Decoupling Property of SUSY Extended Higgs Sectors and Implication for Electroweak Baryogenesis

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One-loop contributions to the Higgs potential at finite temperatures are discussed in the supersymmetric standard model with four Higgs doublet chiral superfields as well as a pair of charged singlet chiral superfields. The mass of the lightest Higgs boson h is determined only by the D-term at the tree-level in this model, while the triple Higgs boson coupling for hhh can receive a significant radiative correction. The same nondecoupling effect can also contribute to realize the sufficient first order electroweak phase transition, which is required for a successful scenario of electroweak baryogenesis. This model can be a new candidate for a model in which the baryon asymmetry of the Universe is explained at the electroweak scale. We also discuss the implication for the measurement of the triple Higgs boson coupling at the ILC.

1 Introduction

The observed baryon asymmetry of the Universe is expressed by $n_b/n_\gamma \simeq (5.1-6.5)\times 10^{-10}$ at the 95 % CL [1], where n_b is the difference in number density between baryons and anti-baryons and n_γ is the number density of photons. It is known that to generate the baryon asymmetry from the baryon symmetric world, the Sakharov's conditions have to be satisfied [2]: 1) Baryon number nonconservation, 2) C and CP violation, 3) Departure from the thermal equilibrium. The electroweak gauge theory can naturally satisfy the above three conditions, where the third condition is satisfied when the electroweak phase transition (EWPT) is of strongly first order. This scenario is often called the electroweak baryogenesis [3]. The scenario is necessarily related to the Higgs boson dynamics. It is directly testable at collider experiments. In the standard model (SM), however, the CP violation by the Cabibbo-Kobayashi-Maskawa matrix is quantitatively insufficient [4]. In addition, from the LEP direct search results the requirement of sufficiently strong first order EWPT requires a light Higgs boson whose mass is too small to satisfy the constraint [5]. A viable model for successful electroweak baryogenesis would be the two Higgs doublet model (THDM) [6]. In Ref. [7], the connection between the first order EWPT and the triple coupling for the SM-like Higgs boson h (the hhh coupling) has been clarified. In the model with sufficiently strong first order EWPT, the hhh coupling constant significantly deviates from the SM prediction due to the same nondecoupling quantum effects of additional scalar bosons. Such nondecoupling effects on the hhh coupling constant have been studied in Ref. [8].

Supersymmetry is a good new physics candidate, which eliminates the quadratic divergence in the one-loop calculation of the Higgs boson mass. The lightest SUSY partner particle with the R parity can naturally be a candidate for the cold dark matter. In the minimal supersymmetric SM (MSSM), there are many studies to realize the electroweak baryogenesis [9, 10, 11]. Currently, this scenario is highly constrained by the experimental data. According to Ref. [10], the strong first order EWPT is possible if $m_h \lesssim 127$ GeV and $m_{\tilde{t}_1} \lesssim 120$ GeV, where h is the lightest Higgs boson and $\tilde{t}_1$ is the lightest stop. To satisfy the LEP bound on m_h, the soft SUSY breaking mass for the left-handed stop should be greater than 6.5 TeV. The most striking feature of this scenario is that the electroweak vacuum is metastable and the global minimum is a charge-color-breaking vacuum. Many studies on the electroweak baryogenesis have been done in such singlet-extended MSSMs; i.e., the Next-to-MSSM [12], the nearly MSSM or the minimal non-MSSM [13], the $U(1)'$-extended MSSM [14], the secluded $U(1)'$-extended MSSM [15, 16], and so on. In the singlet-extended MSSM, however, the vacuum structure is inevitably more complicated than the MSSM, giving rise to the unrealistic vacua in the large portion of the parameter space, especially electroweak baryogenesis-motivated scenario [12, 15].

