A framework for precision 2HDM studies at the ILC and CLIC

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The precision measurements of Higgs boson observables will be critical in the interpretation of the dynamics responsible for electroweak symmetry breaking. The capabilities of the ILC and CLIC for precision Higgs studies are well documented. In this talk, a theoretical framework is presented for interpreting phenomena that can arise in the two-Higgs-doublet model (2HDM). By imposing no constraints on the most general set of 2HDM parameters, the resulting formalism correctly identifies the physical 2HDM observables, which can be determined by model-independent experimental analyses.

1 Introduction

The capabilities of the ILC and CLIC for precision Higgs studies are well documented [1, 2]. Suppose that collider data suggests that electroweak symmetry breaking is a consequence of the dynamics of a two-Higgs doublet model (2HDM). To verify this hypothesis in detail will require precision measurements of numerous Higgs boson observables. However, to perform these measurements within the context of a general 2HDM is challenging, in part due to the numerous parameters that govern the structure of the 2HDM Lagrangian.

Indeed, a generic point of the 2HDM parameter space is incompatible with current phenomenological constraints. For example, without imposing restrictions on the model, the 2HDM generically predicts large Higgs-mediated tree-level flavor changing neutral currents (FCNC) in conflict with experimental observation [3]. It is possible to remove the Higgs-mediated tree-level FCNCs by imposing a simple discrete symmetry (a number of different implementations, reviewed in ref. [4], are possible) or supersymmetry [5, 6]. However, such symmetries are often broken, in which case the effective 2HDM below the symmetry-breaking scale would be a completely general 2HDM. Since there are many different ways to apply the relevant symmetries, one does not know a priori which specific realization of the 2HDM (if any) is the most likely to be realized in nature.

Consequently, phenomenological 2HDM studies tended to be quite model dependent. However, it is preferable to let experiment decide which realization of the 2HDM is relevant. But to do so requires a model-independent treatment of the 2HDM. In such a treatment, the two Higgs doublet fields are indistinguishable, in which case any two linearly-independent combinations of the original two doublet fields can be used to develop the 2HDM Lagrangian. Physical observables cannot depend on this choice of basis for the scalar fields. By identifying the relevant basis-independent (physical) observables, one could experimentally "discover" the presence of any approximate or exact symmetry of the 2HDM needed for the phenomenological consistency of the model.

As a warmup, we consider some aspects of the precision Higgs program in the context of the minimal supersymmetric standard model (MSSM) in Section 2. In Section 3, the basis-independent formalism for the 2HDM is developed and the physical observables (e.g., masses and couplings) are identified. The decoupling limit of the general 2HDM is introduced in

Section 4, and its significance is discussed. Finally, conclusions and lessons for future work are outlined in Section 5.

2 What can we learn from precision MSSM Higgs studies?

At the ILC [7] and CLIC [8], it may be possible to measure the $h^0 b\bar{b}$ coupling to an accuracy of a few percent or less, where h^0 is the lightest CP-even neutral Higgs boson. A deviation from the prediction of the Standard Model at some level is expected in models with an extended Higgs sector. In this section, we consider the MSSM whose Higgs sector is a special case of the 2HDM.

2.1 The wrong-Higgs interactions

In the MSSM, the tree-level Higgs–quark Yukawa Lagrangian is supersymmetry-conserving and is given by:

$$\mathcal{L}_{\text{yuk}}^{\text{tree}} = -\epsilon_{ij} h_b H_d^i \psi_Q^j \psi_D + \epsilon_{ij} h_t H_u^i \psi_Q^j \psi_U + \text{h.c.}$$

Two other possible dimension-four gauge-invariant non-holomorphic Higgs-quark interactions terms, the so-called *wrong-Higgs interactions* [9, 10],

$$H_u^{k*} \psi_D \psi_Q^k \qquad \text{and} \qquad H_d^{k*} \psi_U \psi_Q^k \,,$$

are not supersymmetric (since the dimension-four supersymmetric Yukawa interactions must be holomorphic), and hence are absent from the tree-level MSSM Yukawa Lagrangian.

Nevertheless, the wrong-Higgs interactions can be generated in the effective low-energy theory below the scale of supersymmetry (SUSY)-breaking. In particular, one-loop radiative corrections, in which supersymmetric particles (squarks, higgsinos and gauginos) propagate inside the loop, can generate the wrong-Higgs interactions shown in Fig. 1.

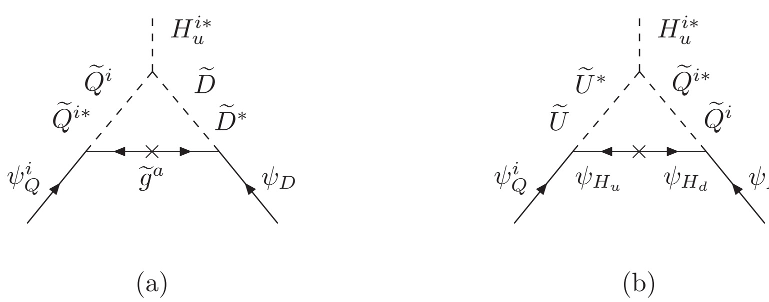

Figure 1: One-loop diagrams contributing to the wrong-Higgs Yukawa effective operators [10]. In (a), the cross (×) corresponds to a factor of the gluino mass M_3. In (b), the cross corresponds to a factor of the higgsino Majorana mass parameter μ. Field labels correspond to annihilation of the corresponding particle at each vertex of the triangle.

