

# “Banana Shape” Bunches and the Luminosity for the ILC\*

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## Abstract

The success of the linear collider depends upon the luminosity achieved at the interaction region. The nominal luminosity of  $2 \times 10^{34}$  [ $\text{cm}^{-2} \text{s}^{-1}$ ] for the current ILC design can be significantly reduced by various reasons such as beam-beam effects at the interaction point, misalignment of the beams or the distortion in the bunch shape due to short-range wakefields. The latter, so-called “banana” effect, can also lead to a significant (10%-15%) luminosity loss even for perfectly aligned bunches. The results discussed in this paper suggest that previously this effect was underestimated for the ILC parameter sets.

## 1 Introduction

The luminosity is the measure of the interaction probability of the colliding beams. The high luminosity at the interaction point is a key issue for the future linear collider program. It can be written as

$$L = \frac{n_b N^2 f_{rep}}{4\pi\sigma_x\sigma_y} \times H_D \quad (1)$$

where  $n_b$  is the number of bunches per train,  $N$  is the number of particles per bunch and  $f_{rep}$  is the repetition rate of bunch trains. The transverse sizes  $\sigma_{x,y}$  of the bunch are determined by the so-called Twiss parameters  $\beta_{x,y}$  of the accelerator lattice and the emittance of the beam  $\epsilon$  as  $\sigma_{x,y} = \sqrt{\beta_{x,y}\epsilon}$ . The parameter  $H_D$  is the pinch enhancement factor, which describes the increase in luminosity due to the extra focusing of the bunch by the field of the opposite bunch. The

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number of particles per bunch  $N$  cannot be increased beyond some limits as the dense bunch population can lead to various bunch-bunch instabilities. Thus for the nominal luminosity of the ILC of order of  $10^{34}$  [ $\text{cm}^{-2} \text{s}^{-1}$ ] very small transverse beam sizes (nanometers) and a beam power of order of 10 MW are required. The production of a beam with the required small transverse characteristics is a challenge. In addition, if the vertical bunch size  $\sigma_y$ <sup>1</sup> is small at the interaction point (IP) so does the vertical beta function  $\beta_y$  but then the beam divergence grows as  $\sqrt{\epsilon/\beta_y}$ . However, if  $\beta_y$  is smaller then the bunch length  $\sigma_z$  this hourglass effect will reduce the luminosity. It had been demonstrated in reference [1] that the use of the special focusing regime, so-called “travelling focus” [2], might overcome the hourglass effect by arranging the tail and the head of the bunches to be focused at proportionally displaced longitudinal position. In principle the use of this scheme could provide additional 30% of luminosity.

Finally the nominal luminosity calculated for an “ideal” case can be significantly reduced in the presence of the orbital/angular misalignments of the beam. The examples of such sensitivity for some parameter sets of the International Linear Collider can be found in reference [3] where the influence of orbital and angular beam-beam offsets were investigated. This paper is an update of reference [3] and evaluates the influence of misalignments and bunch shape distortions for different ILC parameter sets including that of the ‘travelling focus’.

## 2 Luminosity loss due to orbital or angular offset for the ILC.

### 2.1 The new parameter sets for the ILC.

Since the publication of the Reference Design Report (RDR) [4] important changes have been suggested in order to reduce the cost of the ILC. In Table 1 the comparison of these new parameter sets [5] with the former RDR parameter set is given. Three new sets, i.e. “SB2009”, “Low Charge” and “New Low Charge” are based on the reduction of the cost of the machine via the reduction of the charge per bunch train which leads to a smaller spatial extent of the machine and lower power supply. It should be noticed that the production of short bunches will require a two stage bunch compressor while the SB2009 design for the ILC has a one stage bunch compressor.

The “SB2009” is based on the application of the so-called “travelling focus” regime [1]. The alternative “Low Charge” (LC) and “New Low Charge” (J. Gao) parameter sets based on the reduction of the number of particles per bunch and on the reduction of the bunch length, could also provide the luminosity of  $2 \times 10^{34}$  [ $\text{cm}^{-2} \text{s}^{-1}$ ]. In Table 1 the nominal luminosity values were calculated with the guineapig++ simulation code [6] which is C++ version of GUINEAPIG [7].