In this talk, we discuss how the electroweak phase transition can be of sufficiently strong first order in an extended SUSY standard model [17], where a pair of extra doublet chiral superfields H_3 ($Y = -1/2$) and H_4 ($Y = +1/2$) and a pair of charged singlet chiral superfields Ω_1 ($Y = +1$) and Ω_2 ($Y = -1$) are introduced in addition to the MSSM content. A motivation of this model is a SUSY extension of the model to generate the tiny neutrino masses by radiative corrections [18, 19]. In the present model, there is no tree-level F-term contribution to the mass of the lightest Higgs boson, but there can be large one-loop corrections to the triple Higgs boson coupling due to the additional bosonic loop contribution [20]. Similarly to the case of the non-SUSY THDM, these nondecoupling bosonic loop contributions can also make first order phase transition stronger. We here show that the EWPT can be of sufficiently strong first order in this model.

2 Model

In addition to the standard gauge symmetries, we impose a discrete Z_2 symmetry for simplicity. Although the Z_2 symmetry is not essential for our discussion, the symmetry works for avoiding the flavor changing neutral current at the tree level [21, 22, 23, 24]. Furthermore, we assume that there is the R parity in our model. The superpotential is given by

$$\begin{aligned} W &= (y_u)^{ij} U_i^c H_2 \cdot Q_j + (y_d)^{ij} D_i^c H_1 \cdot Q_j + (y_e)^{ij} E_i^c H_1 \cdot L_j \\ &\quad + \lambda_1 \Omega_1 H_1 \cdot H_3 + \lambda_2 \Omega_2 H_2 \cdot H_4 - \mu H_1 \cdot H_2 - \mu' H_3 \cdot H_4 - \mu_\Omega \Omega_1 \Omega_2. \end{aligned} \tag{1}$$

The soft-SUSY-breaking terms are given by

$$\begin{aligned} \mathcal{L}_{\text{soft}} &= -\frac{1}{2}(M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}\tilde{W} + M_3 \tilde{G}\tilde{G}) - \Big\{ (\tilde{M}_{\tilde{q}}^2)_{ij} \tilde{q}_{Li}^\dagger \tilde{q}_{Lj} + (\tilde{M}_{\tilde{u}}^2)_{ij} \tilde{u}_{Ri}^* \tilde{u}_{Rj} \\ &+ (\tilde{M}_{\tilde{d}}^2)_{ij} \tilde{d}_{Ri}^* \tilde{d}_{Rj} + (\tilde{M}_{\tilde{\ell}}^2)_{ij} \tilde{\ell}_{Li}^\dagger \tilde{\ell}_{Lj} + (\tilde{M}_{\tilde{e}}^2)_{ij} \tilde{e}_{Ri}^* \tilde{e}_{Rj} \Big\} - \Big\{ \tilde{M}_{H_1}^2 \Phi_1^\dagger \Phi_1 + \tilde{M}_{H_2}^2 \Phi_2^\dagger \Phi_2 + \\ &+ \tilde{M}_{H_3}^2 \Phi_3^\dagger \Phi_3 + \tilde{M}_{H_4}^2 \Phi_4^\dagger \Phi_4 + \tilde{M}_+^2 \omega_1^+ \omega_1^- + \tilde{M}_-^2 \omega_2^+ \omega_2^- \Big\} - \Big\{ (A_u)^{ij} \tilde{u}_{Ri}^* \Phi_2 \cdot \tilde{q}_{Lj} + \\ &+ (A_d)^{ij} \tilde{d}_{Ri}^* \Phi_1 \cdot \tilde{q}_{Lj} + (A_e)^{ij} \tilde{e}_{Ri}^* \Phi_1 \cdot \tilde{\ell}_{Lj} + (A_1) \omega_1^+ \Phi_1 \cdot \Phi_3 + (A_2) \omega_2^- \Phi_2 \cdot \Phi_4 + \text{h.c.} \Big\}. \end{aligned} \tag{2}$$