If the superpartners are heavy, then one can derive an effective field theory description of the Higgs-quark Yukawa couplings below the scale of SUSY-breaking (M_{SUSY}), where one has integrated out the heavy SUSY particles propagating in the loops. The resulting effective Lagrangian is [9]:

$$\mathcal{L}_{\text{yuk}}^{\text{eff}} = -\epsilon_{ij}(h_b+\delta h_b)\psi_b H_d^i \psi_Q^j + \Delta h_b \psi_b H_u^{k*}\psi_Q^k + \epsilon_{ij}(h_t+\delta h_t)\psi_t H_u^i \psi_Q^j + \Delta h_t \psi_t H_d^{k*}\psi_Q^k + \text{h.c.} \tag{1}$$

In the limit of $M_{\text{SUSY}} \gg m_Z$,

$$\Delta h_b = h_b\left[\frac{2\alpha_s}{3\pi}\mu M_3 \mathcal{I}(M_{\tilde{b}_1}, M_{\tilde{b}_2}, M_g) + \frac{h_t^2}{16\pi^2}\mu A_t \mathcal{I}(M_{\tilde{t}_1}, M_{\tilde{t}_2}, \mu)\right],$$

where, M_3 is the Majorana gluino mass, μ is the supersymmetric Higgs-mass parameter, and $\widetilde{b}_{1,2}$ and $\widetilde{t}_{1,2}$ are the mass-eigenstate bottom squarks and top squarks, respectively. The loop integral is given by:

$$\mathcal{I}(a,b,c) = \frac{a^2b^2\ln(a^2/b^2) + b^2c^2\ln(b^2/c^2) + c^2a^2\ln(c^2/a^2)}{(a^2-b^2)(b^2-c^2)(a^2-c^2)}.$$

In the limit where at least one of the arguments of $\mathcal{I}(a,b,c)$ is large, the loop integral behaves as $\mathcal{I}(a,b,c) \sim 1/\max(a^2,b^2,c^2)$. Thus, in the limit where $M_3 \sim \mu \sim A_t \sim M_{\tilde{b}} \sim M_{\tilde{t}} \sim M_{\text{SUSY}} \gg m_Z$, the one-loop contributions to Δh_b do *not* decouple.

2.2 Phenomenological consequences of the wrong-Higgs Yukawa couplings

A consequence of the wrong-Higgs Yukawa couplings is a $\tan\beta$-enhanced modification of certain physical observables. To see this, we rewrite the Higgs fields in terms of the physical Higgs mass-eigenstates (and the Goldstone bosons):

$$\begin{aligned}
H_d^1 &= \tfrac{1}{\sqrt{2}}(v\cos\beta + H^0\cos\alpha - h^0\sin\alpha + iA^0\sin\beta - iG^0\cos\beta),\\
H_u^2 &= \tfrac{1}{\sqrt{2}}(v\sin\beta + H^0\sin\alpha + h^0\cos\alpha + iA^0\cos\beta + iG^0\sin\beta),\\
H_d^2 &= H^-\sin\beta - G^-\cos\beta,\\
H_u^1 &= H^+\cos\beta + G^+\sin\beta,
\end{aligned}$$

with $v^2 \equiv v_u^2 + v_d^2 = (246\text{ GeV})^2$ and $\tan\beta \equiv v_u/v_d$. The neutral CP-even Higgs mixing angle is denoted by α [6]. Using eq. (1), the b-quark mass is given by

$$m_b = \frac{h_b v}{\sqrt{2}}\cos\beta\left(1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b\tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b),$$

which defines the quantity Δ_b.

In the limit of large $\tan\beta$ the term proportional to δh_b can be neglected, in which case,

$$\Delta_b \simeq (\Delta h_b/h_b)\tan\beta.$$

Thus, Δ_b is $\tan\beta$–enhanced if $\tan\beta \gg 1$. As previously noted, Δ_b survives in the limit of large M_{SUSY}; this effect does not decouple. It can generate measurable shifts in the decay rate for $h^0 \to b\bar{b}$:

$$g_{h^0b\bar{b}} = -\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta}\left[1 + \frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)(1+\cot\alpha\cot\beta)\right].$$

At large $\tan\beta \sim 20$—50, the value of Δ_b can be as large as 0.5 in magnitude and of either sign, leading to a significant enhancement or suppression of the Higgs decay rate to $b\bar{b}$. If $m_{H^\pm} \gg m_Z$ (corresponding to the Higgs decoupling limit treated in section 2.3), then

$$-\frac{\sin\alpha}{\cos\beta} = 1 - \frac{2m_Z^2 \sin^2\beta\cos 2\beta}{m_{H^\pm}^2} + \mathcal{O}\left(\frac{m_Z^4}{m_{H^\pm}^4}\right),$$
$$1 + \cot\alpha\cot\beta = -\frac{2m_Z^2}{m_{H^\pm}^2}\cos 2\beta + \mathcal{O}\left(\frac{m_Z^4}{m_{H^\pm}^4}\right),$$

in which case $g_{h^0 b\bar{b}} = (m_b/v)\left[1 + \mathcal{O}(m_Z^2/m_{H^\pm}^2)\right]$ approaches its SM value.

2.3 The Decoupling Limit of the MSSM Higgs sector

In a significant fraction of the MSSM Higgs sector parameter space, one finds a neutral CP Higgs boson with SM-like tree-level couplings and additional scalar states that are somewhat heavier in mass (of order $m_{H^\pm}$), with small mass splittings of order $m_Z^2/m_{H^\pm}^2$. Below the scale $m_{H^\pm}$, the effective Higgs theory coincides with that of the Standard Model (SM).