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<sup>1</sup>The use of the flat beams with  $\sigma_x \ll \sigma_y$  are typical for the linear collider. Thus the example is given for vertical beam size  $\sigma_y$  and vertical beta function  $\beta_y$ .

Table 1: The ILC parameter sets

|  | RDR                | SB2009             | Low Charge         | New Low Charge     |
|--|--------------------|--------------------|--------------------|--------------------|
| $N_{particles}$                            | $2 \times 10^{10}$ | $2 \times 10^{10}$ | $1 \times 10^{10}$ | $1 \times 10^{10}$ |
| $N_{bunches}$                              | 2625               | 1320               | 5640               | 2625               |
| $\beta_x/\beta_y$ [mm]                     | 20/0.4             | 11/0.2             | 12/0.2             | 8/0.166            |
| $\gamma\epsilon_x$ [ $\mu m$ ]             | 10                 | 10                 | 10                 | 10                 |
| $\gamma\epsilon_y$ [ $\mu m$ ]             | 40                 | 36                 | 30                 | 10                 |
| $\sigma_x$ [nm]                            | 639                | 474                | 495                | 404                |
| $\sigma_y$ [nm]                            | 5.7                | 3.8                | 3.5                | 2.0                |
| $\sigma_z$ [ $\mu m$ ]                     | 300                | 300                | 150                | 166                |
| $D_y$                                      | 19.0               | 38.4               | 10.0               | 24.0               |
| Lumi. $\times 10^{34}$ [ $cm^{-2}s^{-1}$ ] | 1.97               | 1.96               | 1.96               | 2.12               |

## 2.2 Study of the effects of orbital and angular offsets on luminosity.

The nominal luminosity for the ILC should be delivered even in the case of the new parameter sets for reduced beam power. The beam power is directly proportional to the centre-of-mass energy  $E_{CM}$  as

$$P_{beam} = E_{CM} n_b N f_{rep} \quad (2)$$

It follows from Eq. 1 and Eq. 2 that the luminosity is directly proportional to the beam power.

The luminosity is very sensitive to orbital and angular offsets of the interacting bunches. This effect was studied for all three new parameter sets and compared with the former RDR parameter set. The value of luminosity for the different orbital (Fig.1a) and angular (Fig.1b) offsets was normalised with respect to the nominal luminosity and plotted as function of the relative vertical orbital offset  $\Delta y/\sigma_y$  or as a function of the relative vertical bunch divergence  $\alpha_y/\theta_y$  where  $\theta_y = \sqrt{(\epsilon_y/\beta_y)}$ .

It was found that the Low Charge (LC) parameter set is less sensitive to the orbital bunch displacement at the interaction point, while the values for the New Low Charge (J. Gao) set are very close to those of the RDR parameter set. As expected, the travelling focus (SB2009) regime has proved to be more sensitive to the orbital offsets compared to the other sets of parameters. The same parameter sets were used for the luminosity calculations in the presence of angular offsets. For the travelling focus regime the relative luminosity loss could be of order of 60%, while the Low Charge (LC) option gives a relatively small loss of luminosity  $\approx 12\%$ . It can be explained by the fact that for the LC set the disruption parameter  $D_y$  is nearly 4 times smaller than the  $D_y$  parameter for the travelling focus regime. The behavior of the New Low Charge (J. Gao) regime is again close to the RDR parameter set.

