\mathcal{A}(\alpha_s)$ reads

$$\mathcal{A}(\alpha_s) = \Gamma_{\text{cusp}}(\alpha_s) - \beta(\alpha_s) \frac{\partial \Gamma_{\text{soft}}(\alpha_s)}{\partial \alpha_s}, \quad (16)$$

where $\Gamma_{\text{cusp}}(\alpha_s)$ and $\Gamma_{\text{soft}}(\alpha_s)$ are the cusp and the soft anomalous dimensions respectively. The former, together with the coefficient $\mathcal{B}(\alpha_s)$ can be extracted from the asymptotic limit

of the $P_{qq}(\alpha_s, z)$ splitting function [31, 38] as $z \rightarrow 1$

$$P_{qq}(\alpha_s, z) \rightarrow 2 \frac{\Gamma_{\text{cusp}}(\alpha_s)}{(1-z)_+} + 2\mathcal{B}(\alpha_s)\delta(1-z) + \dots \quad (17)$$

The integration contour in eq. (15) runs parallel to the imaginary axis on the right of all singularities of the integrand. From eq. (15) we see that the u -integral in the exponent is regularised by the lower bound $\frac{N_0}{N}$. Such a bound acts as an infrared regulator which prevents the strong coupling constant from being evaluated at non-perturbative scales ($\leq \Lambda_{QCD}$). Then, the contour in eq. (15) should be set away from all the singularities (in particular from the Landau pole). Nevertheless, for resummation purposes we can set the contour on the left of the Landau singularity since it would contribute with a non-logarithmic effect suppressed with some negative power of the center-of-mass energy scale. The inversion of the Laplace transform can be performed analytically by using the residue theorem as shown in [11, 29] and results in

$$R_T(\tau) = \left(1 + \sum_{k=1}^{\infty} C_k \left(\frac{\alpha_s}{2\pi}\right)^k\right) \exp \left[\log \frac{1}{\tau} g_1(\lambda) + g_2(\lambda) + \frac{\alpha_s}{\pi} \beta_0 g_3(\lambda) + \left(\frac{\alpha_s}{2\pi}\right)^3 G_{31} \log \frac{1}{\tau} \right], \quad (18)$$

where

$$\begin{aligned} g_1(\lambda) &= f_1(\lambda), \\ g_2(\lambda) &= f_2(\lambda) - \log \Gamma(1 - f_1(\lambda) - \lambda f_1'(\lambda)), \\ g_3(\lambda) &= f_3(\lambda) + \left(f_1'(\lambda) + \frac{1}{2} \lambda f_1''(\lambda)\right) \left(\psi^{(0)}(1 - \gamma(\lambda))^2 - \psi^{(1)}(1 - \gamma(\lambda))\right) \\ &\quad + f_2'(\lambda) \psi^{(0)}(1 - \gamma(\lambda)) + C_F/\beta_0 (\gamma_E(3/2 - \gamma_E) - \pi^2/6). \end{aligned} \quad (19)$$

The functions $f_i(\lambda)$ as well as the constants C_1 , C_2 and G_{31} are defined in [29], while the C_3 constant term is still analytically unknown. We fit the latter numerically using the fixed order Monte Carlo parton-level generator EERAD3. The fit is performed by subtracting the $\mathcal{O}(\alpha_s^3)$ logarithmic structure from the fixed-order result and taking (numerically) the asymptotic limit $\tau \rightarrow 0$.

EERAD3 is run with a technical cutoff $y_0 = 10^{-5}$ which affects the thrust distribution below $\tau_0 \sim \sqrt{y_0}$. This forbids us from probing the far infrared region and we perform the fit for values of τ larger than τ_0 . Numerical fixed order results are obtained with 6×10^7 points for the leading colour contribution and 10^7 points for the subleading colour structures. Because of the presence of large fluctuations in the Monte Carlo results, each color contribution is fitted separately over an interval where the distribution is stable and the different results are combined to find the numerical value of C_3 . As an alternative approach we first sum up all the color contributions and then fit C_3 . We consider the difference between the two approaches as a systematic error and as final result we obtain

$$C_3 = -1050 \pm 180(\text{stat.}) \pm 500(\text{syst.}). \quad (20)$$

Considering that there is no statistical correlation between different bin errors, as a different

possible estimate of the systematic uncertainty due to the sizeable fluctuation, we varied the fit range observing that it does not alter the result in any significant way outside the quoted systematic error margins. In Fig. 2 we vary the value of C_3 within its error band and we study its impact on the distribution. We observe that the numerical impact of C_3 on the distributions is less than 1.5‰ and it is therefore completely negligible compared the other theoretical uncertainties such that the large relative error range is tolerable for all practical purposes.