In the limit of $m_{H^\pm} \gg m_Z$, the expressions for the tree-level MSSM Higgs masses and CP-even neutral Higgs mixing angle α simplify [11]:

$$m_h^2 \simeq m_Z^2\cos^2 2\beta\,, \qquad m_H^2 \simeq m_A^2 + m_Z^2\sin^2 2\beta\,,$$
$$m_{H^\pm}^2 = m_A^2 + m_W^2\,, \qquad \cos^2(\beta-\alpha) \simeq \frac{m_Z^4\sin^2 4\beta}{4m_{H^\pm}^4}\,. \tag{2}$$

From eq. (2), it follows that (i) the two neutral heavy Higgs states and $H^\pm$ are mass-degenerate up to corrections of $\mathcal{O}(m_Z^2/m_{H^\pm}^2)$; and (ii) $\cos(\beta-\alpha) = 0$ up to corrections of $\mathcal{O}(m_Z^2/m_{H^\pm}^2)$. This is the decoupling limit, since at energy scales below the approximately common mass of the heavy Higgs bosons $H^\pm$ H^0, A^0, the effective Higgs theory is precisely that of the SM. In general, in the limit of $\cos(\beta-\alpha) \to 0$, all the h^0 couplings to SM particles approach their SM limits.

These conclusion remain valid after radiative corrections are taken into account. Even when such effects break the CP-invariance of the tree-level MSSM Higgs sector, the lightest neutral Higgs state possesses CP-even interactions up to small CP-violating corrections of $\mathcal{O}(m_Z^2/m_{H^\pm}^2)$. For example, if we keep only the leading $\tan\beta$-enhanced radiative corrections, then in the approach to the decoupling limit [9],

$$\frac{g_{hVV}^2}{g_{h_{\rm SM}VV}^2} \simeq 1 - \frac{c^2 m_Z^4\sin^2 4\beta}{4m_{H^\pm}^4}\,,$$
$$\frac{g_{htt}^2}{g_{h_{\rm SM}tt}^2} \simeq 1 + \frac{c m_Z^2\sin 4\beta\cot\beta}{m_{H^\pm}^2}\,,$$
$$\frac{g_{hbb}^2}{g_{h_{\rm SM}bb}^2} \simeq 1 - \frac{4c m_Z^2\cos 2\beta}{m_{H^\pm}^2}\left[\sin^2\beta - \frac{\Delta_b}{1+\Delta_b}\right]\,, \tag{3}$$

where $c \equiv 1 + \mathcal{O}(g^2)$ and $\Delta_b \equiv \tan\beta \times \mathcal{O}(g^2)$, with g a generic gauge or Yukawa coupling. The quantities c and Δ_b depend on the MSSM spectrum. The approach to decoupling is fastest for the h^0 couplings to vector boson pairs and slowest for the couplings to down-type quarks [12], as shown in Fig. 2.

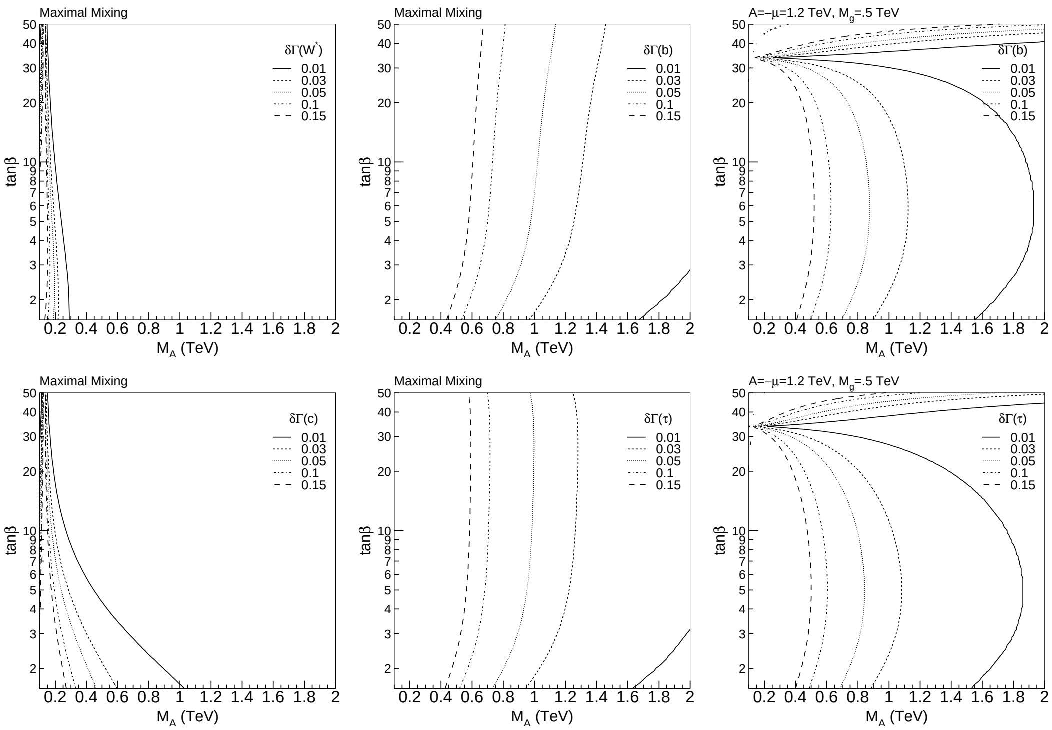

Figure 2: Deviations of Higgs partial widths from their SM values, including the leading one-loop corrections in two different MSSM scenarios, taken from Ref. [12].

3 The general Two-Higgs-Doublet Model (2HDM)

Consider the most general renormalizable 2HDM potential,

$$
\begin{aligned}
\mathcal{V} = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] \\
+\tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
+\left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\} . \qquad (4)
\end{aligned}
$$

In a general 2HDM, Φ_1 and Φ_2 are indistinguishable weak-doublet, hypercharge-one fields. A basis change consists of a global U(2) transformation $\Phi_a \to U_{a\bar{b}}\Phi_b$ (and $\Phi_{\bar{a}}^\dagger = \Phi_{\bar{b}}^\dagger U_{b\bar{a}}^\dagger$). Note that the electroweak gauge-covariant kinetic energy terms of the scalar fields are invariant with respect to U(2), whereas the scalar potential squared-masses and couplings change under U(2) transformations and thus are *basis-dependent* quantities. Physical quantities that can be measured in the laboratory must be basis-independent. Thus, any model-independent experimental study of 2HDM phenomena must employ basis-independent methods for analyzing data associated with 2HDM physics.