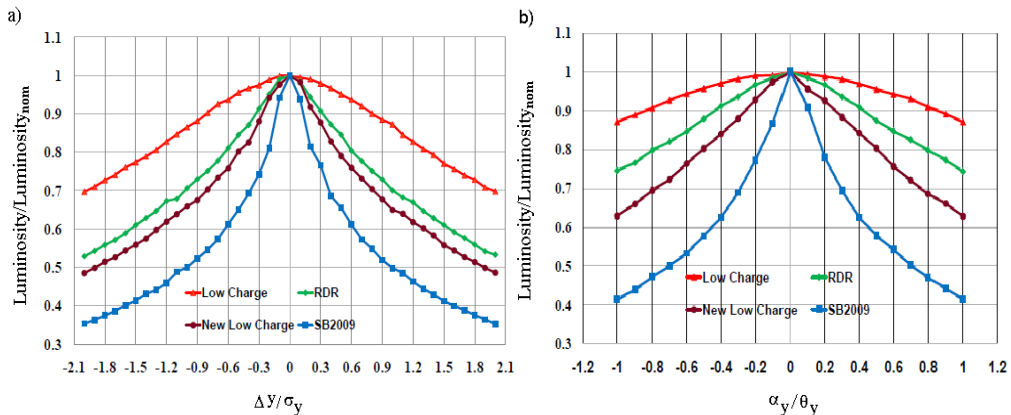


Figure 1: The ILC 500 GeV centre-of-mass energy parameter sets. Scans of effects of orbital a) and angular b) offsets on the normalized luminosity.

### 3 “Banana shape” bunches and the luminosity for the ILC.

#### 3.1 “Banana shape ” bunches.

In the presence of short range wakefields the originally gaussian bunches are distorted. This effect is often referred as “banana shape ” bunches. Despite the relatively small change in the beam emittance the impact on the luminosity can be significant. For the TESLA lattice the effect of “banana shape” bunches was previously studied in [8] and for an emittance growth  $\approx 6\%$  the relative luminosity loss is 30% even without any orbital or angular offsets was reported. This effect can be compensated by a very sophisticated feed-back system. A similar behavior can be confirmed explicitly by the orbital offset scans for the new parameter sets of the ILC and “banana shape” bunches.

In Figure 1 the sensitivity of the Gaussian beams to various orbital and angular offsets is demonstrated for 4 different parameter sets for the ILC. In all four cases the maximum luminosity is achieved at zero orbital and angular offset and the presence of any of such offsets can reduce the luminosity dramatically. In addition, for the Gaussian beam the maximum luminosity value corresponds to the minimum value of beam-beam vertical kick angle. Nevertheless this property does not hold for the distorted bunches. For the non-gaussian beams the maximum luminosity may occur at the non-zero value of orbital (or angular) offset. It is demonstrated in Fig.2 where the maximum attainable luminosity for “banana shaped“ bunches of the SB2009 parameter set is achieved at  $0.5\Delta y/\sigma_y$  fractional orbital offset and  $-16 [\mu rad]$  vertical beam-beam kick angle. Finally, the non-Gaussian shape of bunches can significantly reduce the maximum attainable luminosity even in the absence of orbital offsets. In Fig.3

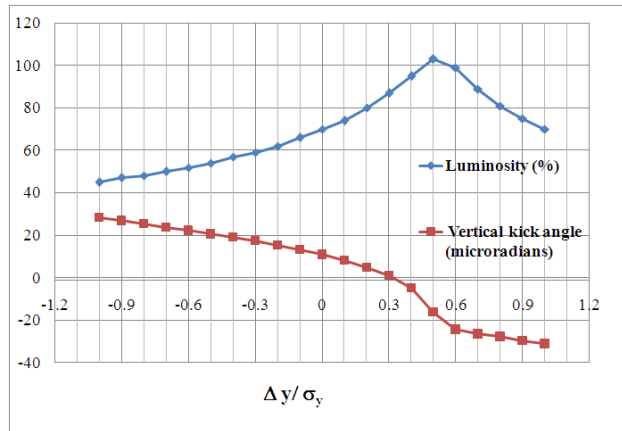


Figure 2: The SB2009 parameter set. Blue: The normalised luminosity for a non-Gaussian ( “banana shaped“ bunch with 1% vertical emittance growth as a function of the orbital offset. Red: The corresponding vertical kick angle to achieve the luminosity given in the blue curve.

the normalized luminosity is given as function of the vertical beam-beam kick angle for Gaussian and non-Gaussian ”banana” beams for the SB2009 parameter set where the combination of two bunches with similar linear orbital tilt (+O/+O) results in  $\approx 11\%$  of luminosity loss.