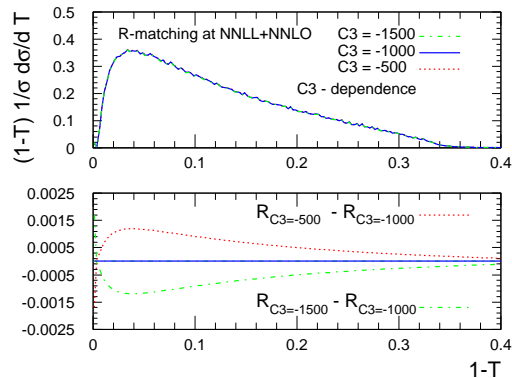


Figure 2: Impact of C_3 variation

2.2 Matching to fixed-order and numerical results

In this section we match the obtained resummed distribution (18) to the NNLO fixed order prediction. The matching formalism must avoid double counting and allow to access theoretical uncertainties. We compare the R -matching and $\log(R)$ -matching scheme described in [14].

In Figure 3 we compare the differential cross section of the new matched NNLL+NNLO results with the old NLL+NNLO derived in [42]. The modification due to the resummation is sizable, leading to a 8% increase of the distribution around the peak region. The effect of the additional resummed subleading logarithms becomes progressively less important towards the multijet region, where the increase is nevertheless of about 5%. It is interesting to note that the matching of NNLO with NNLL resummation shifts the pure NNLO result also in the multijet region (Figure 4). This was not the case for NLL+NNLO, for which the impact of resummation in the region of large τ was negligible. This is another sign of the importance of the NNLL contribution.

The renormalisation scale dependence, which was observed to increase from pure NNLO to NLL+NNLO [42, 45] because of a mismatch in the cancellation of renormalisation scale logarithms, is obtained by varying $0.5 < x_\mu < 2$. It decreases at NNLL+NNLO by 20% in the peak region compared to NLL+NNLO. The magnitude of the scale uncertainty varies between 4% in the 3-jet region and 5% around the peak. In Figure 5 we compare the R -matching and the $\log(R)$ -matching scheme predictions at NNLL+NNLO. The difference between the two matching prescriptions is tiny and lies well below the scale uncertainty. This implies a very good stability of the theoretical predictions under variation of the matching scheme.

One further source of arbitrariness is the choice of the logarithms to be resummed. In fact, it is not clear whether powers of $\alpha_s \log(1/\tau)$ or powers of *e.g.* $\alpha_s \log(2/\tau)$ have to be resummed. The origin of this arbitrariness has to do with how much of the non-logarithmic part of the fixed-order prediction is exponentiated together with the logarithms. We can express this arbitrariness by introducing a new parameter x_L , which rescales the logarithms as [14]: $L \rightarrow \hat{L} = \log(1/(x_L \tau))$.