The most general 2HDM defined by eq. (4) generically contains large tree-level Higgs-mediated FCNCs and CP-violating effects, which are inconsistent with present experimental data over a large range of the 2HDM parameter space. This can be rectified by either (i) fine-tuning of 2HDM parameters to reduce the size of the FCNC and CP-violating effects below the experimentally allowed limits; or (ii) imposing additional symmetries (discrete and/or continuous) on the Higgs Lagrangian to eliminate tree-level Higgs-mediated FCNCs and CP-violation. The latter can distinguish between Φ_1 and Φ_2, in which case a choice of basis acquires physical significance.

However, any experimental analysis that relies on a specific symmetry and corresponding basis choice cannot be used to interpret the data in terms of a more general 2HDM. In contrast, basis-independent methods can be employed to perform model-independent 2HDM analyses. Moreover, such an analysis could in principle be used to experimentally identify and distinguish among possible approximate or exact symmetries. Finally, if these symmetries are badly broken or nonexistent, then the basis-independent methods are the only viable framework for 2HDM studies.

3.1 The basis-independent formalism

The scalar potential can be rewritten in U(2)-covariant notation [13, 14]:

$$\mathcal{V} = Y_{a\bar{b}}\Phi^\dagger_{\bar{a}}\Phi_b + \tfrac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi^\dagger_{\bar{a}}\Phi_b)(\Phi^\dagger_{\bar{c}}\Phi_d)\,, \qquad a,b,c,d = 1,2\,,$$

where $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}}$ and hermiticity implies $Y_{a\bar{b}} = (Y_{b\bar{a}})^*$ and $Z_{a\bar{b}c\bar{d}} = (Z_{b\bar{a}d\bar{c}})^*$. The barred indices help keep track of which indices transform with U and which transform with $U^\dagger$. For example, $Y_{a\bar{b}} \to U_{a\bar{c}}Y_{c\bar{d}}U^\dagger_{d\bar{b}}$ and $Z_{a\bar{b}c\bar{d}} \to U_{a\bar{e}}U^\dagger_{f\bar{b}}U_{c\bar{g}}U^\dagger_{h\bar{d}}Z_{e\bar{f}g\bar{h}}$ under a U(2) transformation.

The vacuum expectation values of the two Higgs fields can be parametrized as

$$\langle\Phi_a\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ \widehat{v}_a \end{pmatrix}, \qquad \text{with} \qquad \widehat{v}_a \equiv e^{i\eta}\begin{pmatrix} \cos\beta \\ e^{i\xi}\sin\beta \end{pmatrix}, \tag{5}$$

where $v = 246$ GeV, $0 \leq \beta \leq \frac{1}{2}\pi$ and η is arbitrary. It is convenient to define $V_{a\bar{b}} \equiv \hat{v}_a\hat{v}^*_{\bar{b}}$, which is an hermitian matrix with orthonormal eigenvectors $\hat{v}_b$ and $\widehat{w}_b \equiv \widehat{v}^*_{\bar{c}}\,\epsilon_{cb}$. (In our conventions, $\epsilon_{12} = -\epsilon_{21} = 1$ and $\epsilon_{11} = \epsilon_{22} = 0$.) In particular, note that $\hat{v}^*_{\bar{b}}\hat{w}_b = 0$. Under a U(2) transformation,

$$\hat{v}_a \to U_{a\bar{b}}\hat{v}_b\,, \qquad \widehat{w}_a \to e^{-i\chi}\,U_{a\bar{b}}\,\widehat{w}_b\,, \tag{6}$$

where $\det U \equiv e^{i\chi}$ is a pure phase. That is, $\widehat{w}_a$ is a pseudo-vector with respect to U(2). One can use $\widehat{w}_a$ to construct a proper second-rank tensor, $W_{a\bar{b}} \equiv \hat{w}_a\hat{w}^*_{\bar{b}} \equiv \delta_{a\bar{b}} - V_{a\bar{b}}$. Moreover $\tan\beta \equiv |\hat{v}_2/\hat{v}_1|$ is basis-dependent. Hence, $\tan\beta$ is *not* in general a physical parameter.

All 2HDM observables must be invariant under a basis transformation $\Phi_a \to U_{a\bar{b}}\Phi_b$. Invariants under a basis transformation $\Phi_a \to U_{a\bar{b}}\Phi_b$ are obtained by combining the tensor quantities Y, Z, $\widehat{v}$ and $\widehat{w}$. All physical observables can be expressed in invariant combinations of real invariants and complex pseudo-invariants:[a]

$$\begin{aligned}
&Y_1 \equiv \mathrm{Tr}(YV)\,, && Y_2 \equiv \mathrm{Tr}(YW)\,, && Y_3 \equiv Y_{a\bar{b}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,,\\
&Z_1 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}V_{d\bar{c}}\,, && Z_2 \equiv Z_{a\bar{b}c\bar{d}}\,W_{b\bar{a}}W_{d\bar{c}}\,, && Z_3 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}W_{d\bar{c}}\,,\\
&Z_4 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{c}}W_{d\bar{a}} && Z_5 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,\widehat{v}^*_{\bar{c}}\,\widehat{w}_d\,, && \\
&Z_6 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{v}_b\,\widehat{v}^*_{\bar{c}}\,\widehat{w}_d\,, && Z_7 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,\widehat{w}^*_{\bar{c}}\,\widehat{w}_d\,. &&
\end{aligned} \tag{7}$$

[a] Pseudo-invariants are useful because one can always combine two such quantities to create an invariant.