### 3.2 The emittance growth. Linearised model.

For the relativistic beam the square of the RMS emittance  $\epsilon$  is given by the determinant of the covariance ( $\sigma$ ) matrix as

$$\epsilon^2 = \det \sigma = \langle y^2 \rangle \langle y'^2 \rangle - \langle yy' \rangle^2 \quad (3)$$

In the presence of additional orbital( $\Delta y$ ) and/or the angular ( $\Delta y'$ ) kicks the beam vertical phase-space is changed according to

$$\begin{aligned} y &= y + \Delta y \\ y' &= y' + \Delta y' \end{aligned} \quad (4)$$

and the new perturbed emittance  $\epsilon_{per}$  can be found as function of the beam Twiss parameters  $\alpha, \beta, \gamma$ , the unperturbed emittance  $\epsilon_0$  and the kicks amplitudes  $\Delta y, \Delta y'$ . The relative emittance growth is given by

$$\frac{\Delta \epsilon}{\epsilon_0} = \frac{\epsilon_{per} - \epsilon_0}{\epsilon_0} \quad (5)$$

For the uncorrelated kicks and the small emittance growth  $\Delta \epsilon / \epsilon_0 \ll 1$  the formula can be derived explicitly as in [9] (see also the Appendix). For example

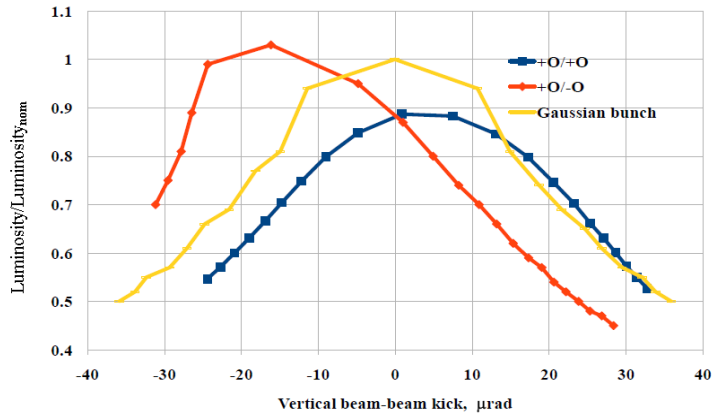


Figure 3: The SB2009 parameter set. The normalized luminosity as a function of vertical kick angle for Gaussian and banana shape bunches with 1% vertical emittance growth.

if only the angular kick  $\Delta y'$  is present the relative emittance growth scales quadratically with the RMS angular kick

$$\frac{\Delta\epsilon}{\epsilon_0} = \frac{\beta_0}{2\epsilon_0} \langle \Delta y'^2 \rangle \quad (6)$$

To study the the impact of "banana" shape on the luminosity the originally Gaussian bunch should be tracked through the linac and the Beam Delivery System (BDS) to the Interaction Point. In [10] it was done by using the orbit tracking codes such as PLACET and MatMerlin. Nevertheless for the quick estimation the distortion of the bunch shape can be introduced by applying the linear (y-z) tilt correlation to the gaussian bunch by "hand". Using the linearised version of spacial and angular kicks in form

$$y = y + k_1 z \quad \text{or} \quad y' = y' + k_2 z \quad (7)$$

and the assumptions that the kicks are uncorrelated, a bunch with the required emittance growth can be generated. The values of the coefficients  $k_1$ ,  $k_2$  for 1% of the relative emittance growth and different ILC parameter sets are given in Table 2. The details of derivation are presented in Appendix A1.