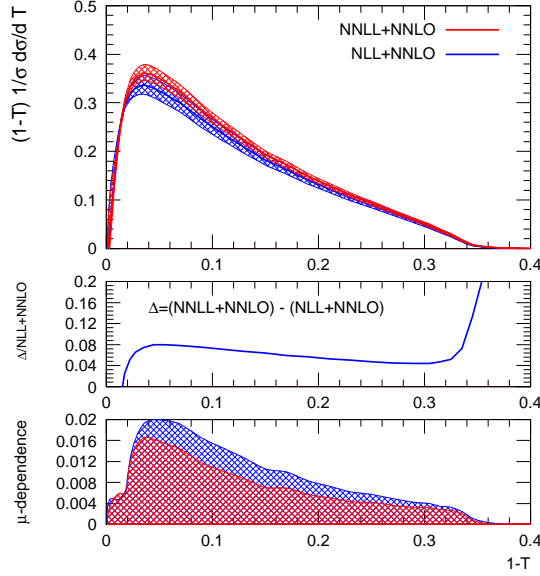


Figure 3: Comparison of the Thrust distributions with NNLL+NNLO and NLL+NNLO accuracy. The plot on the top shows the two distributions, with the uncertainty band due to scale dependence. The curve in the middle shows the difference between NNLL+NNLO and NLL+NNLO normalised to the NLL+NNLO curve. The impact of the resummation at NNLL is an increase in the distribution of order 5-8%. The lowest plot shows the absolute scale dependence of the two curves.

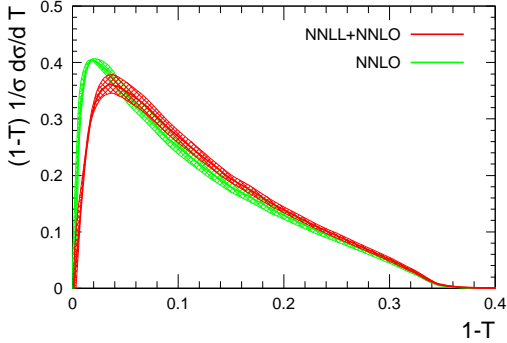


Figure 4: Comparison of the Thrust distribution at NNLO with the matched NNLL+NNLO predictions. The contribution of NNLL resummation is sizable over the full thrust range.

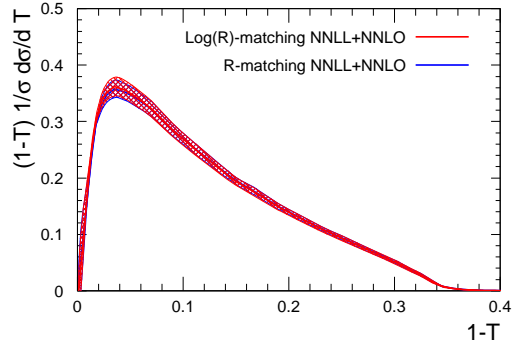


Figure 5: Comparison of the results obtained with the R -matching scheme and the $\log(R)$ -matching scheme. The width of the curve shows the uncertainty related to the scale variation.

We can estimate the related uncertainty by varying the parameter x_L . In Ref. [14] several prescriptions are given on how to set the correct variation range for x_L for different observables. For the sake of simplicity and since we are not performing a fit of the strong

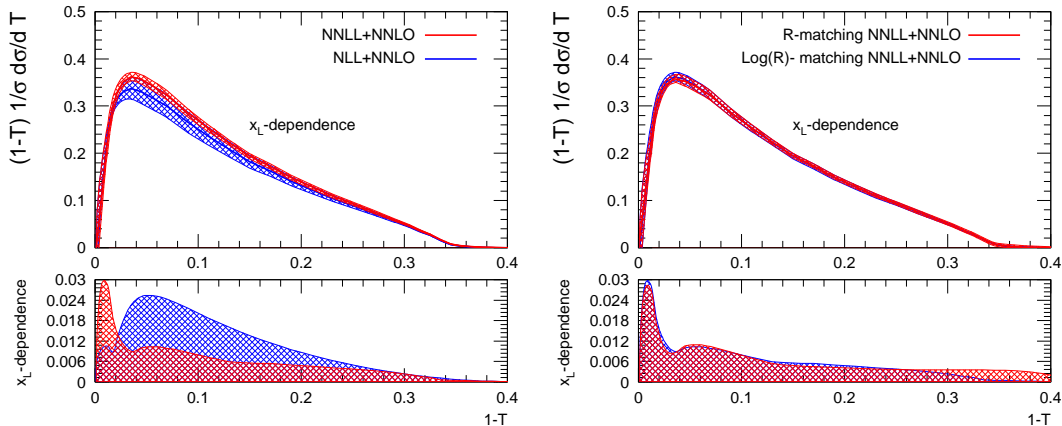


Figure 6: Dependence on the resummed logarithms, determined by varying the parameter x_L . The left plot shows the change in the x_L dependence between NLL+NNLO and NNLL+NNLO. The upper plot shows the distributions with the corresponding uncertainty band, in the lower plot we compare only the uncertainties. In the right plot the x_L dependence using the two different matching schemes is shown.