For example, the charged Higgs boson mass (which is clearly a physical observable) is given by

$$m_{H^\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 v^2 \,.$$

The complex pseudo-invariants listed in eq. (7) transform as

$$[Y_3, Z_6, Z_7] \to e^{-i\chi}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi} Z_5 \,,$$

where χ is defined below eq. (6). As an example, the scalar potential minimum conditions,

$$Y_1 = -\tfrac{1}{2} Z_1 v^2 \quad \text{and} \quad Y_3 = -\tfrac{1}{2} Z_6 v^2 \,,$$

are covariant conditions with respect to U(2) transformations.

3.2 The Higgs basis and Higgs mass eigenstate basis

The three physical neutral Higgs boson mass-eigenstates, denoted by h_1, h_2 and h_3, are determined by diagonalizing a 3×3 real symmetric squared-mass matrix that is defined in a basis where only one of the two neutral Higgs fields has a non-zero vacuum expectation value—the so-called *Higgs basis* [13,15]. The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}. Under a U(2) transformation [15],

$$\theta_{12}\,,\ \theta_{13} \text{ are invariant, and } e^{i\theta_{23}} \to e^{-i\chi}\, e^{i\theta_{23}} \,.$$

One can express the mass eigenstate neutral Higgs field directly in terms of the original shifted neutral fields, $\overline{\Phi}_a^0 \equiv \Phi_a^0 - v\widehat{v}_a/\sqrt{2}$:

$$h_k = \frac{1}{\sqrt{2}} \left[\overline{\Phi}_{\bar{a}}^{0\,\dagger} (q_{k1}\widehat{v}_a + q_{k2} e^{-i\theta_{23}} \widehat{w}_a) + (q_{k1}^* \widehat{v}_{\bar{a}}^* + q_{k2}^* e^{i\theta_{23}} \widehat{w}_{\bar{a}}^*) \overline{\Phi}_a^0 \right] , \tag{8}$$

for $k = 1, \ldots, 4$, where $h_4 \equiv G^0$ is the neutral Goldstone field.

The *invariant* quantities $q_{k\ell}$ are listed in Table 1. Since $\widehat{w}_a e^{-i\theta_{23}}$ is a *proper* U(2)-vector, we see that the neutral mass-eigenstate fields are indeed invariant under basis transformations. Likewise, H^+ and its charge conjugate (as defined below) are U(2)-invariant fields. Inverting eq. (8) yields:

$$\Phi_a = \begin{pmatrix} G^+ \widehat{v}_a + e^{-i\theta_{23}} H^+ \widehat{w}_a \\ \dfrac{v}{\sqrt{2}} \widehat{v}_a + \dfrac{1}{\sqrt{2}} \displaystyle\sum_{k=1}^{4} \left(q_{k1}\widehat{v}_a + q_{k2} e^{-i\theta_{23}} \widehat{w}_a \right) h_k \end{pmatrix} .$$

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

Table 1: The $q_{k\ell}$ are functions of the angles θ_{12} and θ_{13}, where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

3.3 Basis-independent form for the Higgs couplings of the 2HDM

The Higgs boson interactions of the 2HDM can be expressed in terms of the basis-independent $q_{k\ell}$ defined in Table 1. The cubic and quartic vector-scalar couplings were obtained in

Ref. [15]. A slightly more convenient form for these couplings, obtained in Appendix A of Ref. [16], is reproduced below:[b]

$$
\begin{aligned}
\mathscr{L}_{VVH} = {} & \left(g m_W W_\mu^+ W^{\mu\,-} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu\right) q_{k1} h_k \\
& + e m_W A^\mu (W_\mu^+ G^- + W_\mu^- G^+) - g m_Z s_W^2 Z^\mu (W_\mu^+ G^- + W_\mu^- G^+)\,, \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{VVHH} = {} & \left[\tfrac{1}{4} g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] h_k h_k + \left\{\tfrac{1}{2} g^2 W_\mu^+ W^{\mu\,-} + e^2 A_\mu A^\mu \right. \\
& \left. + \frac{g^2}{c_W^2}\left(\tfrac{1}{2} - s_W^2\right)^2 Z_\mu Z^\mu + \frac{2ge}{c_W}\left(\tfrac{1}{2} - s_W^2\right) A_\mu Z^\mu\right\}(G^+ G^- + H^+ H^-) \\
& + \left\{\left(\tfrac{1}{2} e g A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+\right)(q_{k1} G^- + q_{k2} H^-) h_k + \text{h.c.}\right\}, \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{VHH} = {} & \frac{g}{4c_W}\,\epsilon_{jk\ell} q_{\ell 1} Z^\mu h_j \overleftrightarrow{\partial}_\mu h_k - \tfrac{1}{2} g \left\{ i W_\mu^+ \left[q_{k1} G^- \overleftrightarrow{\partial}^\mu h_k + q_{k2} H^- \overleftrightarrow{\partial}^\mu h_k\right] + \text{h.c.}\right\} \\
& + \left[ie A^\mu + \frac{ig}{c_W}\left(\tfrac{1}{2} - s_W^2\right) Z^\mu\right](G^+ \overleftrightarrow{\partial}_\mu G^- + H^+ \overleftrightarrow{\partial}_\mu H^-)\,, \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{VG} = {} & \left[\frac{g^2}{4} W_\mu^+ W^{\mu\,-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] G^0 G^0 + \tfrac{1}{2} g \left(W_\mu^+ G^- \overleftrightarrow{\partial}^\mu G^0 + W_\mu^- G^+ \overleftrightarrow{\partial}^\mu G^0\right) \\
& + \left\{\frac{ieg}{2} A^\mu W_\mu^+ G^- G^0 - \frac{ig^2 s_W^2}{2c_W} Z^\mu W_\mu^+ G^- G^0 + \text{h.c.}\right\} + \frac{g}{2c_W} q_{k1} Z^\mu G^0 \overleftrightarrow{\partial}_\mu h_k\,, \quad (12)
\end{aligned}
$$

where repeated indices $j,k = 1,2,3$ are summed over. Terms quadratic in the scalar fields that contain one or two neutral Goldstone fields are exhibited in eq. (12).