Table 2: The coefficients for the linearized model assuming 1% emittance growth

|                  | RDR                     | SB2009                  | Low Charge              | New Low Charge          |
|------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| $ k_1 $          | $2.6958 \times 10^{-6}$ | $1.7831 \times 10^{-6}$ | $3.3017 \times 10^{-6}$ | $1.5693 \times 10^{-6}$ |
| $ k_2  [m^{-1}]$ | $6.7396 \times 10^{-3}$ | $8.9157 \times 10^{-3}$ | $1.6509 \times 10^{-2}$ | $9.4535 \times 10^{-3}$ |

According to the results reported in [3], where 6% emittance growth was assumed, the relative loss of luminosity was found to be small and a scheme

of compensation via subsequent angular scans was suggested. For the current ILC setting the emittance growth due to the “banana” effect is expected approximately 1% or 2%. Nevertheless the new guineapig++ simulations using

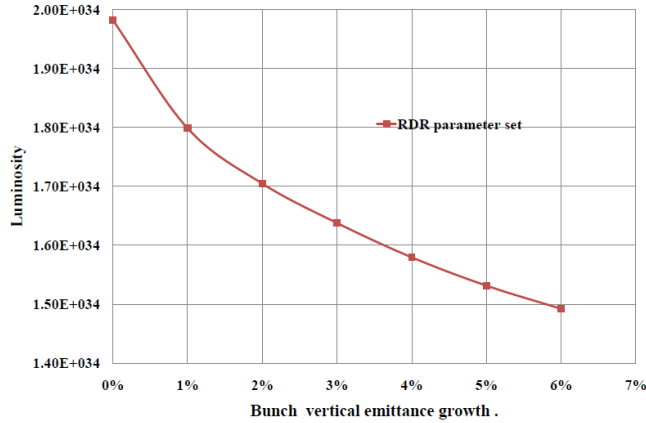


Figure 4: The loss of the nominal luminosity due to emittance growth for perfectly aligned bunches.

the linearised model of emittance growth suggest that for 1% of the emittance growth the luminosity loss can be as significant as 10% -15% even in the case of perfectly aligned bunches. In Fig.4 the luminosity for the RDR parameter set in the absence of any orbital or angular distortion is plotted as a function of the vertical emittance growth. It should be noted that the calculations in [3] correspond in fact to only 0.4% of emittance growth (and not 6%). This explains why the luminosity loss due to “banana” shape bunches was underestimated previously.

### 3.3 Luminosity scans for the banana shape bunches for the ILC

In Table 2 the absolute values of the coefficients  $k_1$  and  $k_2$  are given. In principle the electron and positron bunches can be tilted in both ways, thus there are 16 possible combinations of orbital( $\pm O$ ) and angular( $\pm a$ ) tilted for two interacting bunches.

In Fig.5a the results of orbital offset scans are given for 6 combinations of the orbital ( $y, z$ ) and angular ( $y', z$ ) correlations leading to 1% emittance growth for RDR parameters. The relative luminosity loss is plotted versus the normalized vertical orbital offsets. Surprisingly, the scheme of compensation suggested in [3] still works. The nominal luminosity value can be restored via subsequent angular scan as it seen in Fig.5b.

The results of orbital/angular scans for the travelling focus regime SB2009 are presented in Fig.6a/6b respectively. The polarisation loss due to “banana”