coupling constant, we choose to vary x_L within the canonical interval $0.5 < x_L < 2$, similarly to what is chosen to quantify the renormalisation scale uncertainty. This choice is also close to the nominal range of variation proposed in [14]. The impact of this variation is shown in Figure 6. The left plots show a comparison of the x_L -dependence between NLL+NNLO and NNLL+NNLO predictions. The lower plot allows to quantify the reduction of the uncertainty due to a variation x_L . Apart from the far infrared region, it is observed to decrease by 50% in the peak region. The scale-dependence reduction is smaller towards the multijet region, where the contribution of the logarithmic part becomes less important. The resummation uncertainty at NNLL+NNLO varies between 2% and 3%. In the right plot the same comparison is made at NNLL+NNLO using the R -matching and $\log(R)$ -matching schemes. We observe a similar x_L -dependence in both schemes.

3 Outlook

The recent results on event shape resummation improve the description of existing experimental data. In view of future work at high energy linear colliders and precise determinations of the strong coupling constant, N^2 LL predictions for the remaining Event-Shape observables are necessary. Moreover, an additional source of uncertainty is due to power-behaved hadronisation corrections which get large in the dijet region. Currently there is no deep theoretical understanding of such corrections which constitute an important source of theoretical error. In the past, these were often computed using leading-logarithmic parton shower Monte Carlo programs, which turned out to be clearly insufficient [45] in view of the precision now attained by the perturbative description. Systematic approaches to hadronization within the dispersive model [41, 48] or by using the shape function formalism [25, 47] are offering a more reliable description. Such corrections are quite sizeable at LEP energies (Fig. 7) while they are highly suppressed at future linear colliders energies (Fig. 8). In Fig. 7 we show

what the power-corrected distribution looks like when compared to the pure perturbative answer. Non-perturbative corrections are computed with a dispersive model [48] and both the mean effective coupling α_0 and the strong coupling α_s are obtained by performing a simultaneous fit using ALEPH data at $Q = 91.2$ GeV. Such a fit is purely qualitative since the correlation matrix is degenerate when only one data set is used. To perform a meaningful fit, experimental data over a broader range of energies have to be included. We will address this issue in a future publication.

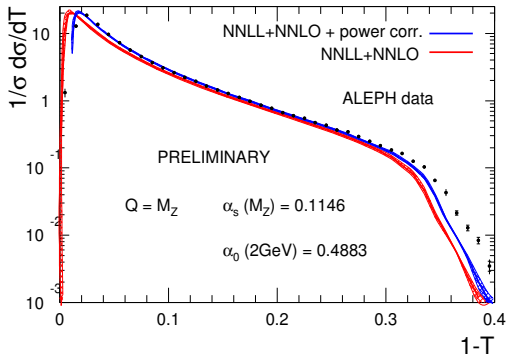


Figure 7: Theoretical prediction with (blue) and without (red) power corrections compared to ALEPH data. The non-perturbative parameter α_0 as well as the strong coupling constant α_s are fitted to experimental data.

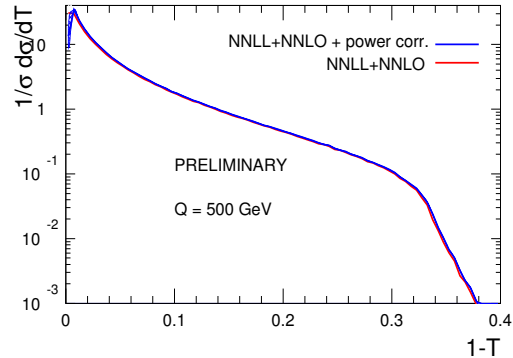


Figure 8: Comparison of the perturbative (red) and power-corrected (blue) distributions at a center of mass energy of 500 GeV. The plot shows how much hadronisation corrections get suppressed at typical future linear collider energies.

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