Likewise, a basis-independent form for the cubic and quartic scalar self-interactions has been obtained in Ref. [15]. The cubic couplings are given by:

$$
\begin{aligned}
\mathcal{V}_3 = {} & \tfrac{1}{2} v\, h_j h_k h_\ell \Big[q_{j1} q_{k1}^* \operatorname{Re}(q_{\ell 1}) Z_1 + q_{j2} q_{k2}^* \operatorname{Re}(q_{\ell 1})(Z_3 + Z_4) + \operatorname{Re}(q_{j1}^* q_{k2} q_{\ell 2} Z_5\, e^{-2i\theta_{23}}) \\
& \qquad + \operatorname{Re}\left([2q_{j1} + q_{j1}^*] q_{k1}^* q_{\ell 2} Z_6\, e^{-i\theta_{23}}\right) + \operatorname{Re}(q_{j2}^* q_{k2} q_{\ell 2} Z_7\, e^{-i\theta_{23}})\Big] \\
& + v\, h_k G^+ G^- \Big[\operatorname{Re}(q_{k1}) Z_1 + \operatorname{Re}(q_{k2}\, Z_6 e^{-i\theta_{23}})\Big] + v\, h_k H^+ H^- \Big[\operatorname{Re}(q_{k1}) Z_3 + \operatorname{Re}(q_{k2}\, Z_7 e^{-i\theta_{23}})\Big] \\
& + \tfrac{1}{2} v\, h_k \Big\{ G^- H^+ \left[q_{k2}^* Z_4 + q_{k2}\, Z_5 e^{-2i\theta_{23}} + 2\operatorname{Re}(q_{k1})\, Z_6 e^{-i\theta_{23}}\right] + \text{h.c.}\Big\}\,,
\end{aligned}
$$

where repeated indices $j,k,\ell = 1,2,3,4$ are summed over. The neutral Goldstone fields are implicitly included by denoting $h_4 \equiv G^0$. In particular, $\operatorname{Re}(q_{k1}) = q_{k1}$ for $k = 1,2,3$,

[b] The results of this section differ from Refs. [15] and [16] by a rephasing of the charged Higgs fields so that the $H^\pm$ are the invariant fields as defined below eq. (8).

whereas $\mathrm{Re}(q_{41})=0$. With the same conventions as above, the quartic scalar couplings are given by:

$$
\begin{aligned}
\mathcal{V}_4 = \tfrac{1}{8} h_j h_k h_l h_m \Big[& q_{j1} q_{k1} q^*_{\ell 1} q^*_{m1} Z_1 + q_{j2} q_{k2} q^*_{\ell 2} q^*_{m2} Z_2 + 2 q_{j1} q^*_{k1} q_{\ell 2} q^*_{m2} (Z_3+Z_4) \\
& +2\mathrm{Re}(q^*_{j1} q^*_{k1} q_{\ell 2} q_{m2} Z_5\, e^{-2i\theta_{23}}) + 4\mathrm{Re}(q_{j1} q^*_{k1} q^*_{\ell 1} q_{m2} Z_6\, e^{-i\theta_{23}}) + 4\mathrm{Re}(q^*_{j1} q_{k2} q_{\ell 2} q^*_{m2} Z_7\, e^{-i\theta_{23}}) \Big] \\
& + \tfrac{1}{2} h_j h_k G^+ G^- \Big[q_{j1} q^*_{k1} Z_1 + q_{j2} q^*_{k2} Z_3 + 2\mathrm{Re}(q_{j1} q_{k2} Z_6\, e^{-i\theta_{23}}) \Big] \\
& + \tfrac{1}{2} h_j h_k H^+ H^- \Big[q_{j2} q^*_{k2} Z_2 + q_{j1} q^*_{k1} Z_3 + 2\mathrm{Re}(q_{j1} q_{k2} Z_7\, e^{-i\theta_{23}}) \Big] \\
& + \tfrac{1}{2} h_j h_k \Big\{ G^- H^+ \big[q_{j1} q^*_{k2} Z_4 + q^*_{j1} q_{k2} Z_5\, e^{-2i\theta_{23}} + q_{j1} q^*_{k1} Z_6\, e^{-i\theta_{23}} + q_{j2} q^*_{k2} Z_7\, e^{-i\theta_{23}} \big] + \mathrm{h.c.} \Big\} \\
& + \tfrac{1}{2} Z_1 G^+ G^- G^+ G^- + \tfrac{1}{2} Z_2 H^+ H^- H^+ H^- + (Z_3+Z_4) G^+ G^- H^+ H^- \\
& + \Big\{ \tfrac{1}{2} Z_5 e^{-2i\theta_{23}} H^+ H^+ G^- G^- + Z_6 e^{-i\theta_{23}} G^+ G^- H^+ G^- + Z_7 e^{-i\theta_{23}} H^+ H^- H^+ G^- + \mathrm{h.c.} \Big\} ,
\end{aligned}
$$

summing over j, k, ℓ, $m = 1,2,3,4$.