From W and $\mathcal{L}_{\text{soft}}$, the Lagrangian is constructed as

$$\mathcal{L} = -\left(\frac{1}{2} \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \psi_{Li} \cdot \psi_{Lj} + h.c. \right) - \left| \frac{\partial W}{\partial \varphi_i} \right|^2 - \frac{1}{2}(g_a)^2 (\varphi_\alpha^* T_{\alpha\beta}^a \varphi_\beta)^2 + \mathcal{L}_{\text{soft}} \ ..., \tag{3}$$

where φ_i and ψ_{Li} are respectively scalar and fermion components of chiral superfields, and $T^a_{\alpha\beta}$ and g_a represent generator matrices for the gauge symmetries and corresponding gauge coupling constants.

The scalar component doublet fields Φ_i are parameterized as

$$\Phi_{1,3} = \begin{bmatrix} \frac{1}{\sqrt{2}}(\varphi_{1,3} + h_{1,3} + ia_{1,3}) \\ \phi^-_{1,3} \end{bmatrix}, \Phi_{2,4} = \begin{bmatrix} \phi^+_{2,4} \\ \frac{1}{\sqrt{2}}(\varphi_{2,4} + h_{2,4} + ia_{2,4}) \end{bmatrix}, \quad (4)$$

where φ_i are classical expectation values, h_i are CP-even, a_i are CP-odd and $\phi^\pm_i$ are charged scalar states. where φ_i are classical expectation values, h_i are CP-even, a_i are CP-odd and $\phi^\pm_i$ are charged scalar states. We use the effective potential method to explore the Higgs sector. At the tree level, the effective potential for the Higgs fields is given by

$$V_0 \ = \sum_{a=1}^{4} \frac{1}{2}\bar{m}_a^2\varphi_a^2 + \frac{1}{2}(B\mu\varphi_1\varphi_2 + B'\mu'\varphi_3\varphi_4 + \text{h.c.}) + \frac{g^2+g'^2}{32}(\varphi_1^2 - \varphi_2^2 + \varphi_3^2 - \varphi_4^2)^2. \quad (5)$$

Using the effective potential, the vacuum is determined by the stationary condition as

$$\left.\frac{\partial V_{\text{eff}}}{\partial \varphi_i}\right|_{\langle\varphi_i\rangle = v_i} = 0. \quad (6)$$

We assume that the Z_2 odd Higgs bosons do not have the vacuum expectation values (VEVs), and we set $\sqrt{v_1^2 + v_2^2} \equiv v$ ($\simeq 246$ GeV) and introduce $\tan\beta = v_2/v_1$. At the tree level, $v_3 = v_4 = 0$ is guaranteed by requiring the nonnegative eigenvalues of $(\partial^2 V_0/\partial\varphi_i\partial\varphi_j)_{\varphi_{i,j}=0}$ $(i,j = 3,4)$, i.e., $\bar{m}_3^2\bar{m}_4^2 - B'^2\mu'^2 \geq 0, \quad \bar{m}_3^2 + \bar{m}_4^2 \geq 0$. In the following, we exclusively focus on the (φ_1, φ_2) space. For the Z_2 even scalar states, we have five physical states as in the MSSM; i.e., two CP-even h and H, a CP-odd A and a pair of charged $H^\pm$ scalar bosons. The tree level mass formulae for these scalar states coincide with those in the MSSM.

We here focus on the one-loop contribution since radiative corrections on the Higgs sector are very important to study the EWPT. The vacuum at the one-loop level is also determined from Eq. (6) with the one-loop corrected effective potential. The one-loop correction to the effective potential at zero temperature is given by

$$V_1(\varphi_1, \varphi_2) \ = \ \sum_i c_i \frac{\bar{m}_i^2}{64\pi^2}\left(\ln\frac{\bar{m}_i^2}{M^2} - \frac{3}{2}\right), \quad (7)$$