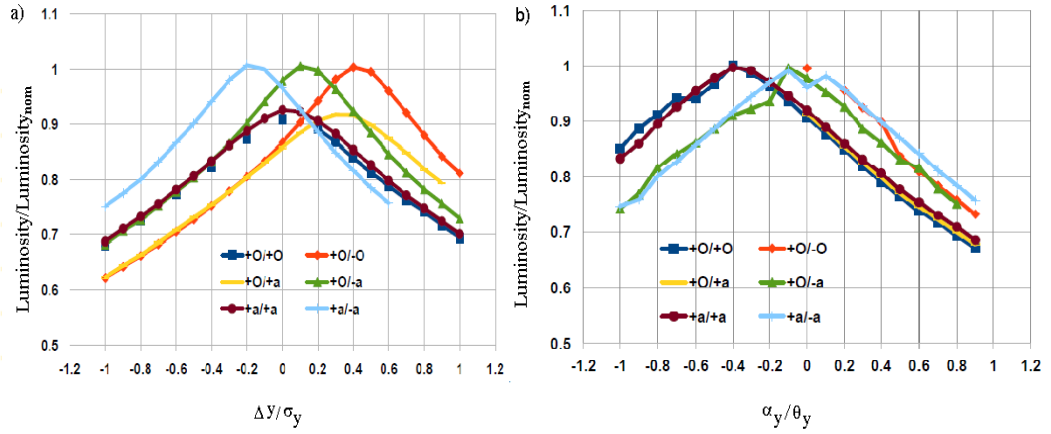


Figure 5: The RDR parameter set scans for different combinations of orbital (O) and angular (a) correlations leading to 1% of emittance growth.

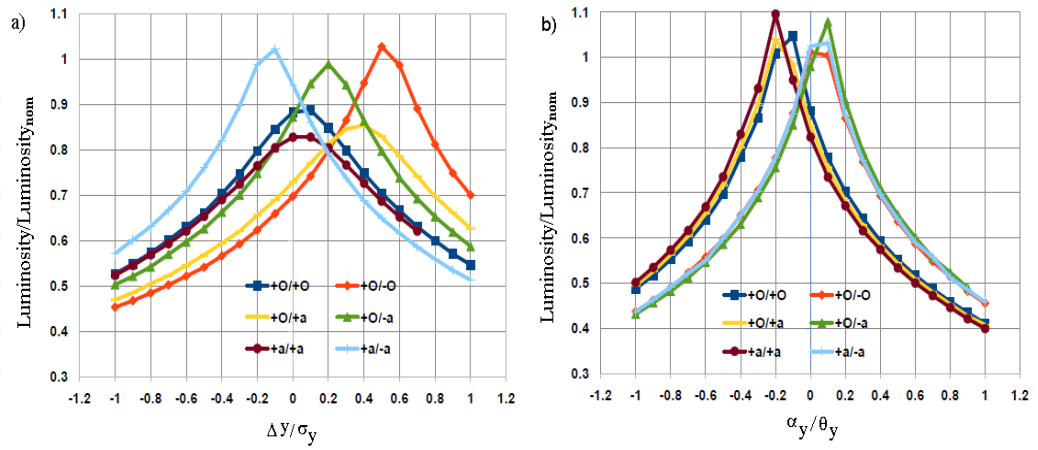


Figure 6: The SB2009 parameter set scans for different combinations of orbital (O) and angular (a) correlations leading to 1% of emittance growth.



shape bunches is even larger, but again can be compensated by angular scan. In Fig.7 the results for the Low Charge parameter set are given. Similar results were obtained for J.Gao set.

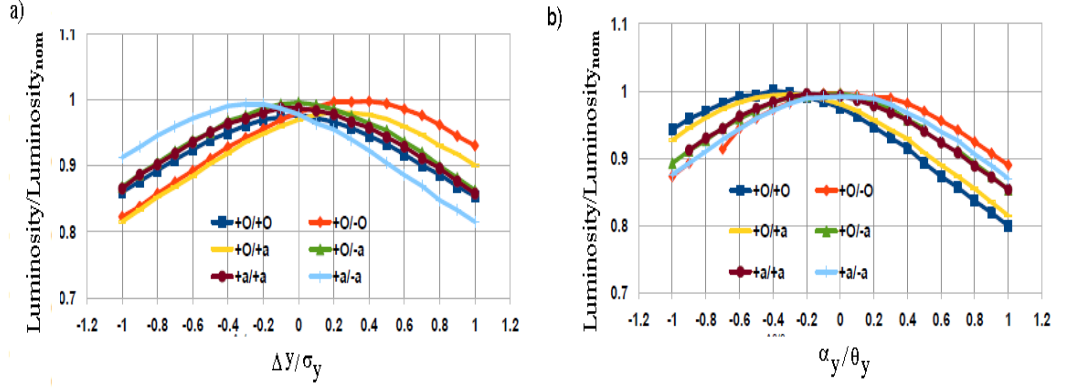


Figure 7: The Low Charge parameter set scans for different combinations of orbital (O) and angular (a) correlations leading to 1% of emittance growth.