The Higgs couplings to quarks and leptons are determined by the Yukawa Lagrangian. In terms of the quark mass-eigenstate fields,

$$
-\mathscr{L}_{\mathrm{Y}} = \overline{U}_L \Phi^{0\,*}_{\bar a} h^U_a U_R - \overline{D}_L K^\dagger \Phi^-_{\bar a} h^U_a U_R + \overline{U}_L K \Phi^+_a h^{D\,\dagger}_{\bar a} D_R + \overline{D}_L \Phi^0_a h^{D\,\dagger}_{\bar a} D_R + \mathrm{h.c.} ,
$$

where $\widetilde{\Phi}_{\bar a} \equiv (\widetilde{\Phi}^0_{\bar a}\,, \widetilde{\Phi}^-_{\bar a}) = i\sigma_2 \Phi^*_{\bar a}$ and K is the CKM mixing matrix. The $h^{U,D}_a$ are 3×3 Yukawa coupling matrices. We can construct invariant and pseudo-invariant matrix Yukawa couplings:

$$
\kappa^Q \equiv \widehat{v}^*_{\bar a} h^Q_a \quad \text{and} \quad \rho^Q \equiv \widehat{w}^*_{\bar a} h^Q_a ,
$$

where $Q=U$ or D. Inverting these equations yields $h^Q_a = \kappa^Q \widehat{v}_a + \rho^Q \widehat{w}_a$. Under a U(2) transformation, κ^Q is invariant, whereas $\rho^Q \to e^{i\chi}\rho^Q$.

By construction, κ^U and κ^D are proportional to the (real non-negative) diagonal quark mass matrices M_U and M_D, respectively, whereas the matrices ρ^U and ρ^D are independent complex 3×3 matrices. In particular,

$$
M_U = \frac{v}{\sqrt{2}} \kappa^U = \mathrm{diag}(m_u\,, m_c\,, m_t)\,, \qquad M_D = \frac{v}{\sqrt{2}} \kappa^{D\,\dagger} = \mathrm{diag}(m_d\,, m_s\,, m_b)\,.
$$

The final form for the Yukawa couplings of the mass-eigenstate Higgs bosons and the Goldstone bosons to the quarks, is given by [15]:

$$
\begin{aligned}
-\mathscr{L}_Y = & \frac{1}{v} \overline{D} \Big\{ M_D (q_{k1} P_R + q^*_{k1} P_L) + \frac{v}{\sqrt{2}} \big[q_{k2} \, [e^{i\theta_{23}} \rho^D]^\dagger P_R + q^*_{k2}\, e^{i\theta_{23}} \rho^D P_L \big] \Big\} D h_k \\
& + \frac{1}{v} \overline{U} \Big\{ M_U (q_{k1} P_L + q^*_{k1} P_R) + \frac{v}{\sqrt{2}} \big[q^*_{k2}\, e^{i\theta_{23}} \rho^U P_R + q_{k2} \, [e^{i\theta_{23}} \rho^U]^\dagger P_L \big] \Big\} U h_k \\
& + \Big\{ \overline{U} \big[K [e^{i\theta_{23}} \rho^D]^\dagger P_R - [e^{i\theta_{23}} \rho^U]^\dagger K P_L \big] D H^+ + \frac{\sqrt{2}}{v} \overline{U} \left[K M_D P_R - M_U K P_L \right] D G^+ + \mathrm{h.c.} \Big\} ,
\end{aligned}
$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. Indeed, the Higgs-fermion Yukawa couplings $\mathscr{L}_Y$ depends only on invariant quantities: the 3×3 matrices M_Q and $\rho^Q e^{i\theta_{23}}$ and the invariant angles θ_{12} and θ_{13}. Note that the unphysical parameter $\tan\beta$ does *not* appear.

The couplings of the neutral Higgs bosons to quark pairs are generically flavor-nondiagonal and CP-violating, since the q_{k2} and the matrices $e^{i\theta_{23}}\rho^Q$ are not generally either pure real or pure imaginary.

3.4 Symmetries of the Higgs-fermion interactions

A general 2HDM exhibits CP-violating neutral Higgs boson couplings to fermions and tree-level FCNCs mediated by neutral Higgs boson exchange. Both effects can be removed by imposing an appropriate symmetry. Once again, a basis-independent formulation of such symmetries is useful (as these could be determined in principle from experimental data).

The conditions for a tree-level CP-conserving neutral Higgs–quark interactions are given by [16]:[c]

$$Z_5(\rho^Q)^2\,,\ Z_6\rho^Q \ \text{ and } \ Z_7\rho^Q \ \text{ are real matrices } (Q = U,\, D).$$

Type-I and Type-II Higgs-quark interactions are defined as follows [17]:

$$\text{Type I: } \epsilon_{\bar{a}\bar{b}} h_a^D h_b^U = \epsilon_{ab} h_{\bar{a}}^{D\,\dagger} h_{\bar{b}}^{U\,\dagger} = 0, \qquad \text{i.e., } h_2^U = h_2^D = 0 \text{ in some basis;}$$

$$\text{Type II: } \delta_{a\bar{b}}\, h_{\bar{a}}^{D\,\dagger} h_b^U = 0, \qquad \text{i.e., } h_1^U = h_2^D = 0 \text{ in some basis,}$$

which can be implemented with a $\mathbb{Z}_2$ symmetry (with appropriate choices for the transformations of the scalar and fermion fields), or with supersymmetry.

Invariant expressions for the Type-I and Type-II conditions are given by [14, 15]:

$$\text{Type I:} \quad \kappa^D \rho^U = \rho^D \kappa^U\,, \qquad\qquad \text{Type II:} \quad \kappa^D \kappa^U + \rho^{D\,\dagger} \rho^U = 0\,,$$

where in both cases, $\rho^Q \propto \kappa^Q = \sqrt{2} M_Q/v$ (for $Q = U,\, D$). Hence, in both cases there are no off-diagonal neutral Higgs–quark couplings. The generalization to Higgs–lepton couplings is straightforward.