where V_1 is regularized in the $\overline{\text{DR}}$-scheme, c_i is the degrees of freedom of the species i, M is a renormalization scale which will be set on m_t^{pole}. For the zero temperature $T = 0$, the one-loop corrected mass matrix for the CP even neutral bosons can be calculated from the effective potential. We here consider the simple case such that $B' = B_\Omega = \mu' = \mu_\Omega = 0$ in order to switch off the mixing effects. The renormalized mass of the lightest Higgs boson h is calculated for $m_A \gg m_Z$ as

$$m_h^2 \simeq m_Z^2\cos^2 2\beta + (\text{MSSM-loop}) + \frac{\lambda_1^4 v^2 c_\beta^4}{16\pi^2}\ln\frac{m^2_{\Omega_2^\pm}m^2_{\Phi_2'^\pm}}{m^4_{\tilde{\chi}_1'^\pm}} + \frac{\lambda_2^4 v^2 s_\beta^4}{16\pi^2}\ln\frac{m^2_{\Omega_1^\pm}m^2_{\Phi_1'^\pm}}{m^4_{\tilde{\chi}_2'^\pm}}, \quad (8)$$

at the leading $\lambda^4_{1,2}$ contributions, where the one-loop contribution in the MSSM is mainly from the top and stop loop diagram[25].

Now we quantify the magnitude of the radiative corrections of the Z_2-odd particles on m_h. The input parameters are fixed as follows [17]:

$$\begin{aligned}
&\tan\beta = 3, m_{H^\pm} = 500 \text{ GeV};\\
&\tilde{M}_{\tilde{q}} = \tilde{M}_{\tilde{b}} = \tilde{M}_{\tilde{t}} = 1000 \text{ GeV}, \mu = M_2 = 2M_1 = 200 \text{ GeV}, A_t = A_b = X_t + \mu/\tan\beta;\\
&\lambda_1 = 2, \mu' = \mu_\Omega = B_\Omega = B' = 0, \overline{m}_+^2 = \overline{m}_3^2 = (500 \text{ GeV})^2, \overline{m}_-^2 = \overline{m}_4^2 = (50 \text{ GeV})^2. \quad (9)
\end{aligned}$$

We note that $m_{\Phi_1'^\pm} < m_{\Phi_2'^\pm}$ and $m_{\Omega_1^\pm} < m_{\Omega_2^\pm}$ in this case. On this parameter set, $m^2_{\Phi_1'^\pm}$, $m^2_{\Omega_1^\pm}$ and $m^2_{\tilde{\chi}_2'^\pm}$ get a significant contribution from λ_2. Then their masses become larger for the greater value of λ_2. Since the mass parameters $\bar{m}_4^2$ and $\bar{m}_-^2$ are taken to be small, large mass values of $m_{\Phi_1'^\pm}$ and $m_{\Omega_1^\pm}$ yield the large nondecoupling effects.

Fig. 1 shows the predicted value of m_h as a function of λ_2 varying $X_t/\tilde{M}_{\tilde{q}} = 2.0, 1.2$ and 0.6 from the top to the bottom [17]. We can see that m_h monotonically decreases as λ_2 increases, which is in contrast with the top/stop loop effects.

The coupling constants λ_1 and λ_2 are free parameters of the model. Its magnitude, however, is bounded from above by the condition that there is no Landau pole below the given cutoff scale Λ. As we are interested in the model where the first order EWPT is sufficiently strong, we allow rather larger values for these coupling constants, and do not require that the model holds until the grand unification scale. A simple renormalization group equation analysis tells us that for assuming $\Lambda = 2$ TeV, 10 TeV or 10^2 TeV, the coupling constant can be taken to be at most $\lambda_2 \sim 2.5$, 2.0 or 1.5, respectively.