## 4 CONCLUSIONS

The study of the new parameter sets confirms that the travelling focus regime is very promising but also very sensitive to the bunch-bunch orbital and angular offsets and requires elaborated feed-back system to deliver the required luminosity. It was also found that the "banana" effect may have significant impact on the luminosity. The results of guineapig++ simulations using a linear model make clear that more investigation should be done. An even more realistic representation of "banana" bunches will be obtained by using a simulation package such as Merlin [10], which can model the wakefields in the linac. Using such generated "banana" shape bunches, the luminosity and relative luminosity loss can be calculated by guineapig++.

## 5 ACKNOWLEDGMENT

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## A The derivation of the coefficients for linearized model of emittance growth

Let's evaluate the relative emittance growth  $\Delta\epsilon/\epsilon_0$  using the linearised version of orbital kicks in form

$$y = y + k_1 z \quad (8)$$

If the kicks are uncorrelated (i.e.  $\langle y k_1 z \rangle = \langle y' k_1 z \rangle = 0$ ) the new perturbed emittance can be written as

$$\begin{aligned} \sigma &= \begin{pmatrix} \langle (y + k_1 z)^2 \rangle & \langle (y + k_1 z) y' \rangle \\ \langle (y + k_1 z) y' \rangle & \langle y'^2 \rangle \end{pmatrix} = \begin{pmatrix} \langle y^2 \rangle + k_1^2 \sigma_z^2 & \langle y y' \rangle \\ \langle y y' \rangle & \langle y'^2 \rangle \end{pmatrix} \\ &= \begin{pmatrix} \epsilon_0 \beta_0 + k_1^2 \sigma_z^2 & -\alpha_0 \epsilon_0 \\ -\alpha_0 \epsilon_0 & \gamma_0 \epsilon_0 \end{pmatrix} \end{aligned} \quad (9)$$

where the expressions for  $\langle y^2 \rangle = \epsilon_0 \beta_0$ ,  $\langle y'^2 \rangle = \gamma_0 \epsilon_0$  and  $\langle y y' \rangle = -\alpha_0 \epsilon_0$  has been used.

The perturbed emittance  $\epsilon_{per}^2 = \det \sigma$ , then from Eq. 9 follows that

$$\epsilon_{per}^2 = \epsilon_0^2 + \frac{k_1^2 \sigma_z^2 \epsilon_0}{\beta_0} \quad (10)$$

and

$$\frac{\epsilon_{per}}{\epsilon_0} = \sqrt{1 + \frac{k_1^2 \sigma_z^2}{\beta_0 \epsilon_0}} \approx 1 + \frac{k_1^2 \sigma_z^2}{2\beta_0 \epsilon_0} \quad (11)$$

The relative emittance growth is normally given in the percents and for small emittance growth

$$\frac{\Delta\epsilon}{\epsilon_0} = \frac{\epsilon_{per} - \epsilon_0}{\epsilon_0} = \frac{\epsilon_{per}}{\epsilon_0} - 1 = \frac{k_1^2 \sigma_z^2}{2\beta_0 \epsilon_0} \quad (12)$$

The values of  $|k_1|$  could be found from Eq. 12 as

$$|k_1| = \sqrt{\frac{\Delta\epsilon}{\epsilon_0} \frac{2\epsilon_0 \beta_0}{\sigma_z^2}} \quad (13)$$

In the similar way the expression can be obtained for the angular offset  $\Delta y' = k_2 z$ :

$$|k_2| = \sqrt{\frac{\Delta\epsilon}{\epsilon_0} \frac{2\epsilon_0}{\beta_0 \sigma_z^2}} \quad (14)$$

The equations Eq. 13 and Eq. 14 allow to get the coefficients for different values of  $\beta_0$  and  $\sigma_z$  and the required relative emittance growth,  $\frac{\Delta\epsilon}{\epsilon_0}$ .

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