The existence of a special basis (up to an overall rephasing of the Higgs fields) in which $h_2^U = h_2^D = 0$ (Type-I) or $h_1^U = h_2^D = 0$ (Type-II) promotes $\tan\beta$ to a physical parameter, where $\tan\beta$ is defined to be the magnitude of the ratio of the neutral Higgs vacuum expectation values in the special basis.

For example, suppose we define the invariant parameters

$$\tan\beta_D \equiv \frac{v}{3\sqrt{2}} \left|\mathrm{Tr}\left(\rho^D M_D^{-1}\right)\right|, \qquad \tan\beta_U \equiv \frac{\sqrt{2}}{3v} \left|\mathrm{Tr}\left([\rho^U]^{-1} M_U\right)\right|.$$

Then, in a Type-II model these two quantities coincide and one can identify the physical parameter $\tan\beta = \tan\beta_D = \tan\beta_U$. Thus, in a model-independent analysis, measuring $\tan\beta_D$ and $\tan\beta_U$ can shed light on the symmetries of the Higgs boson–quark Yukawa couplings.

---

[c] CP symmetry cannot be exact due to the unremovable phase in the CKM matrix that enters via the charged current interactions mediated by either $W^\pm$, $H^\pm$ or $G^\pm$ exchange.

4 The decoupling limit in the general 2HDM

In the decoupling limit, one of the two Higgs doublets of the 2HDM receives a very large mass and is therefore decoupled from the theory. This is achieved when $Y_2 \gg v^2$ and $|Z_i| \lesssim \mathcal{O}(1)$ [for all i]. The effective low energy theory is then a one-Higgs-doublet model, corresponding to the SM Higgs sector.

If we order the neutral scalar masses according to $m_1 < m_{2,3}$ and define the Higgs mixing angles accordingly, then the basis-invariant conditions for the decoupling limit are [15]:

$$|s_{12}| \lesssim \mathcal{O}\left(\frac{v^2}{m_2^2}\right) \ll 1\,, \qquad |s_{13}| \lesssim \mathcal{O}\left(\frac{v^2}{m_3^2}\right) \ll 1\,, \qquad \mathrm{Im}(Z_5\, e^{-2i\theta_{23}}) \lesssim \mathcal{O}\left(\frac{v^2}{m_3^2}\right) \ll 1\,.$$

In the decoupling limit, $m_1 \ll m_2, m_3, m_{H^\pm}$. In particular, the properties of h_1 coincide with the SM-like Higgs boson with $m_1^2 = Z_1 v^2$ up to corrections of $\mathcal{O}(v^4/m_{2,3}^2)$, whereas $m_2 \simeq m_3 \simeq m_{H^\pm}$ with squared mass splittings of $\mathcal{O}(v^2)$. In contrast, far from the decoupling limit, one typically finds that *all* Higgs bosons have a similar mass of $\mathcal{O}(v)$ and *none* are SM-like.

In the exact decoupling limit [$s_{12} = s_{13} = \mathrm{Im}(Z_5\, e^{-2i\theta_{23}}) = 0$], the interactions of h_1 are precisely those of the SM Higgs boson. In particular, the corresponding interactions of h_1 are CP-conserving and flavor-diagonal. In the approach to decoupling limit of a general 2HDM, the CP-violating and flavor-changing neutral Higgs couplings of the SM-like Higgs state h_1 are suppressed by factors of $\mathcal{O}(v^2/m_{2,3}^2)$. In contrast, the corresponding interactions of the heavy neutral Higgs bosons (h_2 and h_3) and the charged Higgs bosons ($H^\pm$) can exhibit both CP-violating and flavor non-diagonal couplings (proportional to the ρ^Q).

The decoupling limit is a generic feature of extended Higgs sectors [18].[d] Thus, the initial observation of a SM-like Higgs boson (at LHC) does not rule out the possibility of an extended Higgs sector in the decoupling regime. In particular, only a precision Higgs program can reveal small deviations from the decoupling limit, which would signal the existence of a new heavy mass scale associated with the heavier Higgs states or a manifestation of other new physics beyond the Standard Model.

5 Conclusions and lessons for future work

If a Higgs boson is discovered at the LHC, with properties that approximate those of the SM Higgs boson, then one must establish a precision Higgs program to determine the structure of the scalar dynamics responsible for electroweak symmetry breaking. A precision Higgs program at the LHC can at best measure the Higgs couplings to gauge bosons and fermions with an accuracy of 10–20% [19]. A precision Higgs program at ILC and/or CLIC can achieve significantly improved accuracies, in some cases by an order of magnitude [7, 8].

Precision measurements of the properties of a SM-like Higgs may reveal small deviations, which can indicate the presence of a non-minimal Higgs sector and/or new physics beyond the Standard Model (characterized by a new mass scale that lies above the scale of electroweak symmetry-breaking). With sufficient precision, the measurement of Higgs couplings can be sensitive to a mass scale of new physics that lies beyond the reach of the collider [cf. Fig. 2].

[d]However, if some terms of the Higgs potential are absent, it is possible that no decoupling limit may exist. In this case, the only way to have very large Higgs masses is to have large Higgs self-couplings.

Basis-independent methods provide a powerful technique for studying the theoretical structure of the two-Higgs doublet model. These methods provide insight into the conditions for CP-conservation (and violation [20]), as well as other exact or approximate symmetries that govern the 2HDM dynamics and can distinguish between the two Higgs doublet fields. If nature suggests an elementary scalar sector with the structure of the 2HDM, then basis-independent techniques will be essential for performing a model-independent analysis to determine the ρ^Q $(Q = U, D)$ and eventually the Z_i. The collider tools for such an analysis have yet to be fully developed.

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