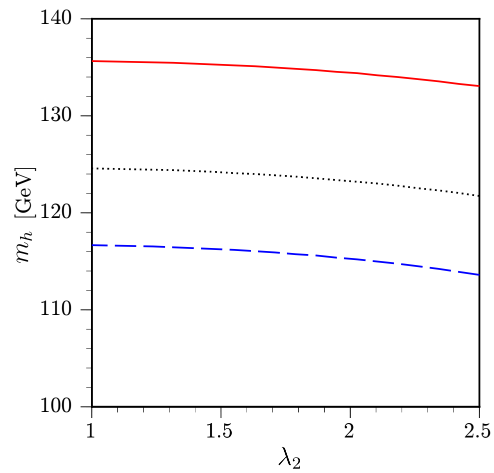

Figure 1: The Z_2-even lightest Higgs boson mass as a function of λ_2. From the top to the bottom, $X_t/\tilde{M}_{\tilde{q}} = 2.0, 1.2$ and 0.6.

3 Electroweak Phase Transition

The nonzero temperature effective potential is

$$V_1(\varphi_1, \varphi_2; T) = \sum_i c_i \frac{T^4}{2\pi^2} I_{B,F}\left(\frac{\bar{m}_i^2}{T^2}\right), \tag{10}$$

where $B(F)$ refer to boson (fermion) and $I_{B,F}$ take the form

$$I_{B,F}(a^2) = \int_0^\infty dx\ x^2 \ln\left(1 \mp e^{-\sqrt{x^2+a^2}}\right). \tag{11}$$

Since the minimum search using $I_{B,F}$ is rather time-consuming, we will alternatively use the fitting functions of them that are employed in Ref. [11].

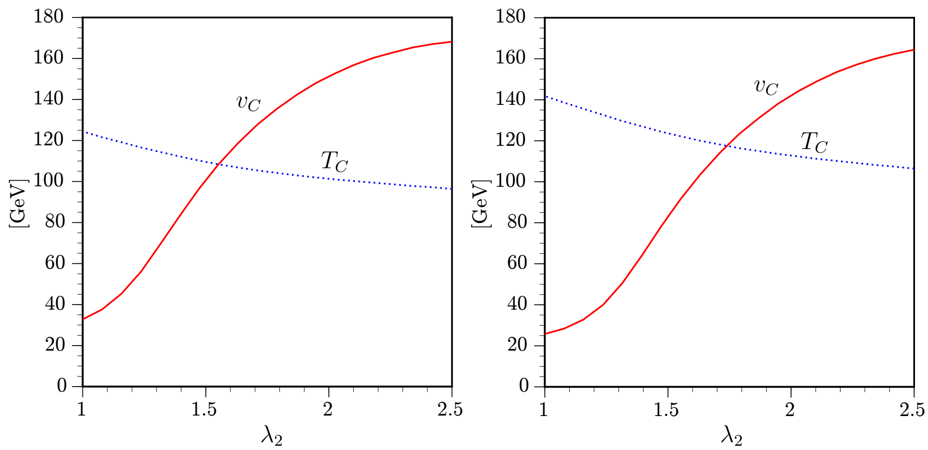

Figure 2: (Left) v_C and T_C vs. λ_2 with $X_t/\tilde{M}_{\tilde{q}} = 0.6$. The other input parameters are the same as in the Fig. 1. The sphaleron decoupling condition (12) can be satisfied for $\lambda_2 \gtrsim 1.6$. (Right) v_C and T_C vs. λ_2 with $X_t/\tilde{M}_{\tilde{q}} = 2.0$. The other input parameters are the same as in the Fig. 1. The sphaleron decoupling condition (12) can be satisfied for $\lambda_2 \gtrsim 1.8$.

For an electroweak baryogenesis scenario to be successful, the sphaleron rate in the broken phase should be smaller than the Hubble constant. This condition is translated into

$$\frac{v_C}{T_C} = \frac{\sqrt{v_1^2(T_C) + v_2^2(T_C)}}{T_C} \gtrsim \zeta, \tag{12}$$

where T_C is the critical temperature, v_C is the Higgs VEV at T_C, and ζ is a $\mathcal{O}(1)$ parameter. To obtain ζ within a better accuracy, the sphaleron energy and zero-mode factors of the fluctuations around the sphaleron must be evaluated. In the SM, the sphaleron energy is simply a function of the Higgs boson mass. As the Higgs boson becomes heavier, the sphaleron energy gets larger as well [26], leading to the smaller ζ [27]. In this model, on the other hand, ζ depends on more parameters. For simplicity, we here take $\zeta = 1$, which is often adopted as a rough criterion in the literature.

In our analysis, T_C is defined as the temperature at which the effective potential has the two degenerate minima. We search for T_C by minimizing

$$V_{\text{eff}}(\varphi_1, \varphi_2; T) = V_0(\varphi_1, \varphi_2) + V_1(\varphi_1, \varphi_2) + V_1(\varphi_1, \varphi_2; T), \tag{13}$$

where the field-dependent masses are modified by adding thermal corrections.

In Fig. 2 (Left), v_C and T_C are plotted as a function of λ_2 in the light h scenario ($X_t/\tilde{M}_{\tilde{q}} = 0.6$) [17]: see Fig. 1. The sphaleron decoupling condition (12) can be fulfilled for $\lambda_2 \gtrsim 1.6$ due to the nondecoupling effects coming from $\phi_1'^{\pm}$ and $\Omega_1^{\pm}$.

We also evaluate v_C and T_C in the heavy h scenario ($X_t/\tilde{M}_{\tilde{q}} = 2.0$) as shown in Fig. 2 (Right) [17]. The sphaleron decoupling condition can be satisfied for $\lambda_2 \gtrsim 1.8$. Though the parameter region is a bit narrower than the light Higgs scenario, the lightest Higgs boson mass as large as 134 GeV is still consistent with the decoupling condition.

4 Phenomenological predictions and Discussions

The nondecoupling effect of the extra Z_2 odd charged scalar bosons on the finite temperature effective potential is an essentially important feature of our scenario in order to realize strong first order phase transition. The same physics affects the triple Higgs boson coupling with a large deviation from the SM (MSSM) prediction as discussed in Ref. [7]. Such deviation in the triple Higgs boson coupling can be 15-70 % [8, 20], and we expect that they can be measured at the future linear collider such as the ILC or the CLIC.

In our model, in order to realize the nondecoupling effect large, the invariant parameters μ' and μ_Ω are taken to be small. Consequently, the masses of extra charginos are relatively as light as 100-300 GeV.

The several comments on the current analysis are in order. I) In the MSSM, it is found that $\zeta \simeq 1.4$ [11], which is 40% stronger than one we impose in our analysis. We emphasize that even if we take $\zeta = 1.4$ in our model a feasible region still exists for the relatively large λ_2, for example, $\lambda_2 \gtrsim 2.2$ even in the heavy h case. The cutoff scale Λ is still around the multi-TeV scale. II) As in the MSSM, the strength of the first order EWPT can get enhanced if the (almost) right-handed stop is lighter than the top quark, enlarging the possible region. III) As in the MSSM, the charginos and the neutralinos can play an essential role in generating the CP violating sources as needed for the bias of the chiral charge densities around the Higgs bubble walls. The Z_2 odd charginos $\tilde{\chi}'^{\pm}_{1,2}$ may also be helpful. IV) We have confirmed that the charge and the Z_2 are not broken at the tree level for our parameter set. V) The potential analysis beyond the tree level will be a future problem. VI) If the Z_2 symmetry is exact, the lightest (neutral) Z_2 odd particle can be a candidate of dark matter. If it is the scalar boson, its properties would be similar to those of the inert doublet model [28]. A Z_2 odd neutralino may also be a candidate for dark matter.

5 Conclusions

We have found that the nondecoupling loop effects of additional charged scalar bosons can make the first order EWPT strong enough to realize successful electroweak baryogenesis. We conclude that this model is a good candidate for successful electroweak baryogenesis.

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